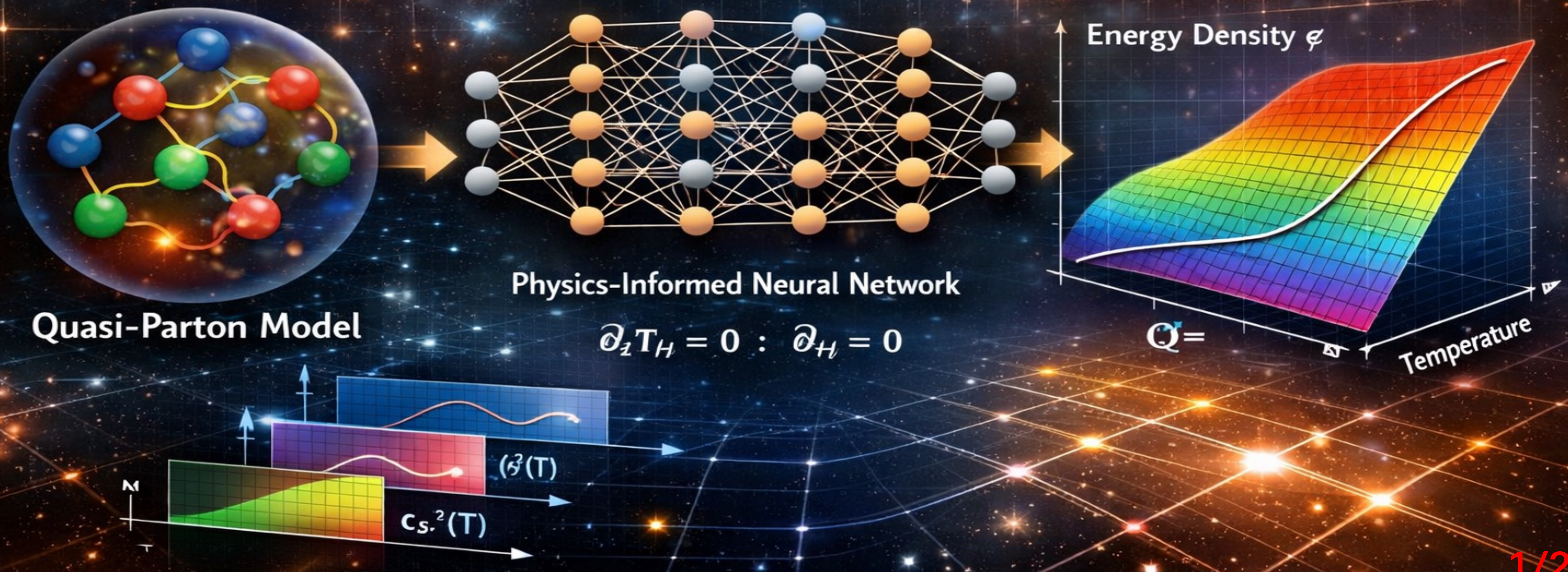
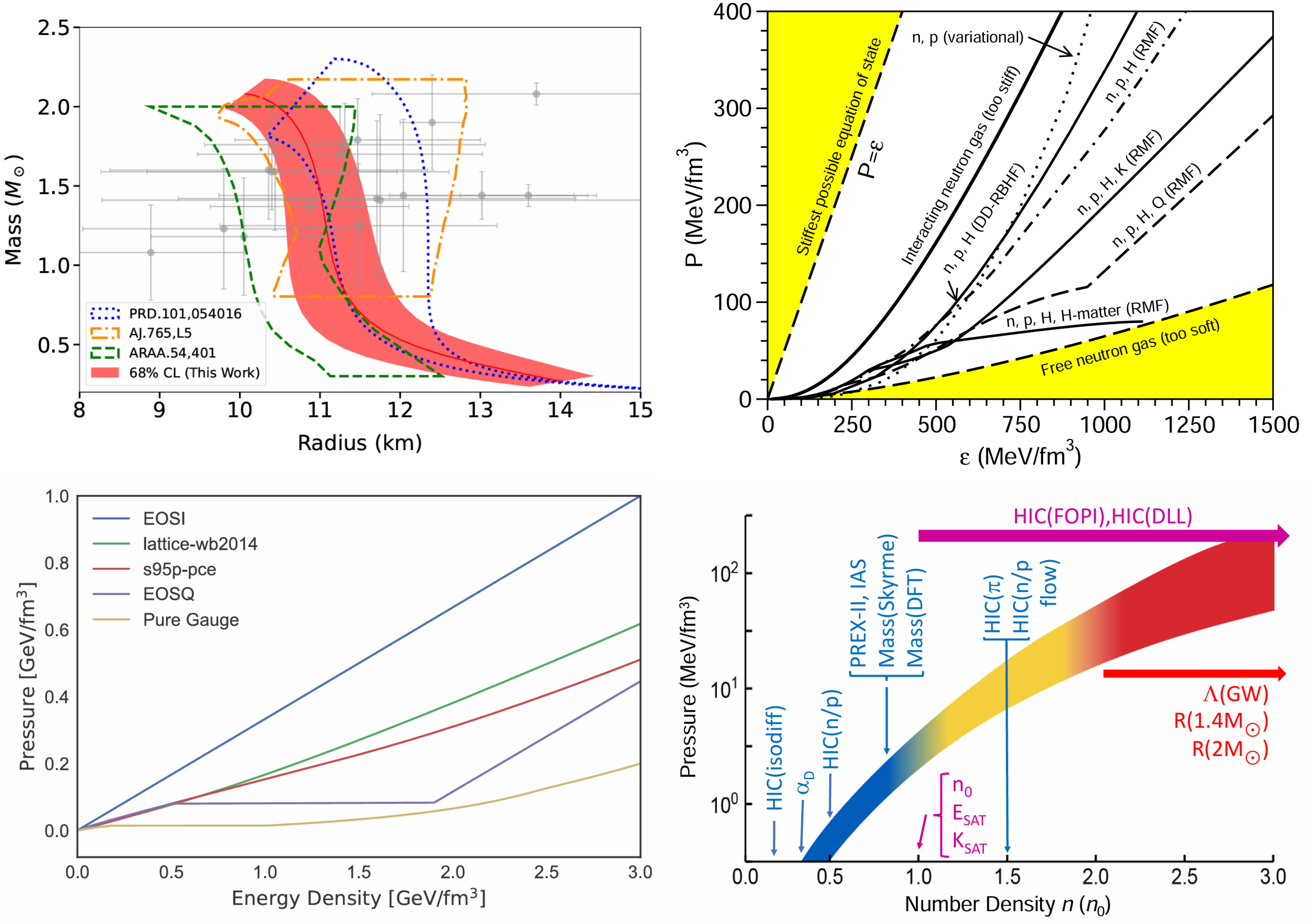
4D QCD EoS from a Quasi-Parton Model via Physics-Informed Neural Network

Fu-Peng Li(FDU&RIKEN-iTHEMS)@KEK



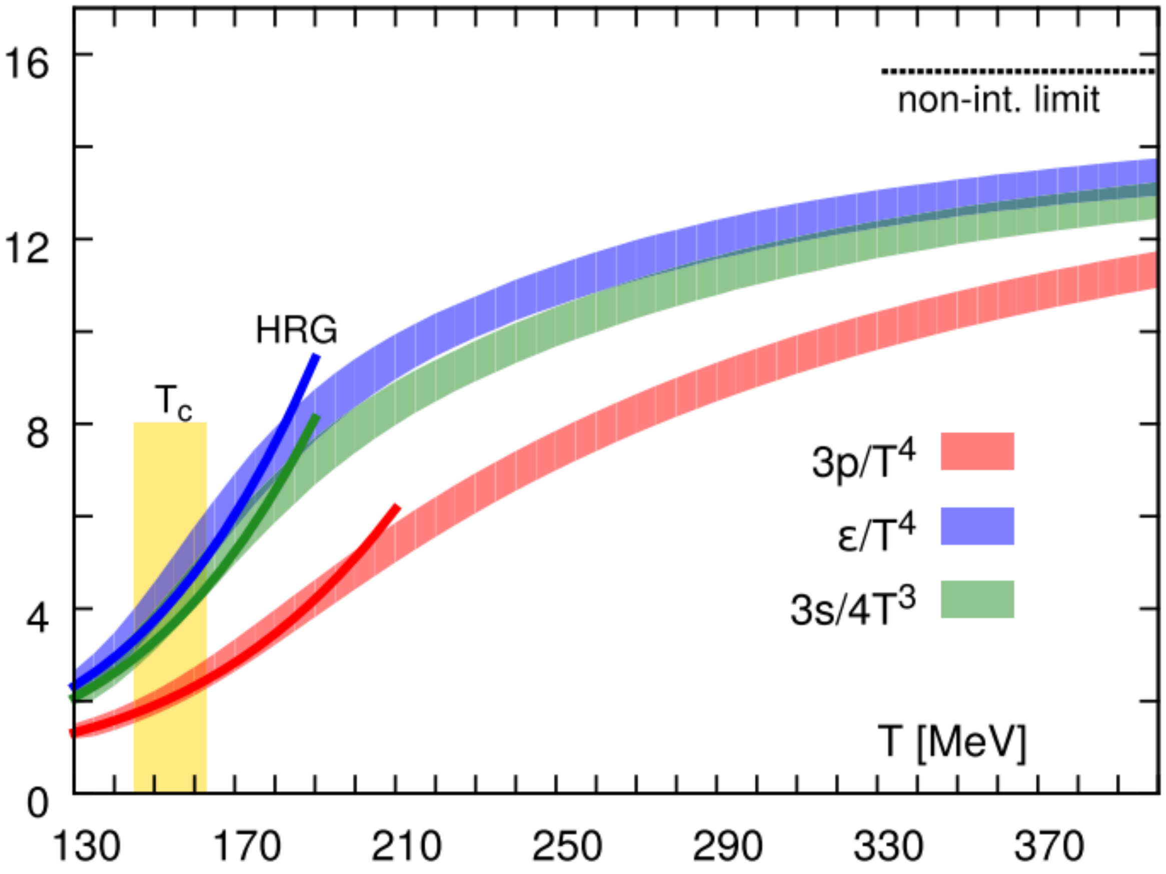
Nuclear EoS

- Crucial for understanding the **evolution of early universe**, **neutron star** and **properties of QGP**
- Also constrains **many-body nuclear interactions** and **non-perturbative QCD**
- One of the **key physical objectives** of heavy-ion collision experiments
- DL for QCD EoS given by Prof. Long-Gang Pang tomorrow.



Shriya Soma, Lingxiao Wang, Shuzhe Shi. et al. Phys.Rev.D 107 (2023) 8, 083028
Tsang, C.Y., Tsang, M.B., Lynch, W.G. et al. Nat Astron 8, 328–336 (2024)
Long-Gang Pang, Hannah Petersen, Xin-Nian Wang. Phys. Rev. C 97, 064918(2018)
F. Weber, R. Negreiros, P. Rosenfield Prog.Part.Nucl. Phys.59:94-113,2007

QCD equation of state

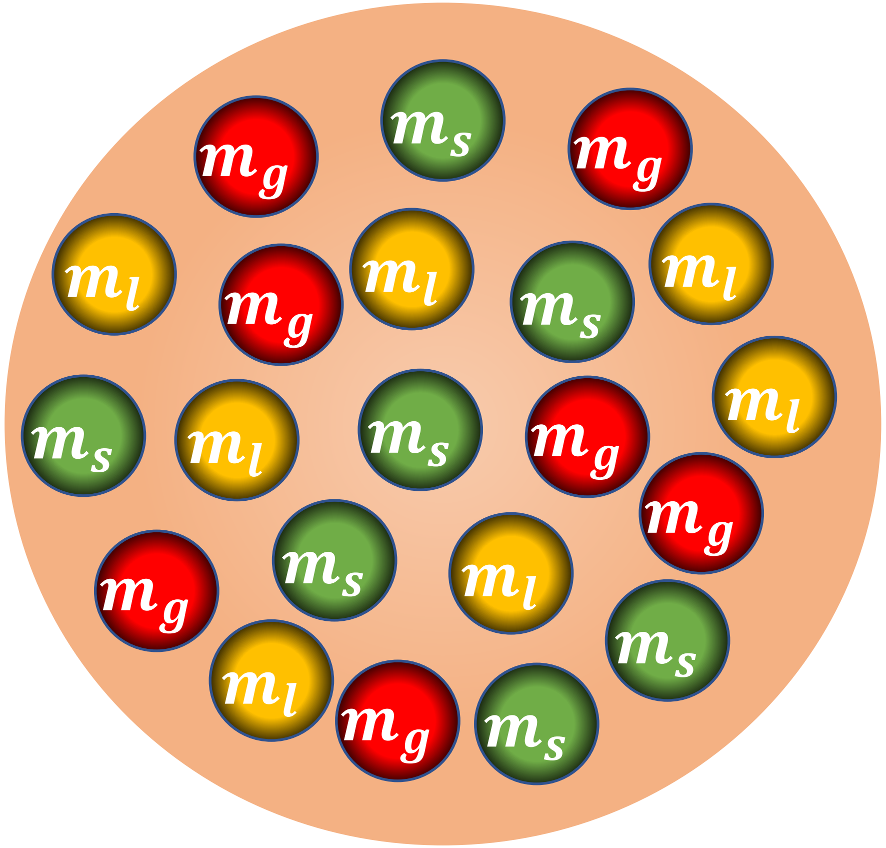


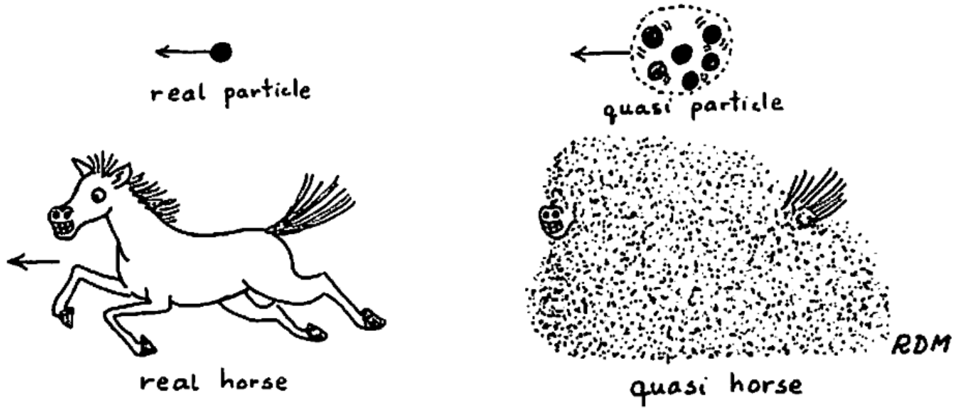
HRG model: the hot and dense QCD matter is considered as non-interaction hadrons.

- T < 200 MeV: QCD equation of state is well described by HRG.
- T > 200 MeV: nuclear matter transitions into the QGP phase.
- **Can we reconstruct?**
- **How?**

HotQCD, Phys Rev D 90, 094503 (2014)

Quasi-particle method





quasi particles of this particular system. Many different types of systems of interacting particles may be described in this manner, and in general we have

$$\text{real particle} + \begin{matrix}\text{'coat' or 'cloud'}\\ \text{of other particles}\end{matrix} = \text{quasi particle.} \qquad (0.1)$$

Sometimes this same equation is stated in a more powerful terminology coming from quantum field theory:

$$\text{'bare' particle} + \begin{matrix}\text{'clothing'}\\ \text{or 'cloud'}\end{matrix} = \begin{matrix}\text{'dressed' or 'clothed'}\\ \text{or 'physical' or}\\ \text{'renormalized' particle.}\end{matrix} \qquad (0.2)$$

- Absorb the interacting potential into the mass.
- We construct a weakly interacting quasi-parton-gas model, which is an effective theory for strongly coupled QGP.

$$H = T + V_{eff} = \frac{P^2}{2M_{real}} + V_{eff}$$

$$H = \frac{P^2}{2M_{quasi}}$$

A Guide to Feynman Diagrams in the Many-Body Problem(1992), R.D.Mattuck

Traditionally *vs* DL

- It will introduce a strong prior bias
- It is intractable to determine two or more unknown functions

prior equation (Parameterization)

$$m(T) = \frac{a}{(t-1)^c} + bt,$$

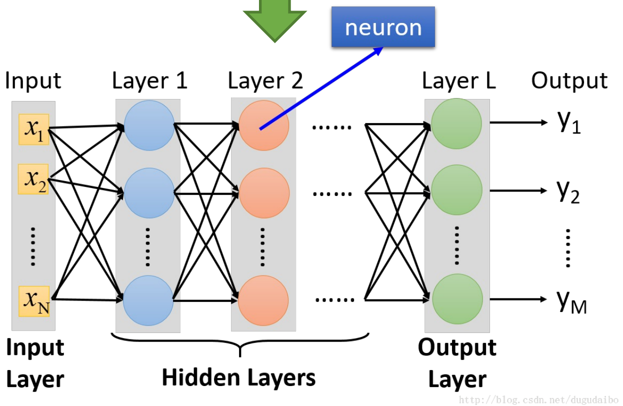
$$g^2(T) = \frac{48\pi^2}{(11N_c - 2N_f)\ln\left[\frac{(aT/T_c+b)^2}{1+ce^{-d(T/T_c)^2}}\right]}$$

fitting →

experimental data

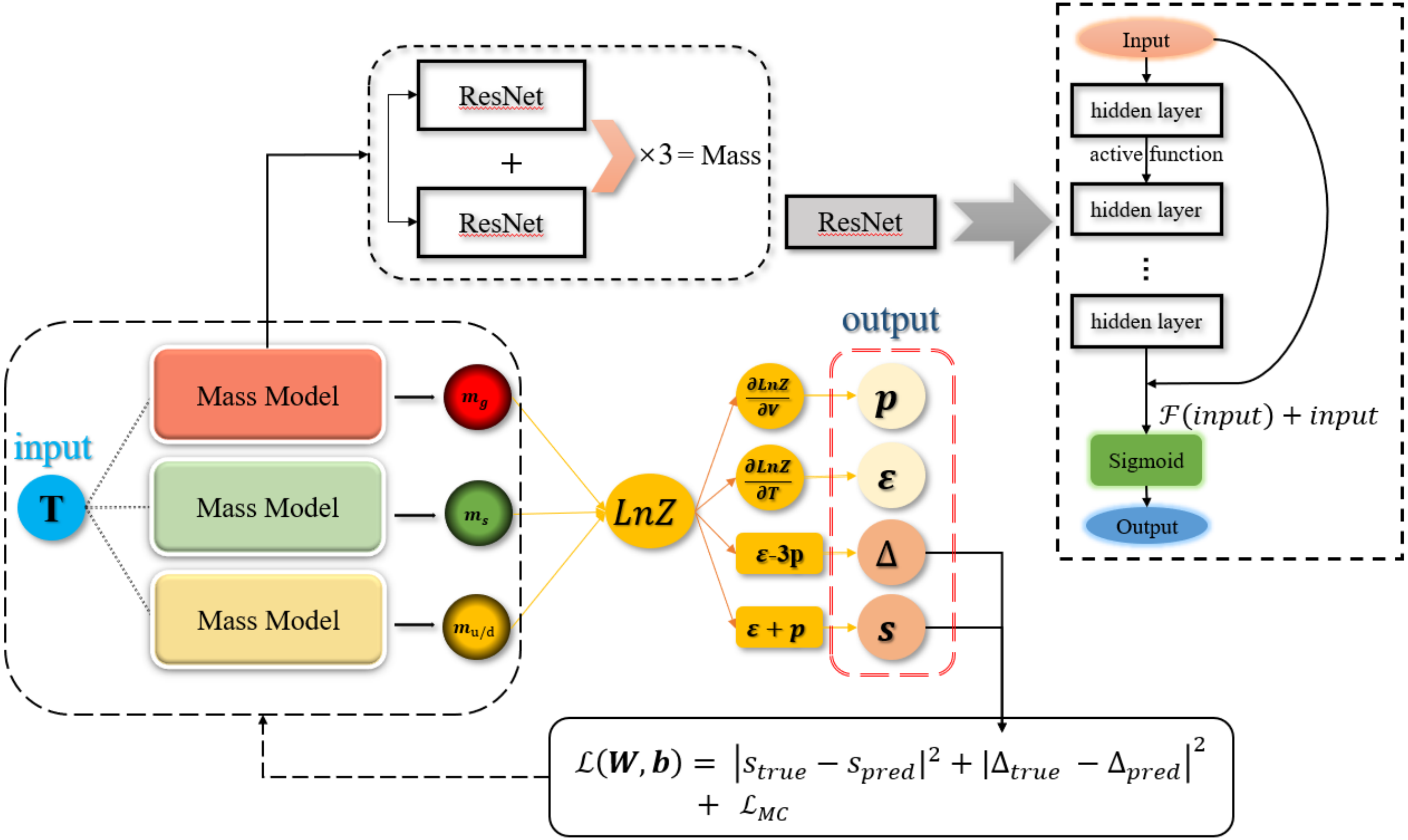
← feed back

DNN



Universal approximation theorem: Multilayer feedforward networks are universal approximators,1989.
Automatic Differentiation: PyTorch,Tensorflow,Jax.

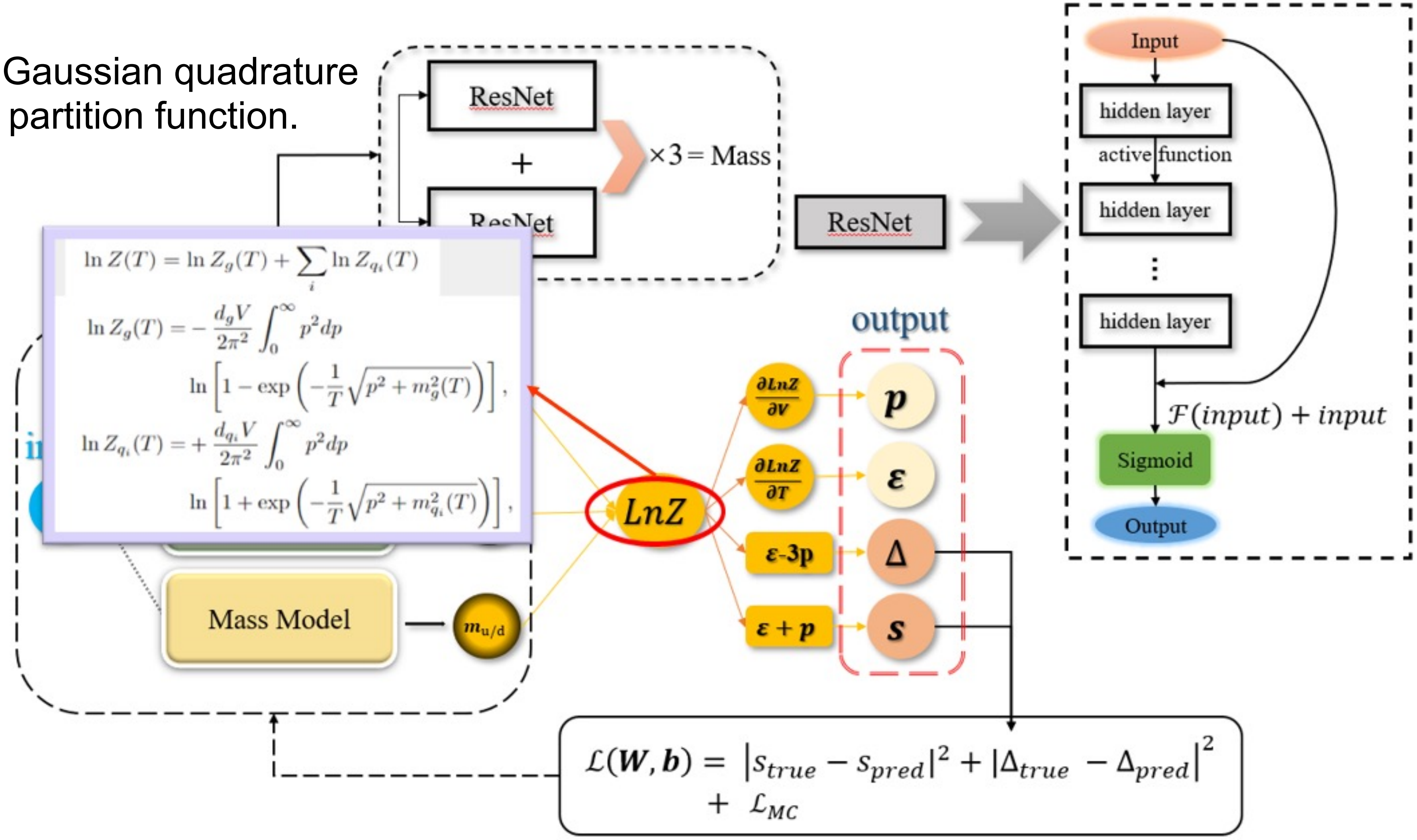
The framework of DNN：



FPL, Hong-Liang Lü, Long-Gang Pang & Guang-You Qin, Phys.Lett.B 844 (2023), 138088

The framework of DNN

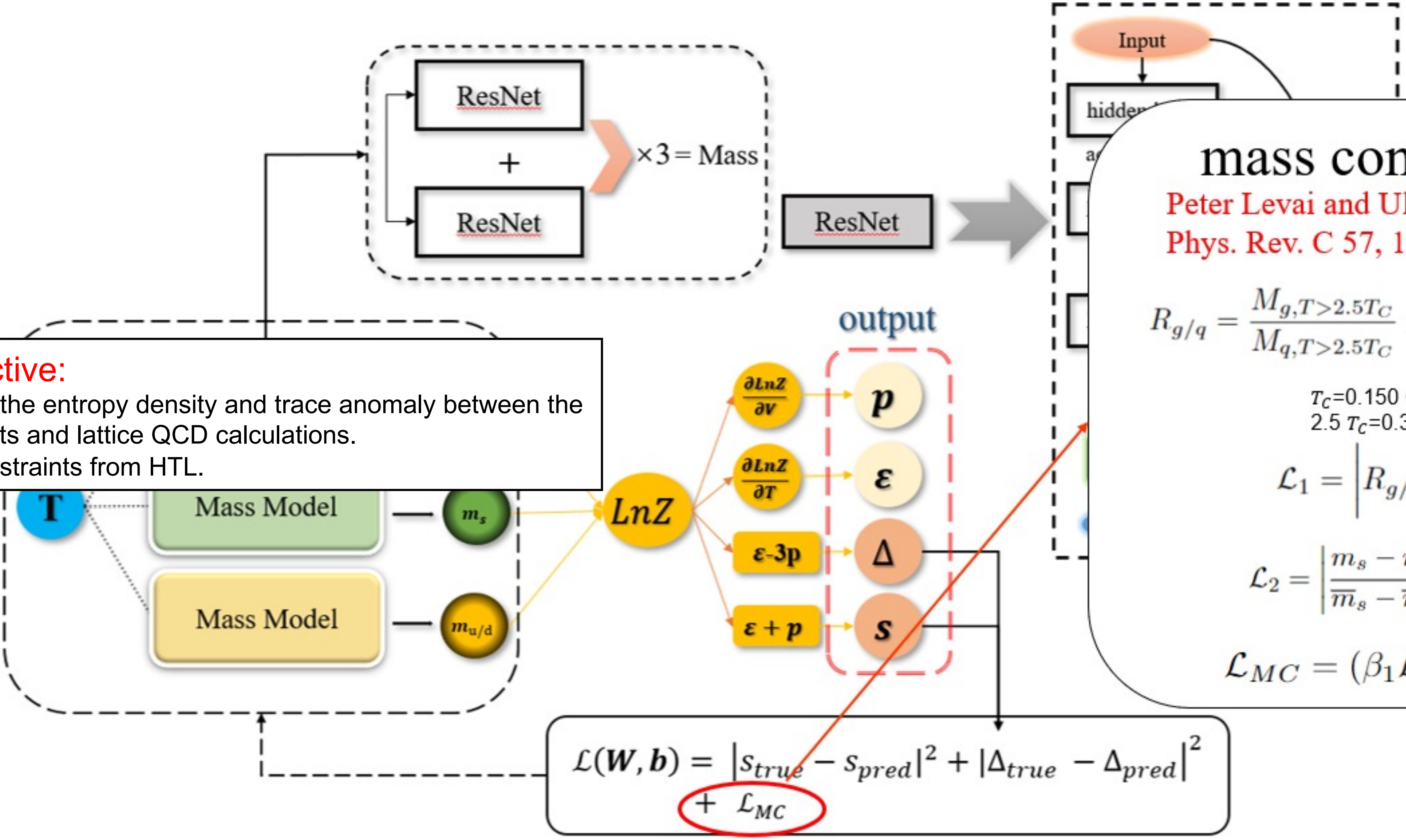
✓ Using 25-point Gaussian quadrature to compute the partition function.



The framework of DNN

Training objective:

1. The RMSE of the entropy density and trace anomaly between the network outputs and lattice QCD calculations.
2. The mass constraints from HTL.



mass constraints

Peter Levai and Ulrich W. Heinz
Phys. Rev. C 57, 1879–1890 (1998)

$$R_{g/q} = \frac{M_{g,T>2.5T_C}}{M_{q,T>2.5T_C}} = \sqrt{\frac{3}{2}\left(\frac{N_C}{3} + \frac{N_f}{6}\right)}$$

T_C=0.150 GeV
2.5 T_C=0.375GeV

$$\mathcal{L}_1 = \left|R_{g/q} - \frac{3}{2}\right|.$$

$$\mathcal{L}_2 = \left|\frac{m_s - m_{u/d}}{\overline{m}_s - \overline{m}_{u/d}} - 1\right|.$$

$$\mathcal{L}_{MC} = (\beta_1 \mathcal{L}_1 + \beta_2 \mathcal{L}_2)^2$$

Result

PHYSICAL REVIEW D **90,** 094503 (2014)

Equation of state in (2 + 1)-flavor QCD

A. Bazavov,[1] Tanmoy Bhattacharya,[2] C. DeTar,[3] H.-T. Ding,[4] Steven Gottlieb,[5] Rajan Gupta,[2] P. Hegde,[4] U. M. Heller,[6] F. Karsch,[7,8] E. Laermann,[8] L. Levkova,[3] Swagato Mukherjee,[7] P. Petreczky,[7] C. Schmidt,[8] C. Schroeder,[9] R. A. Soltz,[9] W. Soeldner,[10] R. Sugar,[11] M. Wagner,[5] and P. Vranas[9]
(HotQCD Collaboration)

Physics Letters B 730 (2014) 99–104

Contents lists available at ScienceDirect

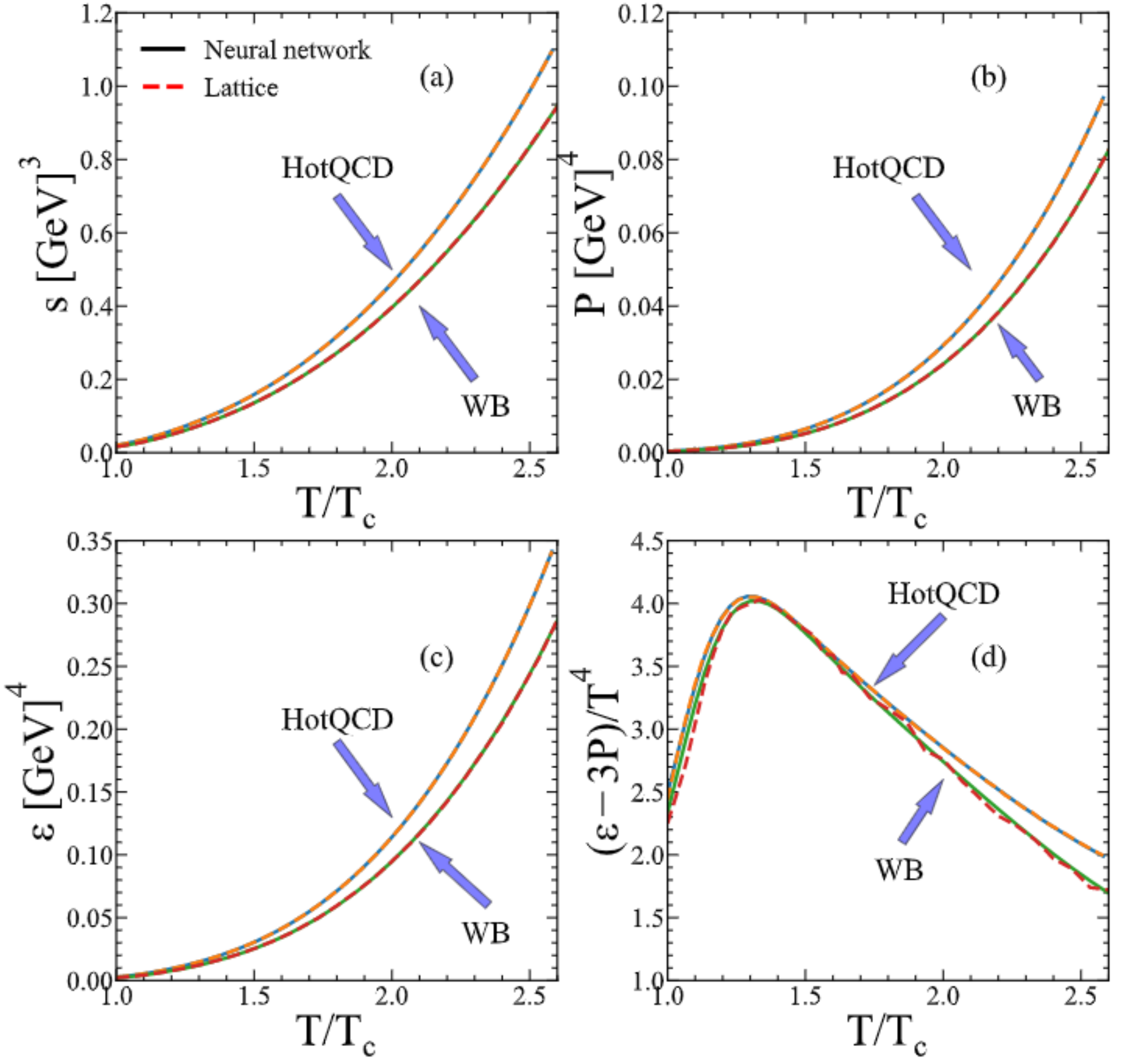
Physics Letters B

www.elsevier.com/locate/physletb

Full result for the QCD equation of state with 2 + 1 flavors

Szabolcs Borsányi[a], Zoltán Fodor[a,b,c], Christian Hoelbling[a], Sándor D. Katz[c,d,*], Stefan Krieg[a,b], Kálmán K. Szabó[a,e]

Training data : HotQCD and WB Lattice QCD.



Result

- QP is based on the mean field theory of equilibrium or near-equilibrium systems

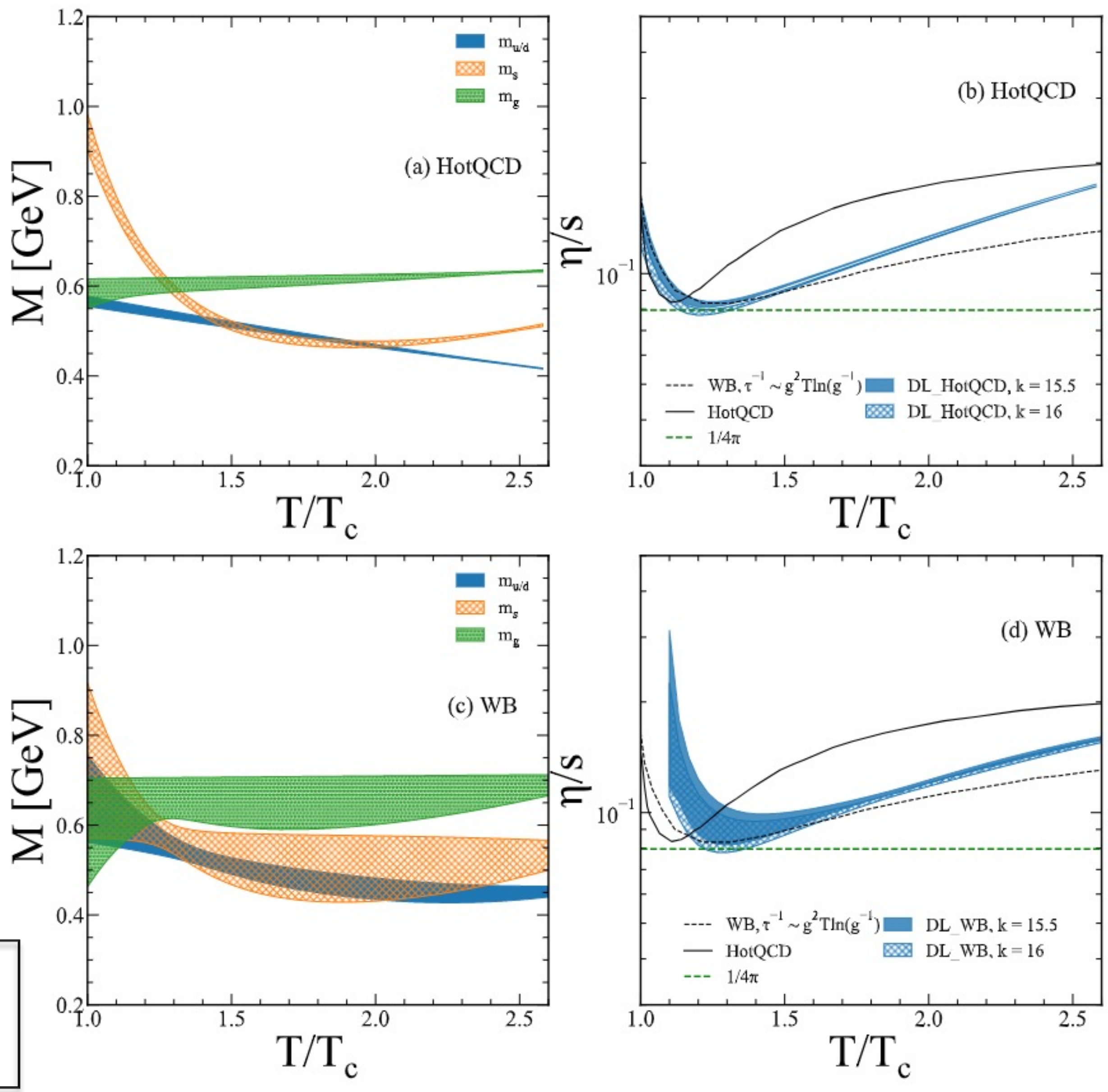
$$g^2 = \frac{m_g^2 + m_{u/d}^2}{\left[\frac{1}{6}(N_C + \frac{1}{2}N_f) + \frac{N_C^2-1}{8N_C}\right]T^2},$$

- The relaxation time τ_i : Assuming that after being perturbed, QP will return to equilibrium from a non-equilibrium state in a specific time.

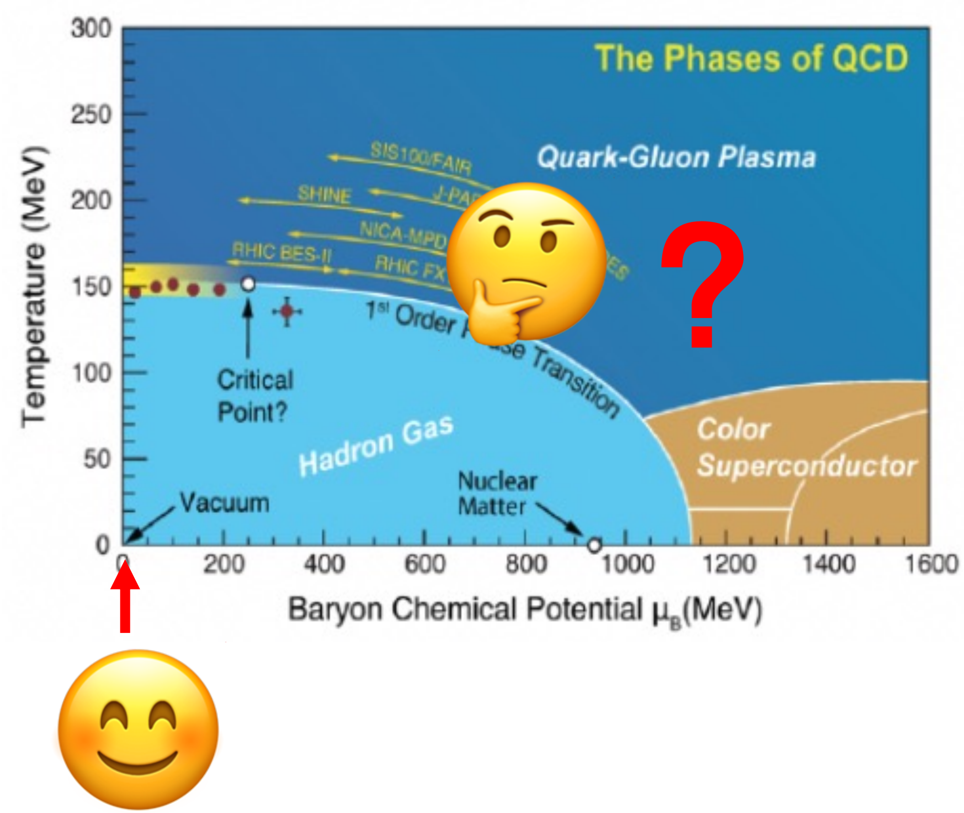
$$\tau_q^{-1} = 2\frac{N_C^2-1}{2N_C}\frac{g^2T}{8\pi}\ln\frac{2k}{g^2}, \tau_g^{-1} = 2N_C\frac{g^2T}{8\pi}\ln\frac{2k}{g^2}.$$

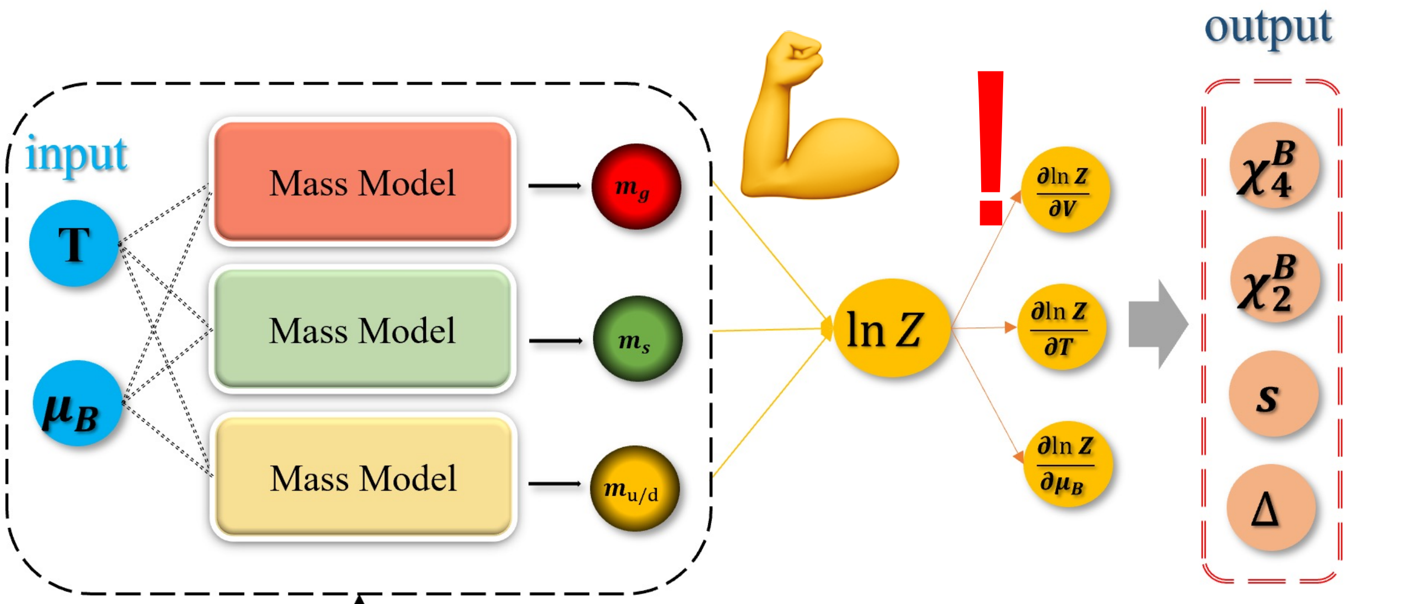
$$\eta = \frac{1}{15T}\sum_i d_i \int \frac{d^3p}{(2\pi)^3}\tau_i \frac{p^4}{E_a^2} f_i(1 \mp f_i)$$

PRD 84, 094004 (2011): $g^2(T) = \frac{48\pi^2}{(11N_C - 2N_f)\ln[\lambda(\frac{T}{T_C} - \frac{T_S}{T_C})]^2}.$



The framework of DNN: $\mu_B > 0$





$$\mathcal{L}(\boldsymbol{W}, \boldsymbol{b}) = |s_{NN} - s_{input}| + \left|\frac{\Delta_{NN} - \Delta_{input}}{T}\right| + |\chi^B_{2,NN} - \chi^B_{2,input}| + |\chi^B_{4,NN} - \chi^B_{4,input}| + \mathcal{L}_{MC}$$

$$\ln Z_g(T,\mu_B) = -\frac{d_g V}{2\pi^2}\int_0^\infty p^2 dp \ln\left[1 - \exp\left(-\frac{1}{T}\sqrt{p^2 + m_g^2(T,\mu_B)}\right)\right],$$

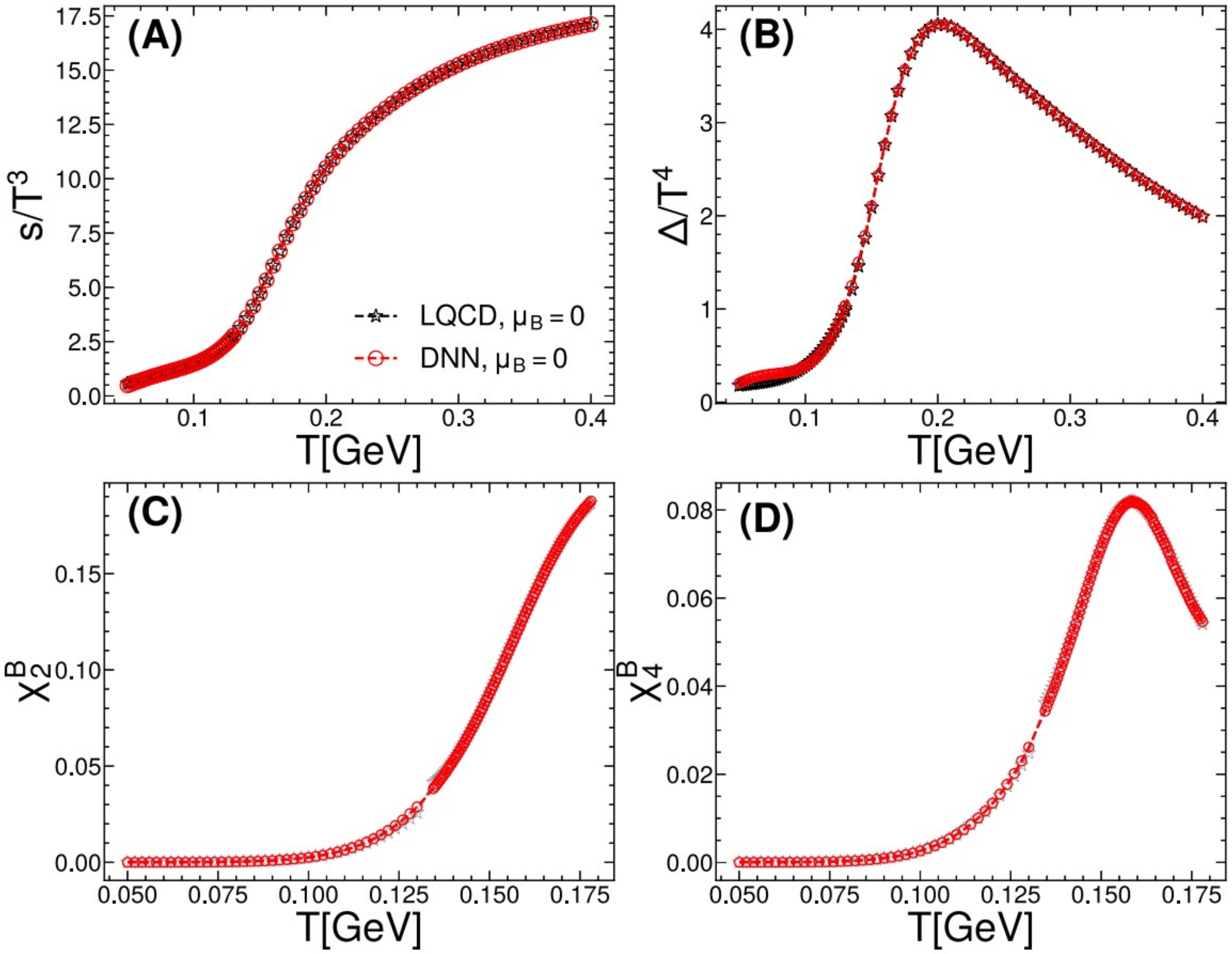
$$\ln Z_{q(\bar{q})_i}(T,\mu_B) = +\frac{d_{q(\bar{q})_i} V}{2\pi^2}\int_0^\infty p^2 dp \ln\left[1 + \exp\left(-\frac{1}{T}\left(\sqrt{p^2 + m^2_{q(\bar{q})_i}(T,\mu_B)} - \mu_{q(\bar{q})_i}\right)\right)\right],$$

$$\mu_u = \frac{1}{3}\mu_B + \frac{2}{3}\mu_Q,$$
$$\mu_d = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q,$$
$$\mu_s = \frac{1}{3}\mu_B - \frac{1}{3}\mu_Q - \mu_S,$$

$$\chi_i^B = \frac{\partial P(T,\hat{\mu}_B)/T^4}{\partial \hat{\mu}_B^i}\Big|_{\hat{\mu}_B=0}, \hat{\mu}_B = \mu_B/T.$$

FPL, Long-Gang Pang & Guang-You Qin, Phys. Lett. B 868 (2025) 139692

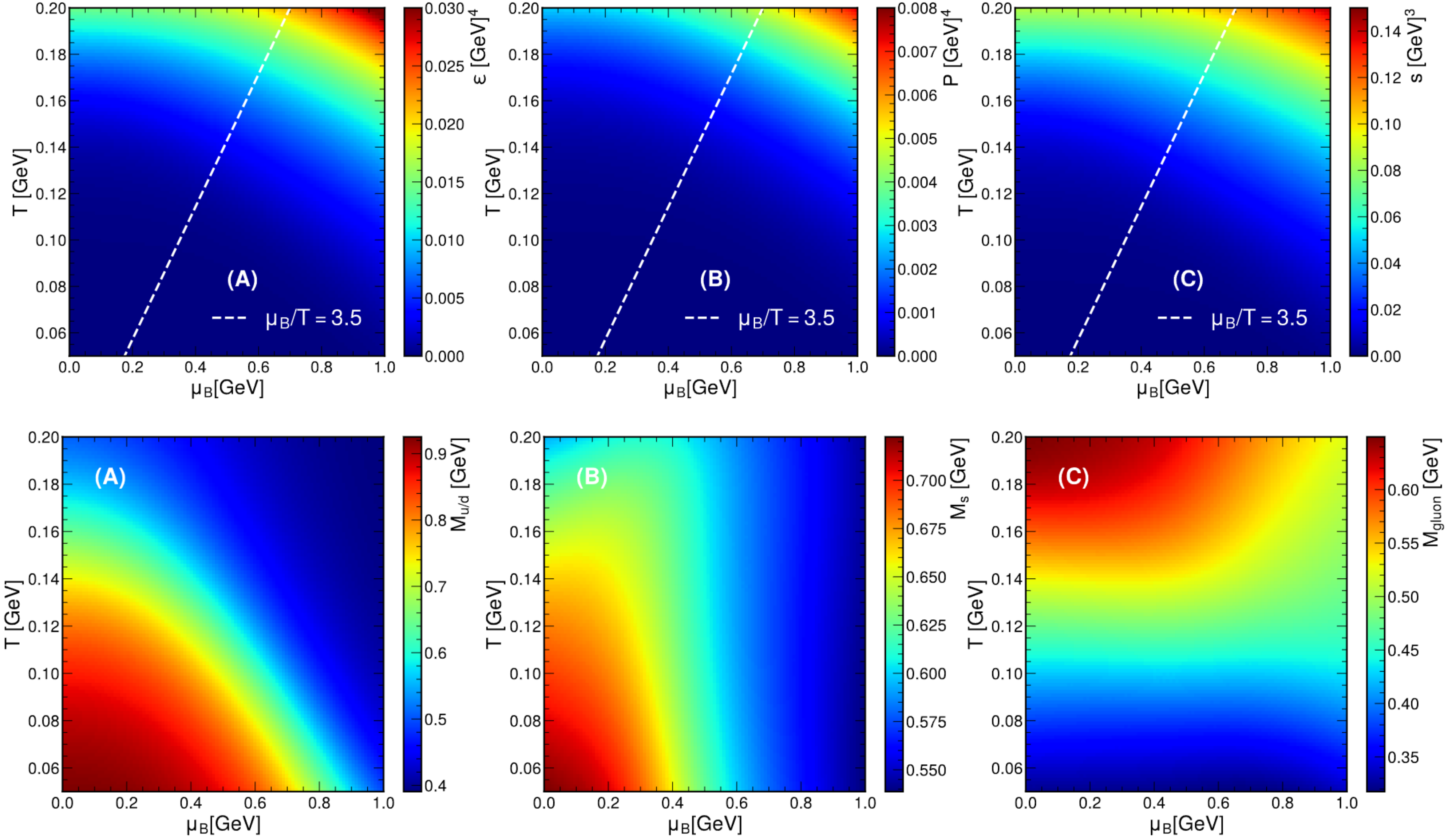
The training data&result



Training data:

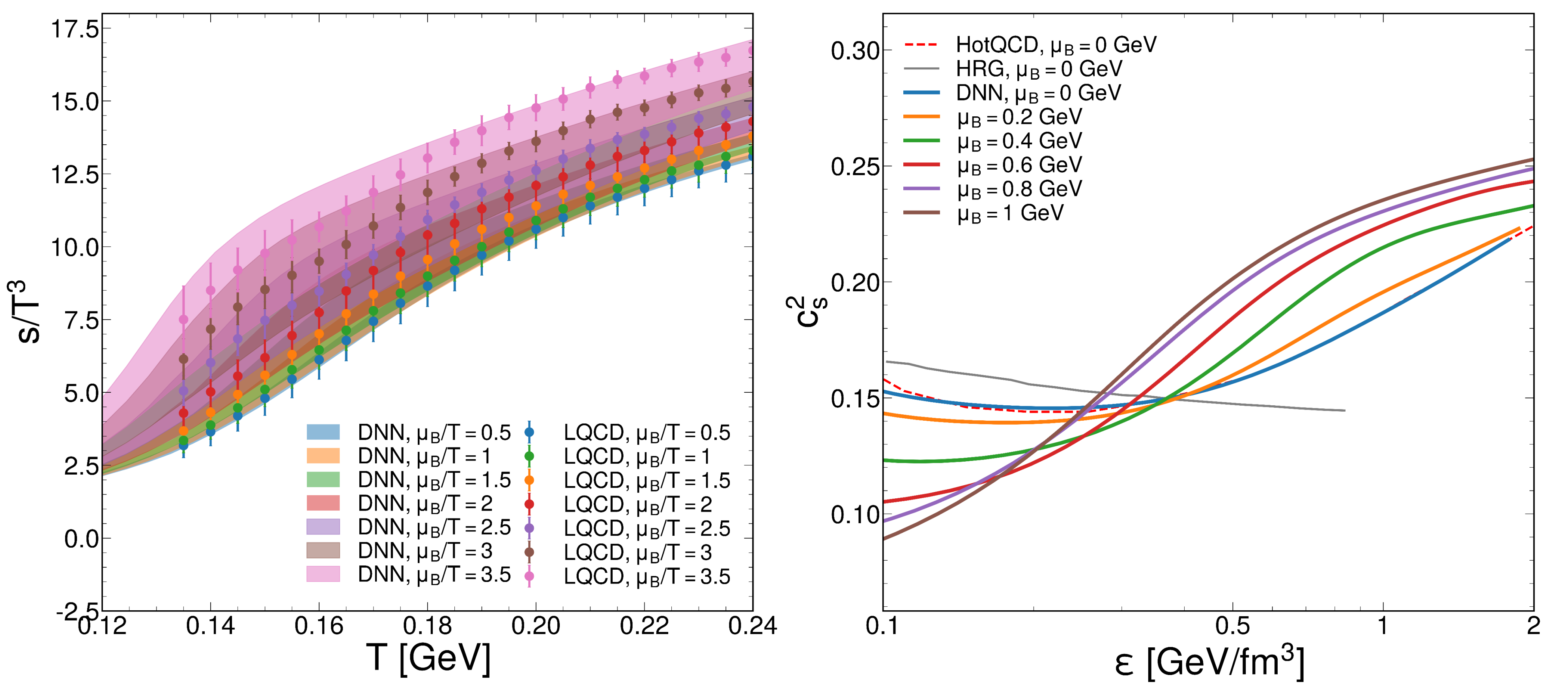
Phys Rev D 95, 054504 (2017)
Phys Rev Lett 118, 182301 (2017)
Phys Rev D 90, 094503 (2014)

Resu

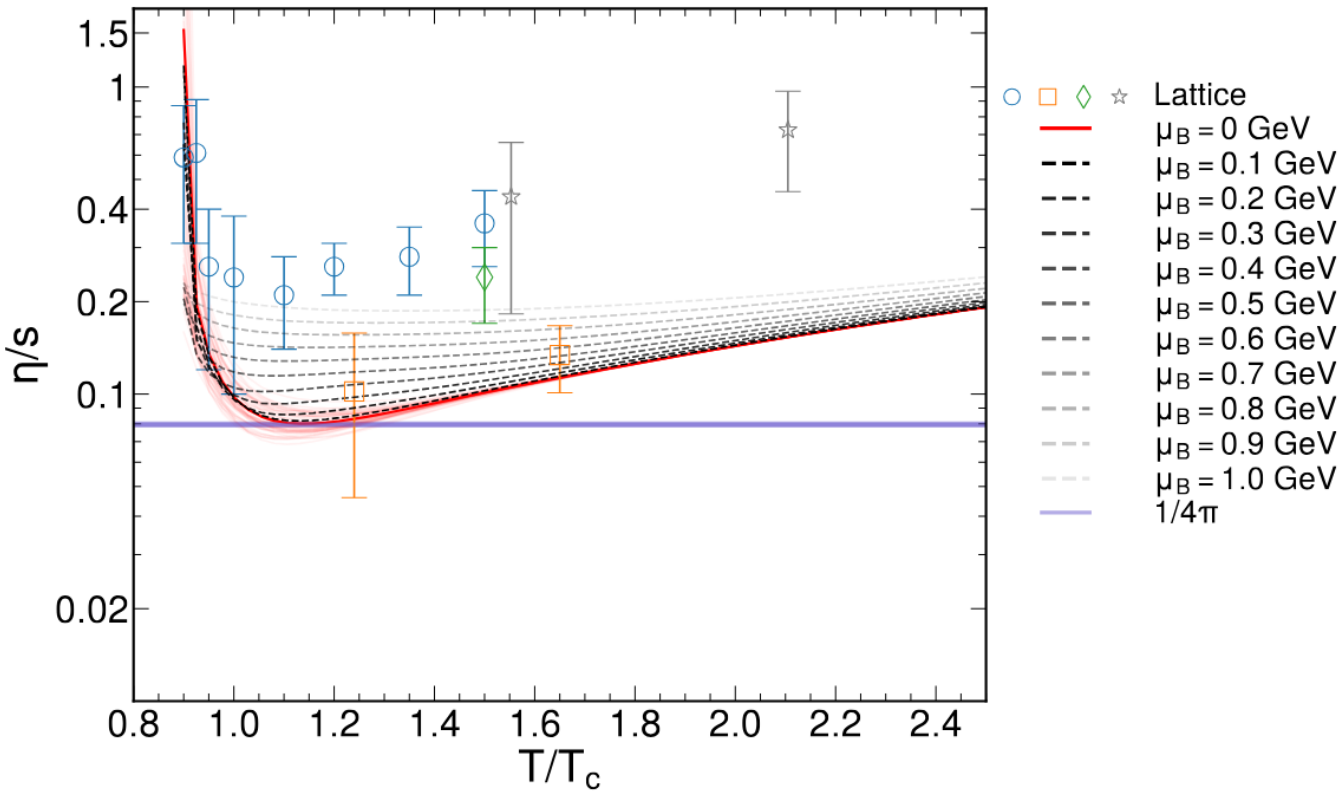


(A)
$\mu_B/T = 3.5$
ε [GeV]^4
(B)
$\mu_B/T = 3.5$
P [GeV]^4
(C)
$\mu_B/T = 3.5$
s [GeV]^3
T [GeV]
μ_B[GeV]
(A)
$M_{u/d}$ [GeV]
(B)
M_s [GeV]
(C)
M_{gluon} [GeV]

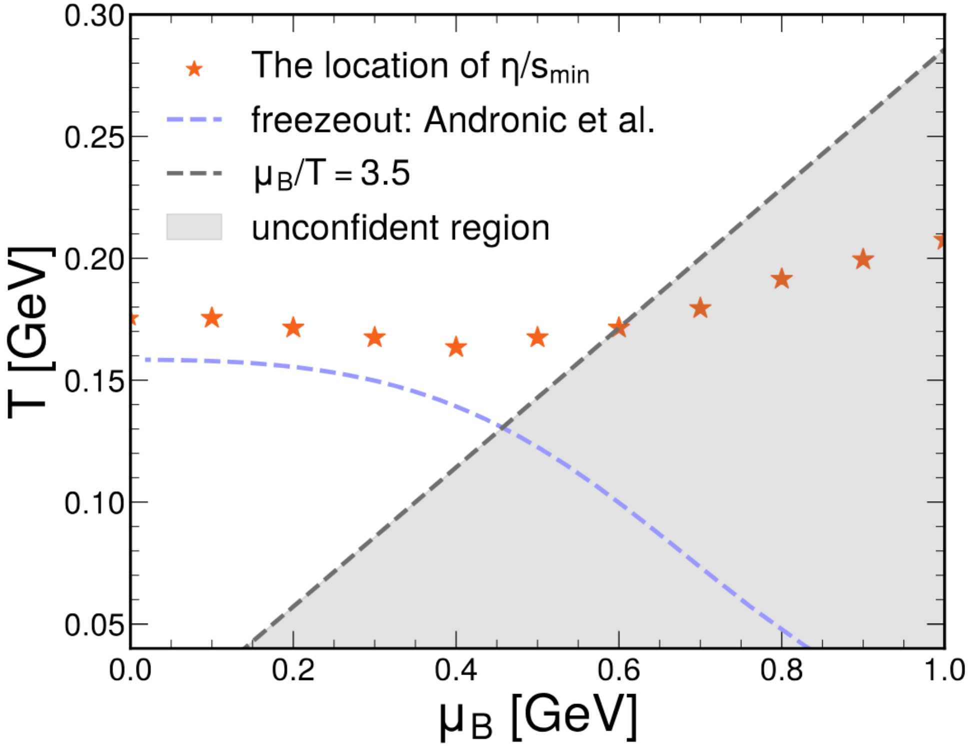
Result


s/T^3
T [GeV]
DNN, μB/T = 0.5
DNN, μB/T = 1
DNN, μB/T = 1.5
DNN, μB/T = 2
DNN, μB/T = 2.5
DNN, μB/T = 3
DNN, μB/T = 3.5
LQCD, μB/T = 0.5
LQCD, μB/T = 1
LQCD, μB/T = 1.5
LQCD, μB/T = 2
LQCD, μB/T = 2.5
LQCD, μB/T = 3
LQCD, μB/T = 3.5
c_s^2
ε [GeV/fm^3]
HotQCD, μB = 0 GeV
HRG, μB = 0 GeV
DNN, μB = 0 GeV
μB = 0.2 GeV
μB = 0.4 GeV
μB = 0.6 GeV
μB = 0.8 GeV
μB = 1 GeV

Result

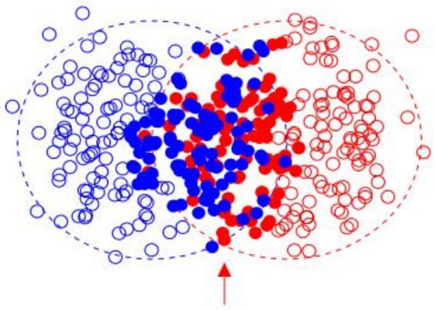


η/s
T/T_c
Lattice
μ_B = 0 GeV
μ_B = 0.1 GeV
μ_B = 0.2 GeV
μ_B = 0.3 GeV
μ_B = 0.4 GeV
μ_B = 0.5 GeV
μ_B = 0.6 GeV
μ_B = 0.7 GeV
μ_B = 0.8 GeV
μ_B = 0.9 GeV
μ_B = 1.0 GeV
1/4π

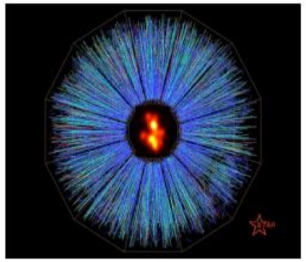


The location of η/s_min
freezeout: Andronic et al.
μ_B/T = 3.5
unconfident region
T [GeV]
μ_B [GeV]

The framework of DNN: 4D DLQPM



$$\nabla_\mu T^{\mu\nu} = 0$$

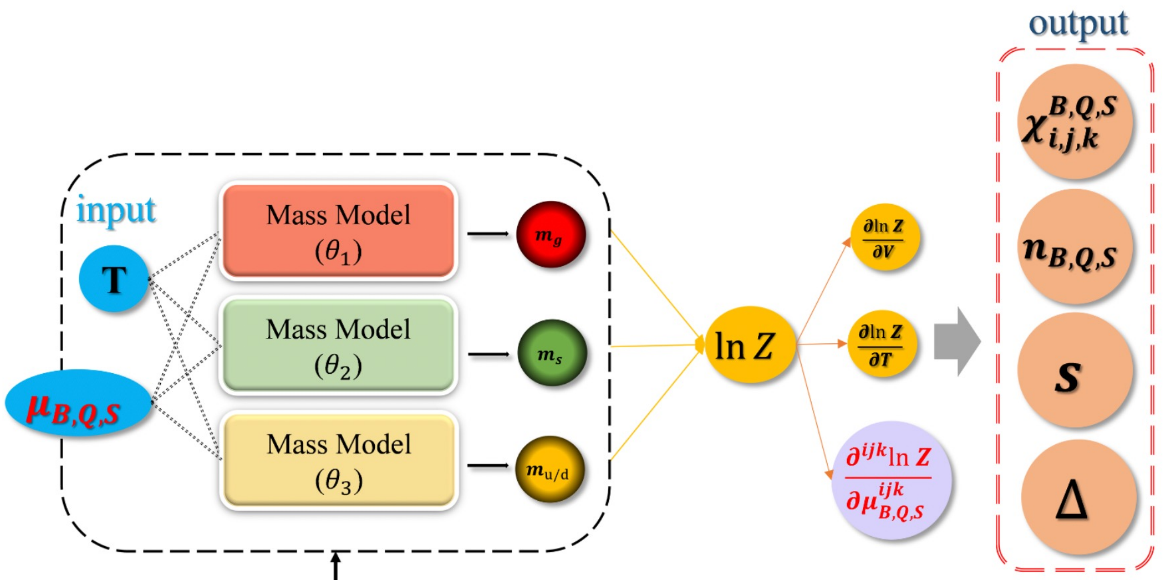


$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

Initial condition

EoS Bulk Viscosity

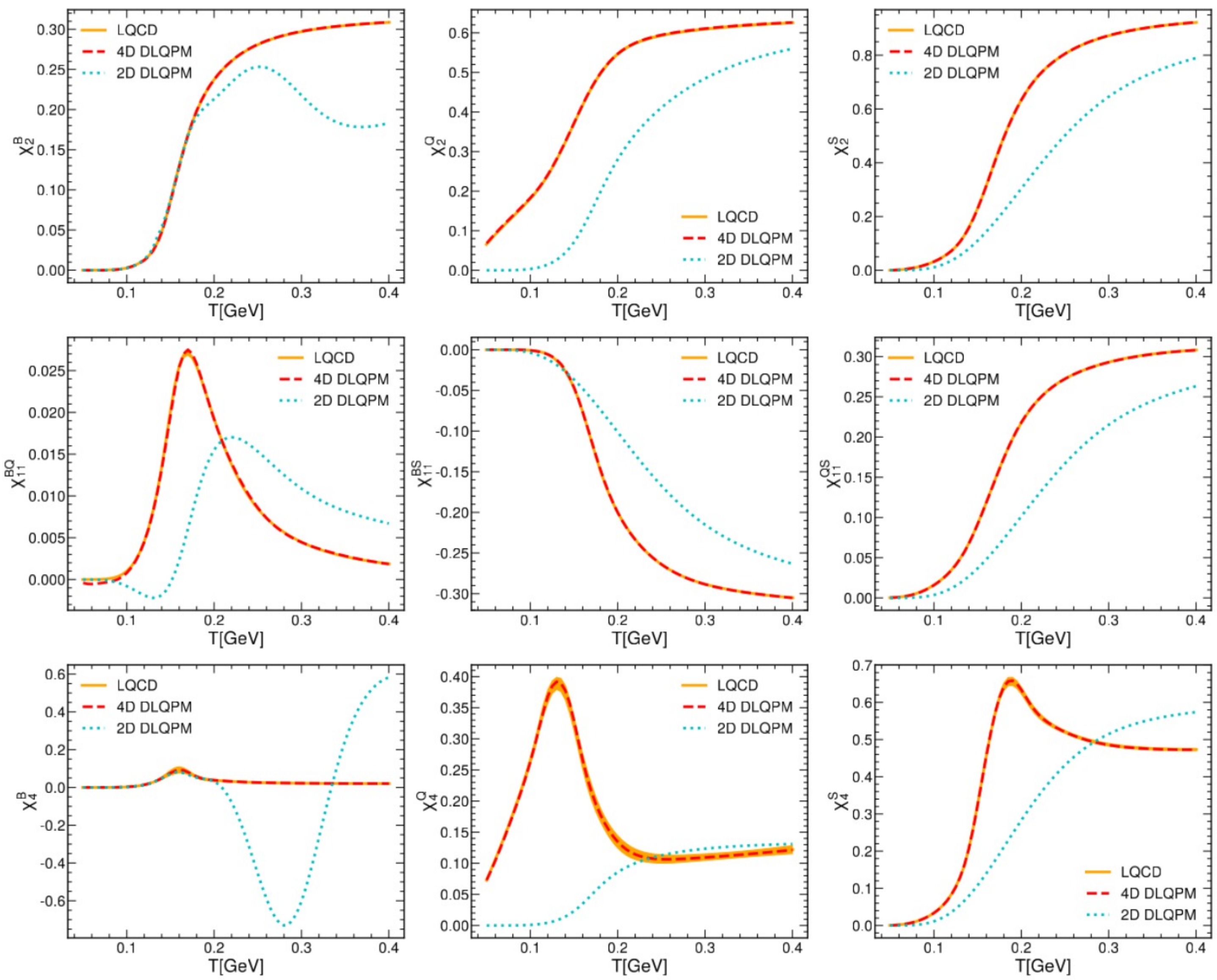
Shear Viscosity



$$\mathcal{L}(\theta_1, \theta_2, \theta_3) = |s_{DNN} - s_{LQCD}| + \left|\frac{\Delta_{DNN} - \Delta_{LQCD}}{T}\right| + |\chi^{B,Q,S}_{i,j,k,DNN} - \chi^{B,Q,S}_{i,j,k,LQCD}| + |n_{B,Q,S}| + | + \mathcal{L}_{MC}$$

FPL, Long-Gang Pang & Guang-You Qin, in preparation

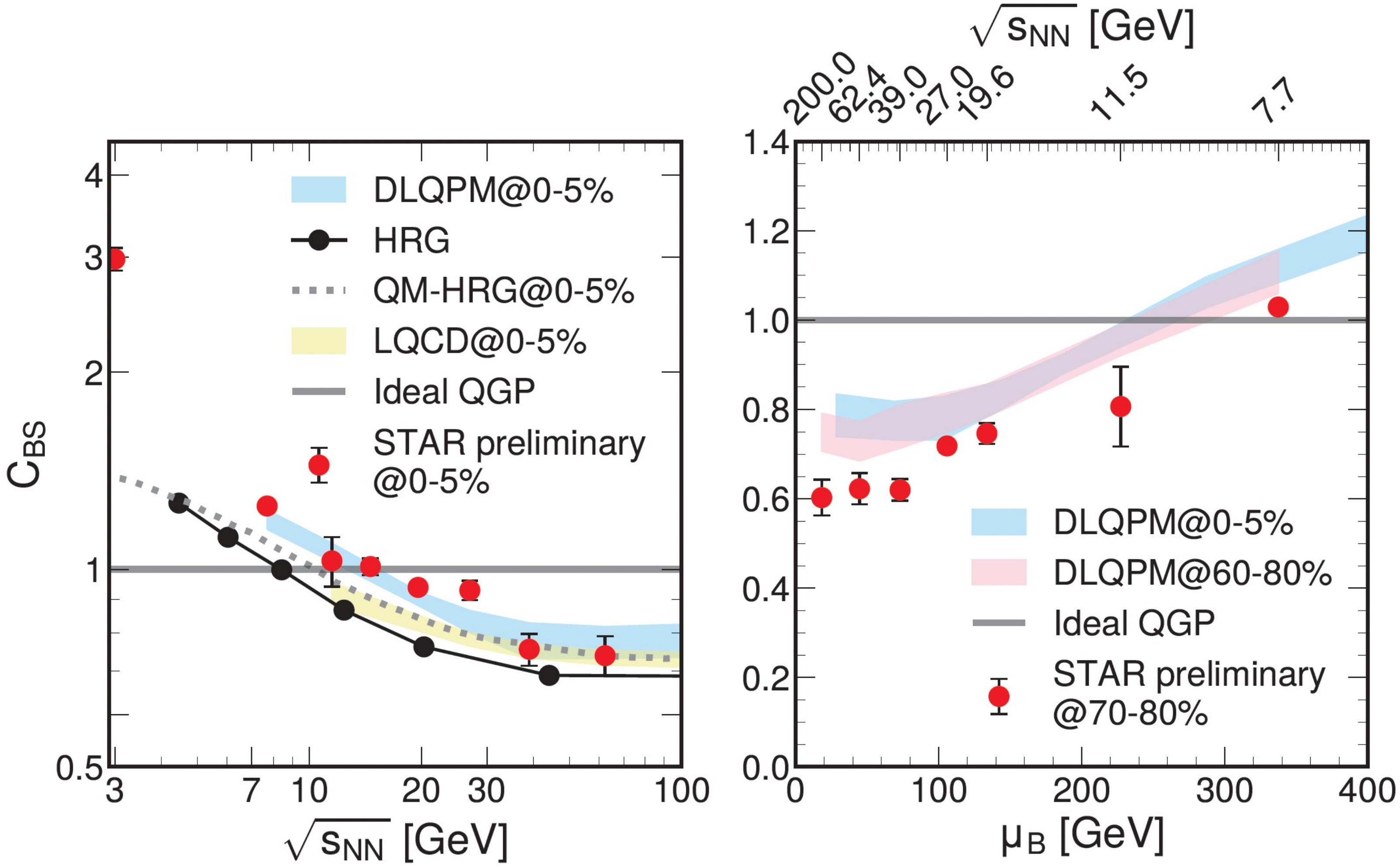
Result



- 2D DLQPM: T and μ_B-dependent masses.
- 4D DLQPM: T and $\mu_{B,Q,S}$-dependent masses.

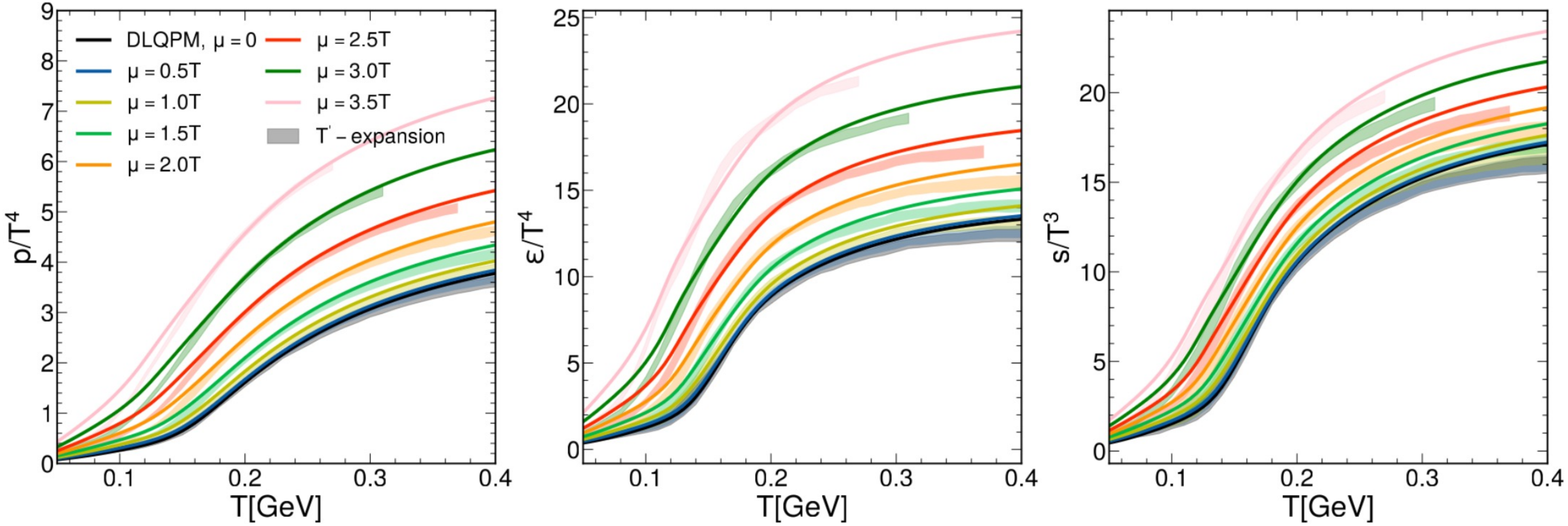
The 4D DLQPM calculations demonstrate consistency with LQCD predictions.

Result



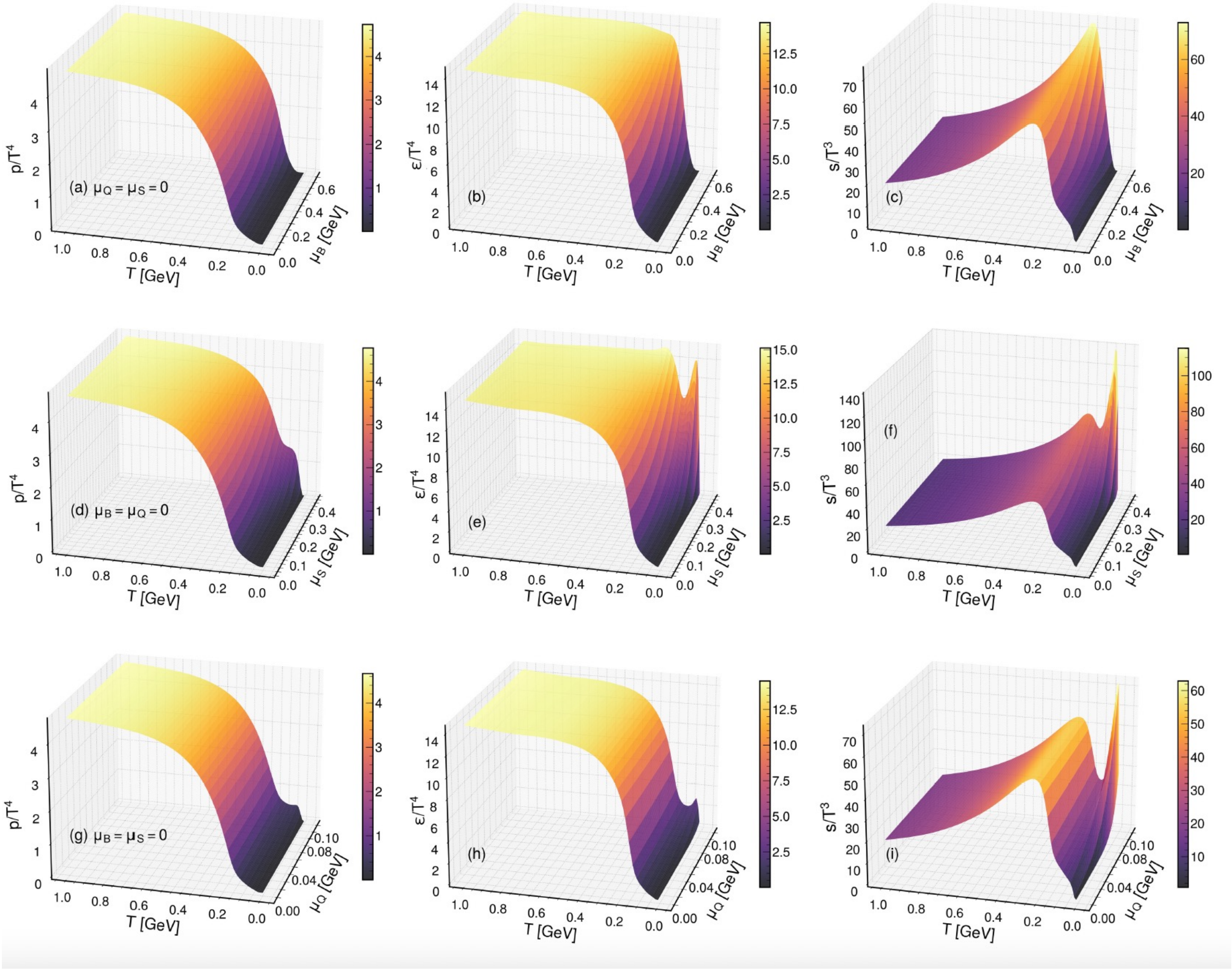
- Our results show agreement with the preliminary STAR data in the energy range of 7.7–100 GeV at 0–5% centrality.
- For large centrality, the DLQPM calculation captures the trend observed in the STAR experimental data but yields values approximately 10% higher especially at low μ_B.

Result



The DLQPM-calculated QCD EoS demonstrates agreement with the T'-expansion results in both temperature-dependent behavior and numerical values.

Result



The outputs of DLQPM can serve as inputs for relativistic hydrodynamic evolution.

Summary

➢ ML/DL methods can be used for inverse problems in NP:

Reconstructing the QCD equation of state

- We use three neural networks to represent the quasi-particles masses can well reproduce the lattice QCD EoS at zero chemical potential.

- We can calculate the entropy density at finite baryon chemical potentials, which is consisted with Lattice QCD result using Taylor expansions.

- The QCD equation of state at finite chemical potential can be used in relativistic hydrodynamics simulations to study the QCD matter produced in the BESII region.

Thank you for your attention!