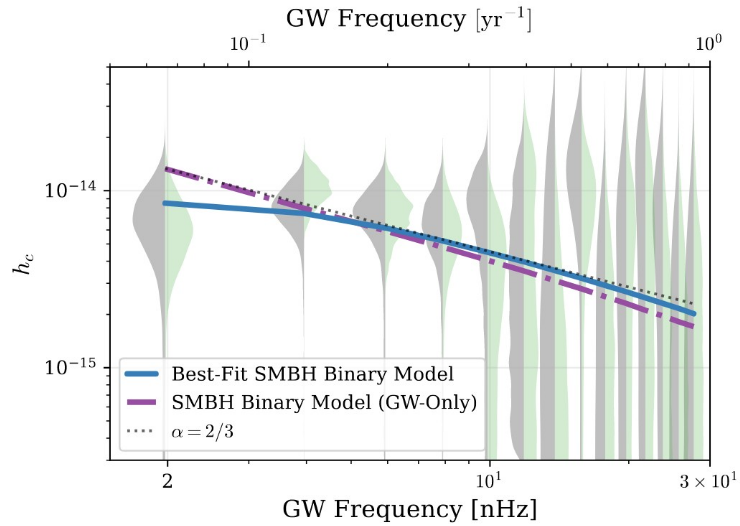
Gravitational waves from coalescing supermassive black holes in a galaxy and AGN formation model based on the CDM cosmology

Masahiro Nagashima (Bunkyo U)
Motohiro Enoki (Tokyo Keizai U)
Taira Oogi (National Institute of Technology, Asahikawa College)
and ν^2GC team ("New Numerical Galaxy Catalog" galaxy formation model)

Enoki, Inoue, Nagashima & Sugiyama (2004) for circular orbit
Enoki & Nagashima (2007) for eccentric orbit
Oogi et al., in preparation

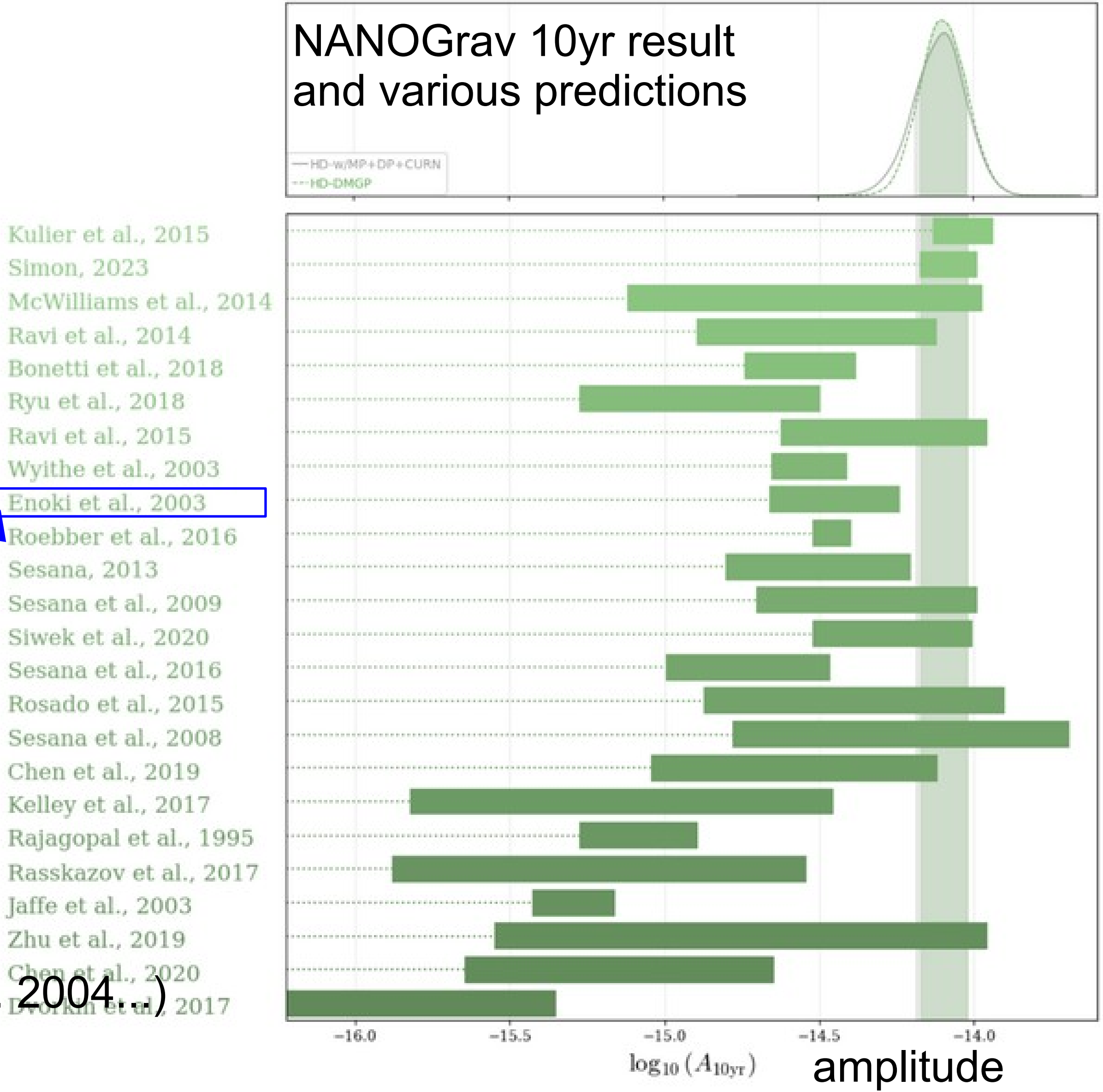
GOAL of this talk:

How did we make this prediction by using a semi-analytic model of galaxy formation?
… we need the density of SMBH binaries and merger rate, which can be taken from our galaxy formation model!



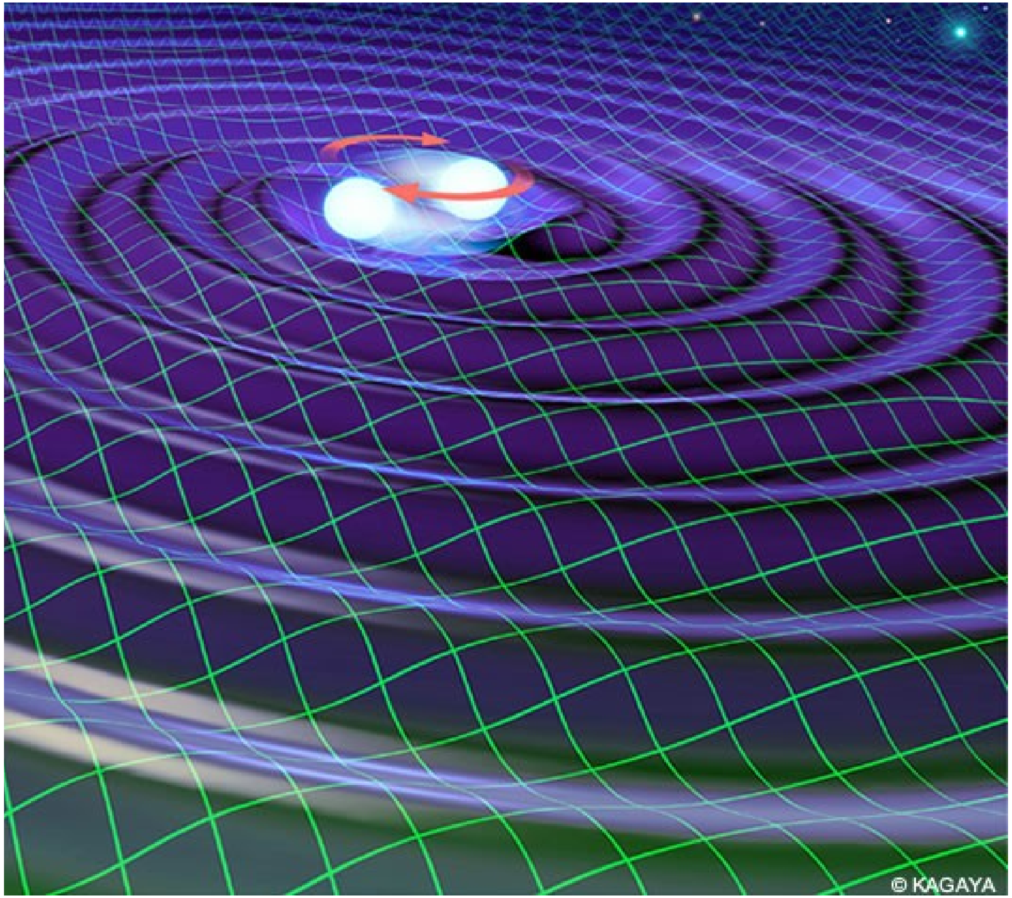
Agazie+23

(actually, Enoki et al. 2004…)



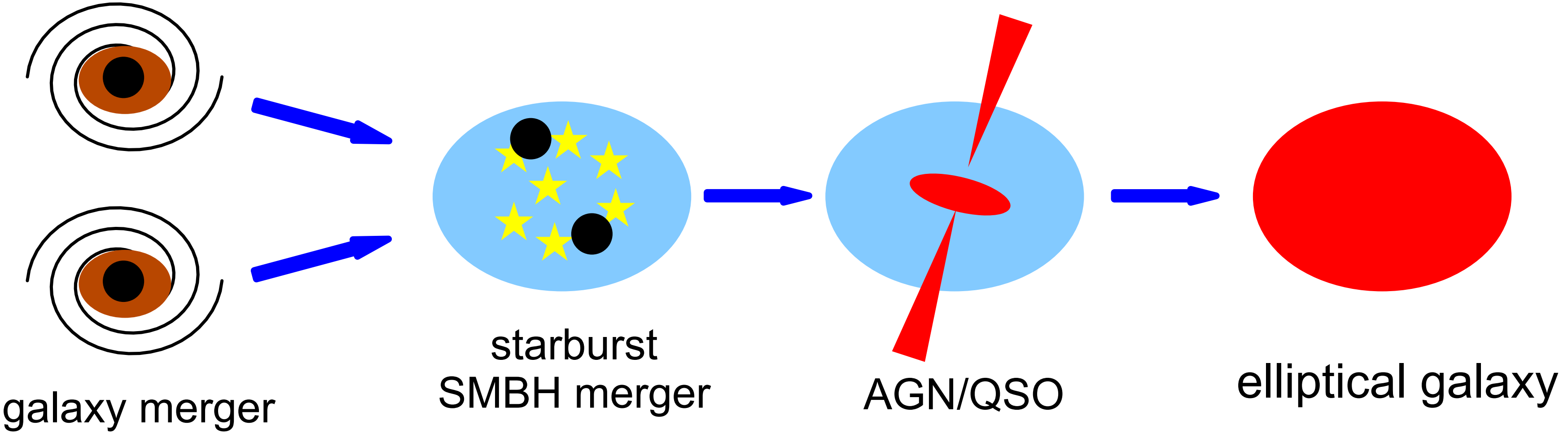
GWs from SMBH binaries

- galaxies have central supermassive black holes (SMBHs)
- galaxies merge together
- then, SMBHs also merge
  - inspiral phase - PTAs
  - GW bursts - LISA



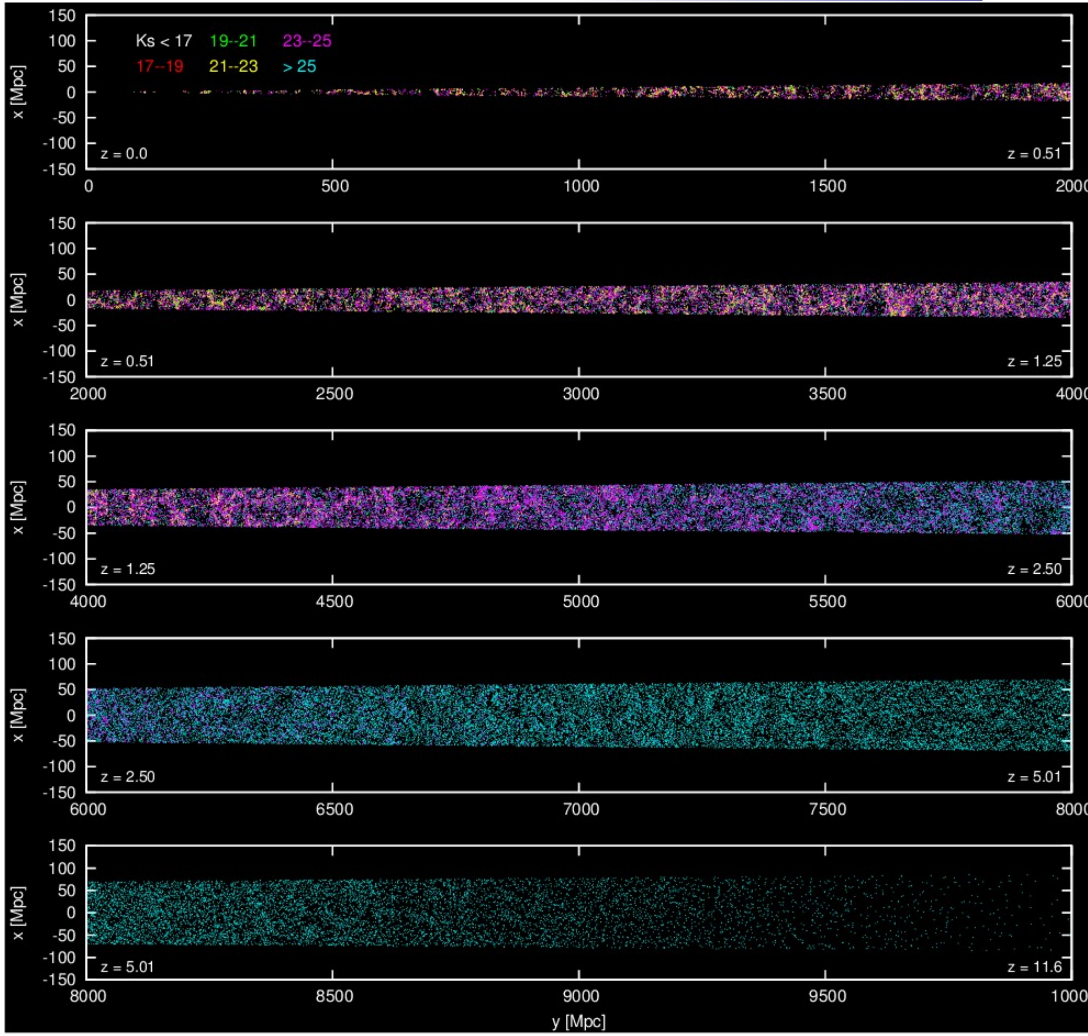
Backgrounds/Statistics

- galaxies form in the cold dark matter (CDM) universe
  - large objects form via mergers of smaller objects
- galaxy mergers often cause active galactic nuclei (AGN) activity
- to obtain the merger rates of SMBHs, we construct a galaxy/AGN formation model in the CDM universe



Backgrounds/Statistics

- combing the galaxy formation model with the GWs from individual SMBH binaries, we obtain the amplitude of the GW background and the statistics of GW bursts in the coalescence



the wedge diagram of galaxy distribution construted by our semi-analytic galaxy formation model, ν^2GC (Makiya+16)

Gravitational Waves
from
SMBH binaries

GWs from individual binary SMBHs

- low frequency (~nHz) GWs from binay SMBHs

$$f \simeq 10^{-4} \left(\frac{r}{R_{\rm S}}\right)^{-3/2} \left(\frac{M}{10^8 M_\odot}\right)^{-1} {\rm Hz} \qquad R_{\rm S} \simeq 10^{-5} \left(\frac{M}{10^8 M_\odot}\right) {\rm pc}$$

  - ensemble of these GWs generates the Gravitational Wave Background (GWB) → *Pulsar Timina Array* $(f \sim {\rm nHz})$

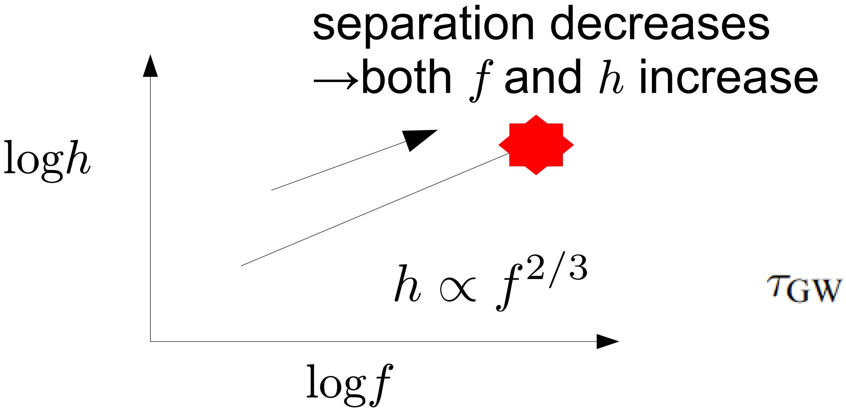
- coalescing SMBHs emit GWs as *GW bursts*

$$E_{\rm GW} \sim \epsilon M_{\rm tot} c^2, \quad \epsilon \sim 0.1 \quad (r \sim R_{\rm S}, \quad f \sim {\rm mHz} \ \to LISA)$$

  - efficiency ϵ is highly unknown, but not important (see later)

GWs from individual binary SMBHs

- for A binary



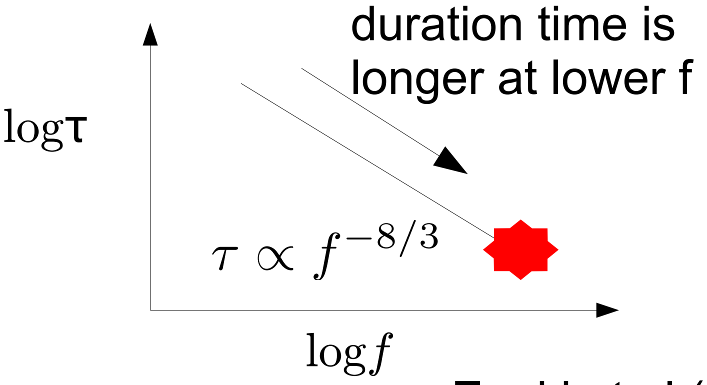
$$h_s(z, f, M_1, M_2) = 4\sqrt{\frac{2}{5}}\frac{(GM_{\rm chirp})^{5/3}}{c^4 D(z)}(2\pi f_p)^{2/3}$$

$$= 3.5\times 10^{-17}\left(\frac{M_{\rm chirp}}{10^8 M_\odot}\right)^{5/3}\left[\frac{D(z)}{\rm Gpc}\right]^{-1}\left[\frac{f(1+z)}{10^{-7}\rm Hz}\right]^{2/3}$$

$$\tau_{\rm GW} \equiv f_p \frac{dt_p}{df_p}$$

$$\tau_{\rm GW,obs}(M_1, M_2, z, f) = \frac{5}{96}\left(\frac{c^3}{GM_{\rm chirp}}\right)^{5/3}(2\pi f_p)^{-8/3}(1+z)$$

$$= 1.2\times 10^4\left(\frac{M_{\rm chirp}}{10^8 M_\odot}\right)^{-5/3}\left(\frac{f}{10^{-7}\rm Hz}\right)^{-8/3}(1+z)^{-5/3}\ {\rm yr}$$



$$\Longrightarrow h_c \propto h\sqrt{\tau} \propto f^{2/3}\times f^{-4/3} \propto f^{-3/2}$$

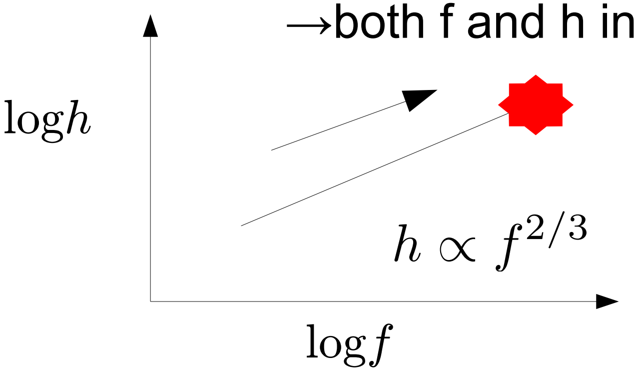
$$\tau \sim 10^{-4}\left(\frac{M}{10^8 M_\odot}\right)\left(\frac{r}{R_S}\right)^4 {\rm yr}$$

Enoki et al.(2004)

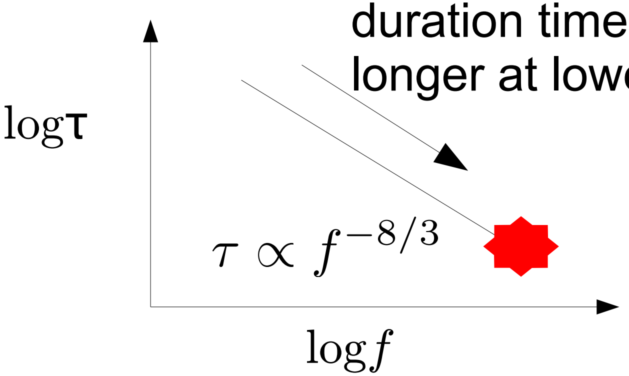
GW Background

- for A binary

separation decreases
→both f and h increase

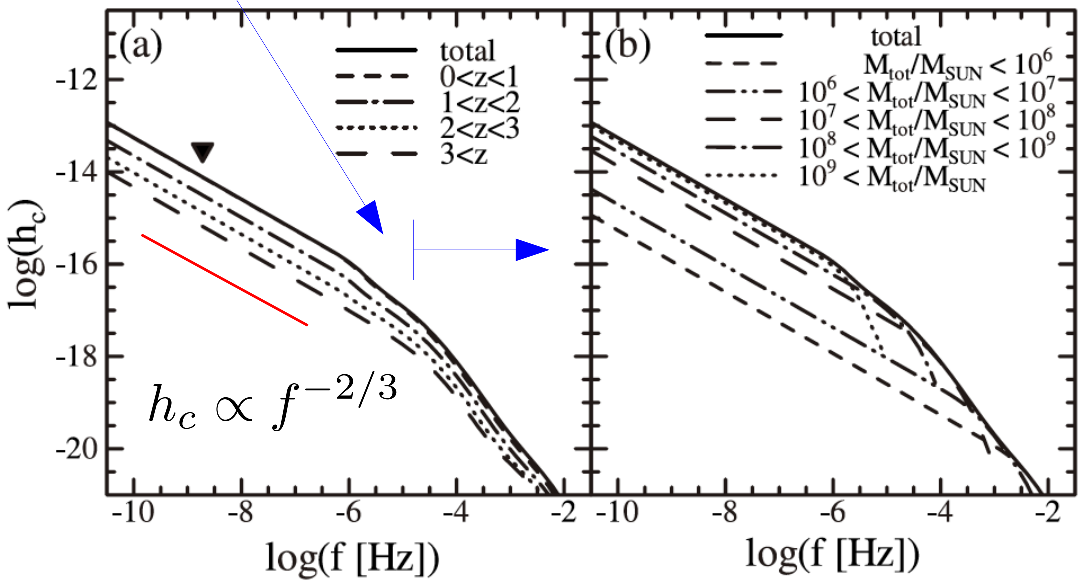


duration time is
longer at lower f



$$\Longrightarrow h_c \propto h\sqrt{\tau} \propto f^{2/3} \times f^{-4/3} \propto f^{-3/2}$$

ISCO (innermost stable circular orbit)



Enoki et al.(2004)

show later again

GW Background

- energy density of GWB (cf. Phinney 2001)

$$\rho_c c^2 \Omega_{gw}(f) = \frac{\pi}{4}\frac{c^2}{G} f^2 h_c^2(f) = \int_0^\infty N(z) \frac{1}{1+z} \left(f_r \frac{dE_{gw}}{df_r} \right)\Bigg|_{f_r = f(1+z)} dz$$

⇨ $$h_c^2(\ln f) = \frac{4G}{\pi c^2 f} \int dM_1 dM_2 dz\, n_c(M_1, M_2, z) \left(\frac{dE_{\rm GW}(M_1, M_2)}{df_r} \right)\Bigg|_{f_r = f(1+z)}$$

number of coalescing SMBHs

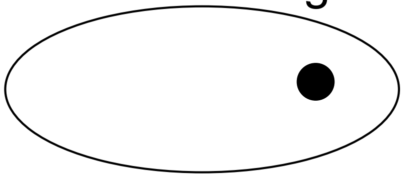
- Now we have got all merger events in the simulation box!!
- … but assuming circular orbits. eccentricity?

Enoki & Nagashima (2007)

eccentric orbit

- energy transportation from lower to higher harmonics

$$\frac{f_p}{f_{p,0}} = \left\{ \frac{1-e_0^2}{1-e^2} \left(\frac{e}{e_0}\right)^{\frac{12}{19}} \left[\frac{1+\frac{121}{304}e^2}{1+\frac{121}{304}e_0^2} \right]^{\frac{870}{2299}} \right\}^{-3/2}$$

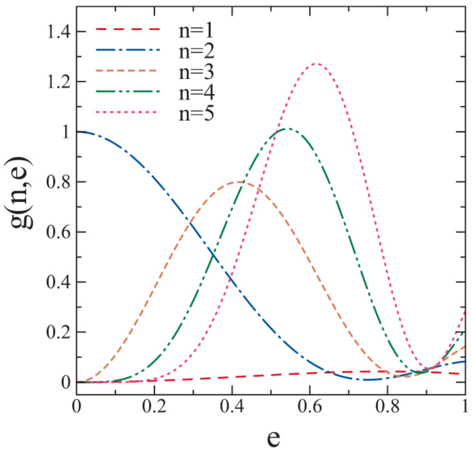


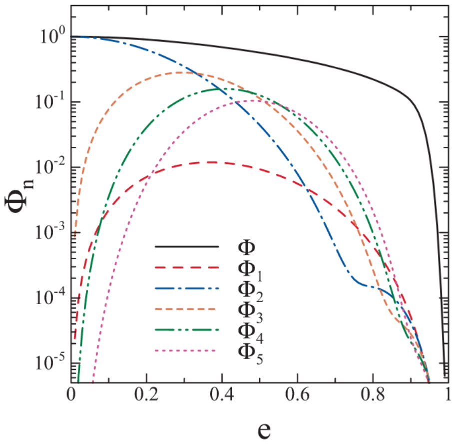
$$h_c^2(\ln f) = \frac{4\pi c^3}{3} \int dM_1 dM_2 dz\, n_c(M_1, M_2, z)(1+z)^{-1/3} \left(\frac{GM_{\rm chirp}}{c^3}\right)^{5/3} (\pi f)^{-4/3} \Phi,$$

$$\Phi \equiv \sum_{n=1}^{\infty} \Phi_n \qquad \Phi_n \equiv \left(\frac{2}{n}\right)^{2/3} \frac{g(n,e)}{F(e)}.$$

$$F(e) \equiv \frac{1+73e^2/24+37e^4/96}{(1-e^2)^{7/2}}. \qquad \sum_{n=1}^{\infty} g(n,e) = F(e)$$

$$g(n,e) \equiv \frac{n^4}{32} \left\{ \left[J_{n-2}(ne) - 2eJ_{n-1}(ne) + \frac{2}{n} J_n(ne) + 2eJ_{n+1}(ne) - J_{n+2}(ne) \right]^2 + (1-e^2) \left[J_{n-2}(ne) - 2eJ_n(ne) + J_{n+2}(ne) \right]^2 + \frac{4}{3n^2} \left[J_n(ne) \right]^2 \right\}, \qquad (2{\cdot}4)$$





evolution of eccentricity

$$\frac{da}{dt} = -\frac{64}{5}\frac{G^3 M_1 M_2 M_{\rm tot}}{c^5 a^3 (1-e^2)^{7/2}}\left(1 + \frac{73}{24}e^2 + \frac{37}{96}e^4\right)$$

$$\frac{de}{dt} = -\frac{304}{15}\frac{G^3 M_1 M_2 M_{\rm tot}}{c^5 a^4 (1-e^2)^{5/2}} e \left(1 + \frac{121}{304}e^2\right)$$

eccentric orbit

show later again

- power at low f significantly decreases

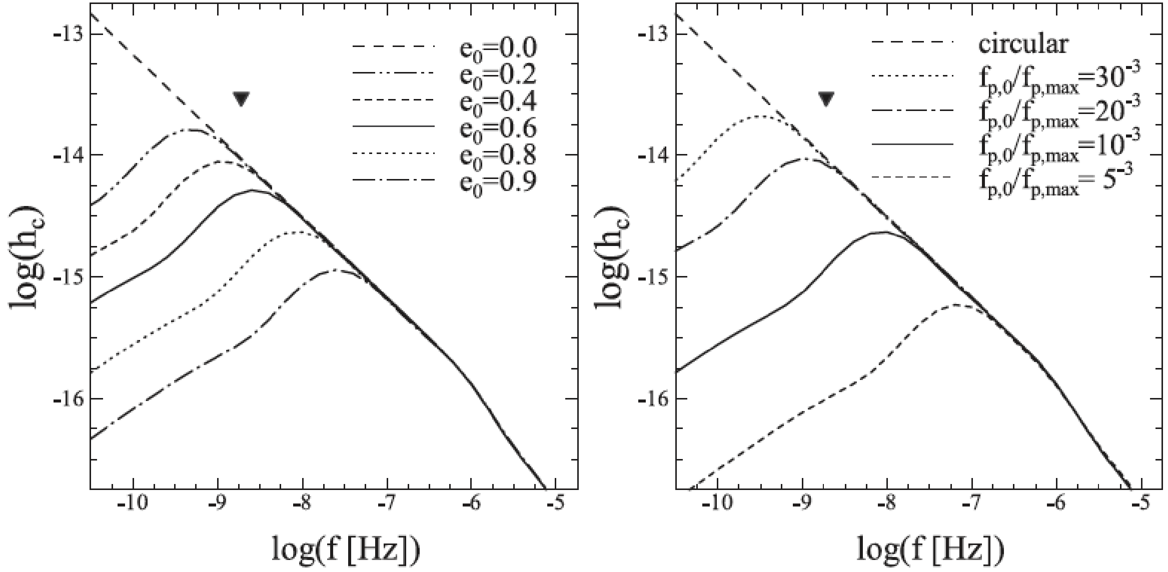
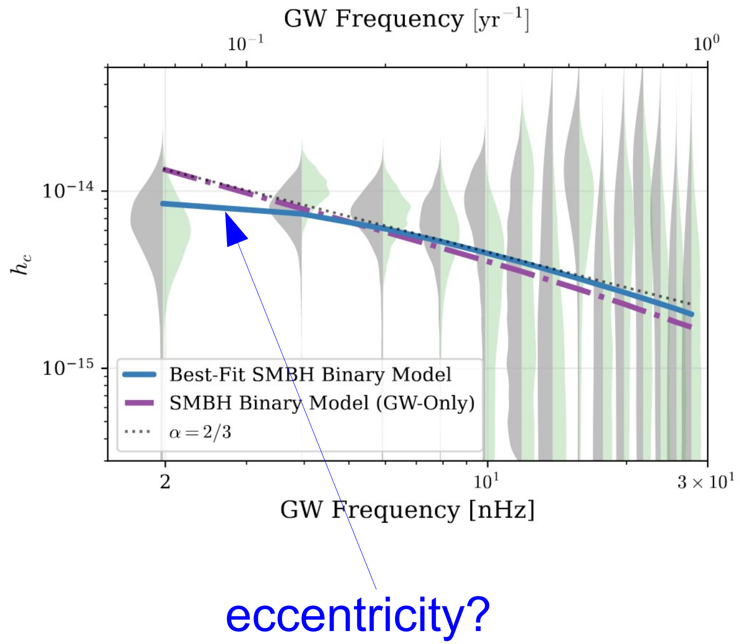


Fig. 4. Characteristic amplitudes of GWBR power spectra over a logarithmic frequency interval, $h_c(\ln f)$, from SMBH binaries. Left panel: Power spectra of GWBR from SMBH binaries with $f_{p,0}/f_{p,\mathrm{max}} = 10^{-3}$ for several initial eccentricities, $e_0 = 0.0, 0.2, 0.4, 0.6, 0.8$ and 0.9. Right panel: Power spectra of GWBR from SMBH binaries for $e_0 = 0.8$ for several initial orbital frequencies, $f_{p,0}/f_{p,\mathrm{max}} = 5^{-3}, 10^{-3}, 20^{-3}$ and 30^{-3}. In each panel, the solid triangle indicates the current limit from pulsar timing measurements.[19]

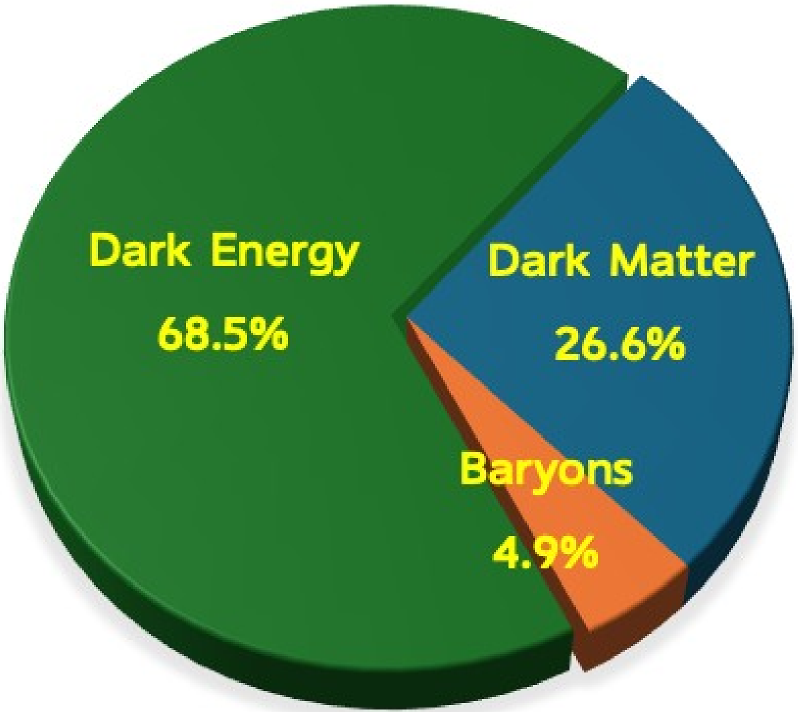


Enoki & Nagashima (2007)

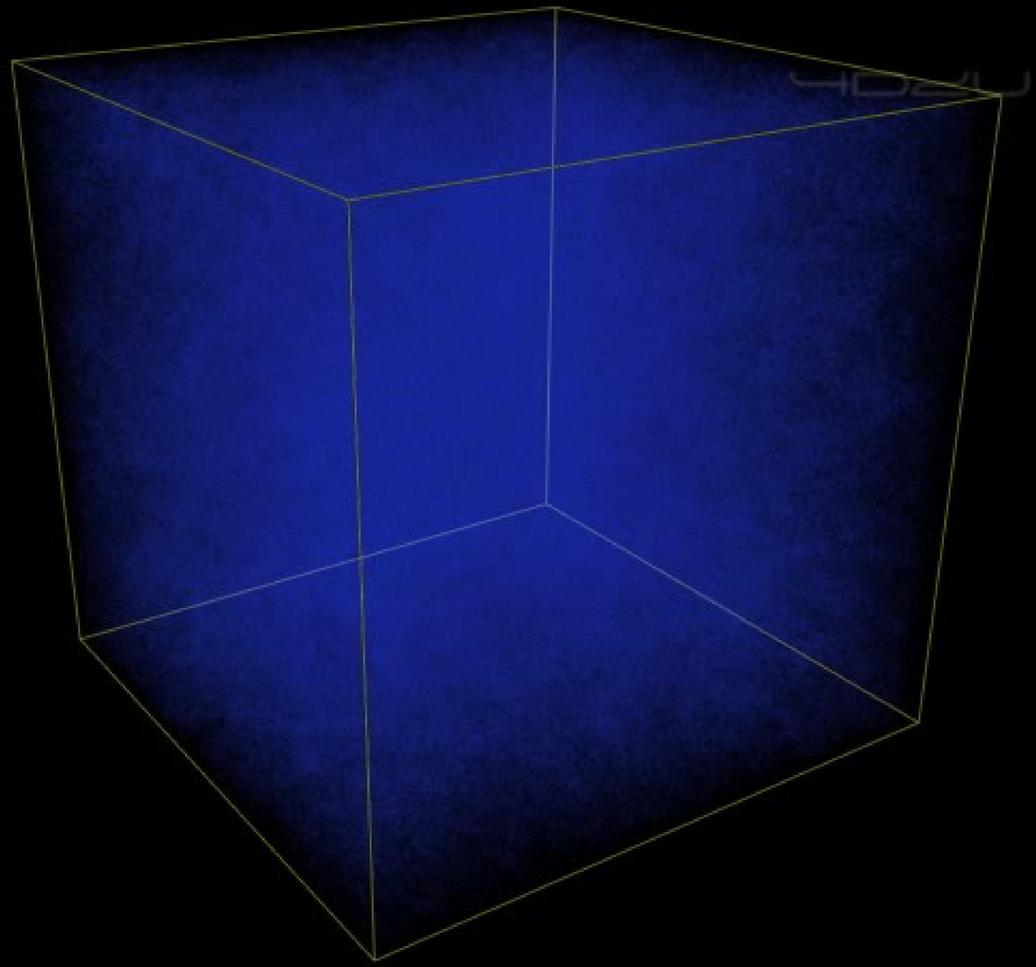
Galaxy Formation

Content of the Universe

- Dark Energy does NOT form any structure
- (Cold) Dark Matter dominates over Baryons
- Structure in the Universe forms primarily based on the gravitation of dark matter
- It is important to follow the structure formation of Dark Matter → *N*-body simulation



Planck 2018



First Generation νGC:
Hierarchical Clustering
in a CDM universe

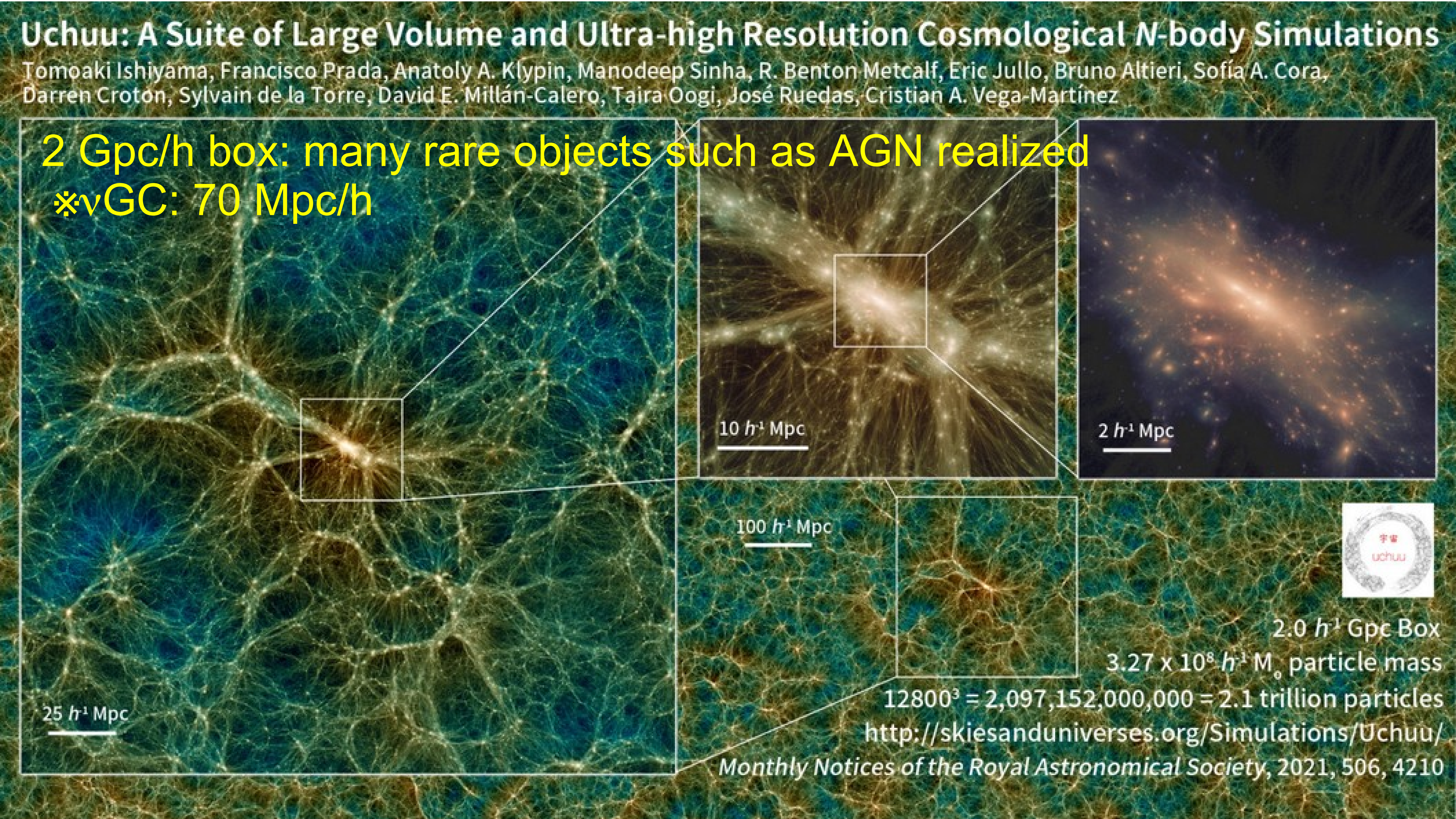
box size: 100 Mpc
blue: dark matter
white: galaxies

Nagashima+2005

https://4d2u.nao.ac.jp/movies/20051001-lss/

2 Gpc/h box: many rare objects such as AGN realized

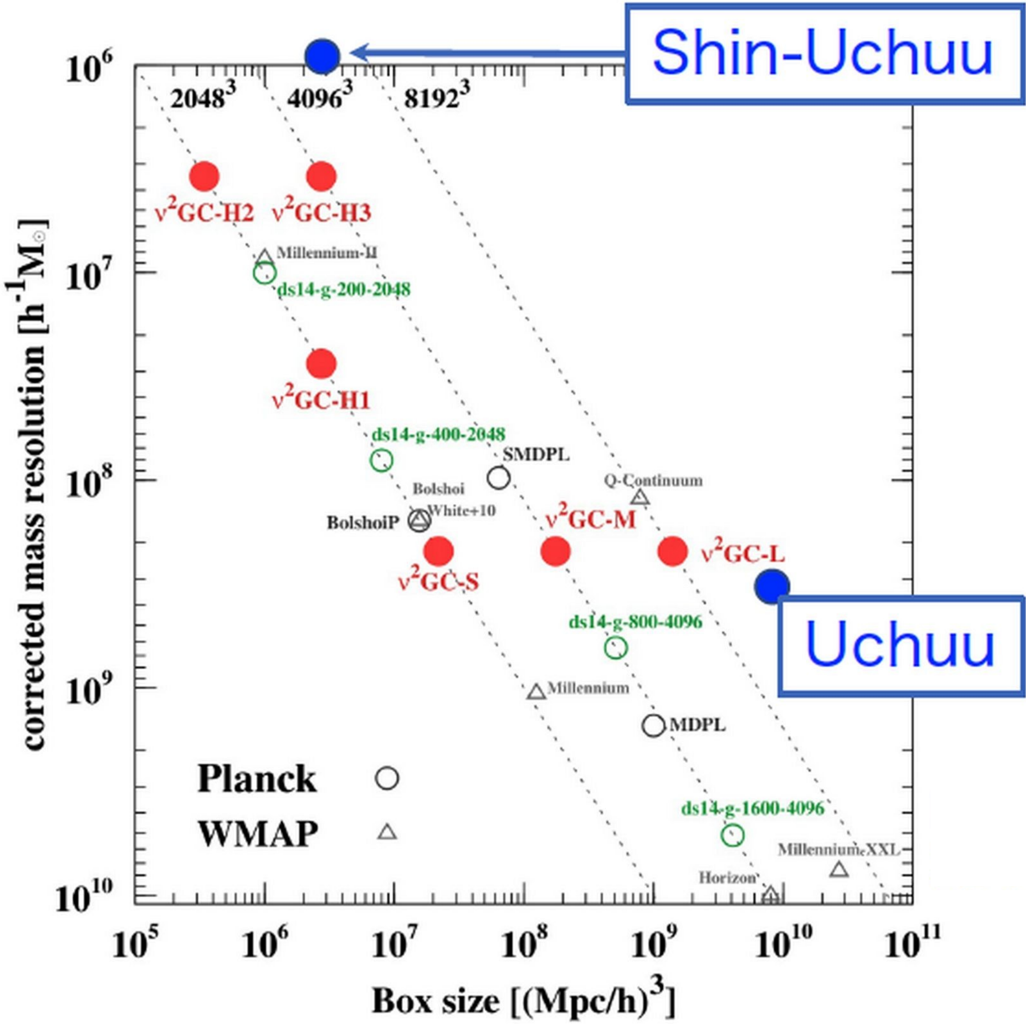
※νGC: 70 Mpc/h



Uchuu simulation suite

Name	N	L $(h^{-1}\,\mathrm{Mpc})$	ε $(h^{-1}\,\mathrm{kpc})$	m_p $(h^{-1}\,\mathrm{M}_\odot)$	M_res $(\mathrm{M}_\odot)$
Uchuu	$12\,800^3$	2000.0	4.27	3.27×10^8	1.93×10^{10}
Mini-Uchuu	2560^3	400.0	4.27	3.27×10^8	
Micro-Uchuu	640^3	100.0	4.27	3.27×10^8	
Shin-Uchuu	6400^3	140.0	0.4	8.97×10^5	5.30×10^7
Phi-4096	4096^3	16.0	0.06	5.13×10^3	3.03×10^5
GLAM	2000^3	2000.0	286	8.60×10^{10}	

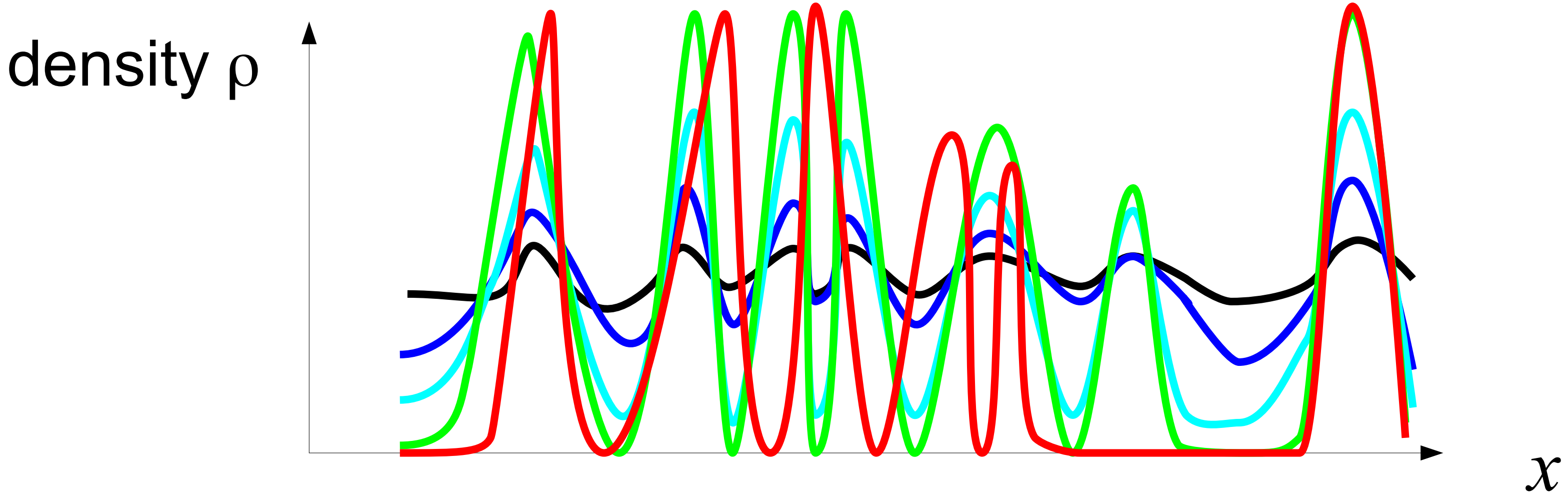
- Halo catalog and merger trees are available
- Uchuu–ν^2GC galaxy and AGN catalog are publicly available



Ishiyama+2015

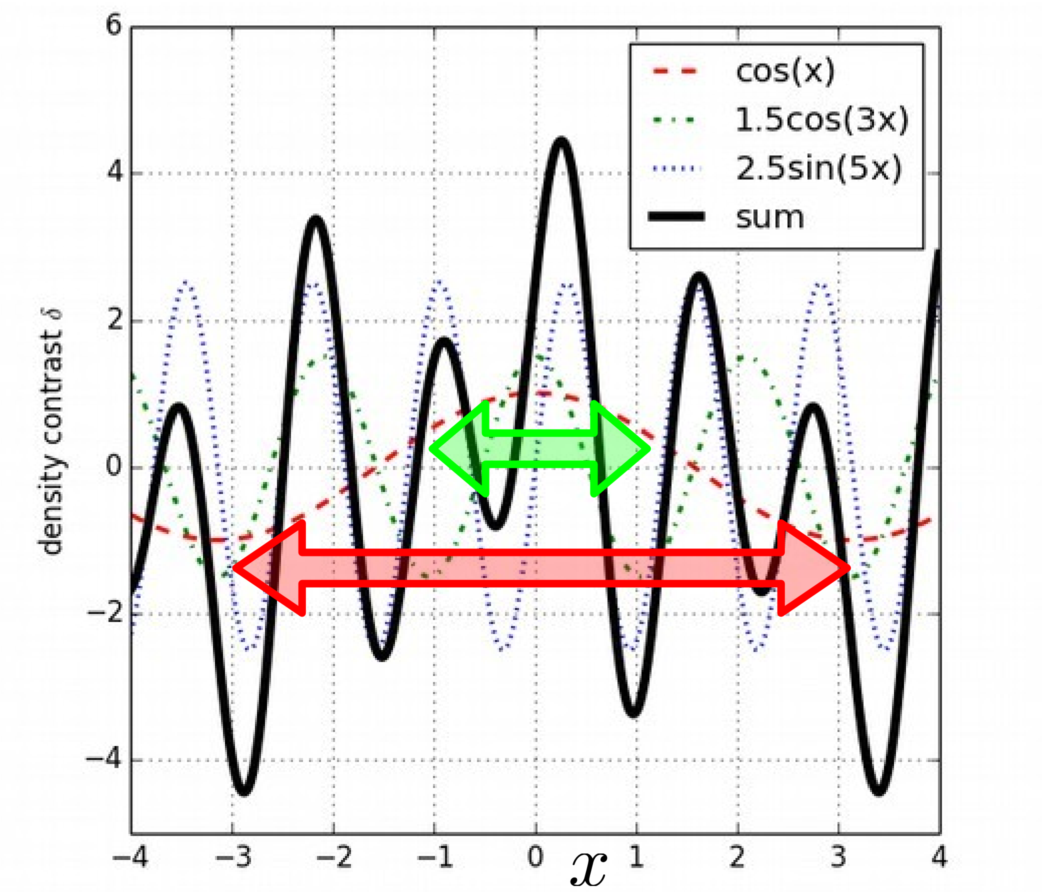
Structure formation

- generation of initial density fluctuations during cosmic inflation by 'classicalization' of quantum fluctuations
- gravitational growth of the density fluctuations
- fluctuations will collapse and get dynamical equilibrium: formation of 'dark haloes'

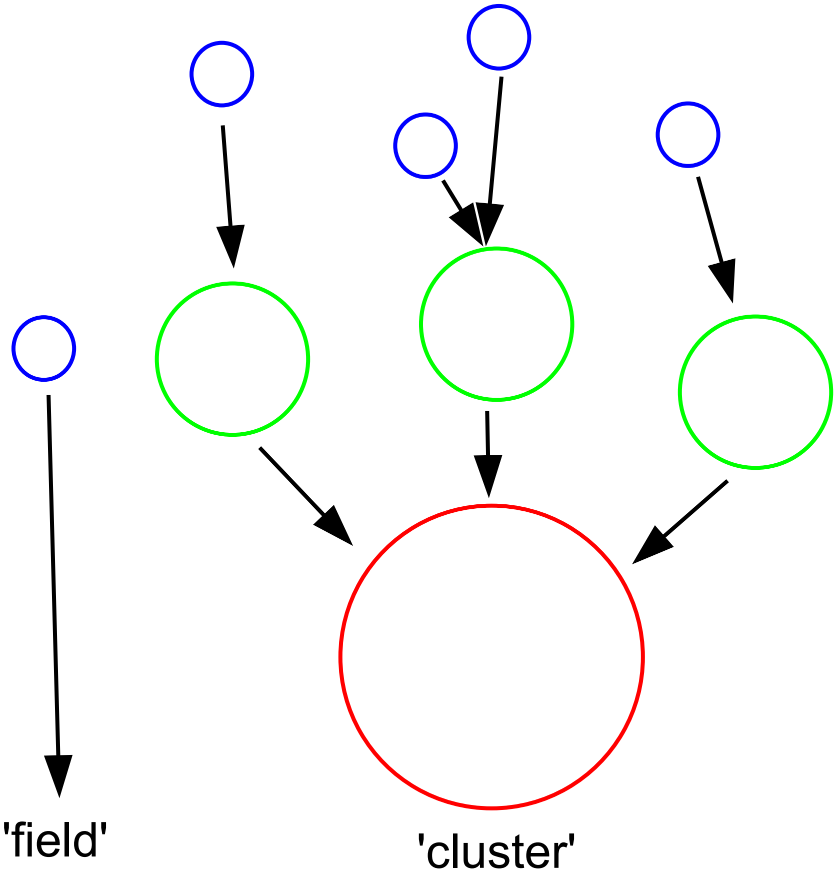


Hierarchical Clustering

simple model of density fluctuations:
larger amplitude in smaller scales

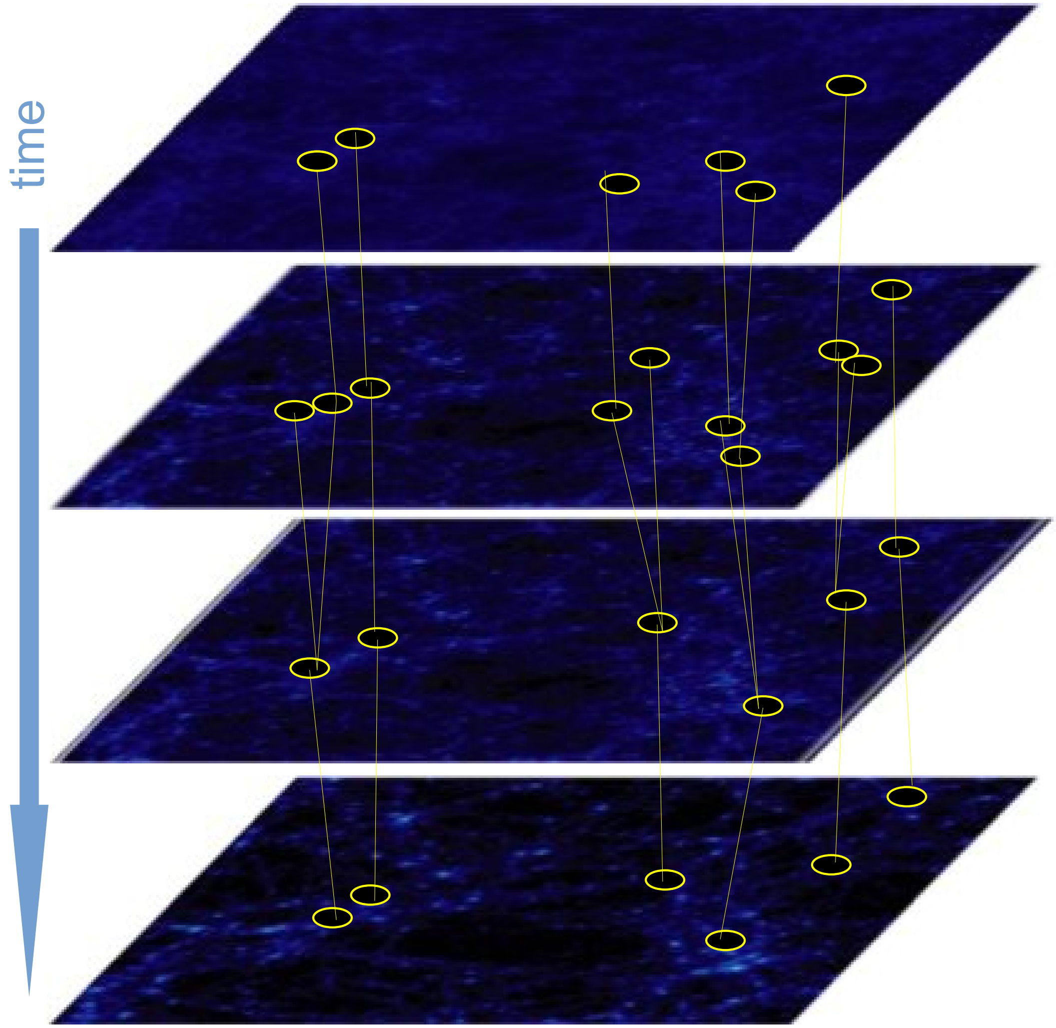


higher density regions collapse first
→biased formation



construction of merger trees

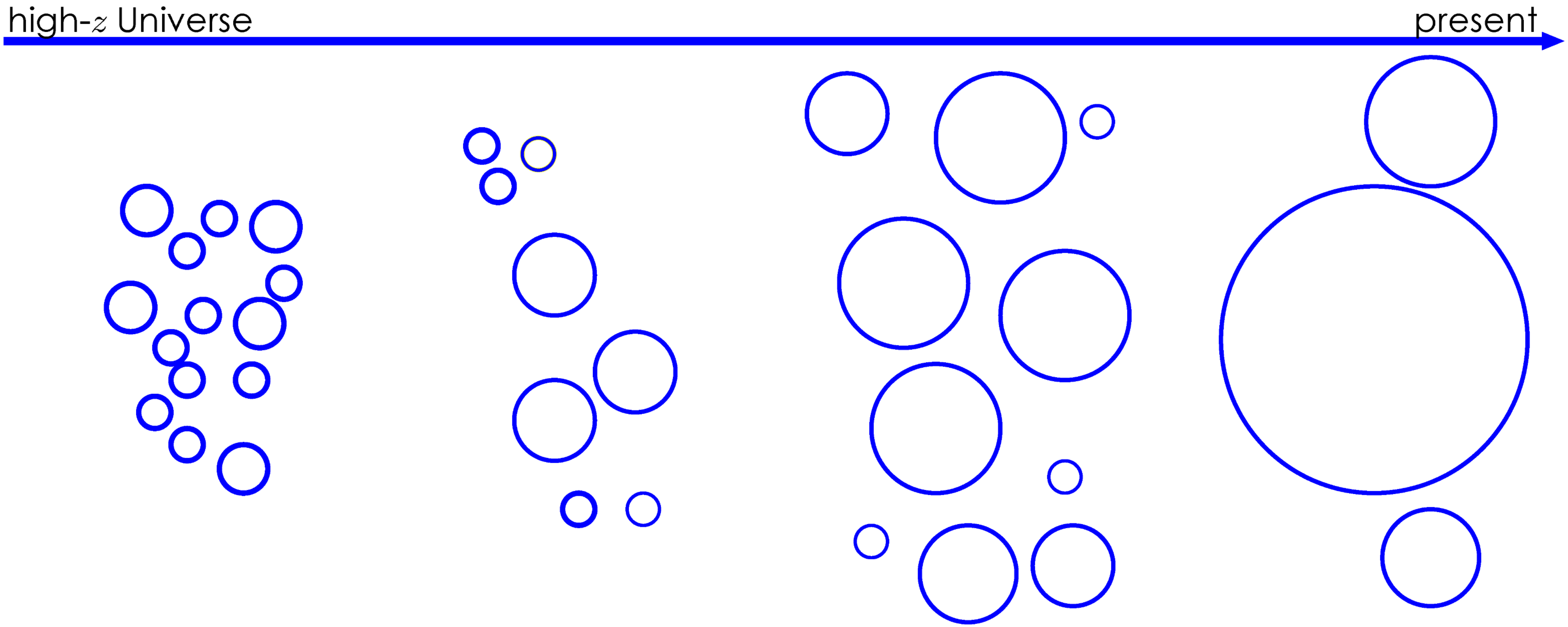
- Running an *N*-body simulation
- Identifying dark halos
- Connecting progenitor/descendant halos
- Merger trees constructed!



N-body snapshots

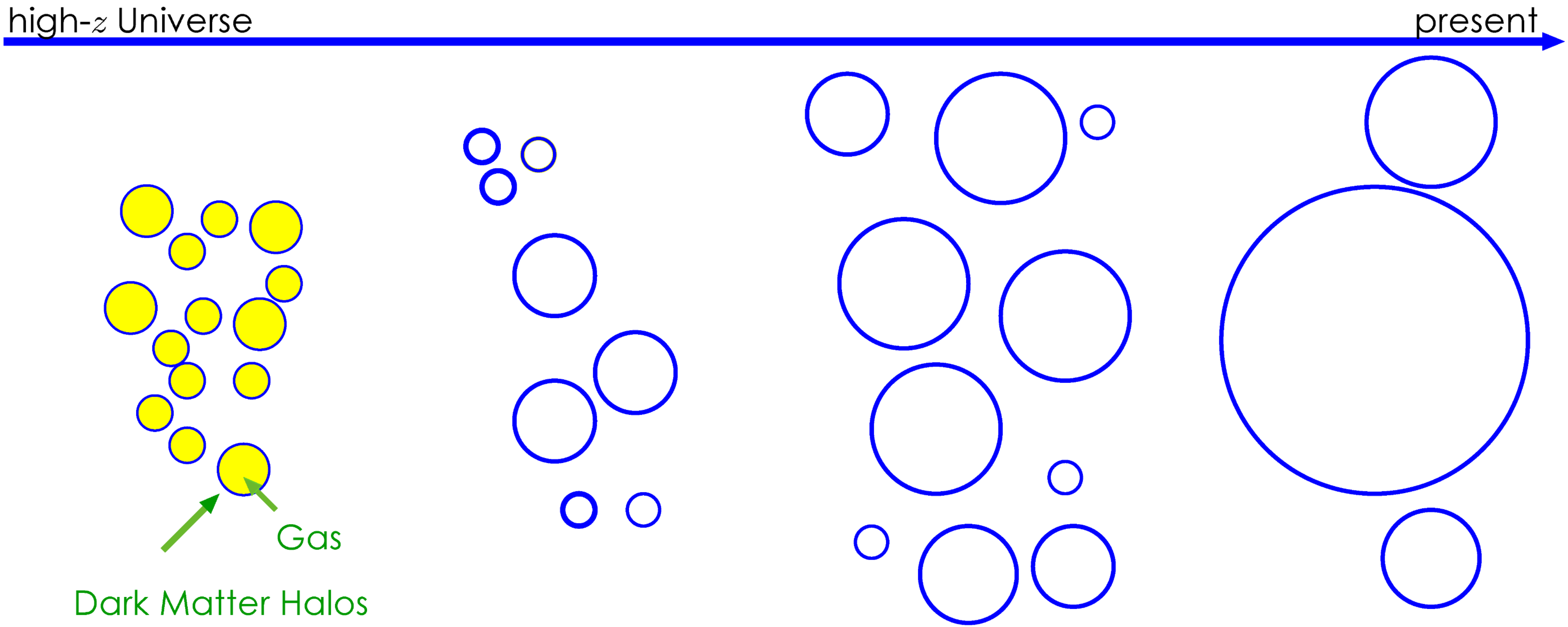
Galaxy Formation in the CDM Universe

high-z Universe → present



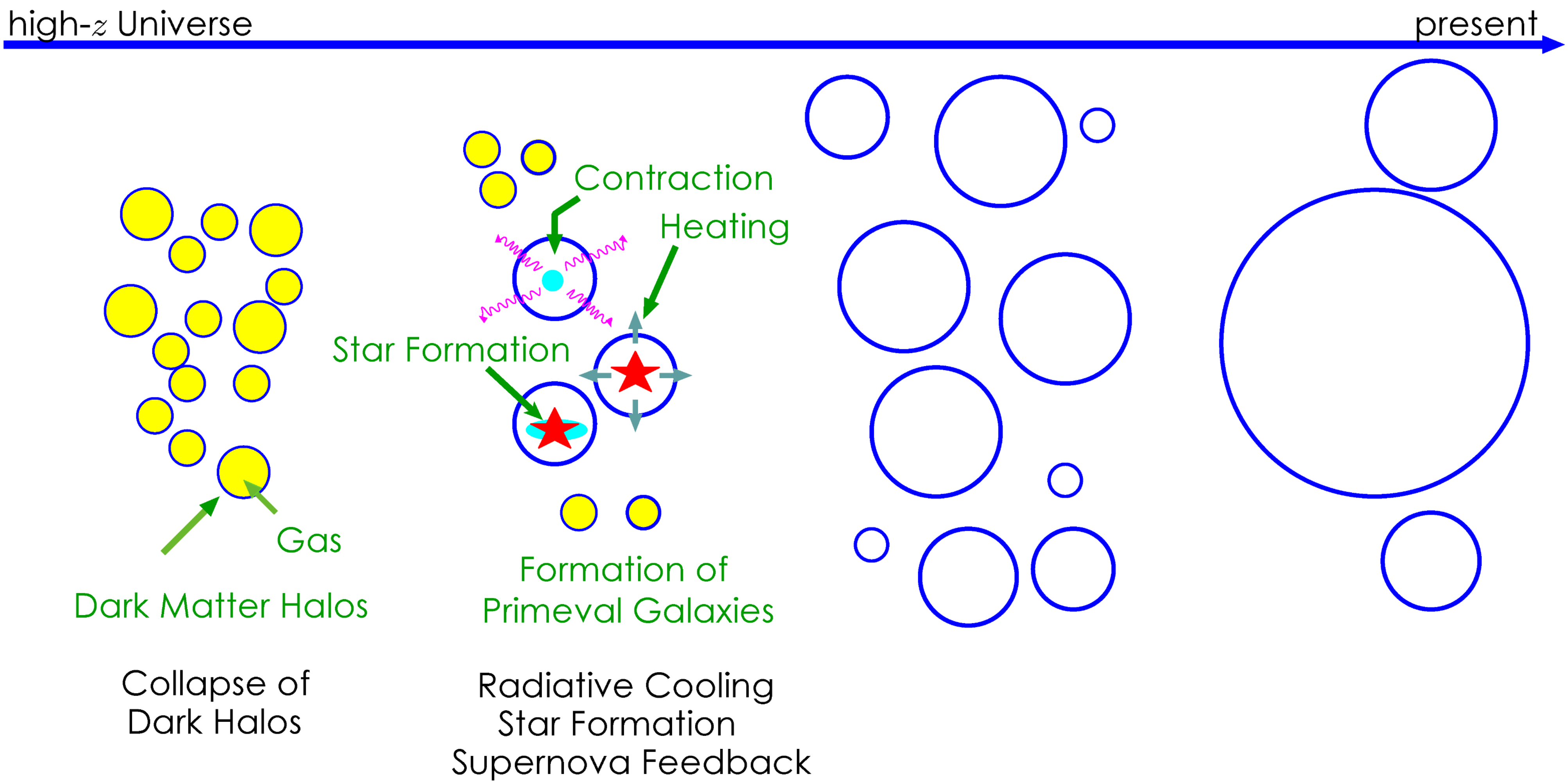
Collapse of
Dark Halos

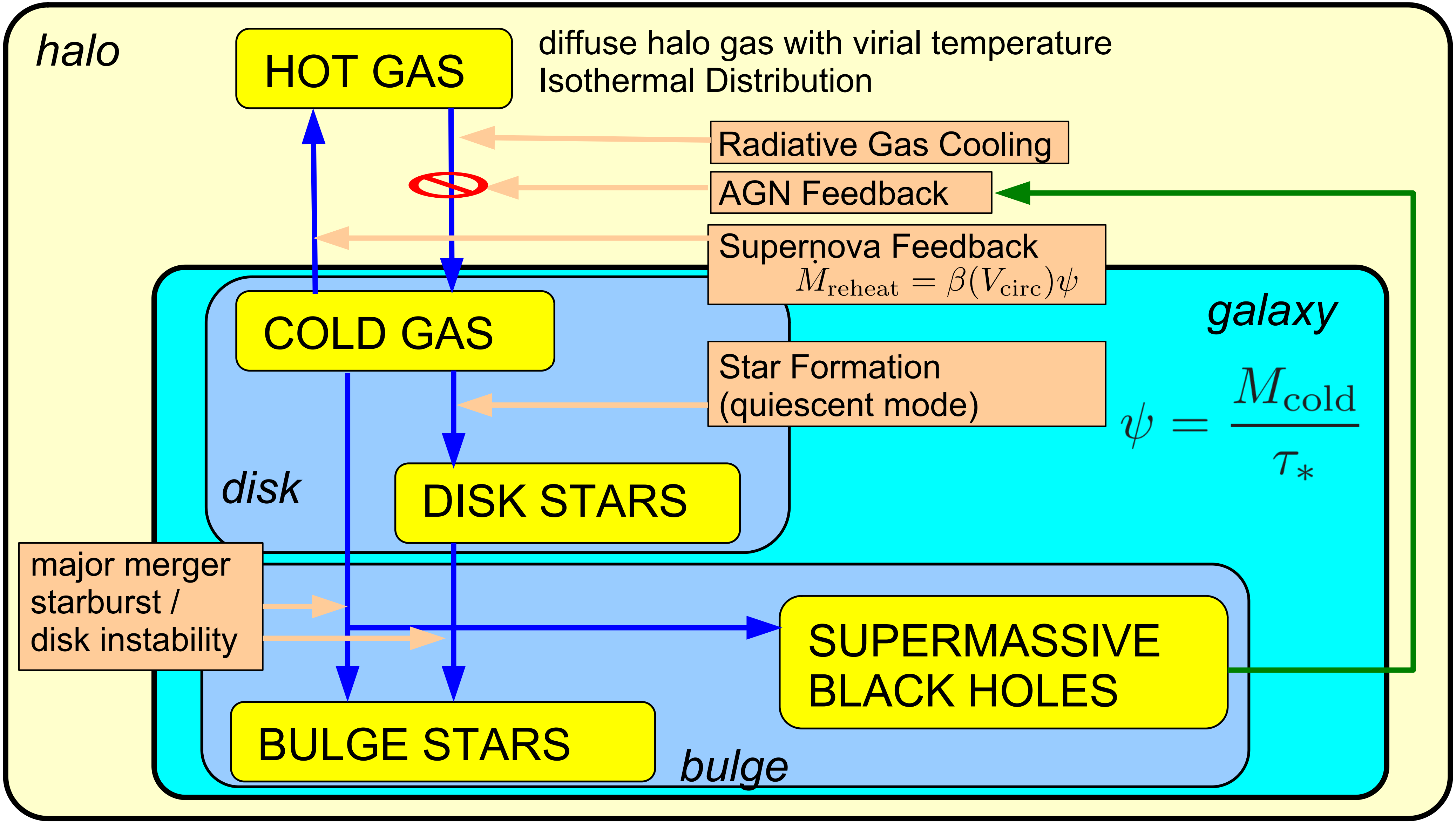
Galaxy Formation in the CDM Universe



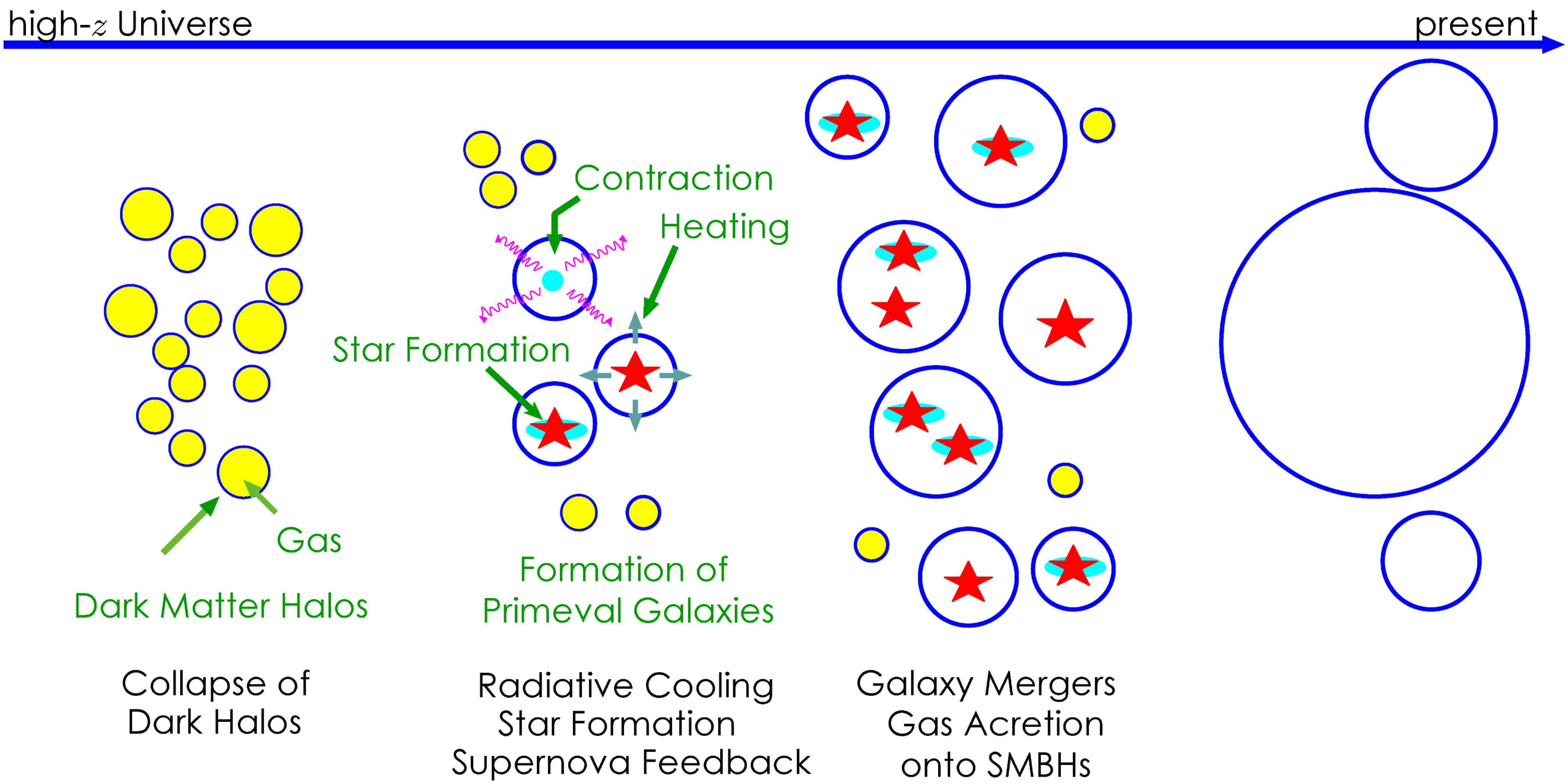
Collapse of
Dark Halos

Galaxy Formation in the CDM Universe




halo
HOT GAS
diffuse halo gas with virial temperature
Isothermal Distribution
Radiative Gas Cooling
AGN Feedback
Supernova Feedback
$\dot{M}_{\rm reheat} = \beta(V_{\rm circ})\psi$
galaxy
COLD GAS
Star Formation
(quiescent mode)
$\psi = \frac{M_{\rm cold}}{\tau_*}$
disk
DISK STARS
major merger
starburst /
disk instability
SUPERMASSIVE
BLACK HOLES
BULGE STARS
bulge

Galaxy Formation in the CDM Universe



Mergers of halos and galaxies

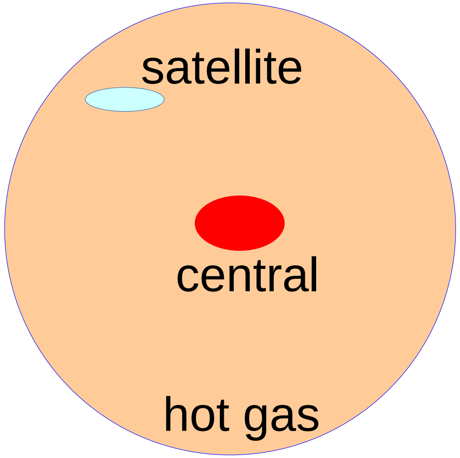
- mergers of dark halos:
  - diffuse hot gas components merge together immediately
  - galaxies are too compact to merge soon
    - the most massive: central, others: satellites
- merger mechanisms of galaxies
  - dynamical friction: satellites fall onto central

$$t_{\rm elapse} > \tau_{\rm fric}$$

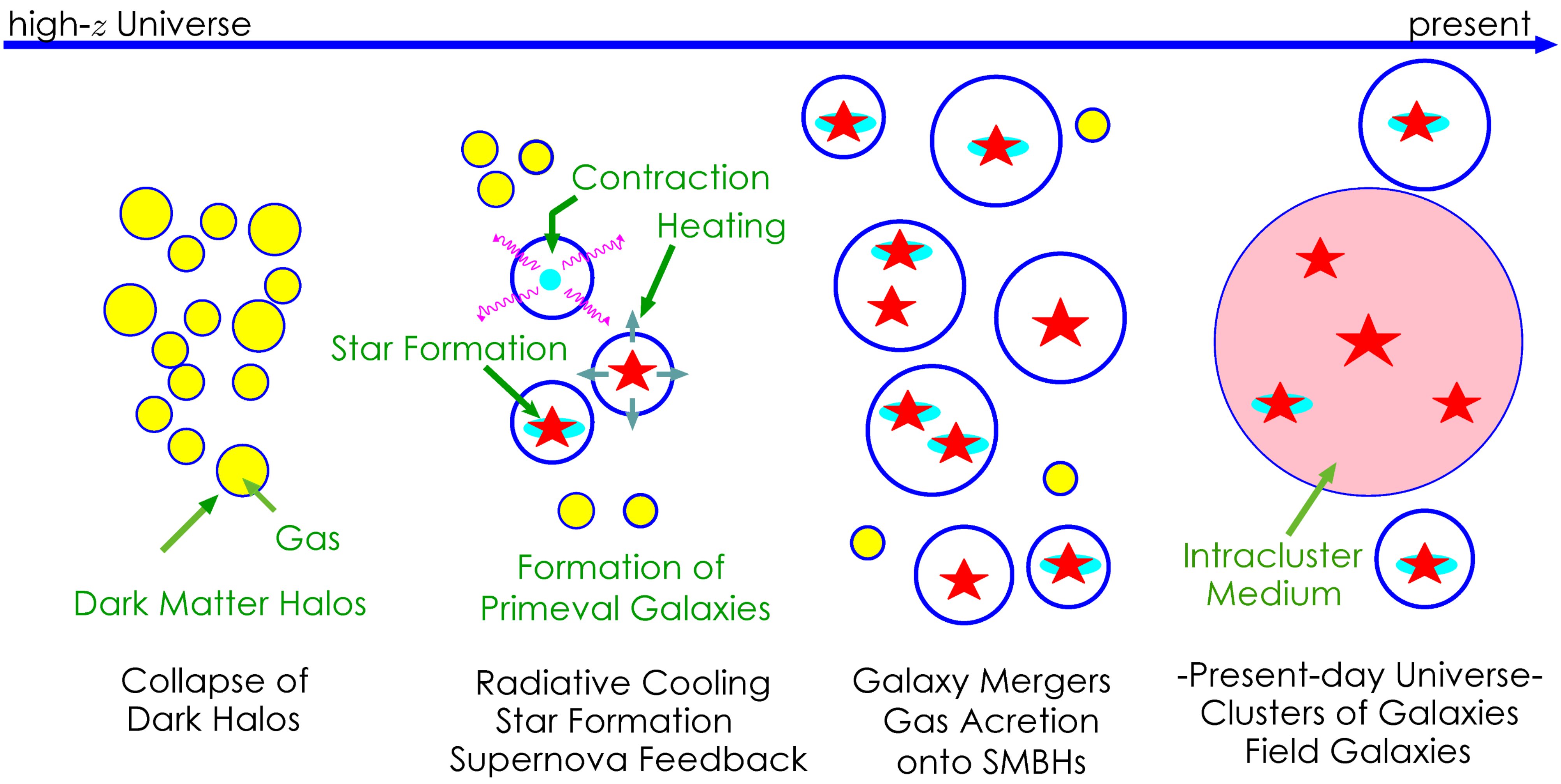
  - random collision: satellite-satellite merger

$$p = \frac{\Delta t}{\tau_{\rm coll}}$$

- when mergers happen:
  - some fraction of gas falls onto bulge

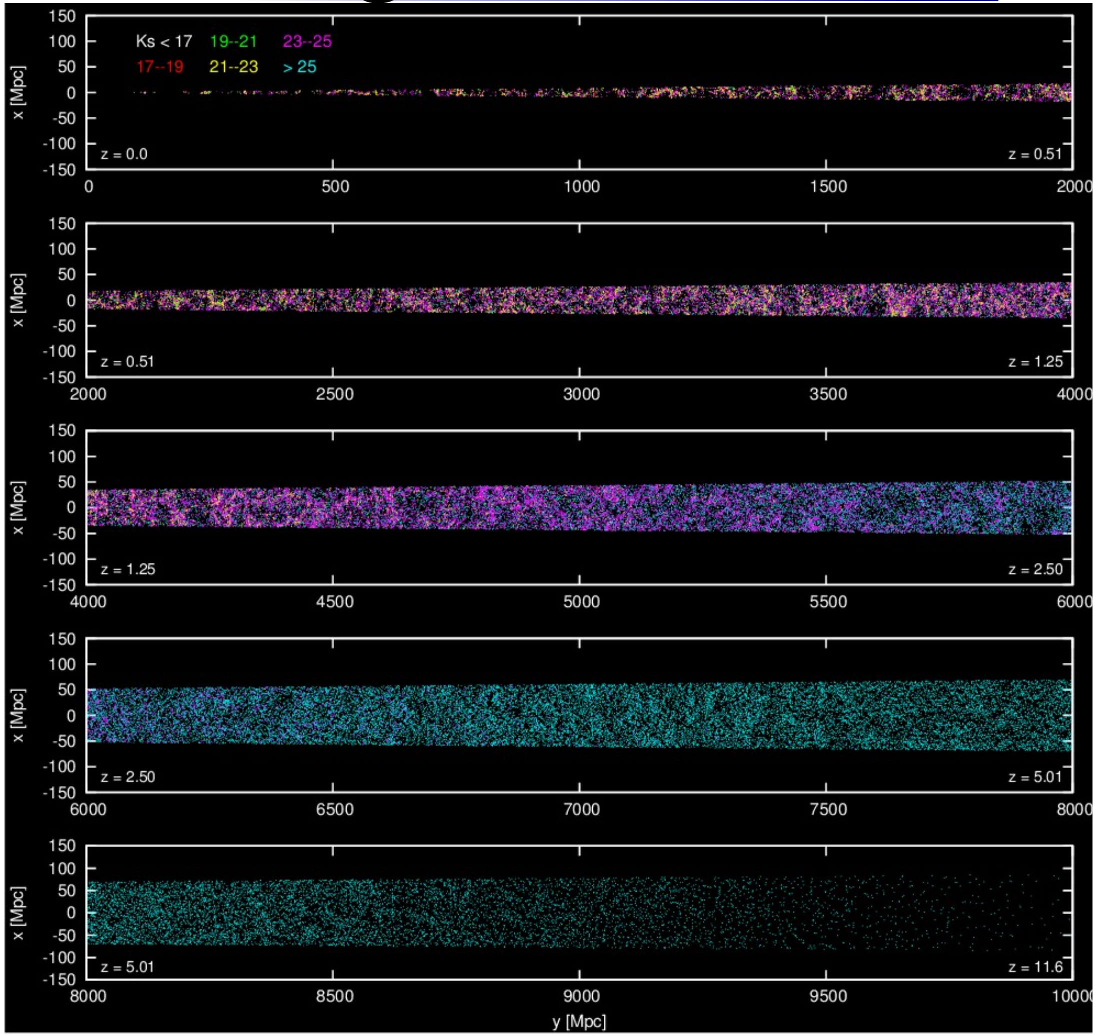


Galaxy Formation in the CDM Universe



Properties of individual galaxies

- Star formation histories of individual galaxies are realized
- Combining population synthesis models, photometric properties are obtained
- Lines such as Hα and OII can be estimated from SFRs and metallicities



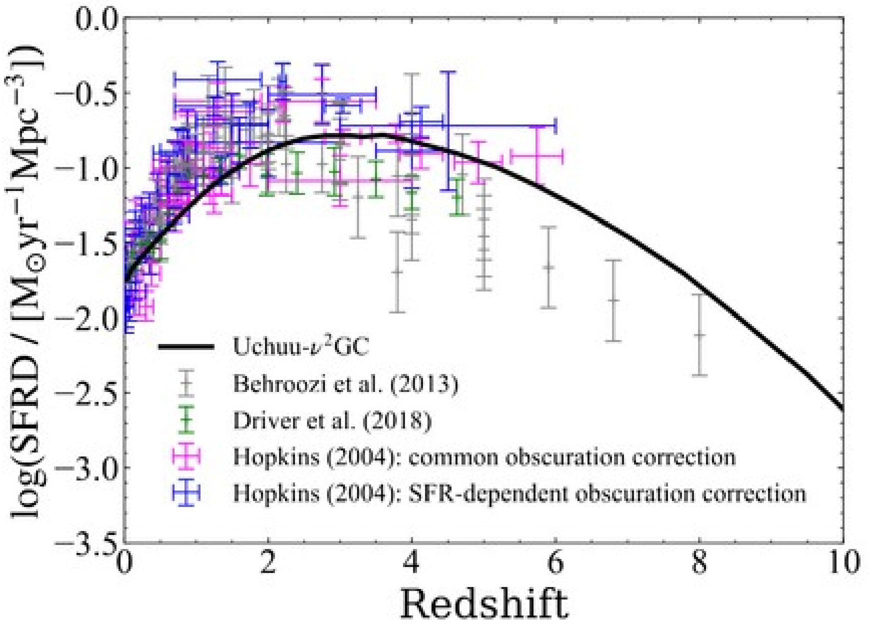
hydrodynamic simulations?

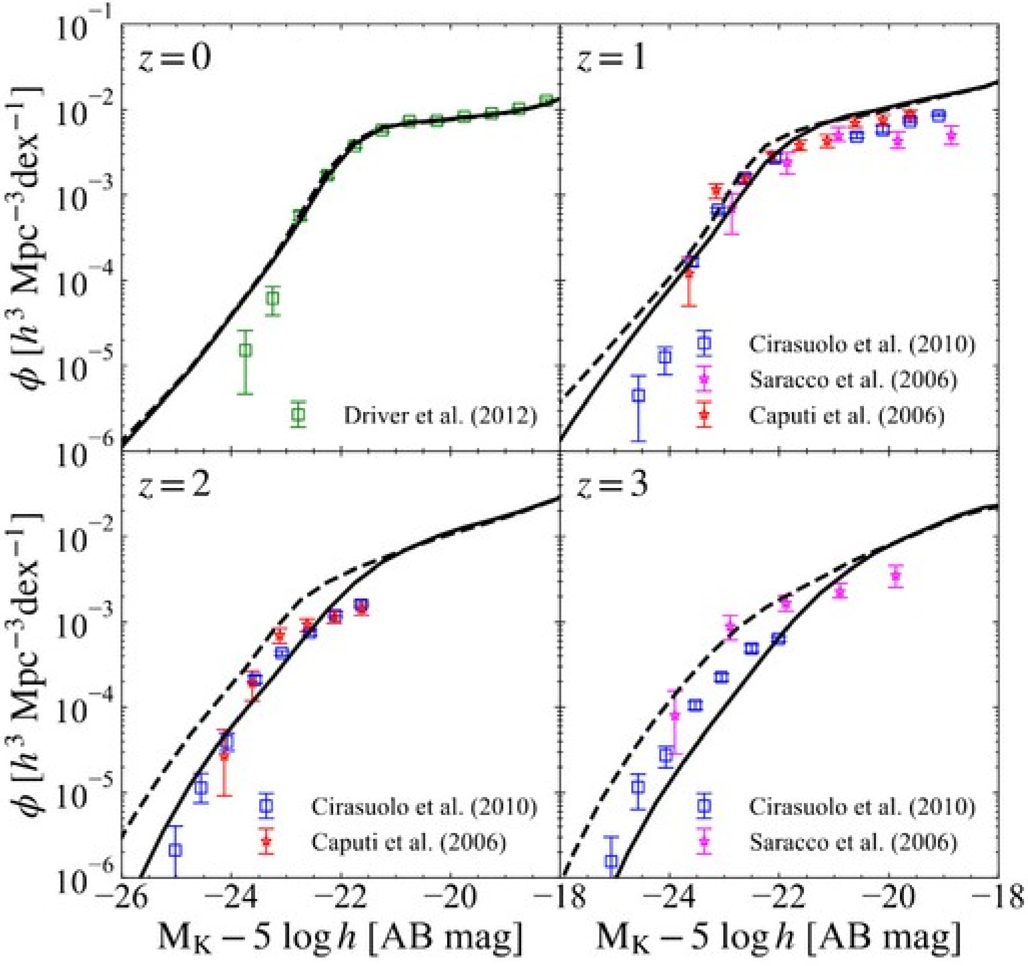
- hydro galaxy formation simulations:
  - solving hydrodynamic equations combined with gravitation
  - severe limitations on resolution
  - therefore box sizes must be small
    - cf. “zoom-in” simulations
- WE NEED AGN/QSOs – constraints on statistics of SMBHs
  - AGN are rare objects (~ 1/100 compared to galaxies), so large boxes are required
  - thus, galaxy formation MODELS are needed: semi-analytic (SA) model of galaxy formation

Some results of Oogi+23

Luminosity functions of galaxies

cosmic star formation history





AGN luminosity

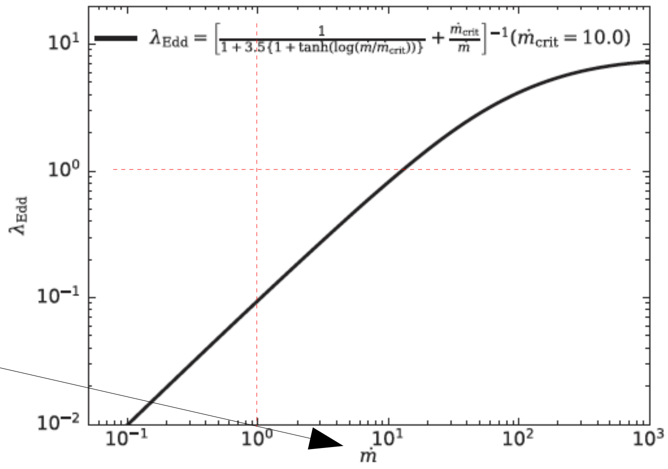
- SMBHs acrete a part of cold gas during galaxy mergers
  - growth of SMBHs

$$\Delta M_{\rm acc} = f_{\rm BH} \Delta M_{\rm star,burst} \qquad \dot{M}_{\rm BH} = \frac{\Delta M_{\rm acc}}{t_{\rm acc}} \exp\left(\frac{t - t_{\rm start}}{t_{\rm acc}}\right)$$

- the accretion rate determines the AGN luminosity
  - INDEPENDENT of SMBH mass ... variety of Eddington ratio

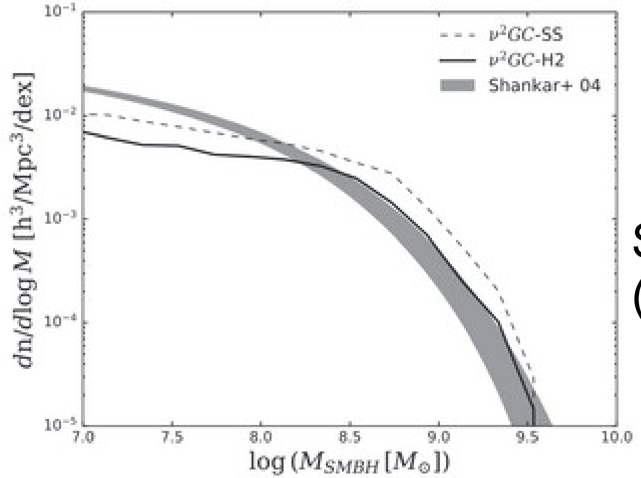
$$\lambda_{\rm Edd} \equiv \frac{L_{\rm bol}}{L_{\rm Edd}}$$

$$\dot{m} \equiv \frac{\dot{M}}{\dot{M}_{\rm Edd}} = \frac{\dot{M}}{L_{\rm Edd}/c^2}$$



Shirakata+2019b

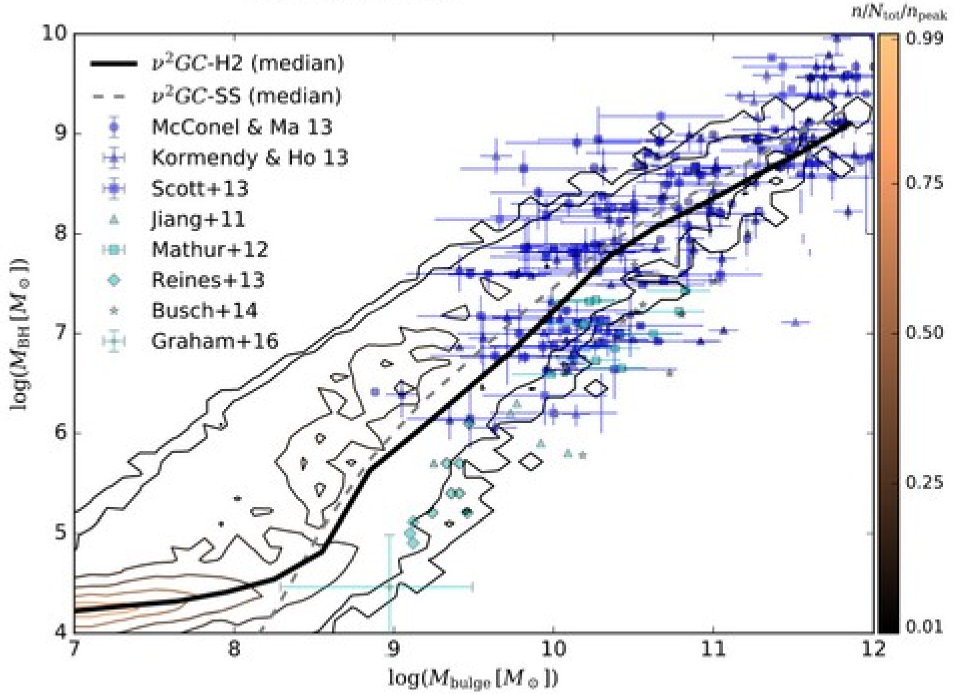
AGN luminosity functions in hard X-ray

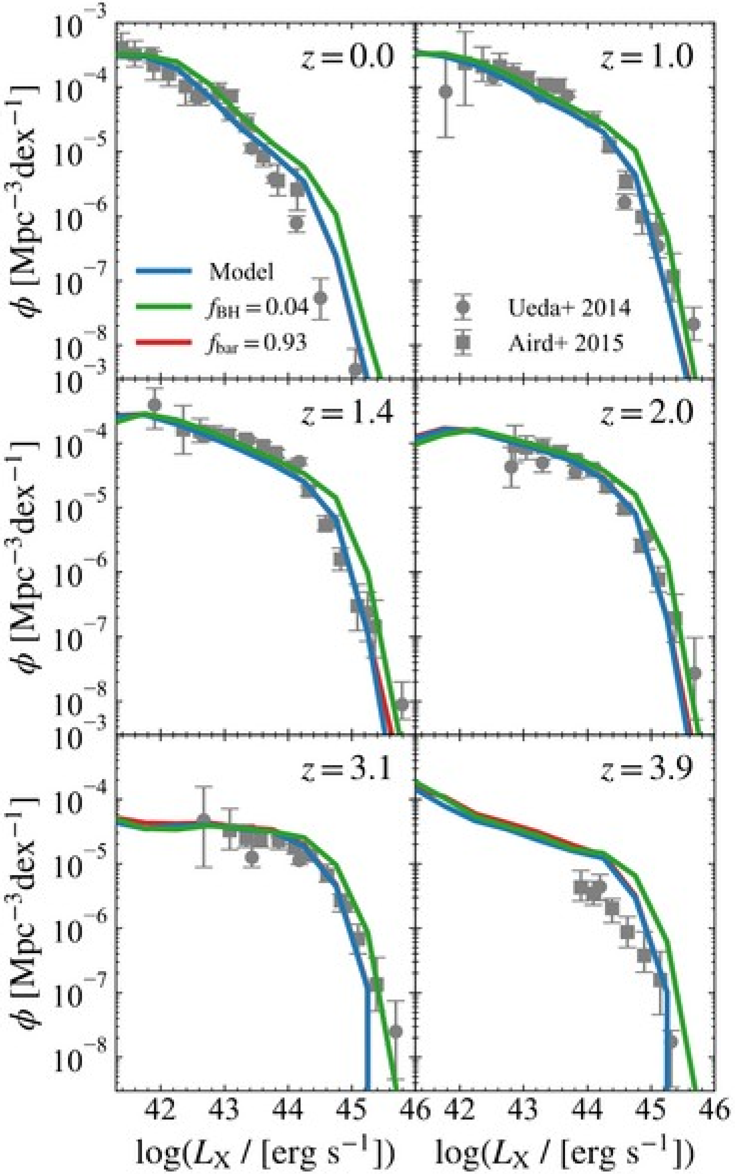


SMBH mass functions (z=0)

(Shirakata+19)

$M_{\rm bulge}$-$M_{\rm BH}$ relation





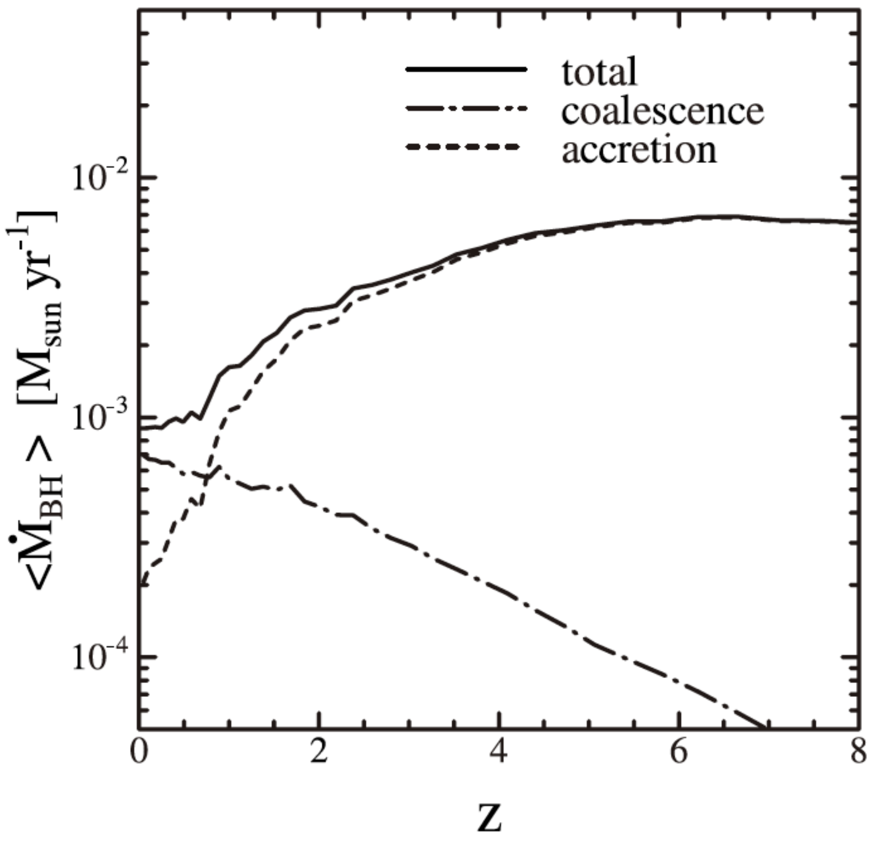
Gravitational Waves

reliability

- our model tightly constraints the number density of SMBHs via:-
  - AGN luminosity functions, which provide the number of gas-accreting SMBHs
  - M_{BH}-M_{bulge} relations, which provide the mass of SMBHs
  - galaxy luminosity functions, which provide the total number of SMBHs
- combining different redshift results, we obtain:-
  - merger rates of galaxies
  - merger rates of SMBHs, assuming all SMBHs merge together immediately → predicting maximal values

Growth rate of SMBHs

- $\Delta M = M_{\rm acc}$ or $M_{\rm BH,small}$
- Gas fraction in the Universe is decreasing with time, because gas turns into stars
- Growth rate of SMBHs is almost determined by gas accretion (high-z) and mergers (low-z)
- GWs are emitted when mergers



Enoki et al.(2004)

eccentric orbit

- power at low f significantly decreases

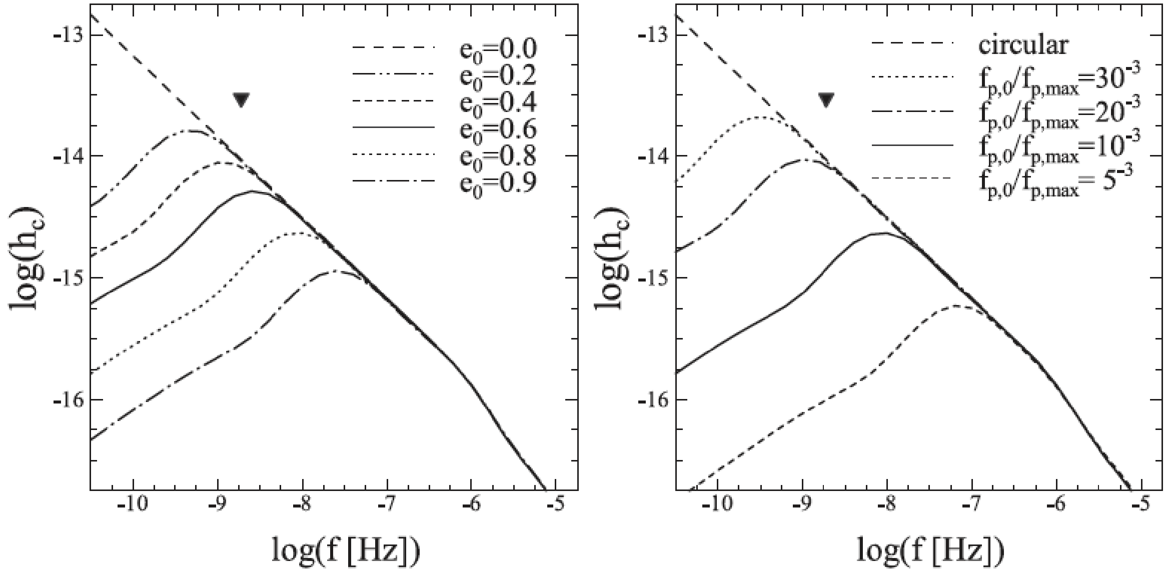
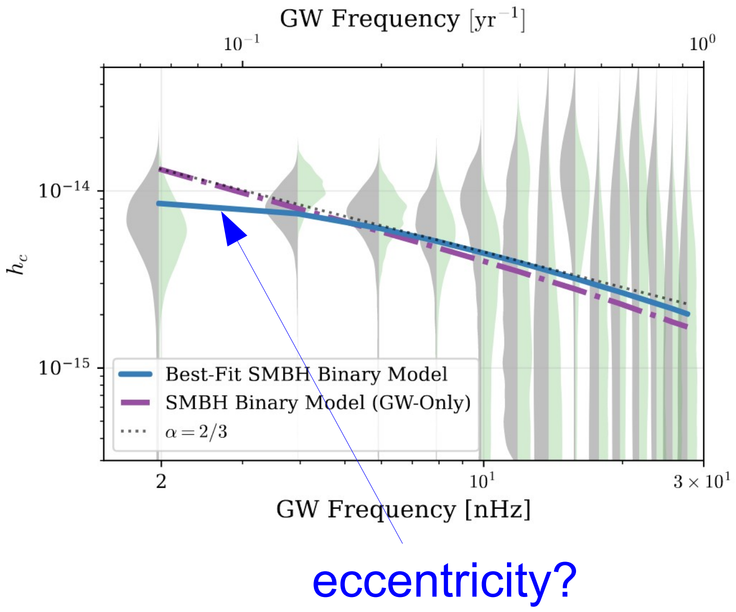


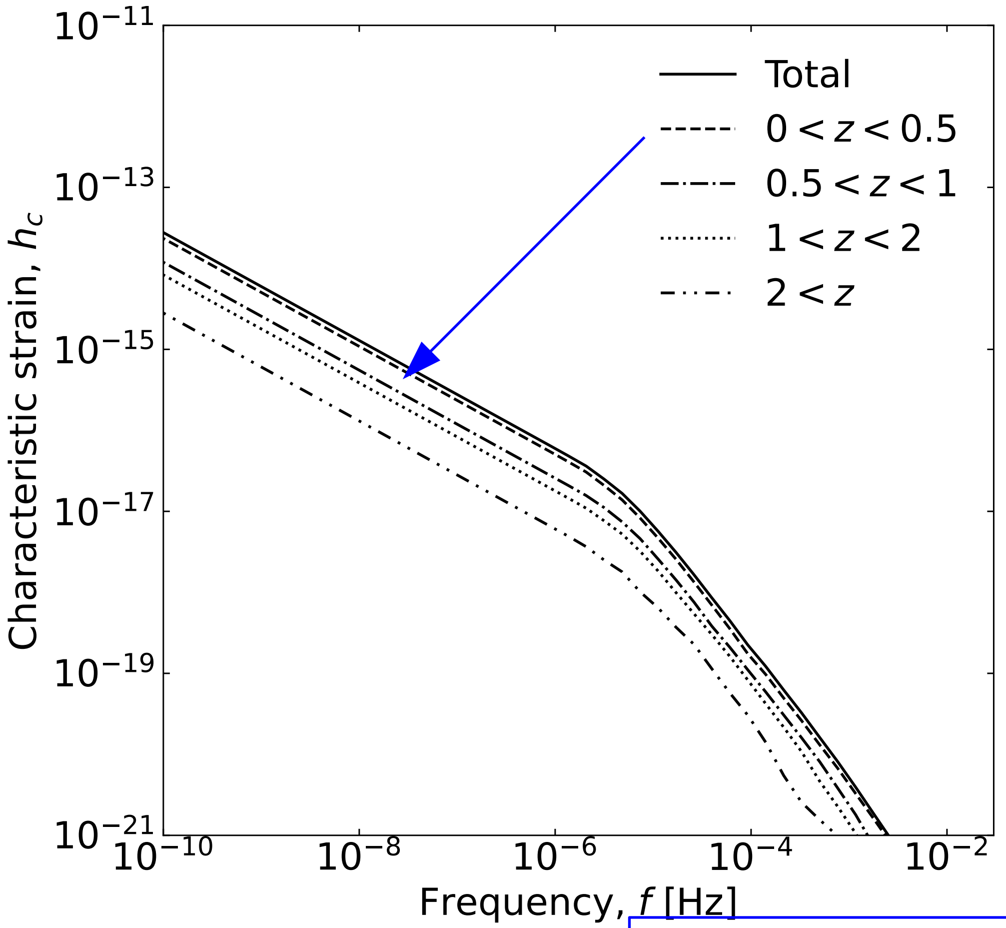
Fig. 4. Characteristic amplitudes of GWBR power spectra over a logarithmic frequency interval, $h_c(\ln f)$, from SMBH binaries. Left panel: Power spectra of GWBR from SMBH binaries with $f_{p,0}/f_{p,\max} = 10^{-3}$ for several initial eccentricities, $e_0 = 0.0, 0.2, 0.4, 0.6, 0.8$ and 0.9. Right panel: Power spectra of GWBR from SMBH binaries for $e_0 = 0.8$ for several initial orbital frequencies, $f_{p,0}/f_{p,\max} = 5^{-3}, 10^{-3}, 20^{-3}$ and 30^{-3}. In each panel, the solid triangle indicates the current limit from pulsar timing measurements.[19]

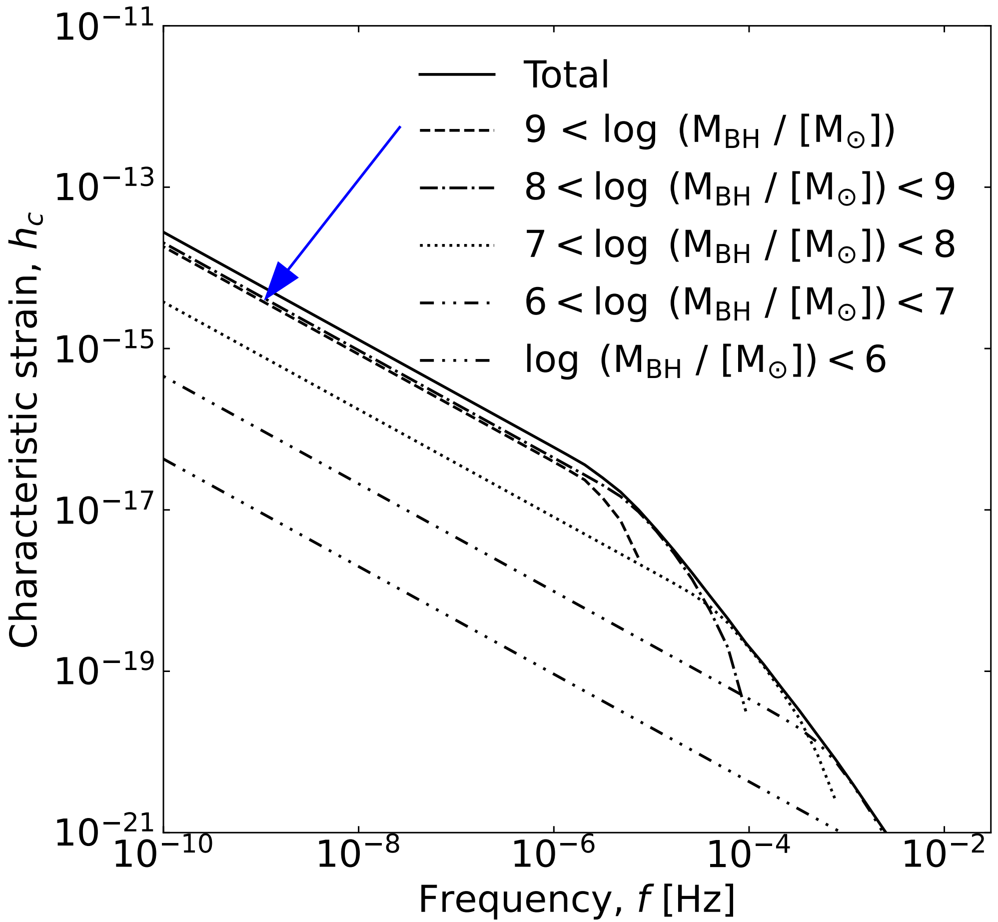


Enoki & Nagashima (2007)

$$\frac{r_0}{r_{\mathrm{ISCO}}} = \left(\frac{f_{p,0}}{f_{p,\max}}\right)^{-2/3}$$

current results (circular;preliminary)

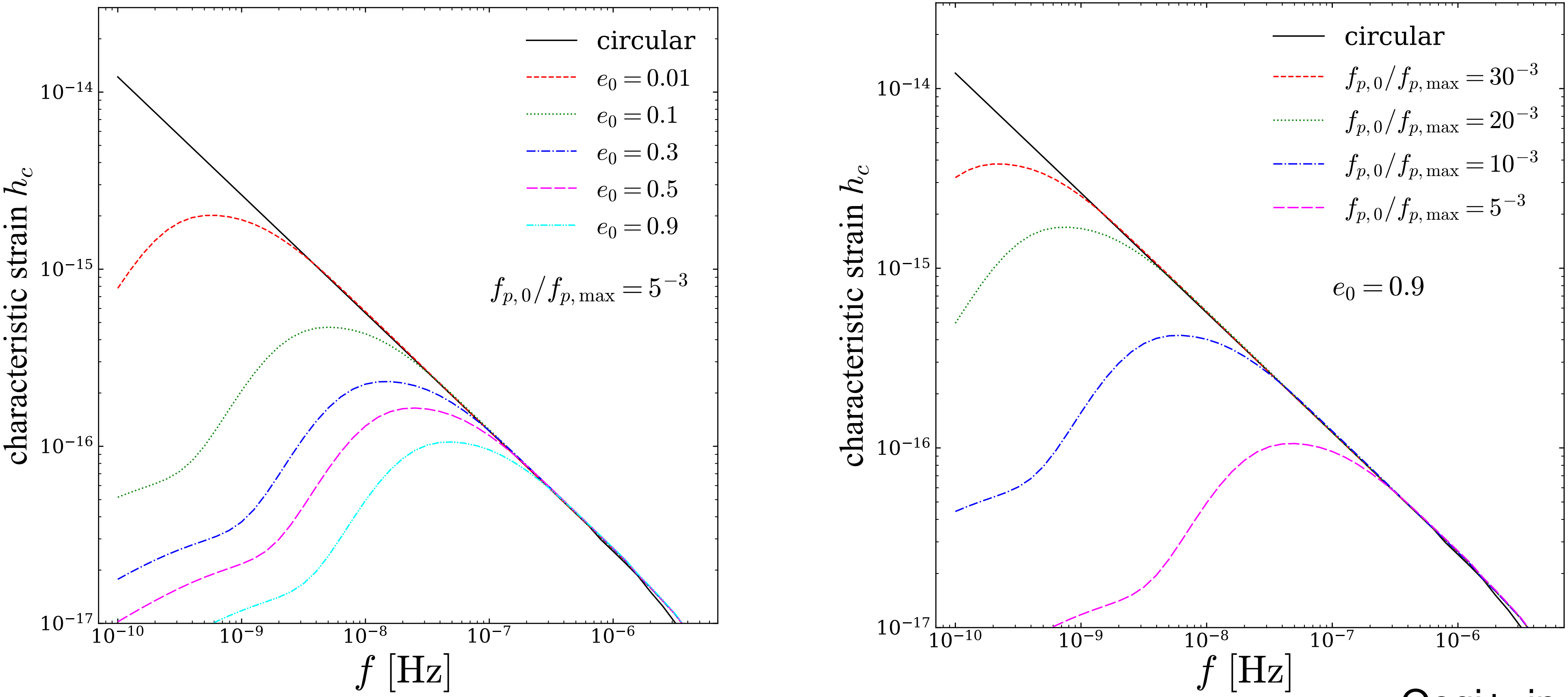




massive BHs at low-z are important !

Oogi+, in prep.

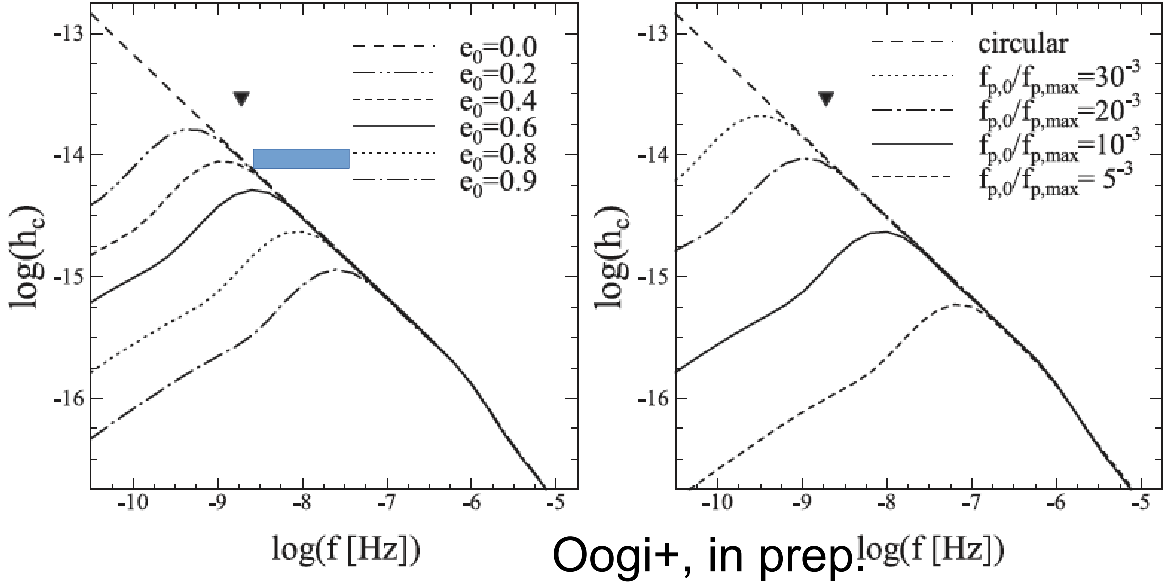
eccentricity (preliminary)



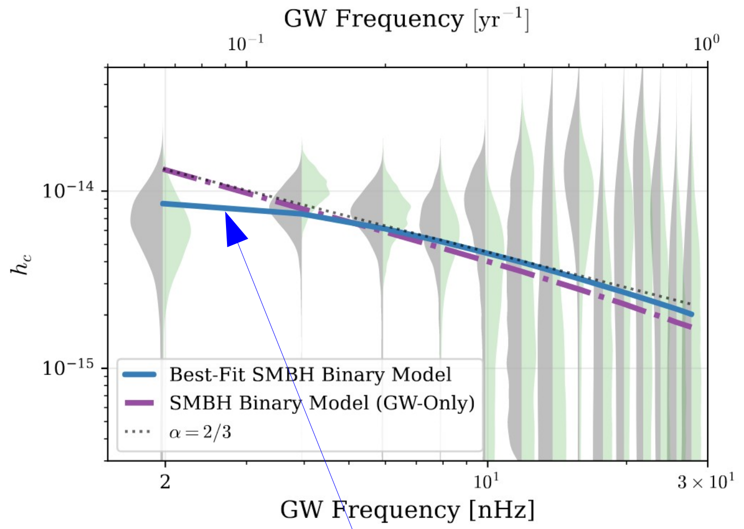
Oogi+, in prep.

eccentric orbit

- power at low f significantly decreases



Oogi+, in prep.



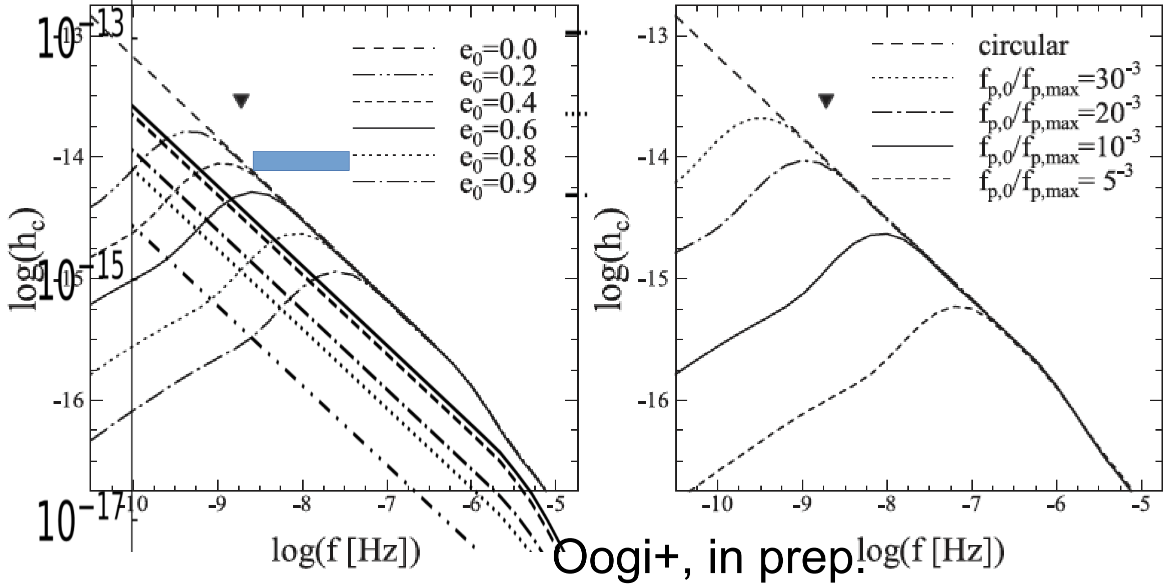
eccentricity?

Fig. 4. Characteristic amplitudes of GWBR power spectra over a logarithmic frequency interval, $h_c(\ln f)$, from SMBH binaries. Left panel: Power spectra of GWBR from SMBH binaries with $f_{p,0}/f_{p,\rm max} = 10^{-3}$ for several initial eccentricities, $e_0 = 0.0, 0.2, 0.4, 0.6, 0.8$ and 0.9. Right panel: Power spectra of GWBR from SMBH binaries for $e_0 = 0.8$ for several initial orbital frequencies, $f_{p,0}/f_{p,\rm max} = 5^{-3}, 10^{-3}, 20^{-3}$ and 30^{-3}. In each panel, the solid triangle indicates the current limit from pulsar timing measurements.[19)]

3~5 times smaller than observation!

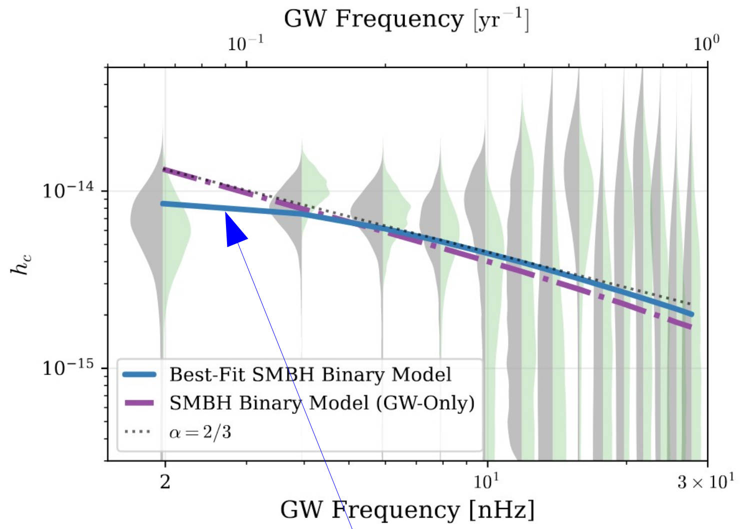
eccentric orbit

- power at low f significantly decreases



Oogi+, in prep.

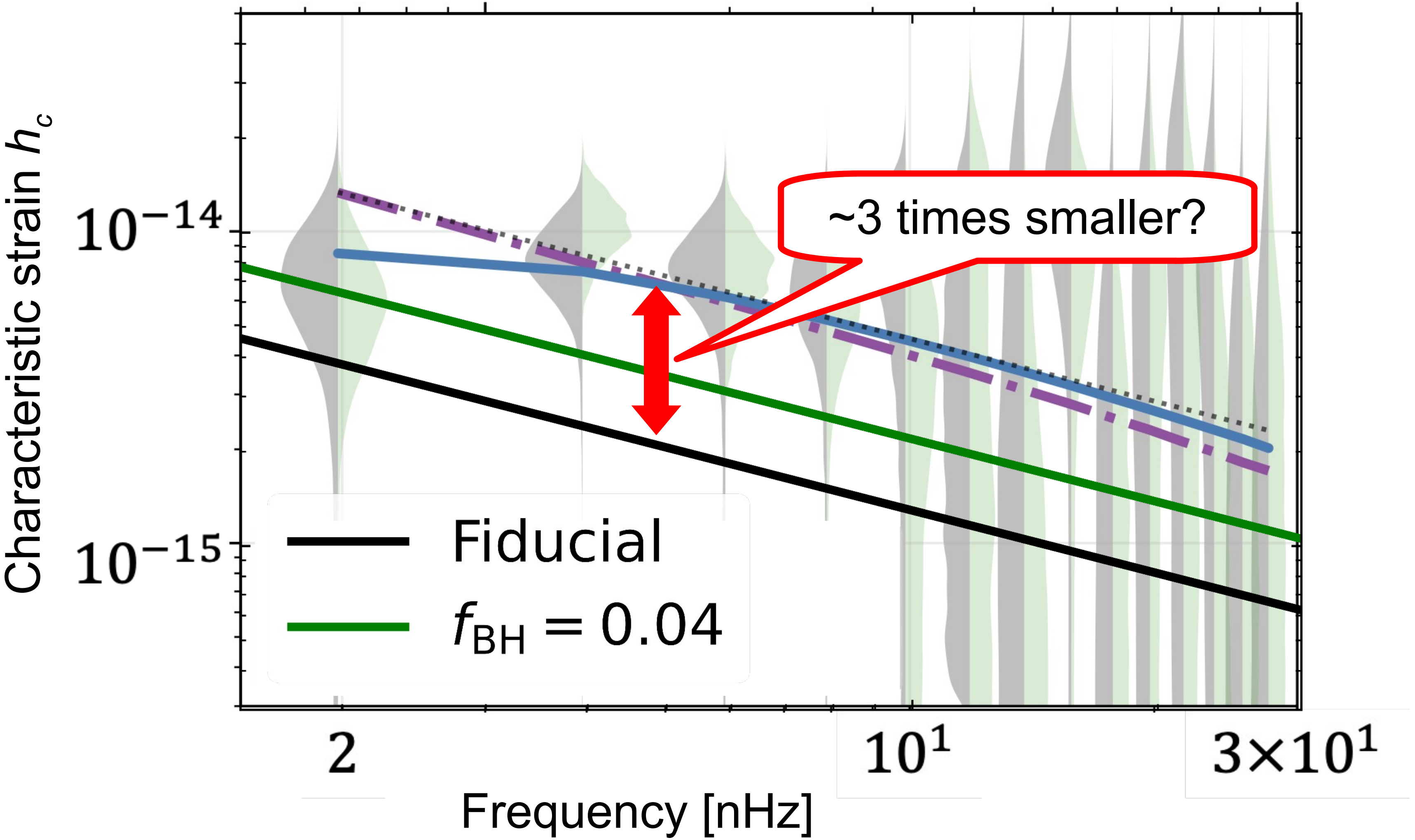
Fig. 4. Characteristic amplitudes of GWBR power spectra over a logarithmic frequency interval, $h_c(\ln f)$, from SMBH binaries. Left panel: Power spectra of GWBR from SMBH binaries with $f_{p,0}/f_{p,\max} = 10^{-3}$ for several initial eccentricities, $e_0 = 0.0, 0.2, 0.4, 0.6, 0.8$ and 0.9. Right panel: Power spectra of GWBR from SMBH binaries for $e_0 = 0.8$ for several initial orbital frequencies, $f_{p,0}/f_{p,\max} = 5^{-3}, 10^{-3}, 20^{-3}$ and 30^{-3}. In each panel, the solid triangle indicates the current limit from pulsar timing measurements.[19)]



eccentricity?

3~5 times smaller than observation!

comparison with the NANOGrav

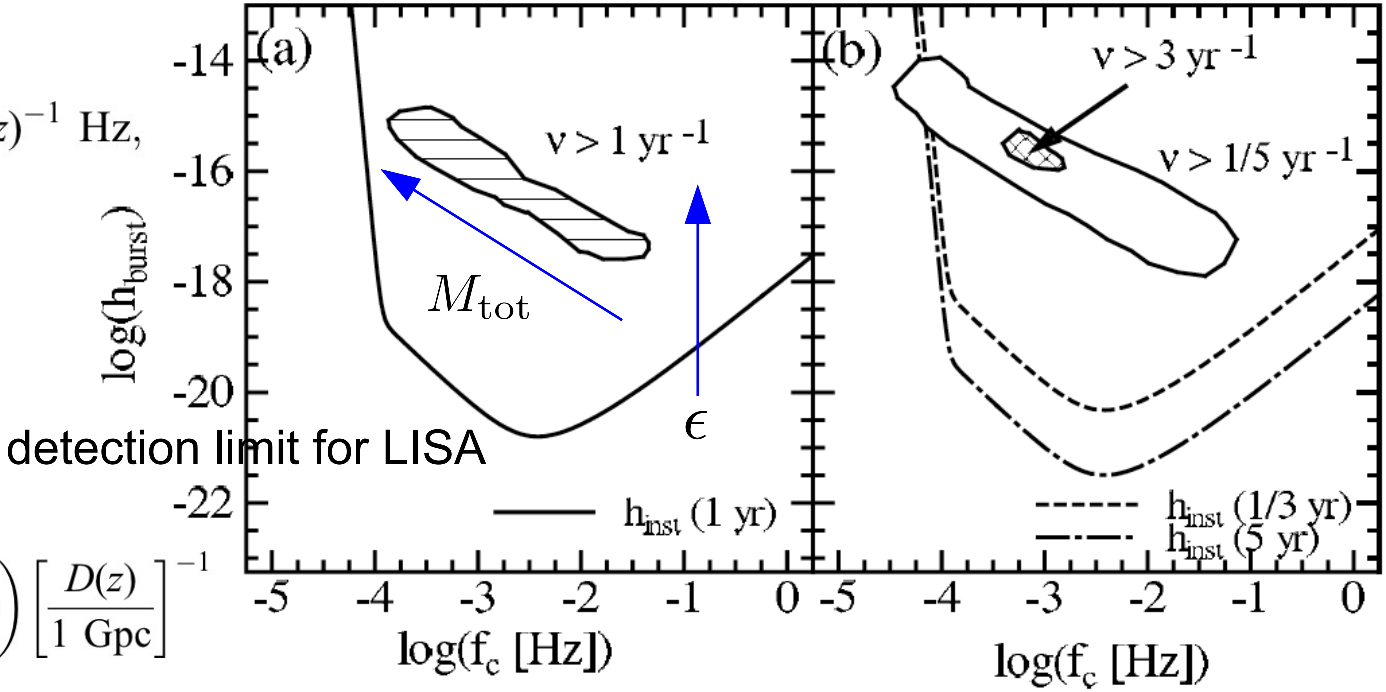


GW bursts

- proportional to merger rate of galaxies $\nu_{\rm burst}(h_{\rm burst}, f_c) = \int n_{\rm burst}(h_{\rm burst}, f_c, z) \frac{dV}{dt_0} dz.$
- lowering ϵ decreases h, NOT event rate $E_{\rm GW} \sim \epsilon M_{\rm tot} c^2, \quad \epsilon \sim 0.1$

$$f_c = \frac{c^3}{3^{3/2} G M_{\rm tot}(1+z)}$$
$$= 3.9 \times 10^{-4} \left(\frac{M_{\rm tot}}{10^8\ M_\odot}\right)^{-1} (1+z)^{-1}\ {\rm Hz},$$

Enoki+04

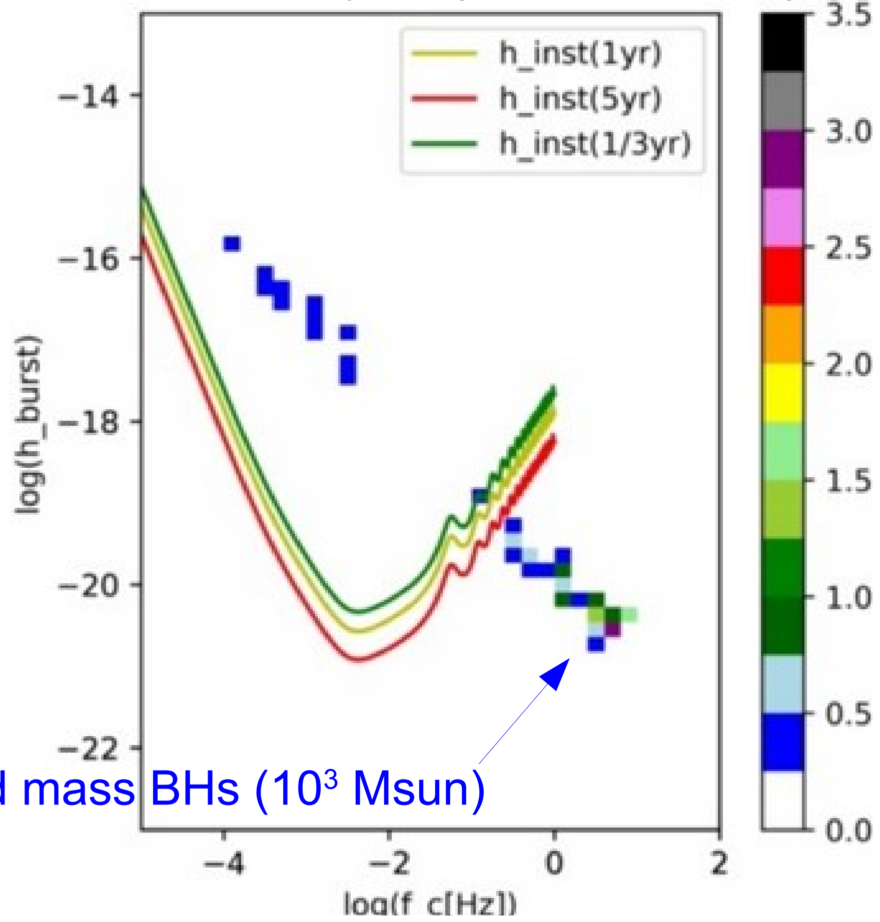


$$h_{\rm burst} = 7.8 \times 10^{-16} \left(\frac{\epsilon}{0.1}\right)^{1/2} \left(\frac{M_{\rm tot}}{10^8\ M_\odot}\right) \left[\frac{D(z)}{1\ {\rm Gpc}}\right]^{-1}$$

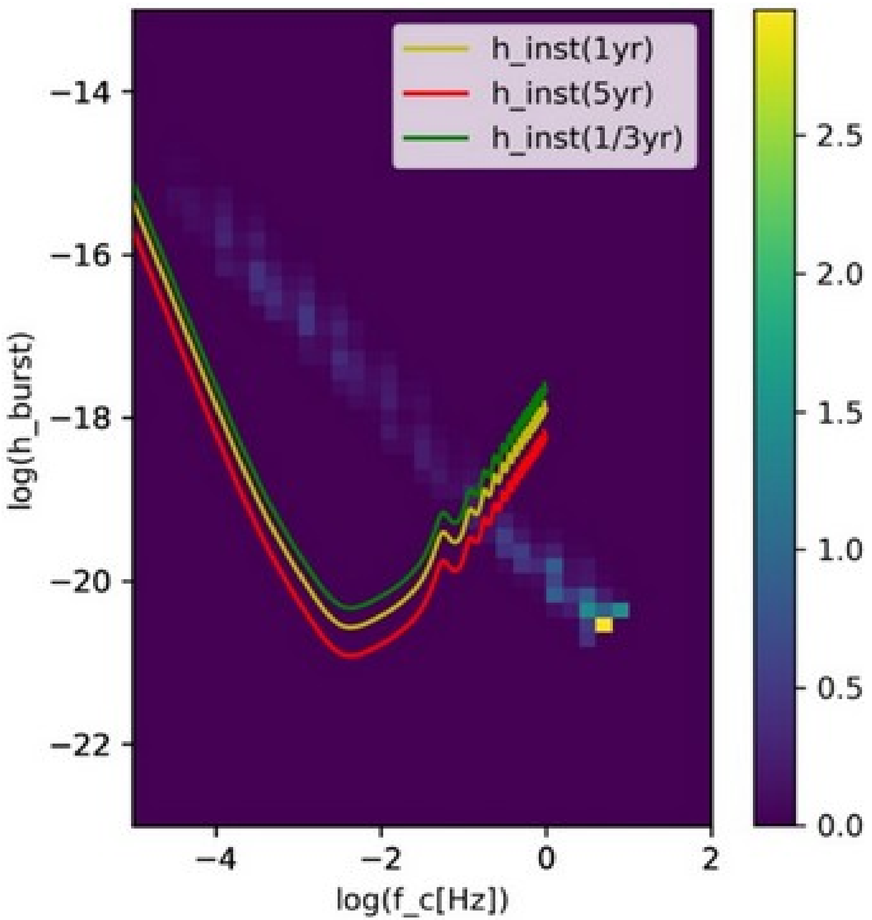
current results (preliminary)

- 1 for 2-3 yrs (decreased!)

Kobayashi (2023), graduation thesis

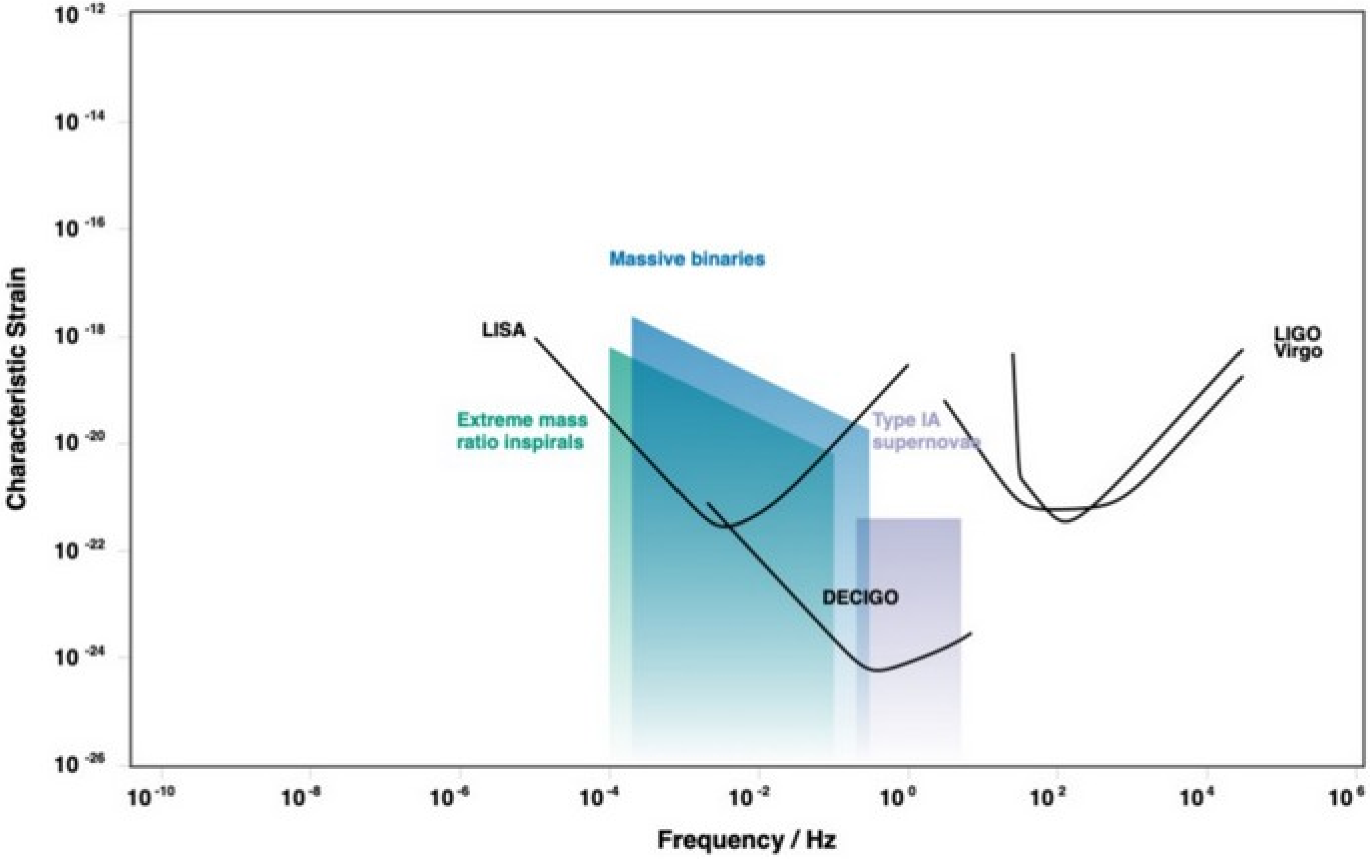


seed mass BHs (10^3 Msun)

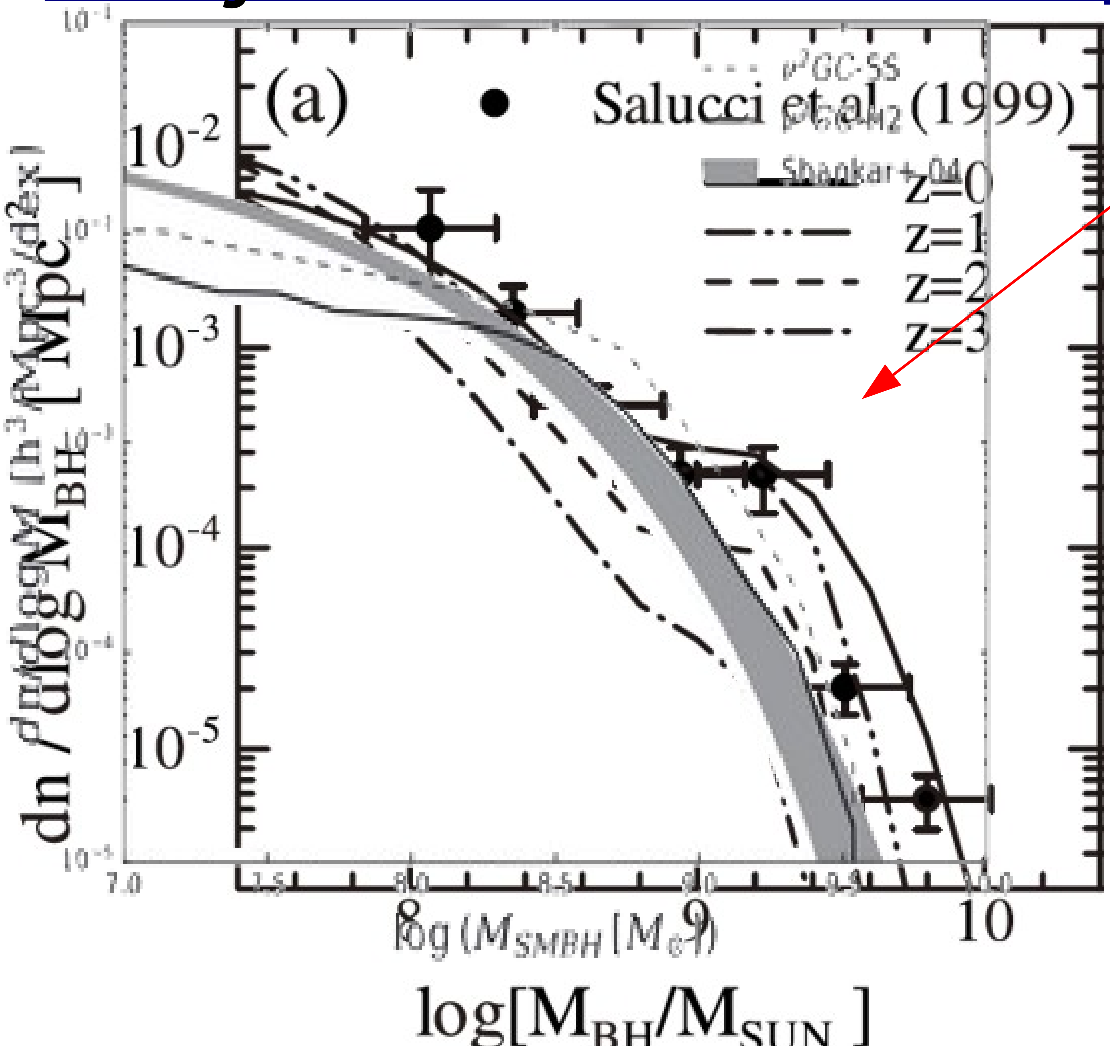


seed mass and DECIGO

- DECIGO will clarify the seed mass of massive BHs…?



Why overestaimted?



- Old observation (Salucci+99) overestimated the mass of massive SMBHs
- This leads to overestimation of the merger rates of SMBHs, then the amplitude of GWs

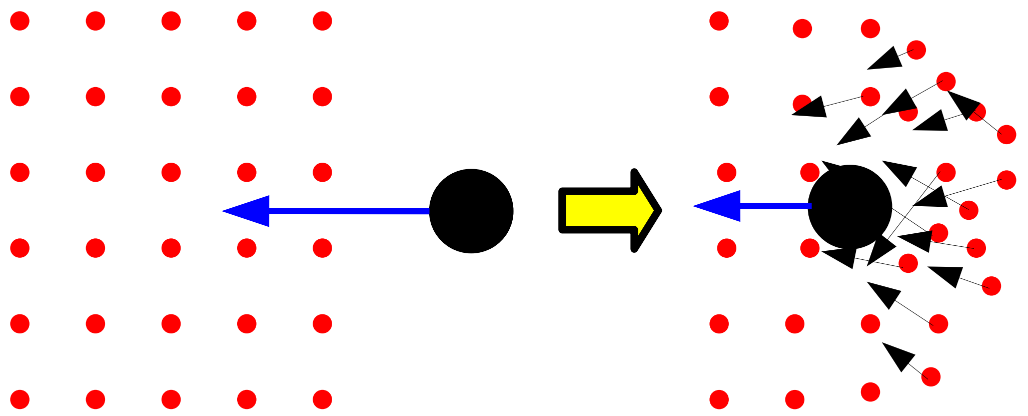
SMBH mass functions
Salucci+99: symbols
Shankar+04: shaded region

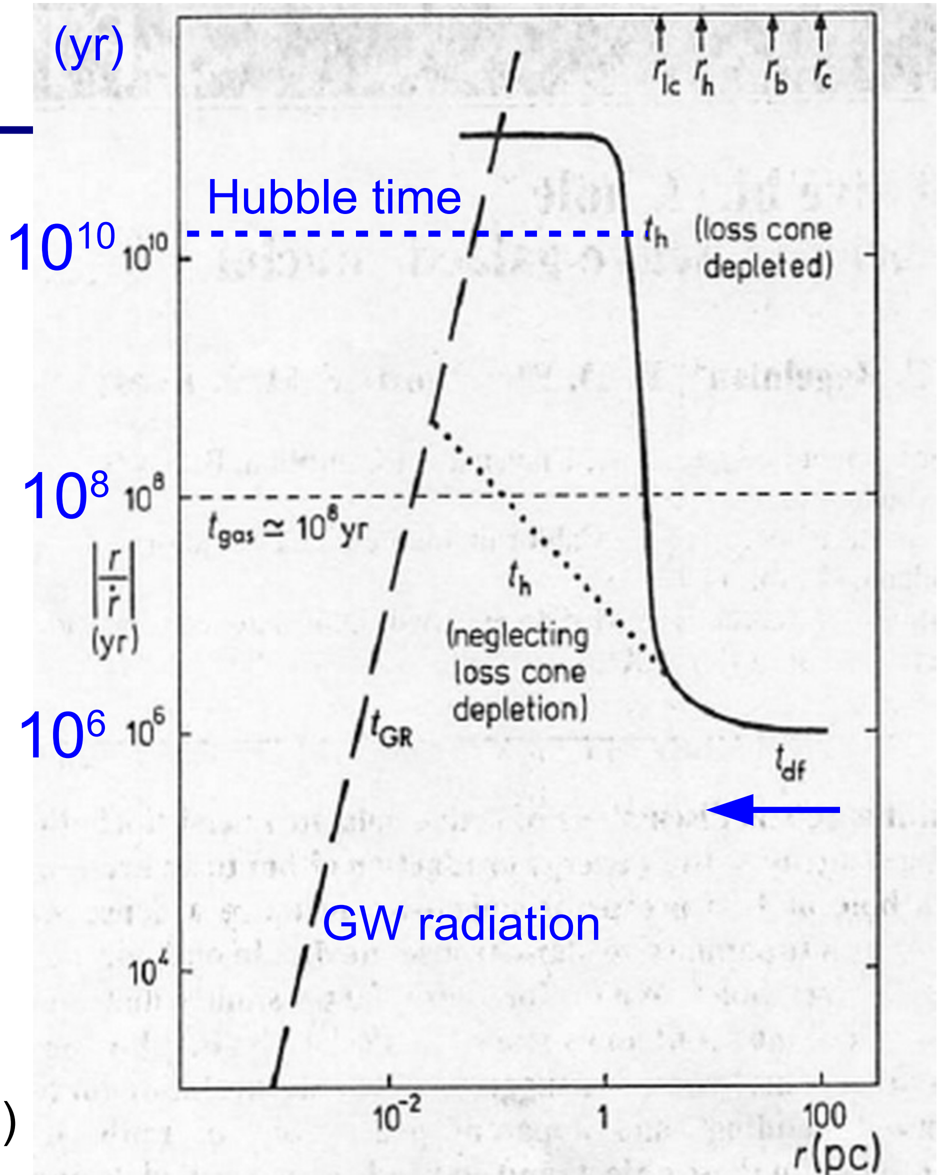
Summary

- Current semi-analytic models include formation processes of SMBHs, so we can pick out each merger event of SMBHs.
- Our semi-analytic model predicts a slightly lower amplitude of the GWB compared to the NANOGrav results. The difference might be solved by (1) obs. data lowered with future data, and/or (2) new physics.
- LISA will detect at least one event per 3 yrs.
- We assume that all binary SMBHs merge together. In case that significant fraction of binaries does not go into coalescence, by, e.g. kicked out, the predicted amplitude decreases.

timescale

dynamical friction:
slow-speed stars deccelerate the BH





Begelman, Blandford & Rees (1980)