Parity-violating scalar trispectrum from helical primordial magnetic fields

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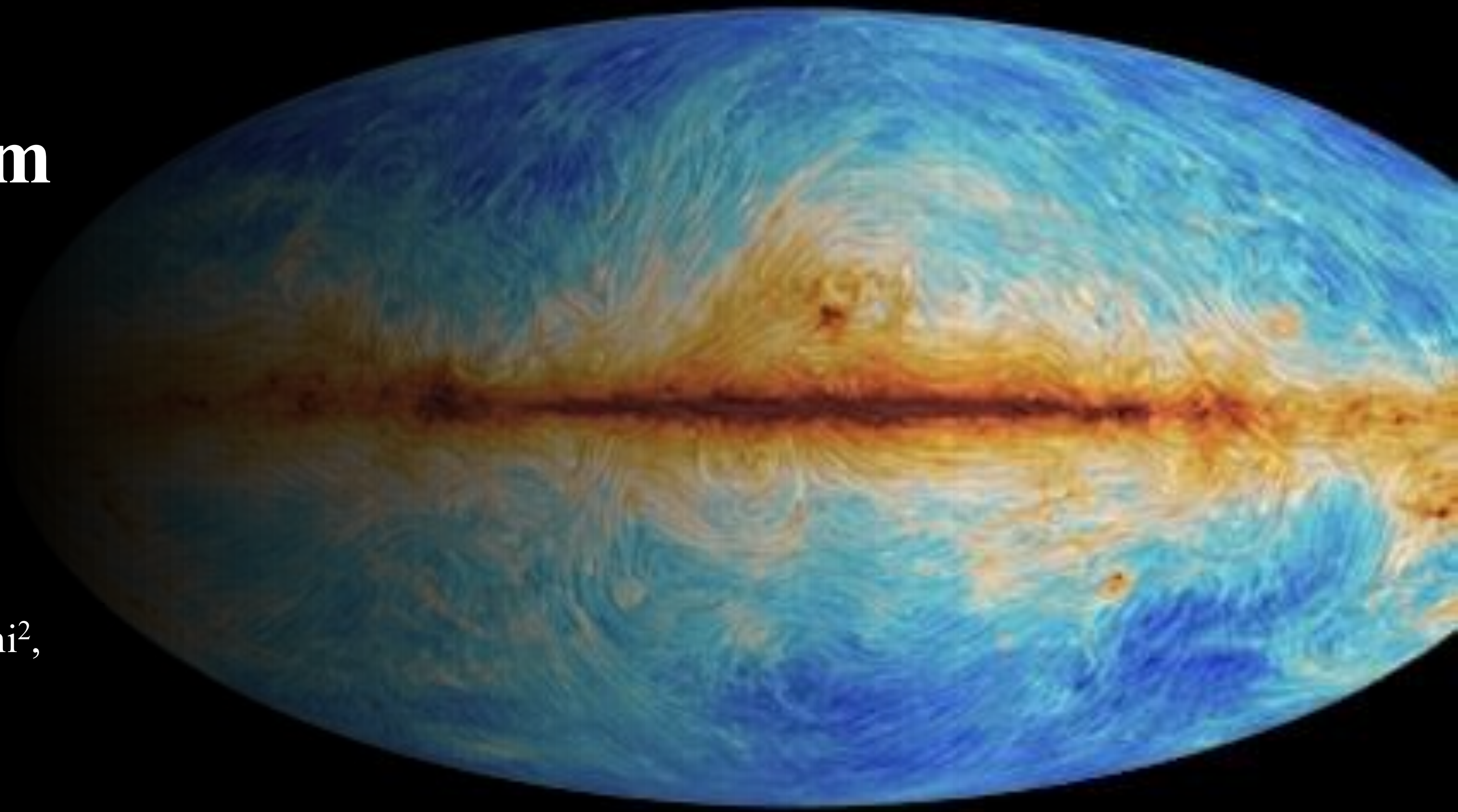


KEK-Cosmo’25

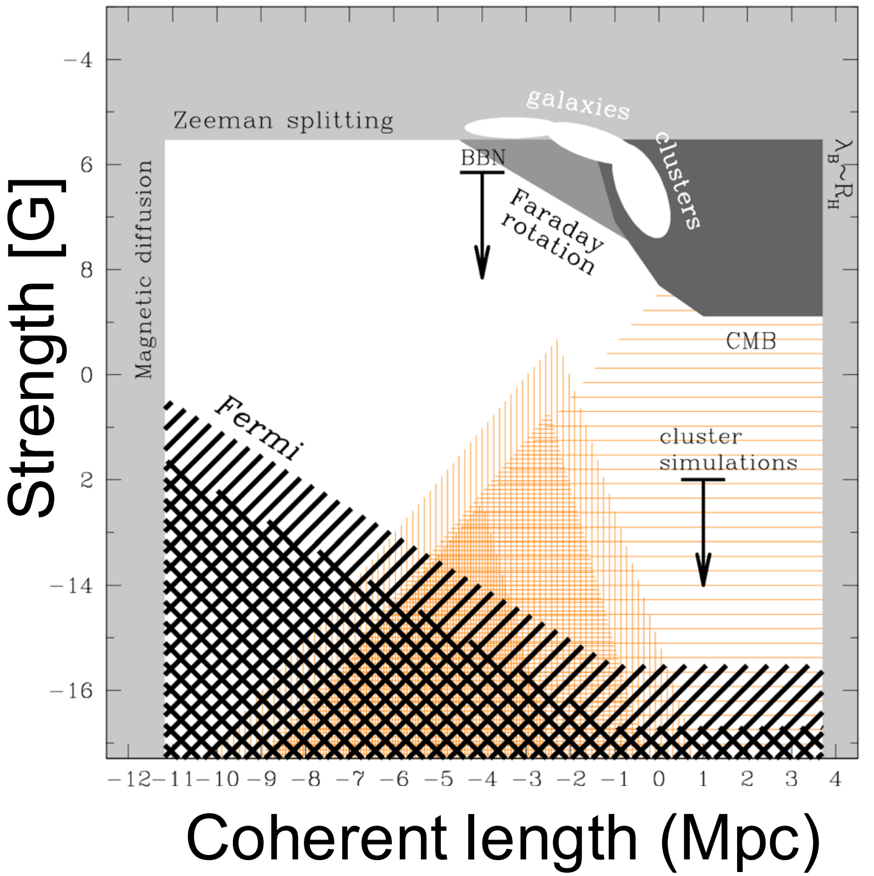
Based on：arXiv:2505.16844

Origin of Cosmic Magnetic Fields

Magnetic fields of $\mathcal{O}(\mu\mathrm{G})$ are observed in galaxies and galaxy clusters

Void regions : $B \geq 3 \times 10^{-16}\mathrm{G}$

Neronov & Vovk, science.1184192 (2010)



Mechanism A seed field is amplified by dynamo

- ☐ Astrophysical origin (Biermann battery)
  - generated in the plasma during the primordial galaxy formation
  - small coherence
- ☐ **Cosmological origin (Primordial magnetic fields; PMFs)**
  - generated in the early universe (e.g., inflation)
  - large coherent length

Generation of Helical PMFs

Helical magnetic field : Parity-odd part of magnetic field

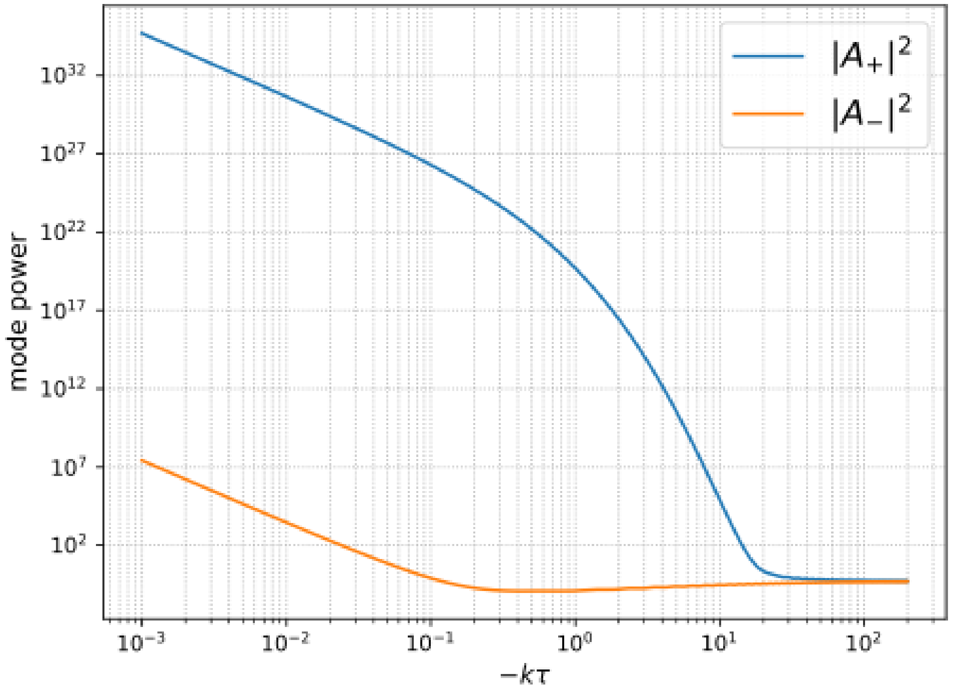
Ratra model *Axion-like coupling* (parity-odd coupling)

$$\mathcal{L} = I^2(\tau)\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\gamma}{8}\epsilon_{\mu\nu\rho\lambda}F^{\mu\nu}F^{\rho\lambda}\right) \qquad I(\tau) = a^n(\tau)$$

Chiara Caprini & Lorenzo Sorbo, JCAP10(2014)056
Tomohiro Fujita & Ruth Durrer, JCAP09(2019)008

EoM of gauge field

$$\tilde{A}''_\pm + \left(k^2 \pm 2\xi\frac{k}{\tau} - \frac{n(n+1)}{\tau^2}\right)\tilde{A}_\pm = 0$$



→ Only one helicity mode grows exponentially

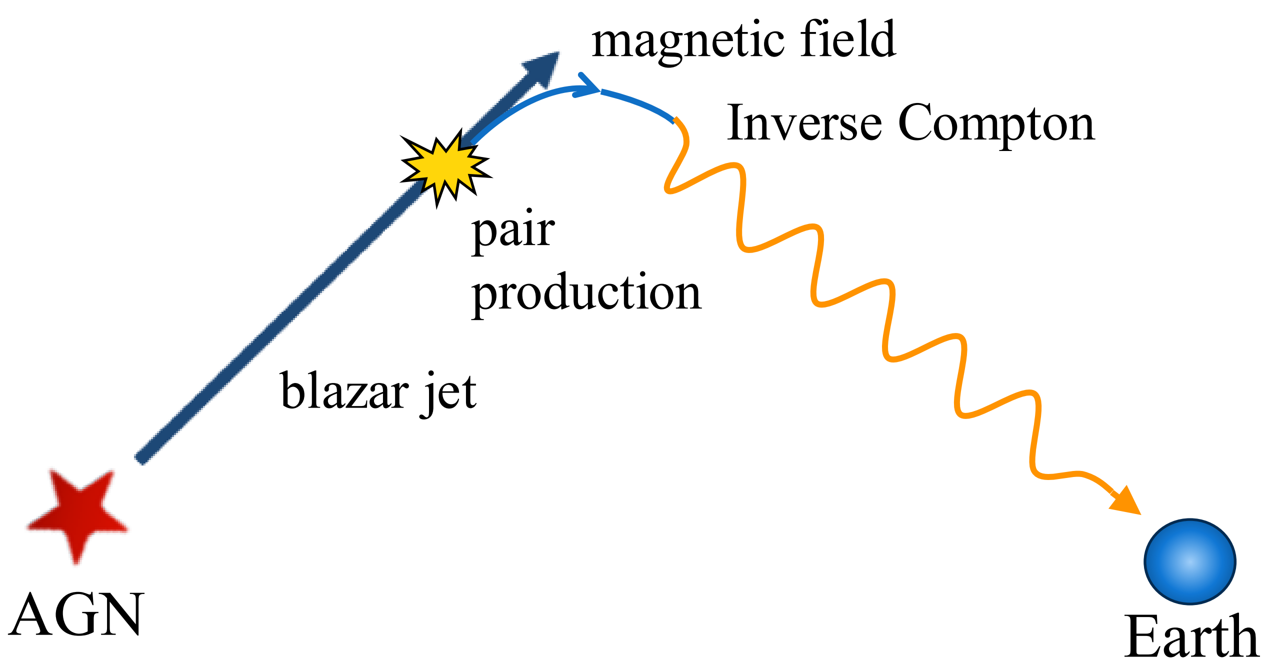
✓ *Ratra* : adjust the tilt of magnetic fields

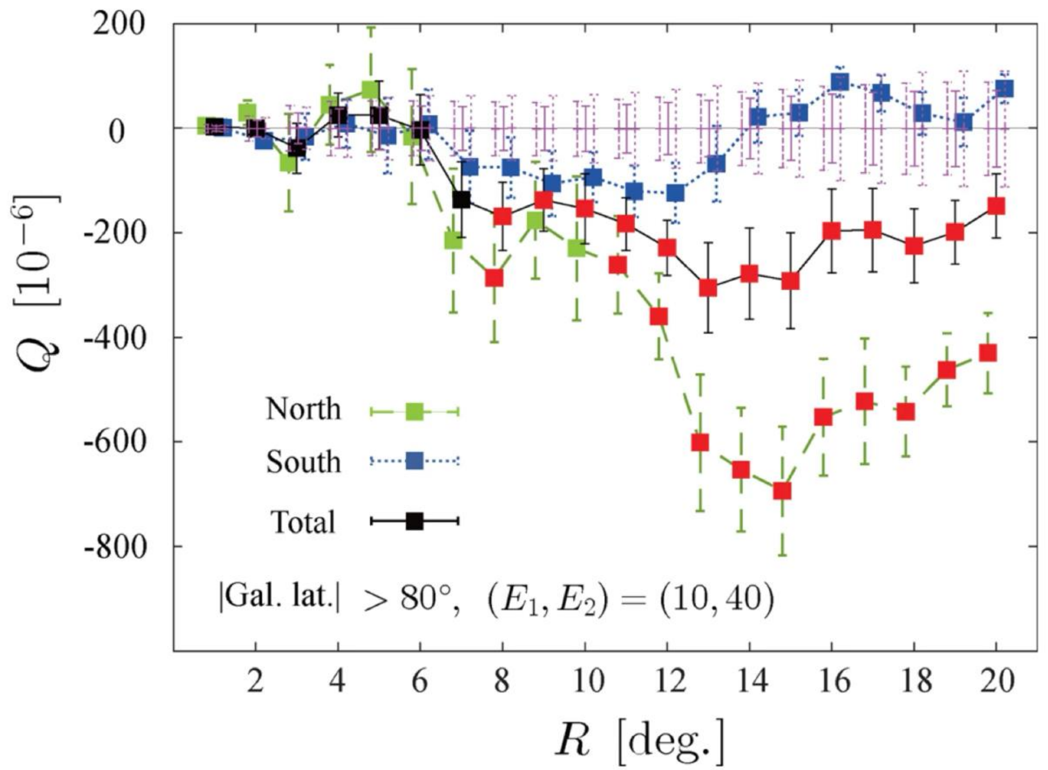
✓ *Axion-like coupling* : inject the helicity to PMFs

Helical PMFs enables us to
approach the PMF generation & parity violation

Observational Implication of Helical PMF

Hiroyuki Tashiro et al., MNRASL 445, L41–L45 (2014)





Implication of the possibility
that extragalactic magnetic field is helical

Non-zero Q with $\gtrsim 3\sigma$ significance

⇨ Is the origin of magnetic field in void helical?

Aim Explore helical PMFs phenomenologically thorough parity violation

Passive scalar mode

J. Richard Shaw & Antony Lewis, Phys. Rev. D 81, 043517 (2010)

<u>Energy momentum tensor from PMF</u>

anisotropic stress

$$T^i_j \supset \frac{1}{4\pi a^4}\left(\frac{1}{2}B^2(\boldsymbol{x})\delta^i_j - B^i(\boldsymbol{x})B_j(\boldsymbol{x})\right) \propto a^{-4}{\Pi_B}^i{}_j$$

Curvature perturbation from PMF anisotropic stress

$$\zeta(\tau) = \zeta(\tau_B) - \frac{1}{3}R_\gamma \Pi_B(\boldsymbol{k})\left[\ln(\tau/\tau_B) + \frac{\tau_B}{2\tau} - \frac{1}{2}\right]$$

R_γ : Energy density ratio between photon and relativistic particles

Curvature perturbation on superhorizon scale

Passive scalar mode

J. Richard Shaw & Antony Lewis, Phys. Rev. D 81, 043517 (2010)

Energy momentum tensor from PMF

anisotropic stress

$$T^i_j \supset \frac{1}{4\pi a^4}\left(\frac{1}{2}B^2(\boldsymbol{x})\delta^i_j - B^i(\boldsymbol{x})B_j(\boldsymbol{x})\right) \propto a^{-4}{\Pi_B}^i{}_j$$

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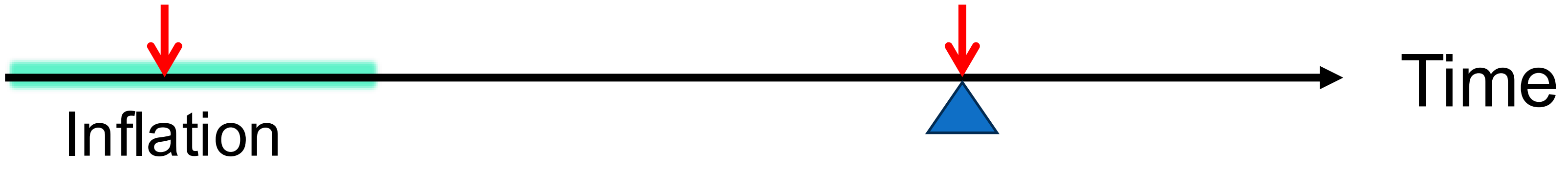
τ_B : PMF generation

τ_ν : neutrino decoupling



Neutrino anisotropic stress compensates PMF anisotropic stress

Passive scalar mode

J. Richard Shaw & Antony Lewis, Phys. Rev. D 81, 043517 (2010)

Energy momentum tensor from PMF

anisotropic stress

$$T^i_j \supset \frac{1}{4\pi a^4}\left(\frac{1}{2}B^2(\boldsymbol{x})\delta^i_j - B^i(\boldsymbol{x})B_j(\boldsymbol{x})\right) \propto a^{-4}{\Pi_B}^i{}_j$$

Curvature perturbation from PMF anisotropic stress

$$\zeta(\tau) = \zeta(\tau_B) - \frac{1}{3}R_\gamma \Pi_B(\boldsymbol{k})\left[\ln(\tau/\tau_B) + \frac{\tau_B}{2\tau} - \frac{1}{2}\right]$$

[After neutrino decoupling]

Passive scalar mode:

$$\zeta_B(\boldsymbol{k}) \simeq -\frac{1}{3}R_\gamma \Pi_B(\boldsymbol{k})\ln\left(\frac{\tau_\nu}{\tau_B}\right)$$

$$\equiv \mathcal{T}\,\Pi_B(\boldsymbol{k})$$

Scalar Observable Sensitive to Parity Violation

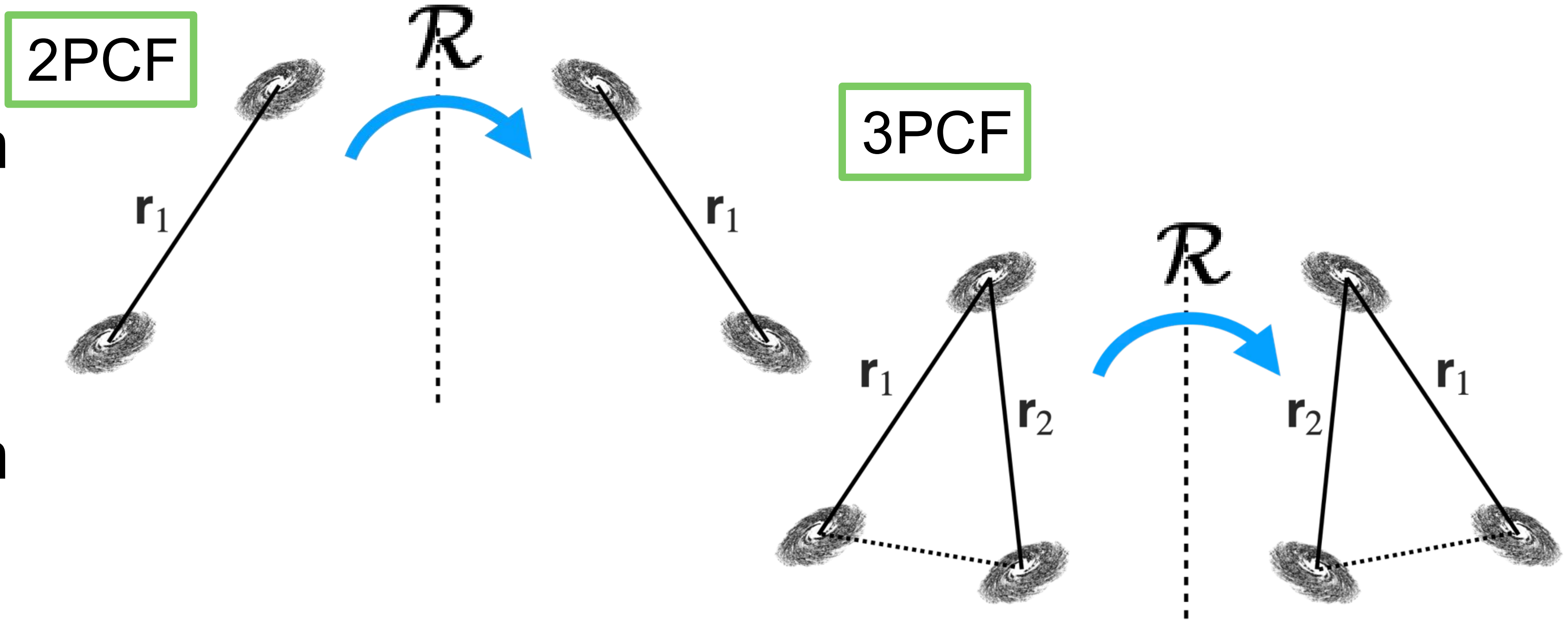
Parity symmetry : physical law is invariant under spatial inversion

- 2-pt correlation function (2PCF) :

  Parity = Rotation

- 3-pt correlation function (3PCF) :

  Parity = Rotation



2PCF & 3PCF are not statistics sensitive purely to parity violation

4-Point Correlation Function

Parity symmetry : physical law is invariant under spatial inversion

- 2-pt correlation function (2PCF) :

  Parity = Rotation

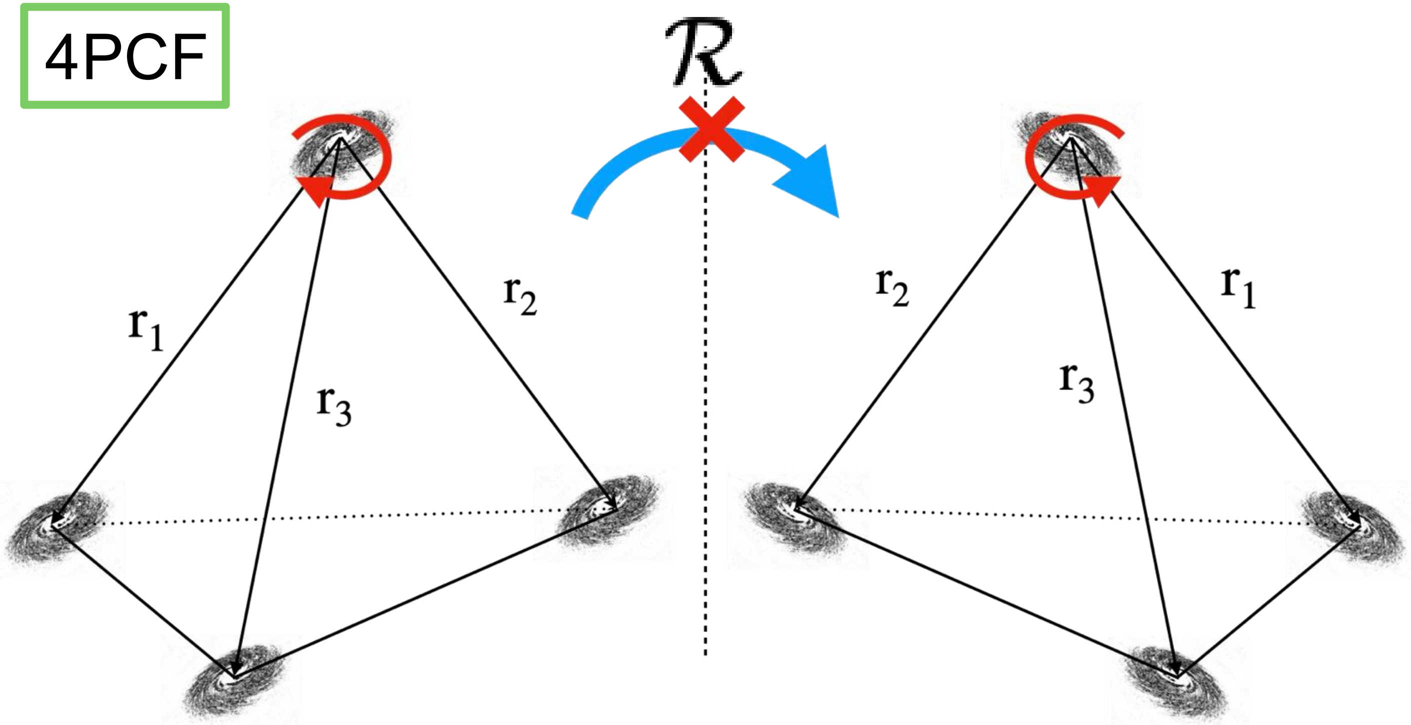
- 3-pt correlation function (3PCF) :

  Parity = Rotation

- 4-pt correlation function (4PCF) :

  Parity ≠ Rotation

4PCF



Tetrahedron cannot be rotated into its mirror image

4PCF (Trispectrum) : the lowest order statistic sensitive to parity violation for scalar fields

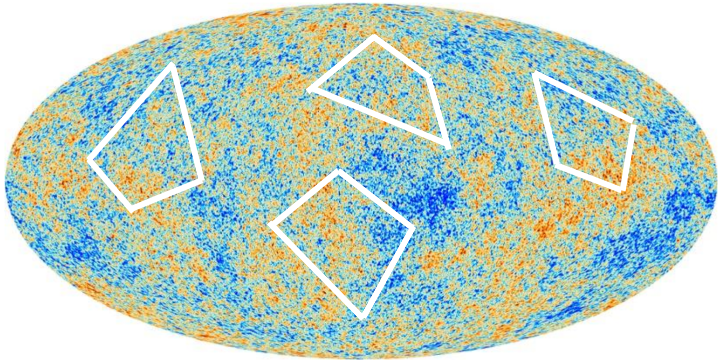
Observational Searches for Parity-Odd 4PCF

Jiamin Hou et al., MNRAS, 522.5701H (2023)
Oliver H. E. Philcox, PRL 131.181001 (2023)
Oliver H. E. Philcox, PRD 111, 123534 (2025)
Zachary Slepian et al., arXiv:2508.09133 (2025)
Jiamin Hou et al., arXiv:2512.20132 (2025)

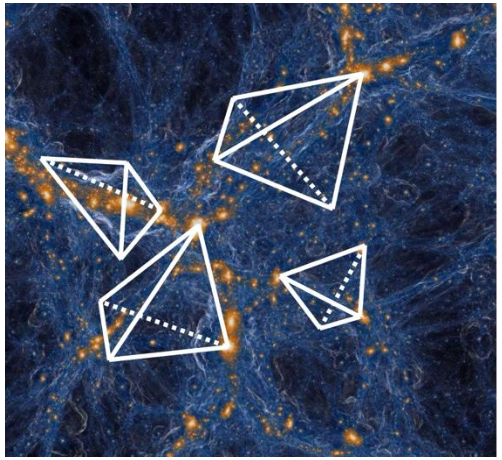
Parity-odd 4PCF has been measured in CMB&LSS

CMB Planck : constraints on the amplitude of 4PCF

$$10^{-4}\tau_{\rm NL}^{1,\rm odd} = -1.5 \pm 1.6 \quad (95\%\text{C.L.})$$



LSS
- BOSS : maximally $\boldsymbol{7.1\sigma}$ for parity-odd modes
- DESI : $\boldsymbol{2\sigma, \sim 4 - 10\sigma}$ (systematics-limited)



<u>Physical model of parity-violating trispectrum</u>

- axion inflation / spectator axion
  (Matthew A. Reinhard et al. (2024), Tomohiro Fujita et al. (2024))
- Chiral scalar-tensor theory of gravity
  (Tommaso Moretti et al. (2025))

Parity Transformation of Trispectrum

General formalism of the passive scalar mode trispectrum

$$\langle \zeta_B(\boldsymbol{k}_1)\zeta_B(\boldsymbol{k}_2)\zeta_B(\boldsymbol{k}_3)\zeta_B(\boldsymbol{k}_4)\rangle_c \equiv (2\pi)^3\delta_D(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3+\boldsymbol{k}_4)\mathcal{T}^4 T_{\Pi_B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)$$

Parity transformation in Fourier space：Inversion of wave vectors

Reality condition for curvature perturbation

$$\zeta_B(-\boldsymbol{k}) = \zeta_B^*(\boldsymbol{k})$$

⇨ Parity transformation of the trispectrum

$$T_{\Pi_B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) \to T_{\Pi_B}(-\boldsymbol{k}_1,-\boldsymbol{k}_2,-\boldsymbol{k}_3,-\boldsymbol{k}_4) = (T_{\Pi_B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4))^*$$

- ✓ Real part of the trispectrum $\mathrm{Re}\left[T_{\Pi_B}\right]$ → Parity-even
- Imaginary part of the trispectrum $\mathrm{Im}\left[T_{\Pi_B}\right]$ → Parity-odd

Parity violation for scalar quantity appears in “imaginary part of the trispectrum”

Statistical Properties of PMF

Anisotropic stress of PMFs

$$\Pi_{B\,ij}(\boldsymbol{k}) = -\frac{1}{4\pi\rho_{\gamma,0}}\int \frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3} B_i(\boldsymbol{k}')B_j(\boldsymbol{k}-\boldsymbol{k}')$$

PMFs power spectrum

Non-helical — Helical

$$\langle B_i(\boldsymbol{k})B_j(\boldsymbol{k}')\rangle = \frac{(2\pi)^3}{2}\delta_{\mathrm{D}}(\boldsymbol{k}+\boldsymbol{k}')\left[P_{ij}(\hat{\boldsymbol{k}})P_B(k) + i\epsilon_{ijl}\hat{k}_l P_H(k)\right]$$

Levi-Civita tensor

$$\equiv \frac{(2\pi)^3}{2}\delta_{\mathrm{D}}(\boldsymbol{k}+\boldsymbol{k}')\mathcal{P}_{ij}(\boldsymbol{k})$$

Assumption: Power-law

Helical-to-non-helical ratio

$$P_B(k) = A_B k^{n_B}\,,\; P_H(k) = r_H A_B k^{n_H}$$

$$|r_H| \leq 1$$

$$A_B = \frac{4\pi^2 r^{n_B+3} B_r^2}{\Gamma\left(\frac{n_B+3}{2}\right)}$$

Analytic formalism of the trispectrum

Analytic expression of passive scalar mode trispectrum

$$T_{\Pi_B}(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4) = \frac{1}{(4\pi\rho_{\gamma,0})^4} O^{(0)}_{ab}(\hat{\boldsymbol{k}}_1) O^{(0)}_{cd}(\hat{\boldsymbol{k}}_2) O^{(0)}_{ef}(\hat{\boldsymbol{k}}_3) O^{(0)}_{gh}(\hat{\boldsymbol{k}}_4) \times \int \frac{\mathrm{d}^3\boldsymbol{k}'_1}{(2\pi)^3} \Big[\mathcal{P}_{ac}(\boldsymbol{k}'_1)\mathcal{P}_{be}(\boldsymbol{k}_1-\boldsymbol{k}'_1)\mathcal{P}_{dg}(\boldsymbol{k}'_1+\boldsymbol{k}_2)\mathcal{P}_{fh}(-\boldsymbol{k}'_1-\boldsymbol{k}_2-\boldsymbol{k}_4) + \mathcal{P}_{ac}(\boldsymbol{k}'_1)\mathcal{P}_{bg}(\boldsymbol{k}_1-\boldsymbol{k}'_1)\mathcal{P}_{de}(\boldsymbol{k}'_1+\boldsymbol{k}_2)\mathcal{P}_{fh}(\boldsymbol{k}'_1+\boldsymbol{k}_2+\boldsymbol{k}_3) + \mathcal{P}_{ae}(\boldsymbol{k}'_1)\mathcal{P}_{bg}(\boldsymbol{k}_1-\boldsymbol{k}'_1)\mathcal{P}_{ch}(\boldsymbol{k}'_1-\boldsymbol{k}_1-\boldsymbol{k}_4)\mathcal{P}_{df}(-\boldsymbol{k}'_1-\boldsymbol{k}_3) \Big]$$

$$\Pi_B(\boldsymbol{k}) \equiv O^{(0)}_{ij}(\hat{\boldsymbol{k}})\Pi_{Bij}(\boldsymbol{k}),\ \ O^{(0)}_{ij}(\hat{\boldsymbol{k}}) \equiv \frac{\delta_{ij}}{3} - \hat{k}_i\hat{k}_j$$

! Many arguments & k-Integral → Numerical calculation is challenging

→

1. Perform k-integral analytically using <u>an approximation</u>
2. Consider the simplified configuration of wave vectors

Pole Approximation

Assuming nearly scale-invariant power spectrum : $n_B = n_H = -2.9$

⇨ Dominant contribution: $k' \sim 0$

✓ Pole approximation enables us to perform k-integrals **analytically**

Trispectrum after pole approximation

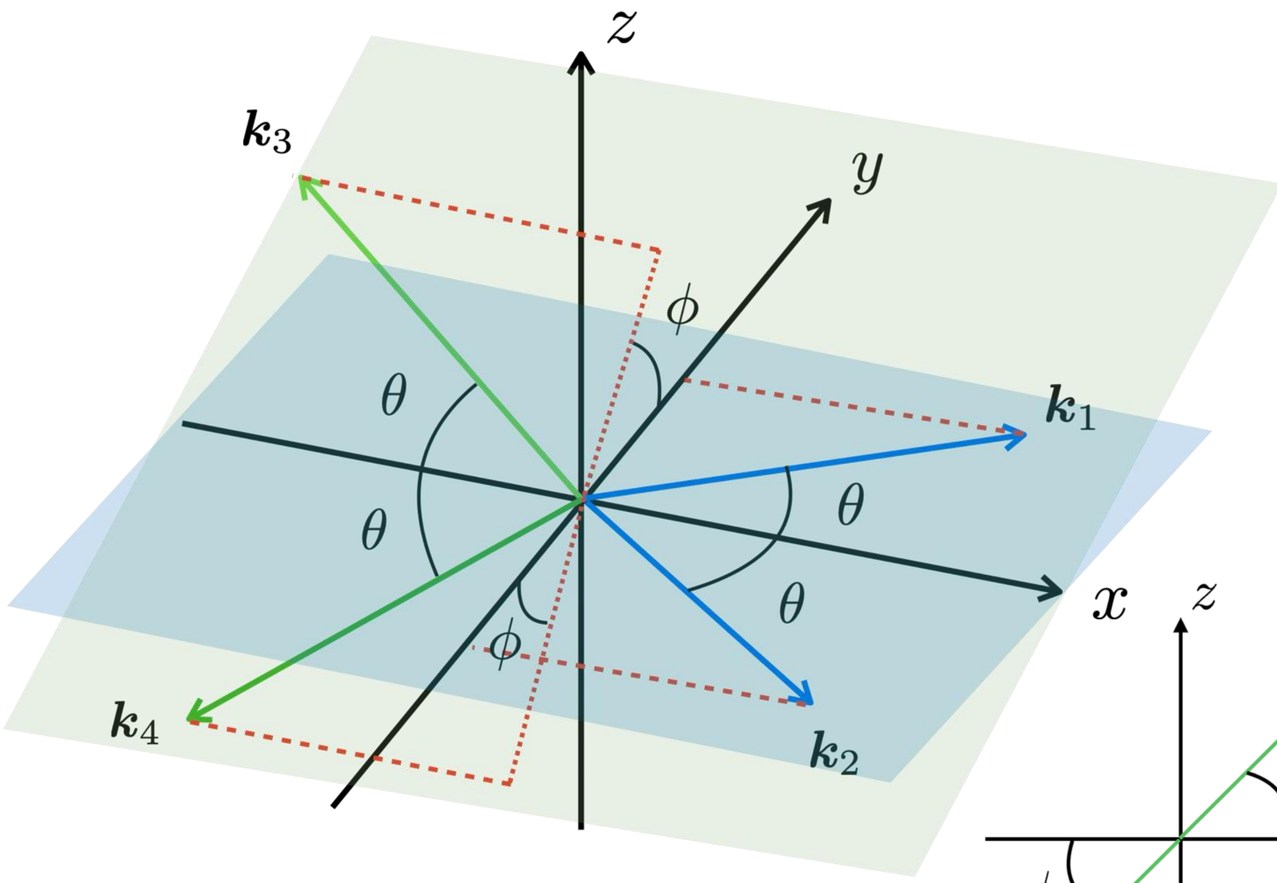
$$T_{\Pi_B}(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{k}_3, \boldsymbol{k}_4) \simeq \frac{\mathcal{A}_B}{(4\pi\rho_{\gamma,0})^4} O^{(0)}_{ab}(\hat{\boldsymbol{k}}_1) O^{(0)}_{cd}(\hat{\boldsymbol{k}}_2) O^{(0)}_{ef}(\hat{\boldsymbol{k}}_3) O^{(0)}_{gh}(\hat{\boldsymbol{k}}_4)$$

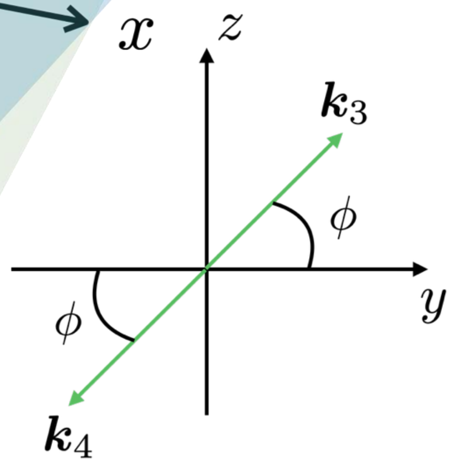
$$\times \left[\delta_{ac} \mathcal{P}_{be}(\boldsymbol{k}_1) \mathcal{P}_{dg}(\boldsymbol{k}_2) \mathcal{P}_{fh}(\boldsymbol{k}_1 + \boldsymbol{k}_3) + (11\,\mathrm{perms.}) \right]$$

$$\int \frac{\mathrm{d}^3\boldsymbol{k}'}{(2\pi)^3} \mathcal{P}_{ab}(\boldsymbol{k}') = \mathcal{A}_B \delta_{ab}, \; \mathcal{A}_B \equiv \frac{2}{3} \int \frac{k^2 \mathrm{d}k'}{2\pi^2} P_B(k) = \frac{A_B}{6\pi^2} k_*^{n_B+3} \Gamma\left(\frac{n_B+3}{2}\right)$$

Exact Equilateral Configuration

Tomohiro Fujita et al., JCAP05 (2024) 127





Momentum conservation：

$$\boldsymbol{k}_1 + \boldsymbol{k}_2 + \boldsymbol{k}_3 + \boldsymbol{k}_4 = \boldsymbol{0}$$

4 wave vectors

$$\boldsymbol{k}_1 = k(\cos\theta, \sin\theta, 0)$$

$$\boldsymbol{k}_2 = k(\cos\theta, -\sin\theta, 0)$$

$$\boldsymbol{k}_3 = k(-\cos\theta, \sin\theta\cos\phi, \sin\theta\sin\phi)$$

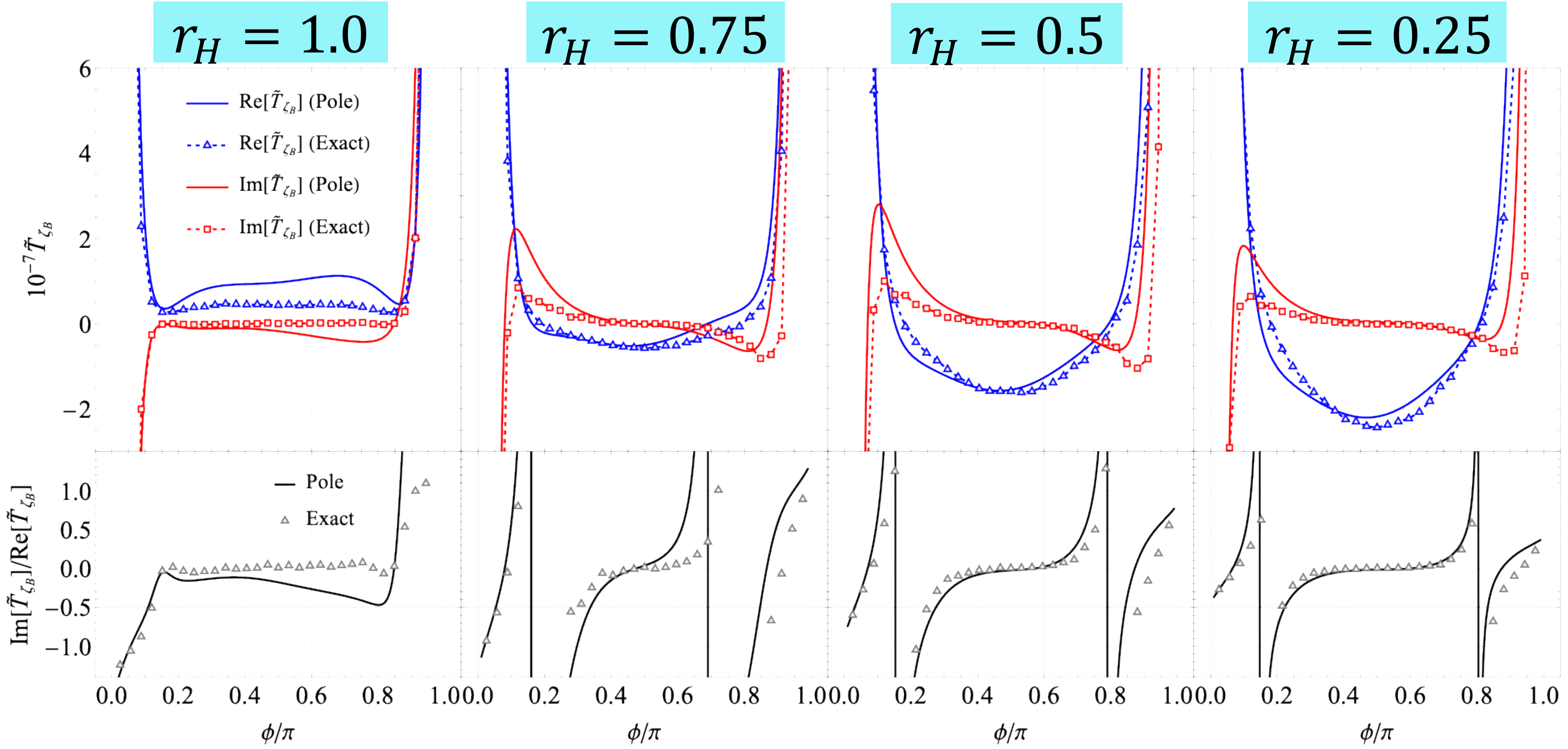
$$\boldsymbol{k}_4 = k(-\cos\theta, -\sin\theta\cos\phi, -\sin\theta\sin\phi)$$

$$0 < \theta < \pi/2\,,\ 0 \le \phi < \pi$$

Parameters : $\{k, \theta, \phi\}$

How the trispectrum depends on tetrahedral shape?

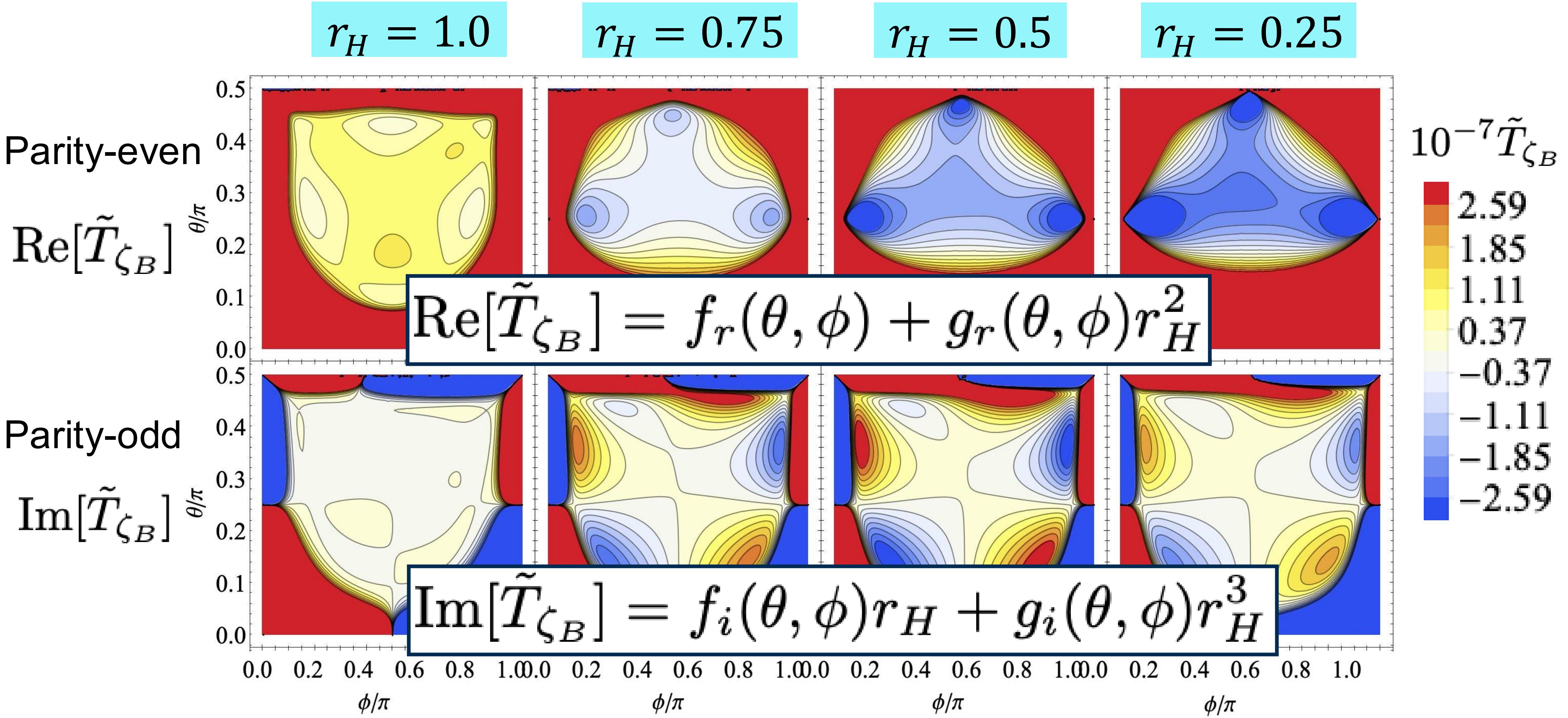
Pole Approximation Trispectrum (ϕ dependence)



Trispectrum has the peak in collapsed limit ($\phi \to 0, \pi$) $|\boldsymbol{k}_1 + \boldsymbol{k}_4| \ll k_1, k_2, k_3, k_4$

➡ **Pole approximation reproduce the peak in collapsed limit**

Pole Approximation Trispectrum Shape

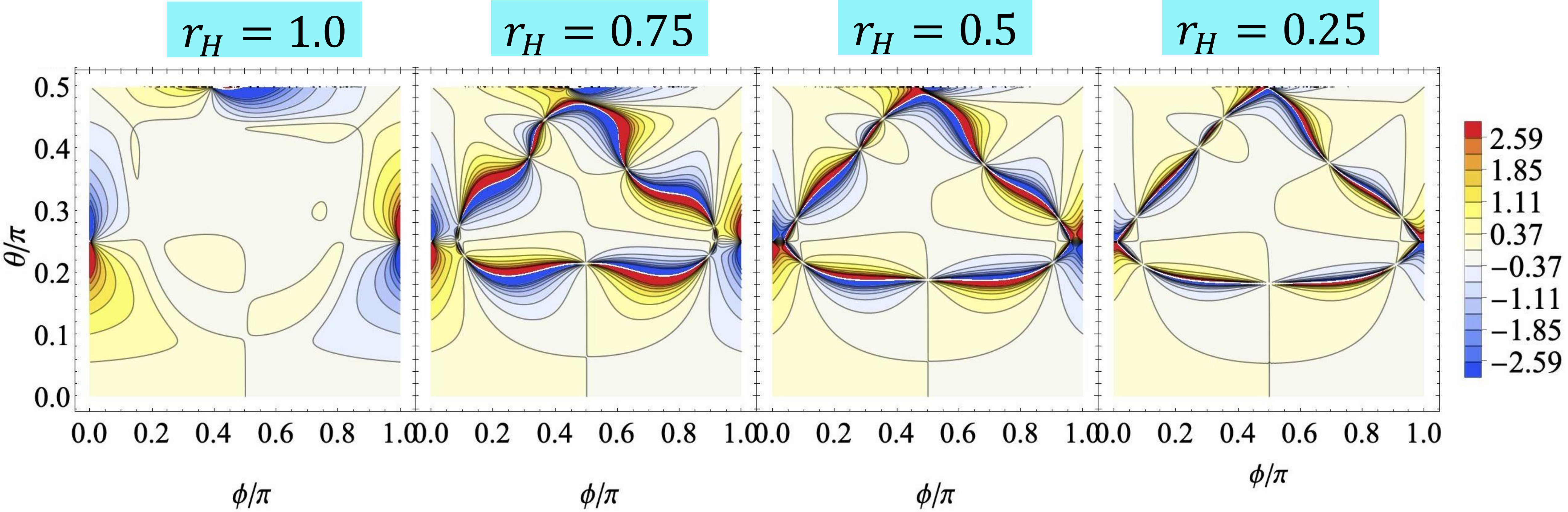


$$\mathrm{Re}[\tilde{T}_{\zeta_B}] = f_r(\theta,\phi) + g_r(\theta,\phi) r_H^2$$

$$\mathrm{Im}[\tilde{T}_{\zeta_B}] = f_i(\theta,\phi) r_H + g_i(\theta,\phi) r_H^3$$

Ratio of Imaginary to Real Part of Trispectrum

Imaginary/Real



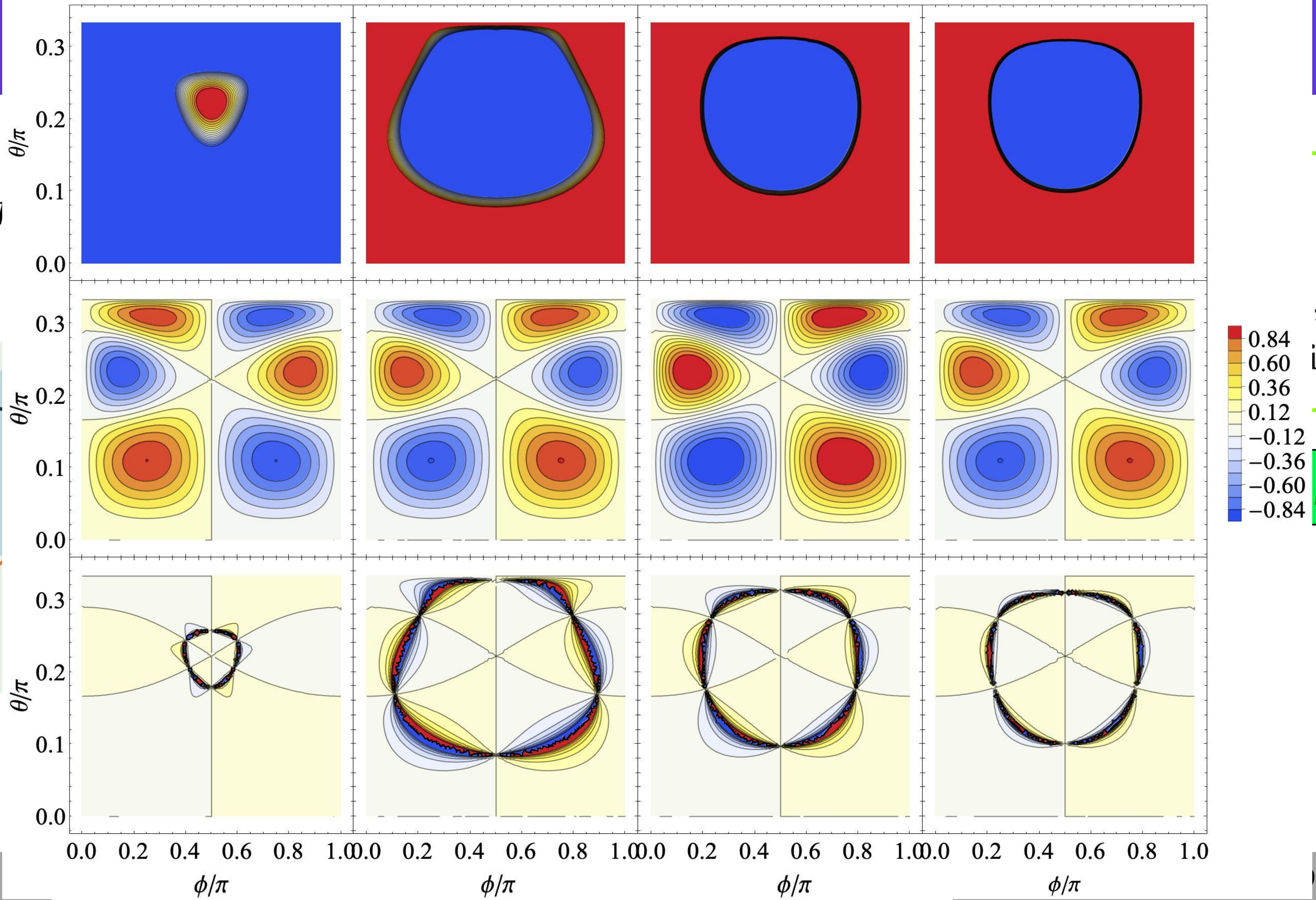
Qualitative change in the structure from $r_H = 1.0$ to 0.75

Triangle-like structure is the feature of scale-invariant helical PMF

$\mathrm{Re}[\tilde{T}_{\zeta_B}]$

$\mathrm{Im}[\tilde{T}_{\zeta_B}]$

$$\frac{\mathrm{Im}[\tilde{T}_{\zeta_B}]}{\mathrm{Re}[\tilde{T}_{\zeta_B}]}$$

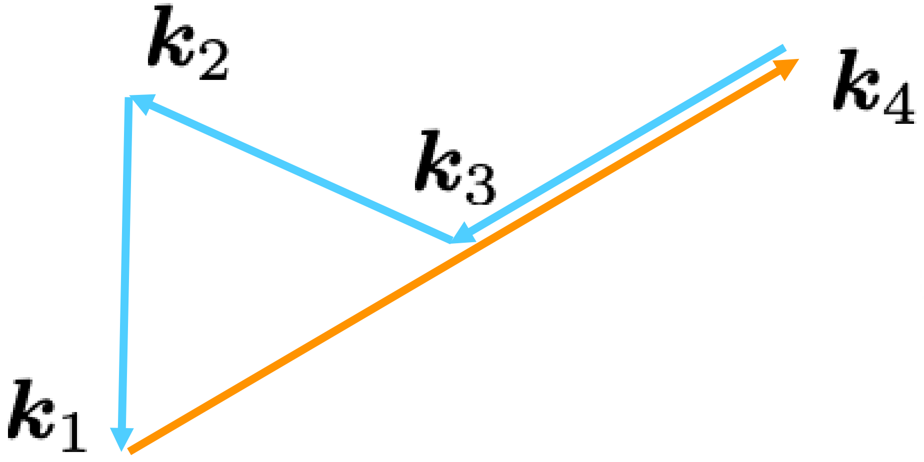


,

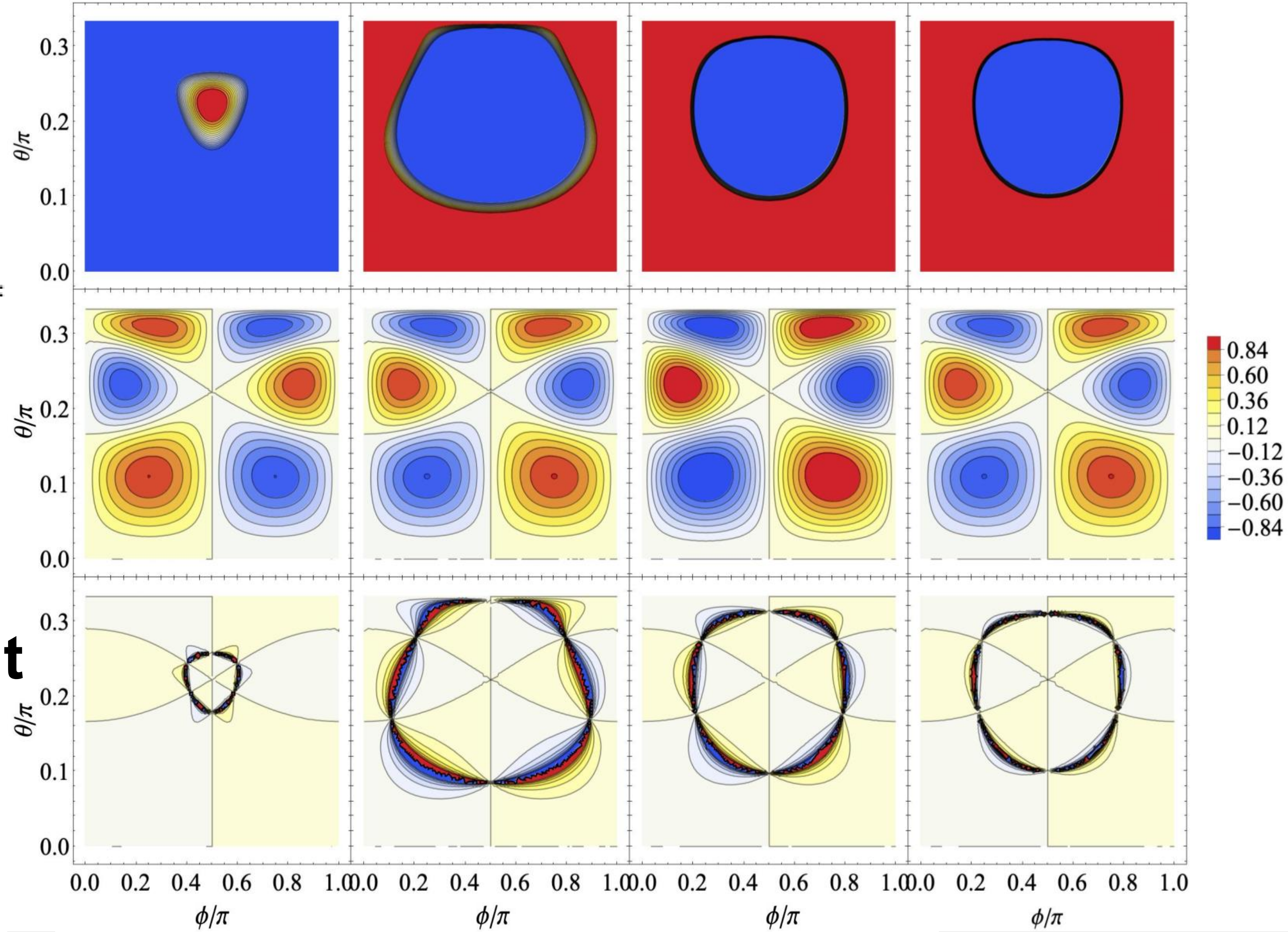
in ϕ)

k

Quasi Equilateral Configuration



Cannot take collapsed limit



Implication from CMB Trispectrum

Template of trispectrum

M. Shiraishi, Phys.Rev.D 94 (2016) 083503
O.H.E. Philcox, Phys.Rev.D 111 (2025) 123534

$$[T_\zeta(\boldsymbol{k}_1,\boldsymbol{k}_2,\boldsymbol{k}_3,\boldsymbol{k}_4)]_{d_1^{\rm odd}} \supset -i\left\{\hat{\boldsymbol{k}}_{12}\cdot(\hat{\boldsymbol{k}}_1\times\hat{\boldsymbol{k}}_3)\right\}\left[-(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_{12})+(\hat{\boldsymbol{k}}_3\cdot\hat{\boldsymbol{k}}_{12})+(\hat{\boldsymbol{k}}_1\cdot\hat{\boldsymbol{k}}_3)\right]$$

$$\times\, d_1^{\rm odd} P_\zeta(k_1)P_\zeta(k_3)P_\zeta(k_{12})$$

Ratio to our template

$$|d_1^{\rm odd}| = \mathcal{T}^4\frac{\mathcal{A}_B}{(4\pi\rho_{\gamma,0})^4}\frac{1}{81}|r_H|\frac{P_B(k_1)P_B(k_3)P_B(k_{12})}{P_\zeta(k_1)P_\zeta(k_3)P_\zeta(k_{12})}$$

$$\approx 10\,|r_H|\left(\frac{B_r}{\rm nG}\right)^8$$

$$P_\zeta(k) = 2\pi^2 A_{\rm s} k^{-3}$$

Constraints from *Planck*

$$|d_1^{\rm odd}| \lesssim 10^3$$

$$B_{1\,\rm Mpc} = 5\,{\rm nG}$$

$$|r_H| \lesssim 10^{-4}$$

Summary

Summary

- Derived an analytic formulation of the trispectrum of passive scalar mode induced by helical PMF by **pole approximation**
- Found pole approximation is useful for nearly scale-invariant PMF
- Predicted the **parity-odd signal of the trispectrum** under the different helical-to-non-helical ratio in the exact equilateral configuration and quasi equilateral configuration
- Presented the observational implication of helical-to-non-helical ratio $r_H \lesssim 10^{-4}$ with $B_{1\,\mathrm{Mpc}} = 5\,\mathrm{nG}$ from CMB trispectrum

Prospects

- Calculation of the CMB trispectrum in multipole space and estimation of S/N for future experiments (e.g., Simons Observatory, LiteBIRD)
- Computation of the trispectrum of galaxy shape generated by the passive mode from helical PMFs

Appendix

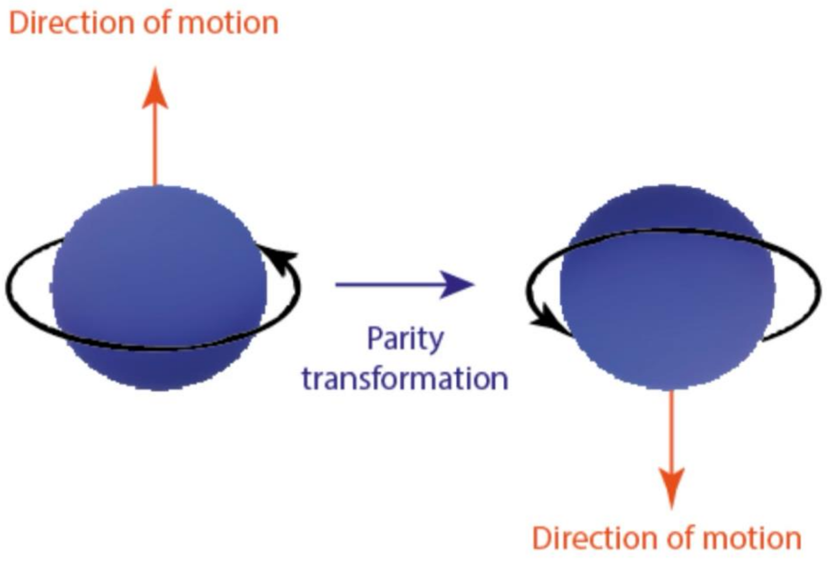
What is Parity?

- Parity transformation = Inversion of all spatial coordinates

  Mirror reflection + Rotation

- **Parity symmetry** of physical states :

  Physical law is invariant
  under the parity transformation



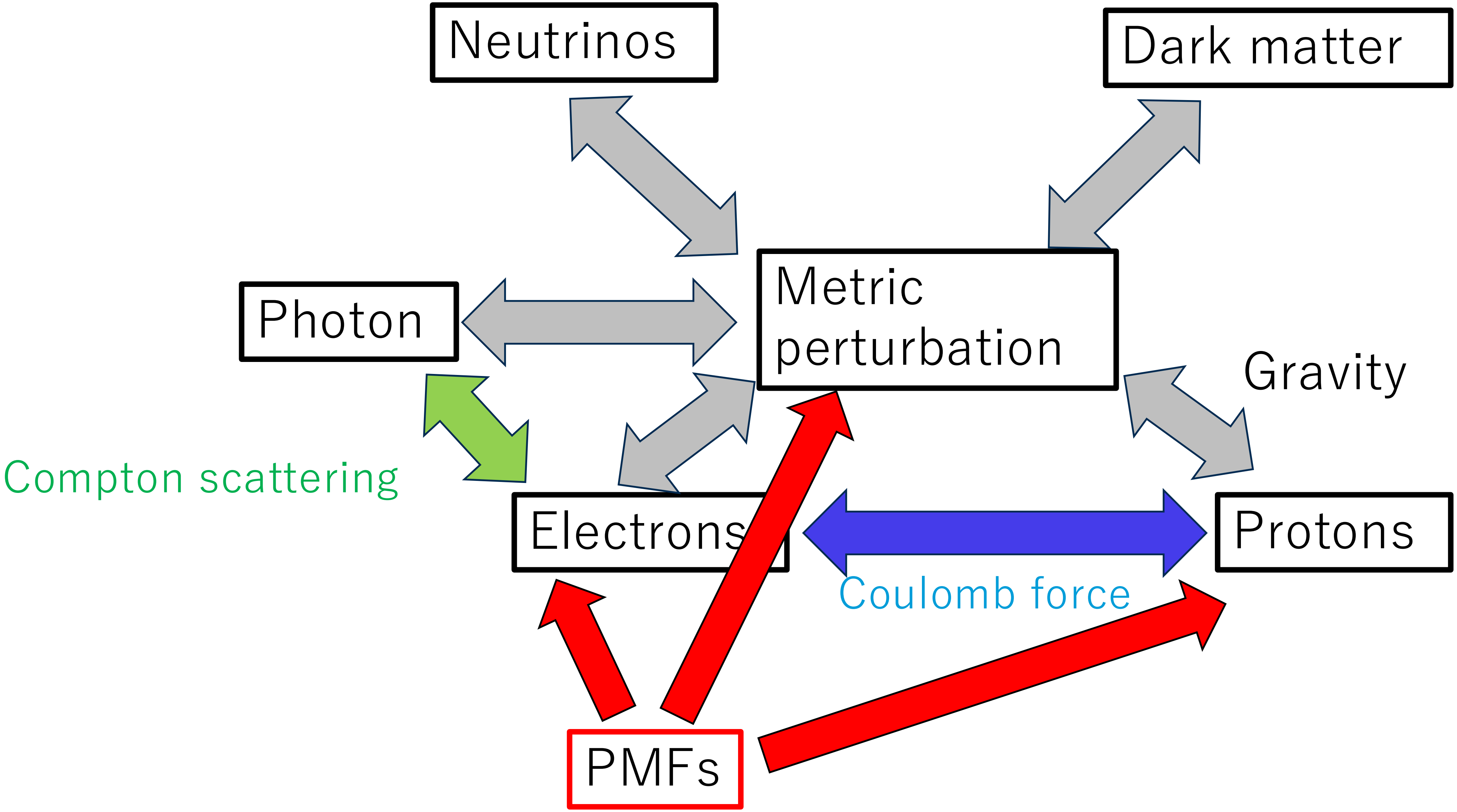
GR conserves parity symmetry

⇕

Some **parity-violating** theories:

- ➢ Axion-like coupling: $\mathcal{L}_{int} = g(\chi) F_{\mu\nu} \tilde{F}^{\mu\nu}$
- ➢ Chern-Simons-like interaction: $\mathcal{L}_{int} = f(\Phi) R^{\lambda}_{\ \sigma\mu\nu} \tilde{R}^{\sigma\mu\nu}_{\lambda}$

Is our universe parity-symmetric?


Neutrinos
Dark matter
Photon
Metric perturbation
Gravity
Compton scattering
Electrons
Protons
Coulomb force
PMFs

Exact Trispectrum

Pattern A

$$\langle \Pi_{Bab}(\boldsymbol{k}_1)\Pi_{Bcd}(\boldsymbol{k}_2)\Pi_{Bef}(\boldsymbol{k}_3)\Pi_{Bgh}(\boldsymbol{k}_4)\rangle_{\rm A}$$

$$= \frac{1}{(4\pi\rho_{\gamma,0})^4}\left(\prod_{i=1}^{4}\int \mathrm{d}^3\boldsymbol{k}'_i\right)\delta_{\rm D}(\boldsymbol{k}'_2+\boldsymbol{k}'_1-\boldsymbol{k}_1)\delta_{\rm D}(\boldsymbol{k}'_3+\boldsymbol{k}'_4-\boldsymbol{k}_2)\delta_{\rm D}(-\boldsymbol{k}'_2-\boldsymbol{k}'_4-\boldsymbol{k}_3)\delta_{\rm D}(-\boldsymbol{k}'_1-\boldsymbol{k}'_3-\boldsymbol{k}_4)$$

$$\times\frac{1}{2^4}\left[\mathcal{P}_{ac}(\boldsymbol{k}'_1)\mathcal{P}_{be}(\boldsymbol{k}'_2)\mathcal{P}_{dg}(\boldsymbol{k}'_3)\mathcal{P}_{fh}(\boldsymbol{k}'_4)+(a\leftrightarrow b\ \ \text{or}\ \ c\leftrightarrow d\ \ \text{or}\ \ e\leftrightarrow f\ \ \text{or}\ \ g\leftrightarrow h)\right]$$

Pattern B

$$\langle \Pi_{Bab}(\boldsymbol{k}_1)\Pi_{Bcd}(\boldsymbol{k}_2)\Pi_{Bef}(\boldsymbol{k}_3)\Pi_{Bgh}(\boldsymbol{k}_4)\rangle_{\rm B}$$

$$= \frac{1}{(4\pi\rho_{\gamma,0})^4}\left(\prod_{i=1}^{4}\int \mathrm{d}^3\boldsymbol{k}'_i\right)\delta_{\rm D}(\boldsymbol{k}'_2+\boldsymbol{k}'_1-\boldsymbol{k}_1)\delta_{\rm D}(\boldsymbol{k}'_3+\boldsymbol{k}'_4-\boldsymbol{k}_2)\delta_{\rm D}(-\boldsymbol{k}'_2-\boldsymbol{k}'_4-\boldsymbol{k}_3)\delta_{\rm D}(-\boldsymbol{k}'_1-\boldsymbol{k}'_3-\boldsymbol{k}_4)$$

$$\times\frac{1}{2^4}\left[\mathcal{P}_{ac}(\boldsymbol{k}'_1)\mathcal{P}_{bg}(\boldsymbol{k}'_2)\mathcal{P}_{de}(\boldsymbol{k}'_3)\mathcal{P}_{fh}(\boldsymbol{k}'_4)+(a\leftrightarrow b\ \ \text{or}\ \ c\leftrightarrow d\ \ \text{or}\ \ e\leftrightarrow f\ \ \text{or}\ \ g\leftrightarrow h)\right]$$

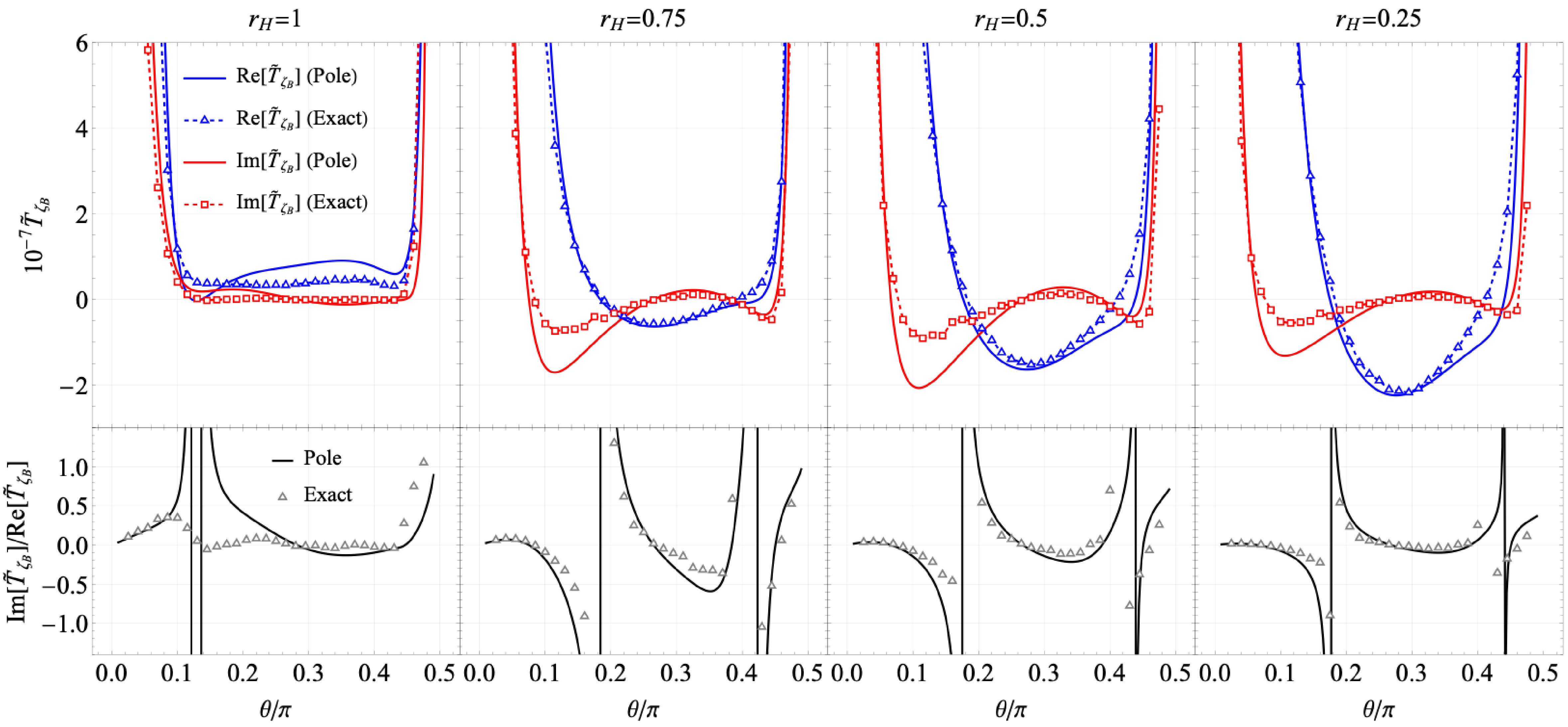
Pattern C

$$\langle \Pi_{Bab}(\boldsymbol{k}_1)\Pi_{Bcd}(\boldsymbol{k}_2)\Pi_{Bef}(\boldsymbol{k}_3)\Pi_{Bgh}(\boldsymbol{k}_4)\rangle_{\rm C}$$

$$= \frac{1}{(4\pi\rho_{\gamma,0})^4}\left(\prod_{i=1}^{4}\int \mathrm{d}^3\boldsymbol{k}'_i\right)\delta_{\rm D}(\boldsymbol{k}'_2+\boldsymbol{k}'_1-\boldsymbol{k}_1)\delta_{\rm D}(\boldsymbol{k}'_3+\boldsymbol{k}'_4-\boldsymbol{k}_2)\delta_{\rm D}(-\boldsymbol{k}'_1-\boldsymbol{k}'_4-\boldsymbol{k}_3)\delta_{\rm D}(-\boldsymbol{k}'_2-\boldsymbol{k}'_3-\boldsymbol{k}_4)$$

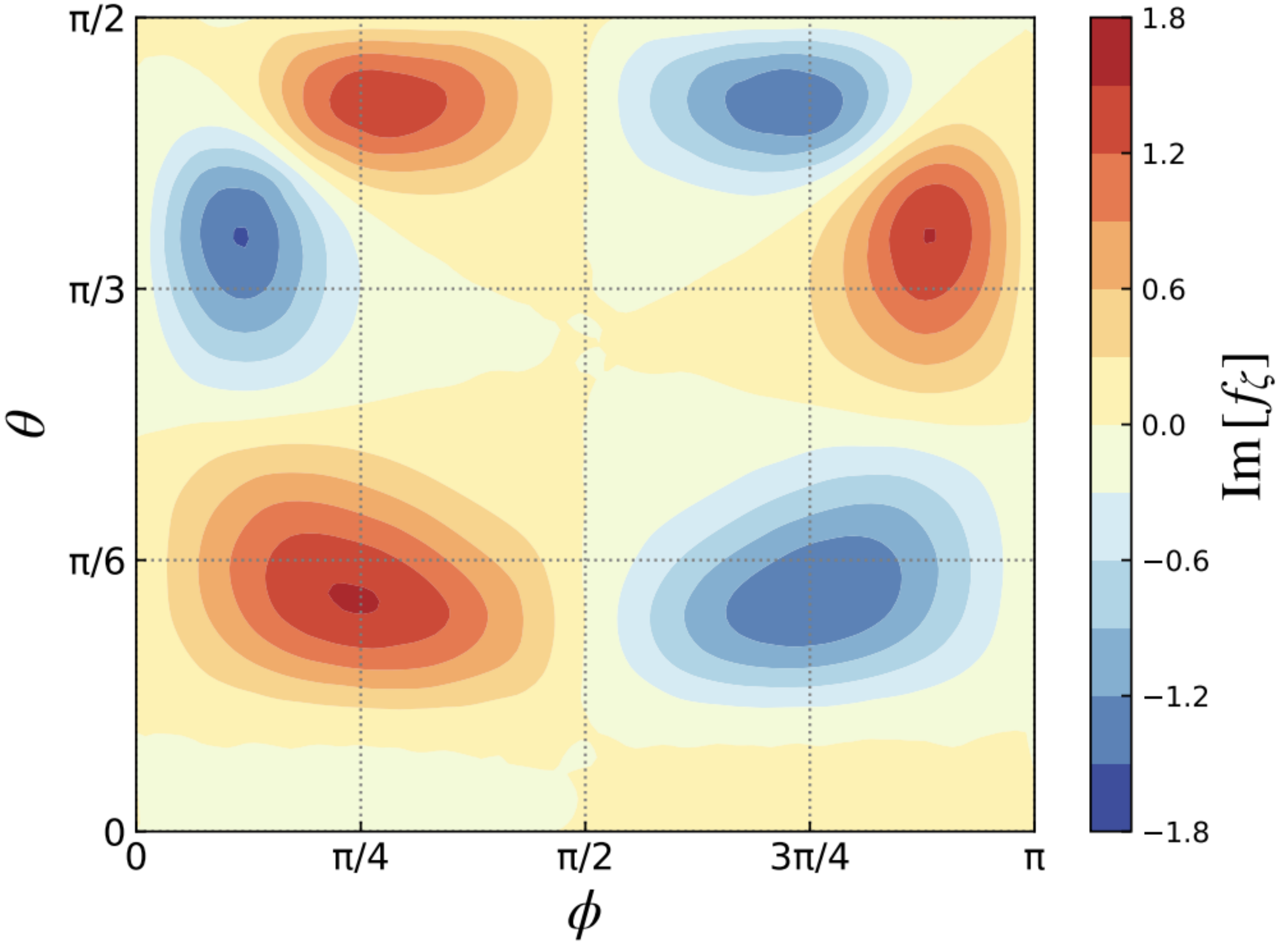
$$\times\frac{1}{2^4}\left[\mathcal{P}_{ae}(\boldsymbol{k}'_1)\mathcal{P}_{bg}(\boldsymbol{k}'_2)\mathcal{P}_{ch}(\boldsymbol{k}'_3)\mathcal{P}_{df}(\boldsymbol{k}'_4)+(a\leftrightarrow b\ \ \text{or}\ \ c\leftrightarrow d\ \ \text{or}\ \ e\leftrightarrow f\ \ \text{or}\ \ g\leftrightarrow h)\right]$$

Pole approx. vs Exact Integration ~θ dependence~



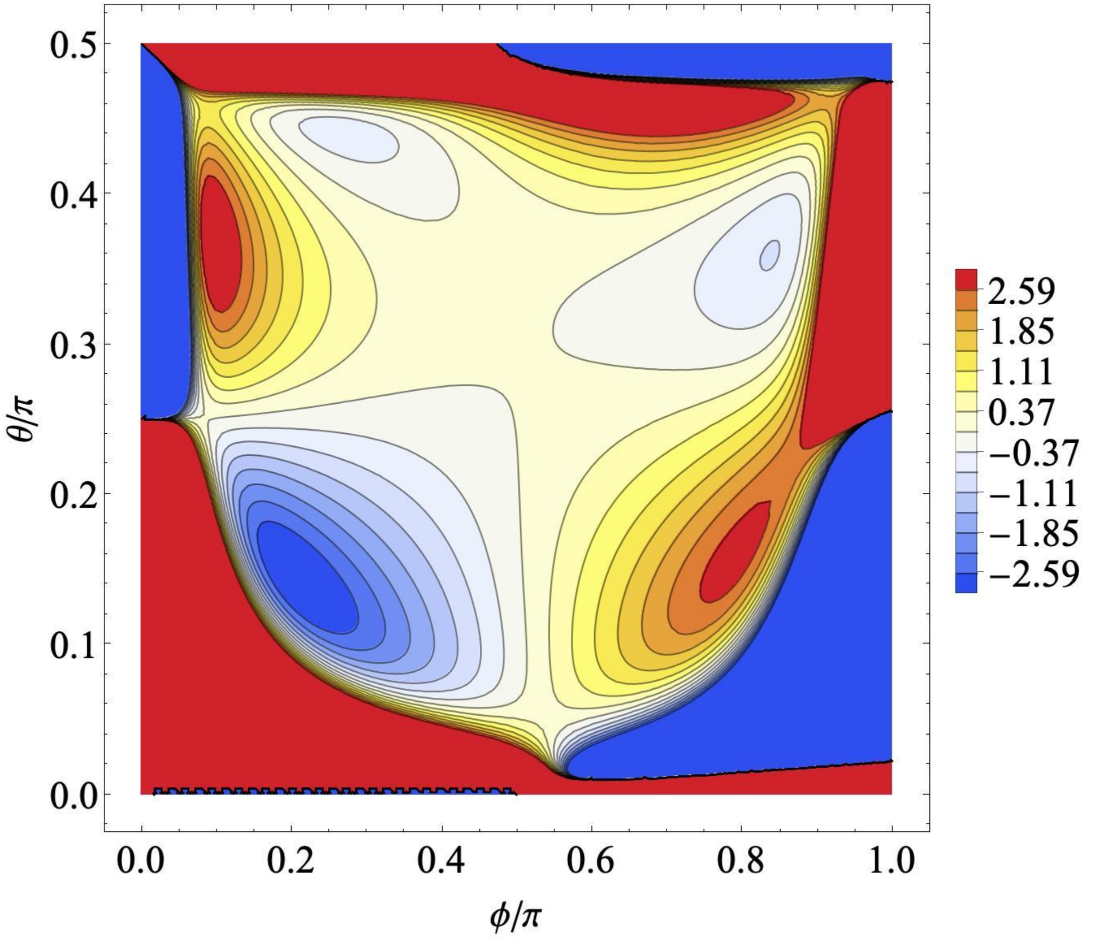
Comparison of Rolling axion model

Rolling axion model



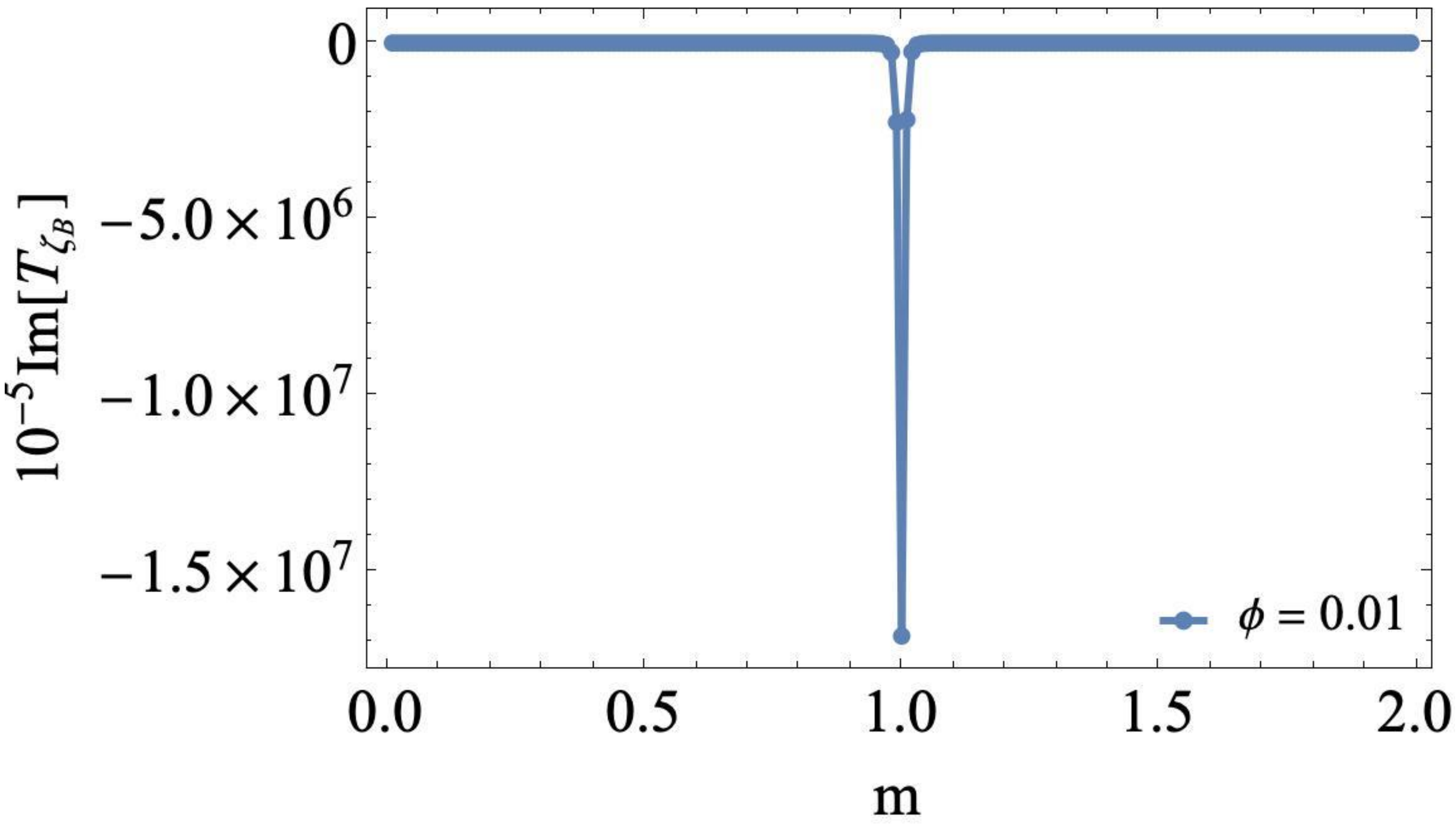
Tomohiro Fujita et al., JCAP05 (2024) 127

Our helical PMF model



Quasi Equilateral Configuration

$$k_4 = mk$$
$$\phi = 0.01$$



$m \neq 1$ ➡ The trispectrum has no peak in collapsed limit

Quasi Equilateral Configuration