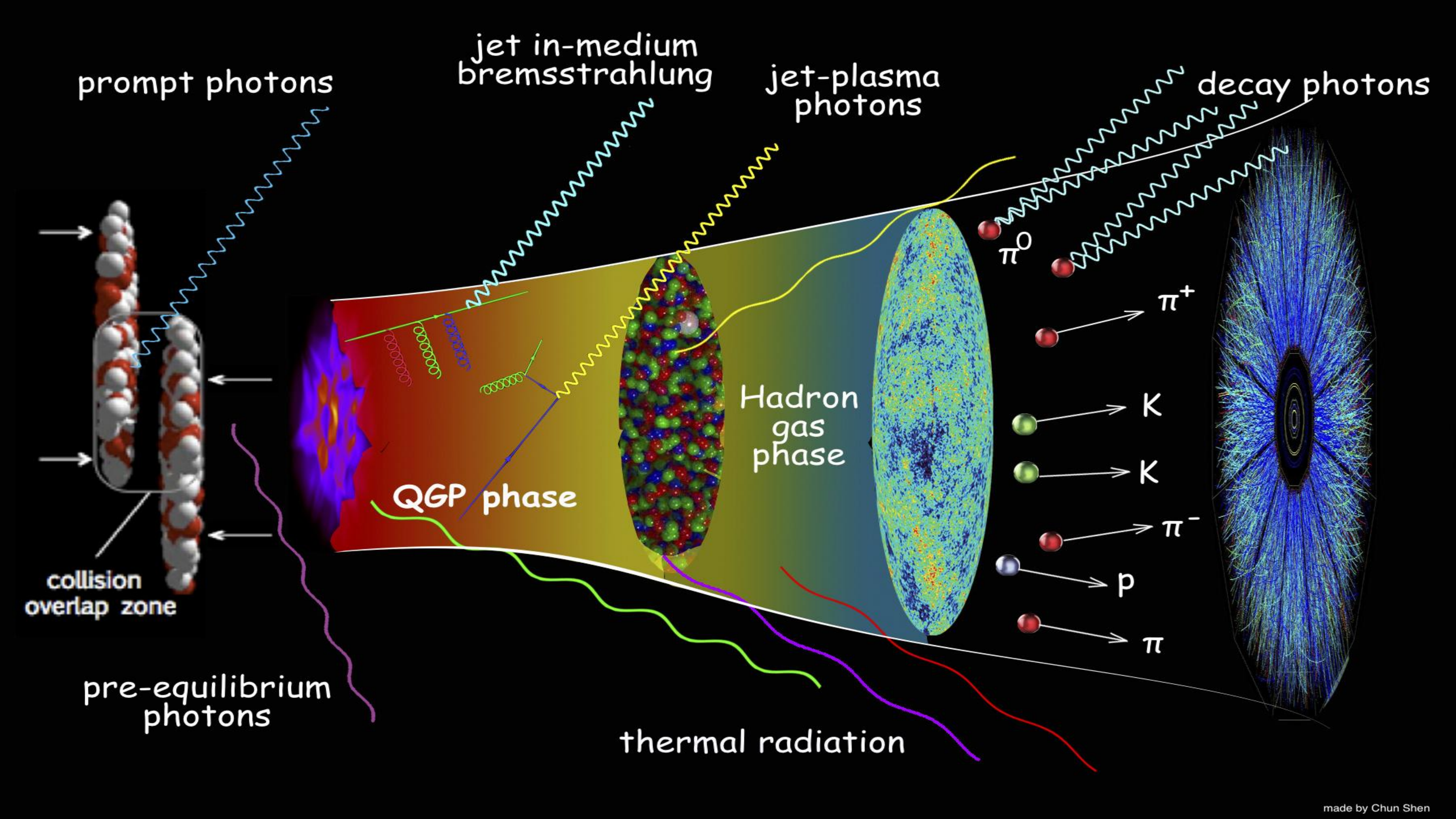
Deep Learning QCD Equation of State

Long-Gang Pang 庞龙刚

Central China Normal University

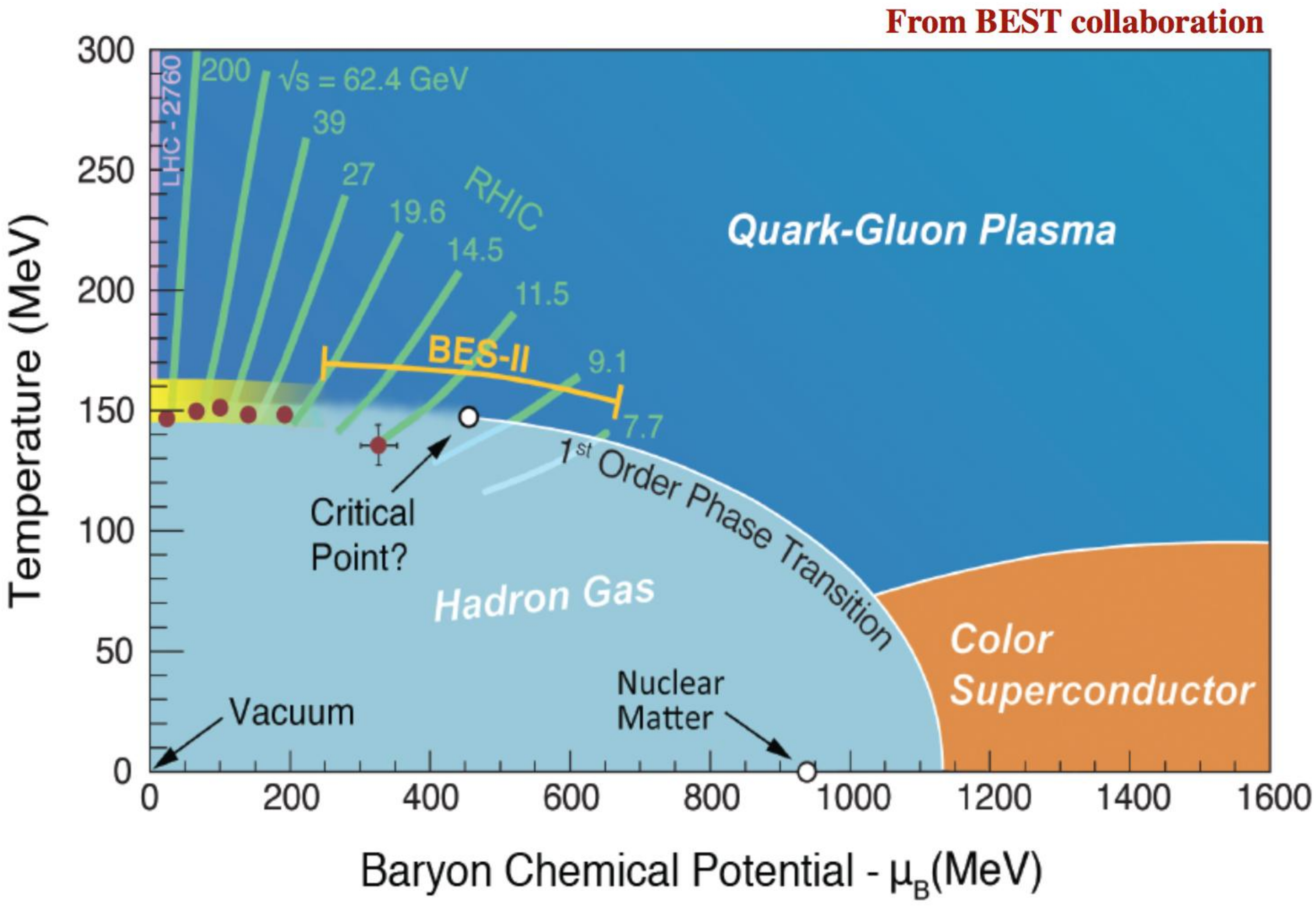
The 2nd "AI+HEP in East Asia" workshop,

KEK，2026-01-22



prompt photons
jet in-medium bremsstrahlung
jet-plasma photons
decay photons
collision overlap zone
QGP phase
Hadron gas phase
π^0
π^+
K
K
π^-
p
π
pre-equilibrium photons
thermal radiation
made by Chun Shen

QCD matter under extreme conditions



High T, zero/near-zero μB region

- Smooth crossover from QGP to HRG.
- Early universe; experiments at the LHC (Large Hadron Collider) at CERN, or top RHIC energy HIC.

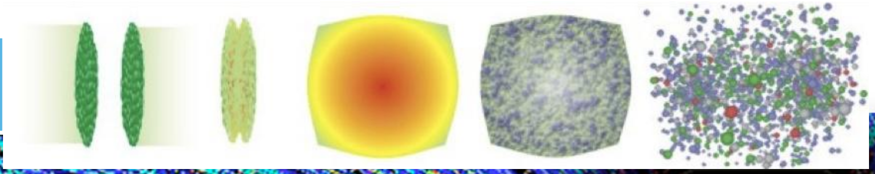
Intermediate T, high μB region

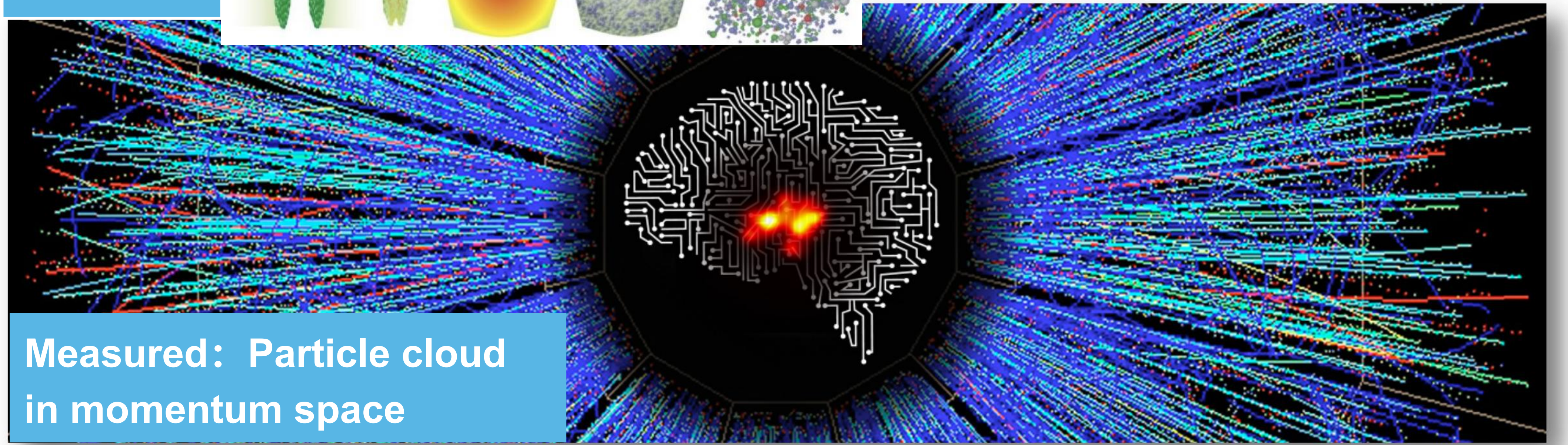
- First-order phase transition (with an endpoint: the critical point).
- Medium to low-energy heavy-ion collisions (e.g., the RHIC Beam Energy Scan program); future facilities like HIAF, CBM, FAIR, and NICA.

HIC： Typical inverse problem

QGP Lifetime： $10^{-23}s$

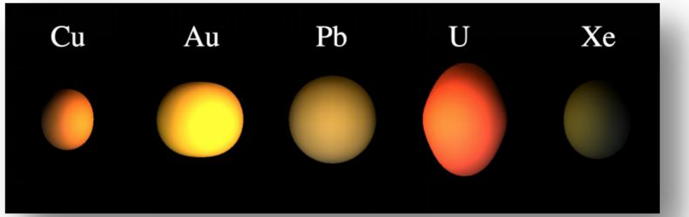
Forward



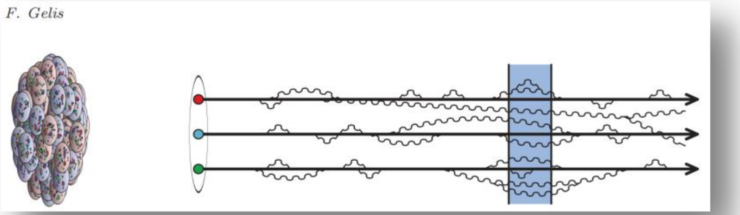


Measured： Particle cloud in momentum space

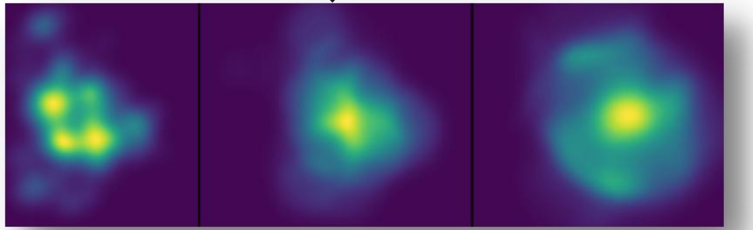
Inverse



(1) Nuclear Structure

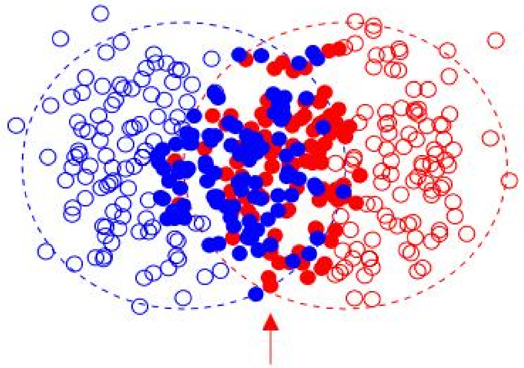


(2) Initial Parton Distribution

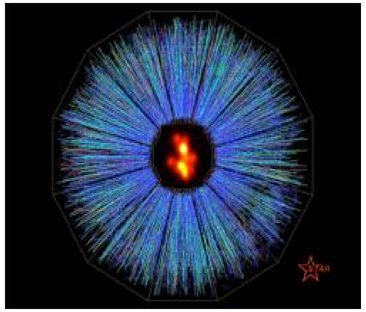


(3) QGP properties and EoS

One theoretical model: relativistic hydro



$$\nabla_\mu T^{\mu\nu} = 0$$



$$T^{\mu\nu} = (\varepsilon + P + \Pi)u^\mu u^\nu - (P + \Pi)g^{\mu\nu} + \pi^{\mu\nu}$$

Initial condition

EoS Bulk Viscosity Shear Viscosity

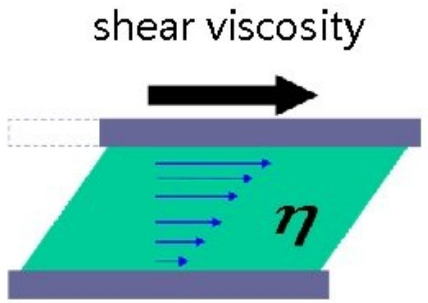
CLVisc:

1. CCNU-LBNL Viscous Hydro, CCNU = Central China Normal University
2. A 3+1D viscous hydro parallized on GPU using OpenCL

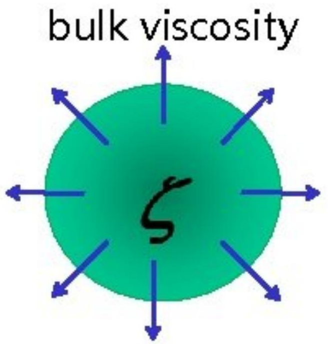
Purpose: Describe the non-equilibrium space-time evolution of hot QCD matter
Feature: **100 times faster** than using a single core CPU.

L.G. Pang, Q. Wang and X. N. Wang, PRC 86 (2012) 024911
L.G. Pang, B.W. Xiao, Y. Hatta, X.N.Wang, PRD 2015
L.G. Pang，H.Petersen, XN Wang, PRC97(2018)no.6,064918

Dissipation



resistance to flow gradients



resistance to expansion

credit：HuiChao Song

First order：acausal

$$\Pi = -\zeta\theta - \tau_\Pi \left[u^\lambda \nabla_\lambda \Pi + \frac{4}{3}\Pi\theta \right]$$

$$\pi^{\mu\nu} = \eta_v \sigma^{\mu\nu} - \tau_\pi \left[\Delta^\mu_\alpha \Delta^\nu_\beta u^\lambda \nabla_\lambda \pi^{\alpha\beta} + \frac{4}{3}\pi^{\mu\nu}\theta \right]$$
$$-\lambda_1 \pi_\lambda^{\langle\mu}\pi^{\nu\rangle\lambda} - \lambda_2 \pi_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda} - \lambda_3 \Omega_\lambda^{\langle\mu}\Omega^{\nu\rangle\lambda},$$

Expansion rate: $\theta \equiv \nabla_\mu u^\mu,$

Shear viscous tensor: $\sigma^{\mu\nu} \equiv 2\nabla^{\langle\mu}u^{\nu\rangle} \equiv 2\Delta^{\mu\nu\alpha\beta}\nabla_\alpha u_\beta,$

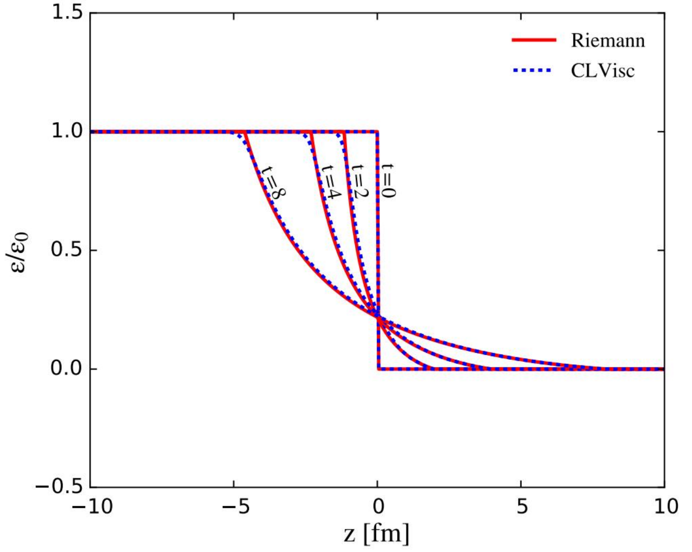
Vorticity tensor: $\Omega^{\mu\nu} \equiv \frac{1}{2}\Delta^{\mu\alpha}\Delta^{\nu\beta}(\nabla_\alpha u_\beta - \nabla_\beta u_\alpha),$

Double projection operator: $\Delta^{\mu\nu\alpha\beta} \equiv \frac{1}{2}(\Delta^{\mu\alpha}\Delta^{\nu\beta} + \Delta^{\mu\beta}\Delta^{\nu\alpha}) - \frac{1}{3}\Delta^{\mu\nu}\Delta^{\alpha\beta},$

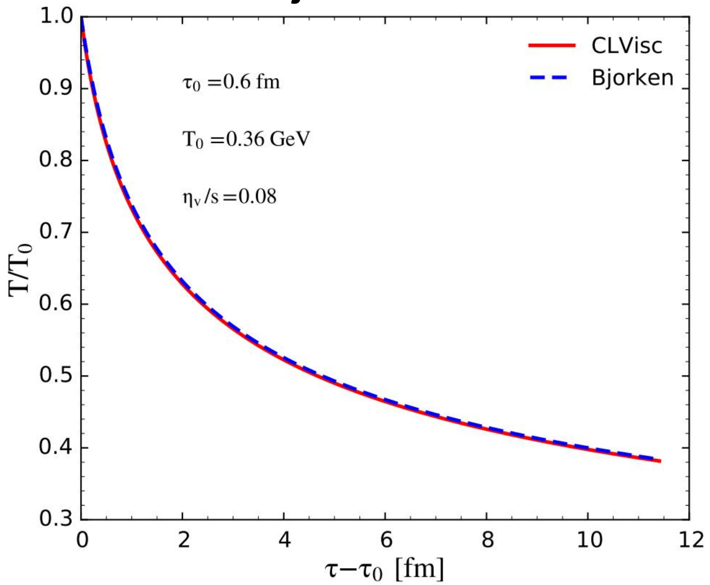
Projection operator: $\Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu$

Numerical vs Analytical solutions

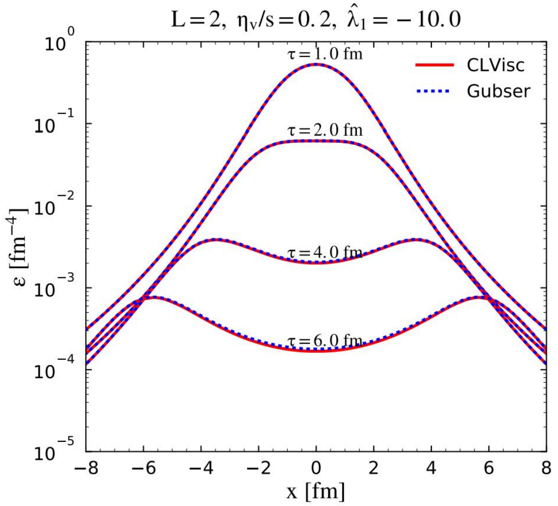
Riemann Solution

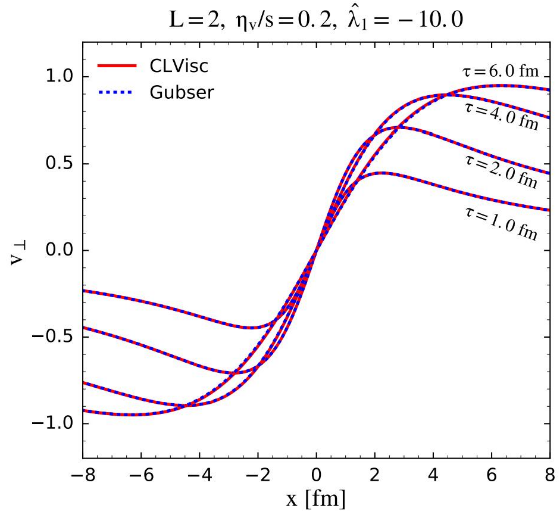


Bjorken Solution



Viscous Gubser Sol.





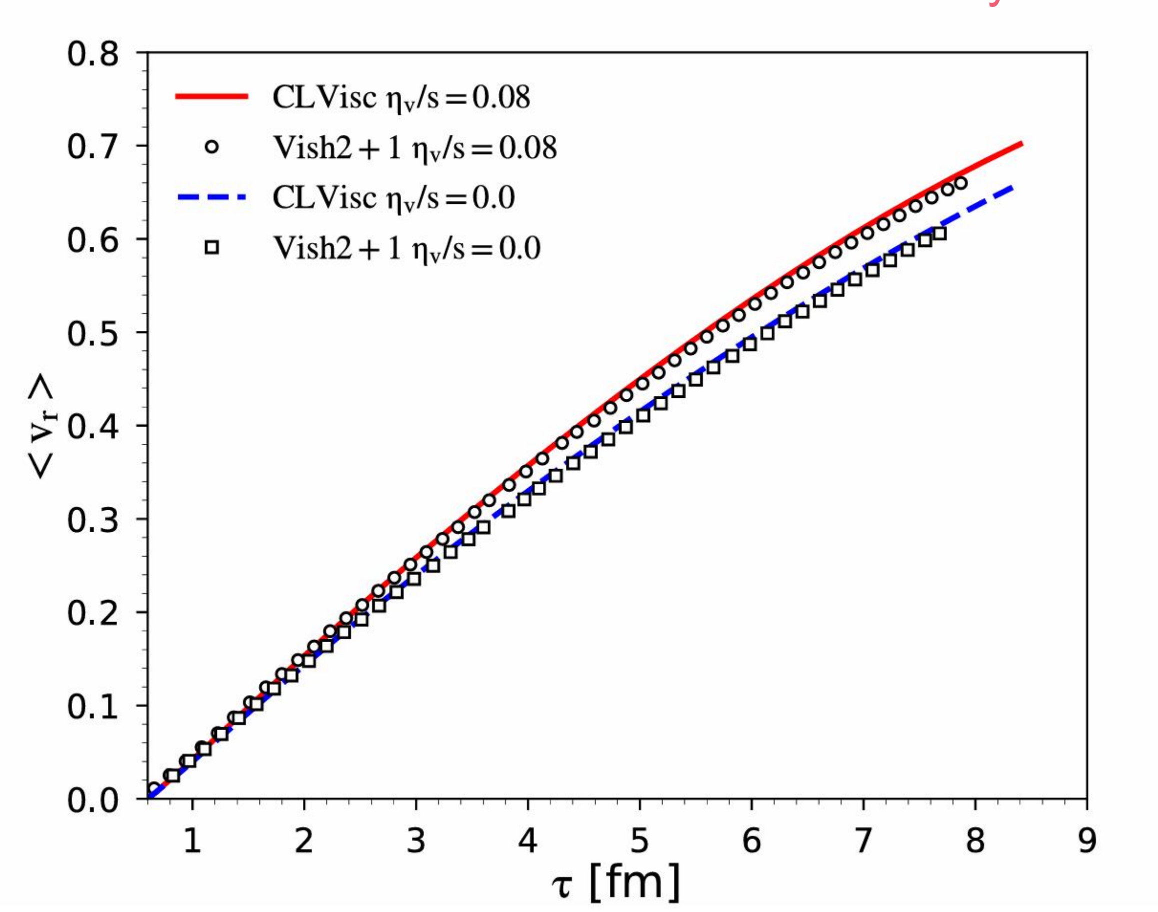
Constructing Analytical Solutions:

- **Select a specific spacetime metric.**
- **Make symmetry assumptions:** Assume a homogeneous energy density distribution.
- **Obtain the flow velocity:** This yields u^μ = (1, 0, 0, 0) in the chosen coordinates.
- **Transform back to flat spacetime:** This results in a non-zero flow velocity profile.

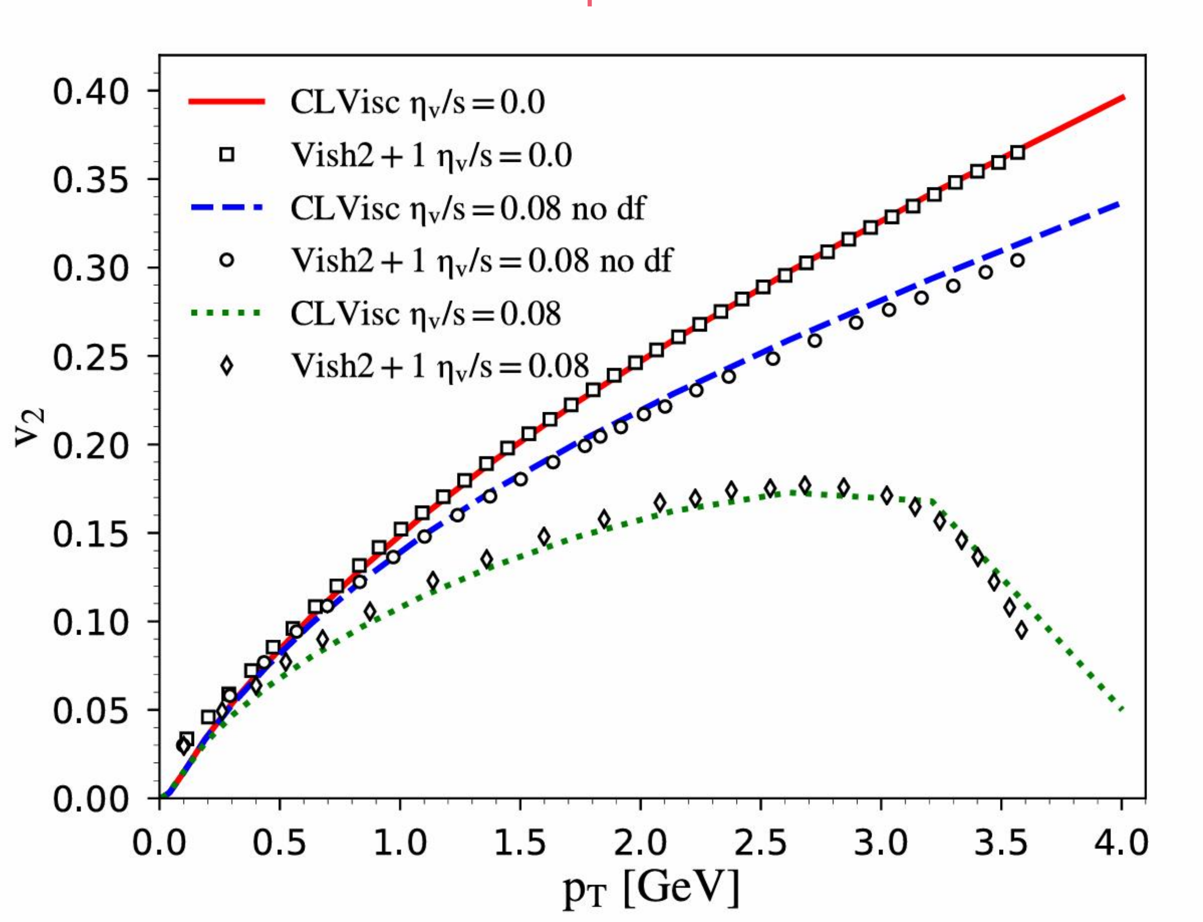
L.G. Pang， H.Petersen, XN Wang, PRC97(2018)no.6,064918

Relativistic fluid from different groups

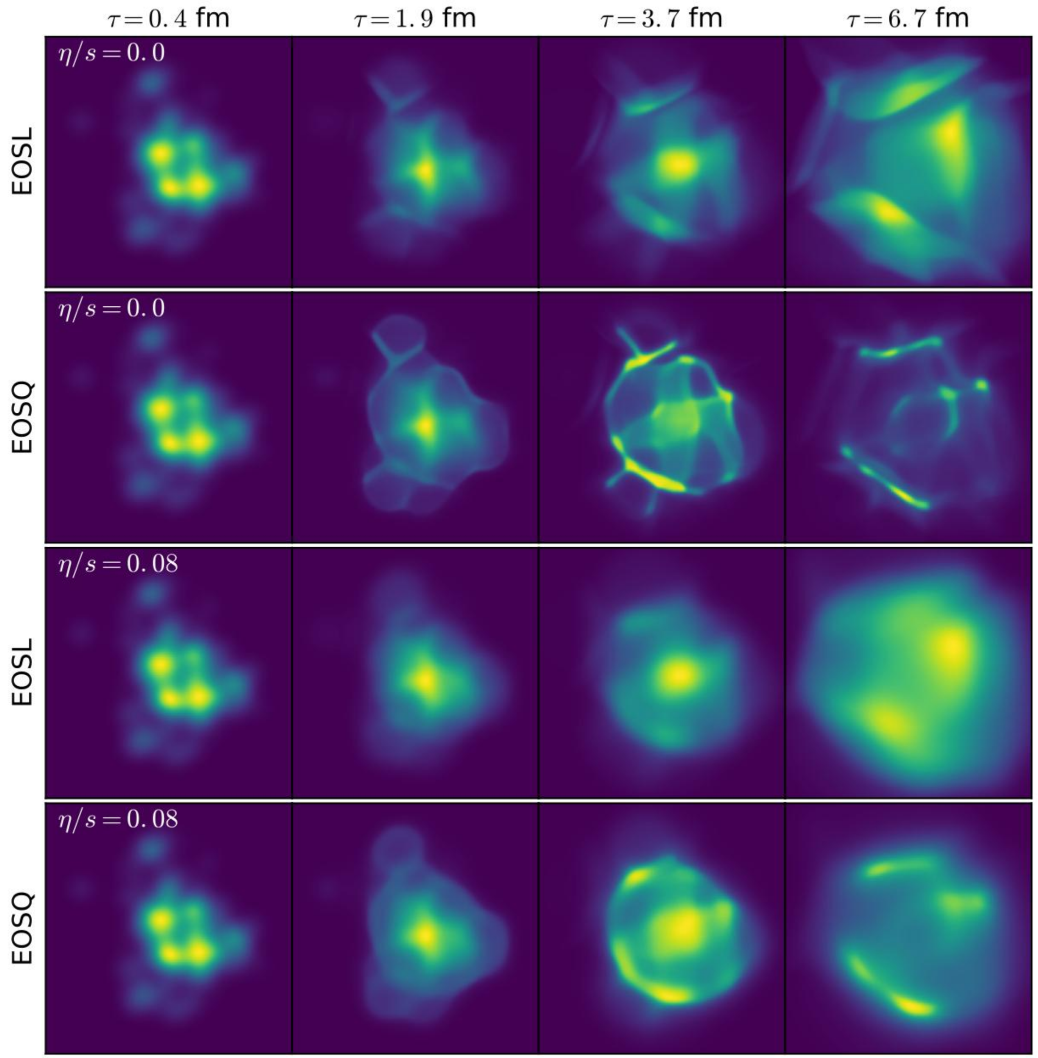
Mean transverse velocity



Elliptic flow



CLVisc for different EoS and η/s



η/s = 0 (shear viscosity over entropy density)
Lattice QCD EoS (smooth cross over)

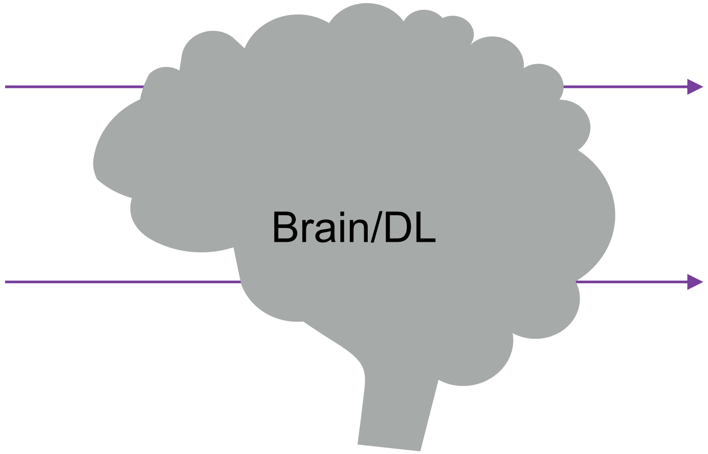
η/s = 0 (ideal hydro)
First order phase transition

η/s = 0.08 (viscous hydro)
Lattice QCD EoS (smooth cross over)

η/s = 0.08
First order phase transition

It is unknown **whether the information of EoS survives the complex dynamical evolution of HICs** and exists in each single event of the final state output.

DL for inverse/variational problems in HICs



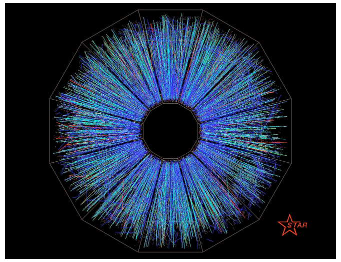
Dog

Cat

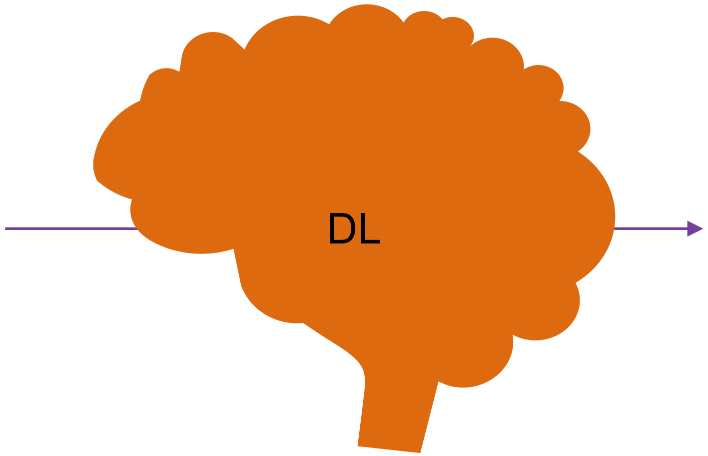
Human brains are not optimized for processing high-dimensional scientific data. Deep neural network can be trained:

(i) to identify optimal feature combinations

(ii) to represent variational functions



$\rho(p_T, \Phi)$

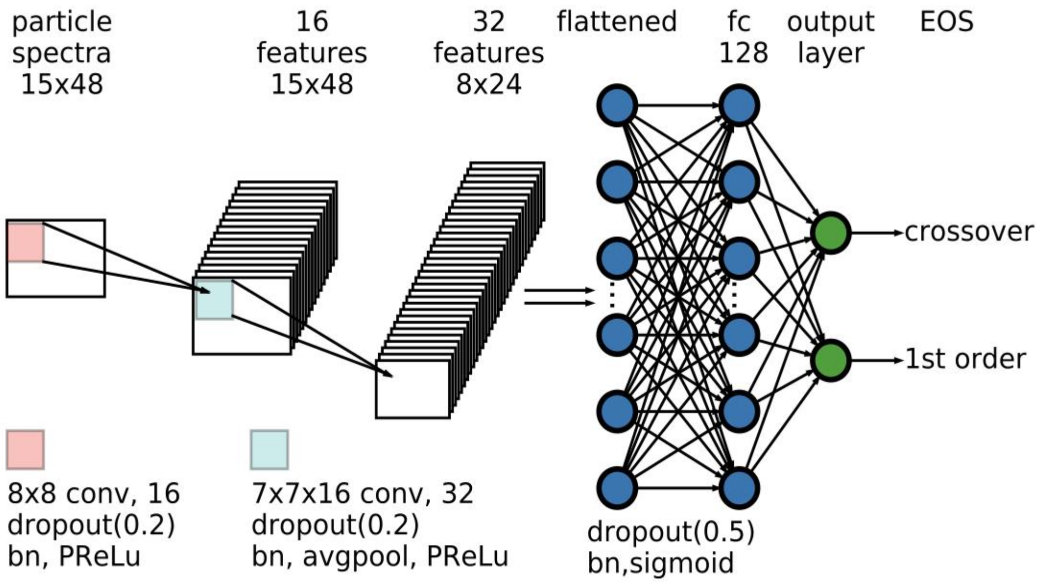


Lattce QCD EoS (crossover)

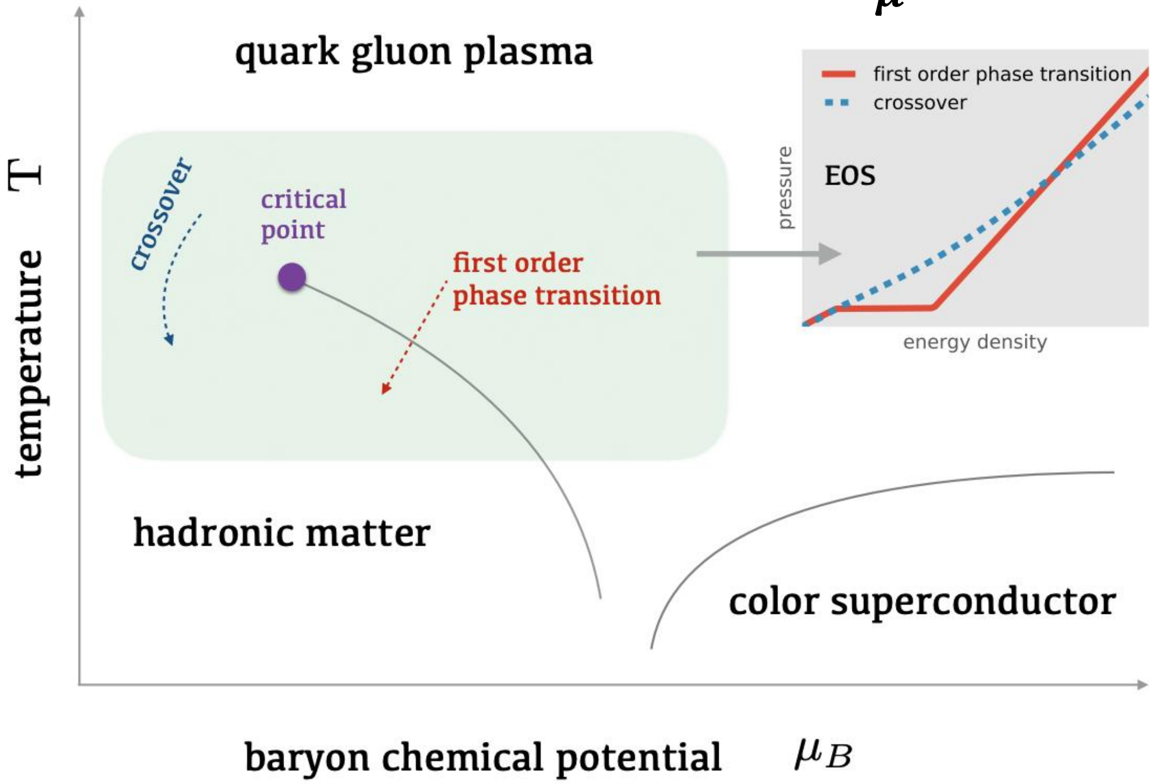
EOSQ (First order phase transition)

EoS for different phase transition types

$$\nabla_\mu T^{\mu\nu} = 0$$



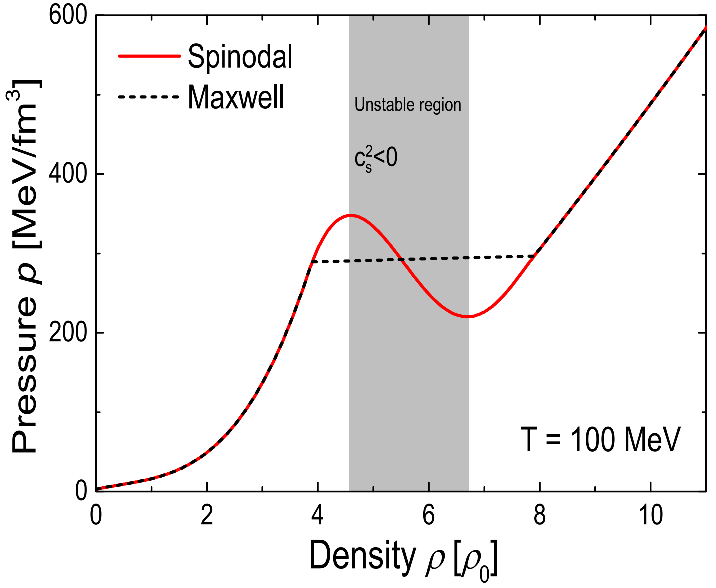
$$l(\theta) = \underbrace{-\frac{1}{N}\sum_{i=1}^{N}[y_i \log \hat{y}_i + (1-y_i)\log(1-\hat{y}_i)]}_{\text{cross entropy loss}} + \underbrace{\lambda||\theta||_2^2}_{\text{L2 regularization}}$$

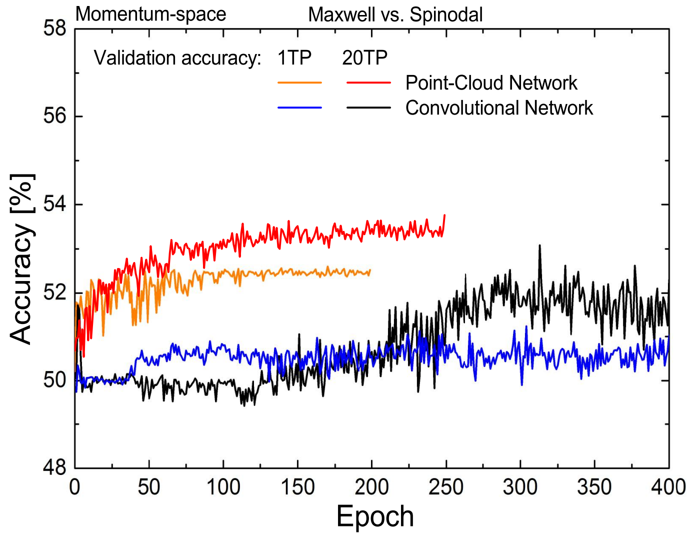


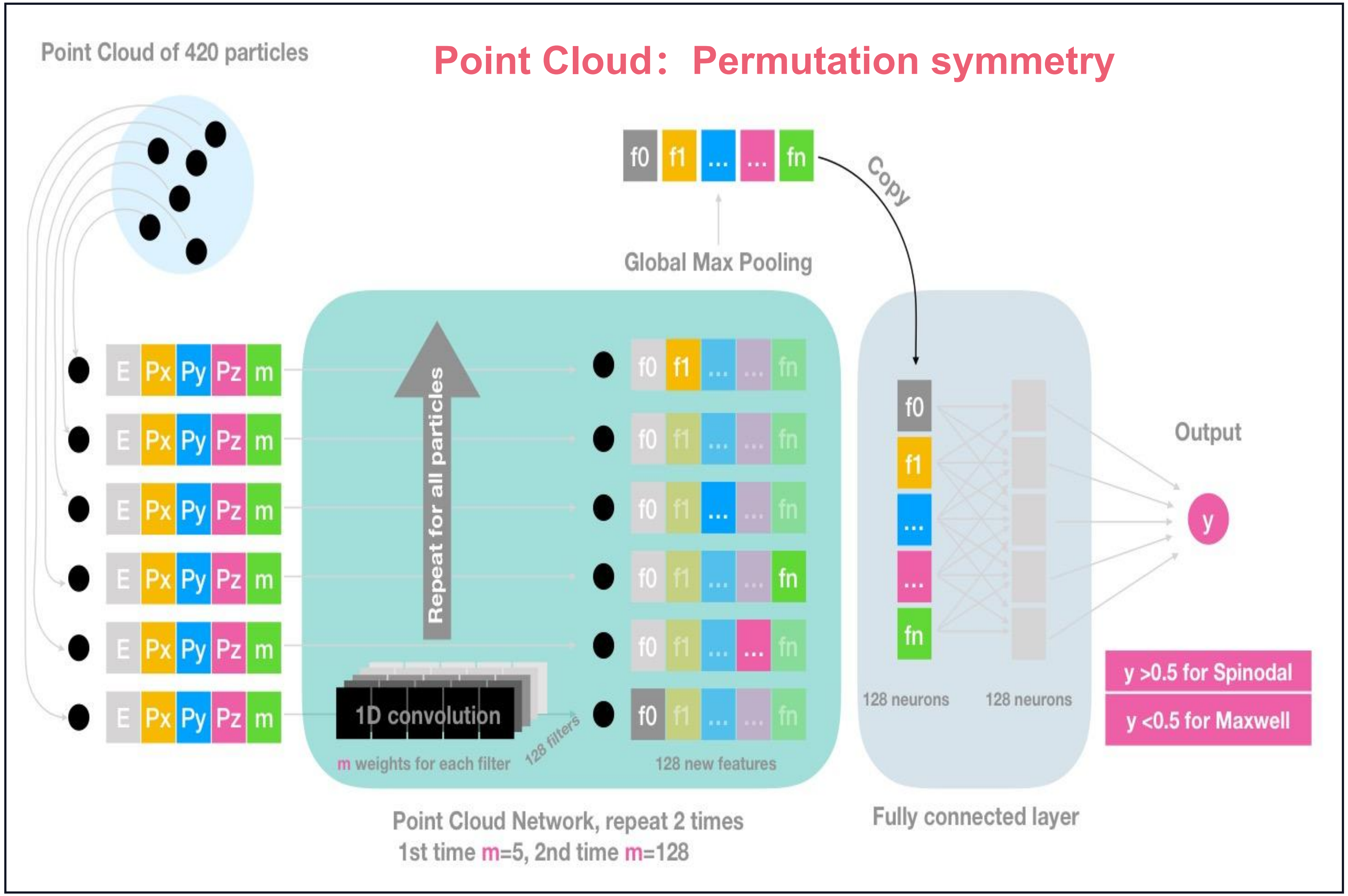
DL helps to decode the information of QCD phase transition in the QCD EoS (>93% accuracy).

Nature Communications 2018, **LG. Pang**, K.Zhou, N.Su, H.Petersen, H. Stoecker, XN. Wang.

Spinodal vs Maxwell 1st order phase transition

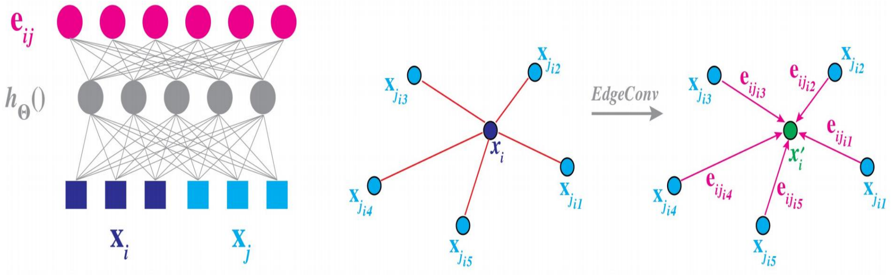


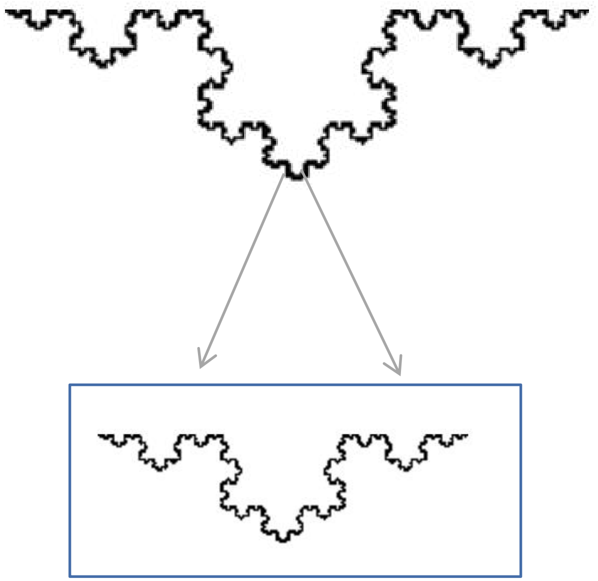




J. Steinheimer, L.G. Pang, K. Zhou, V. Koch and J. Randrup, JHEP 12 (2019) 122

Looking for self similarity in momentum space

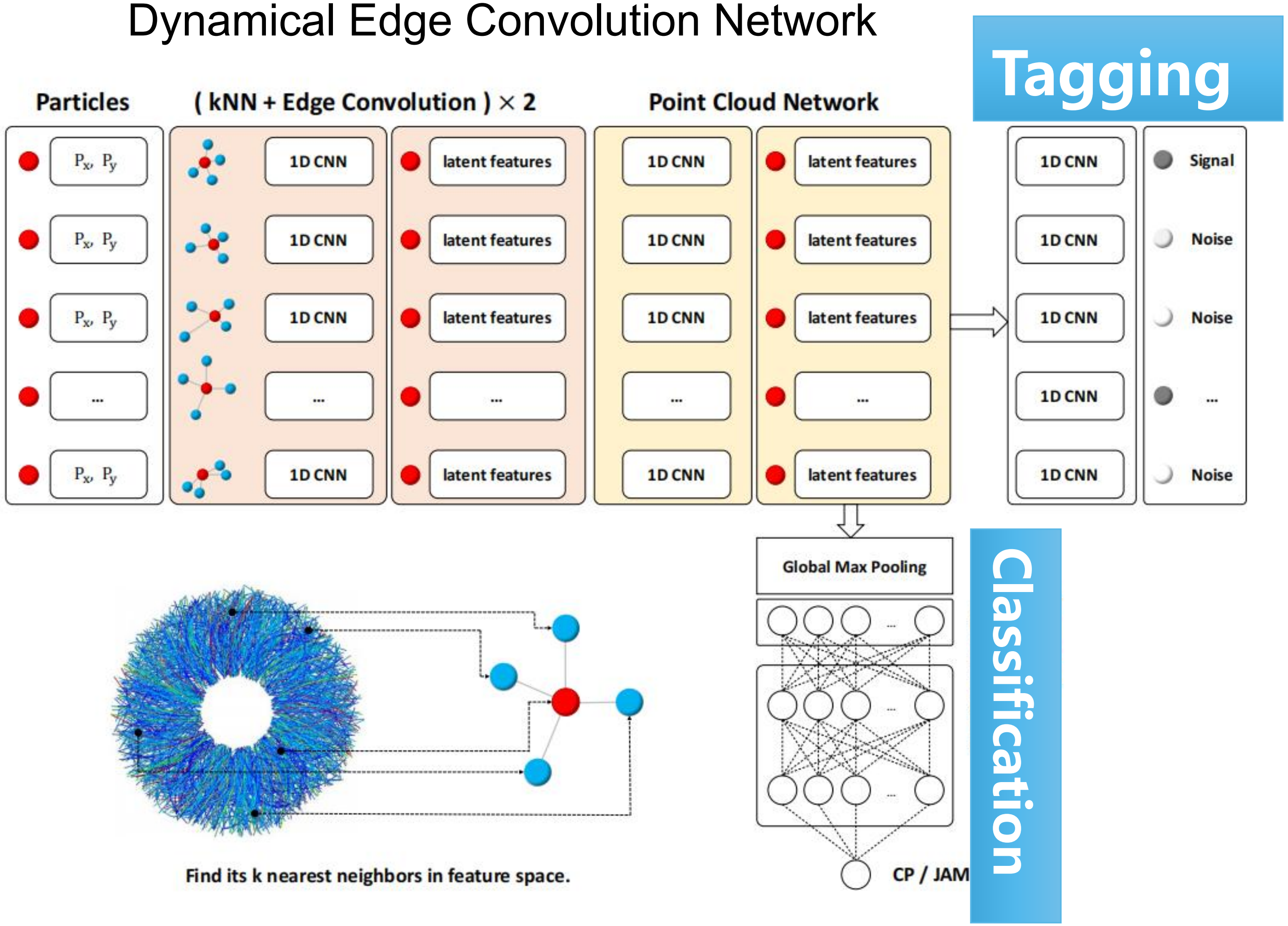




Self similarity, scaling invariance

Capture local and long range correlations

Dynamical Edge Convolution Network



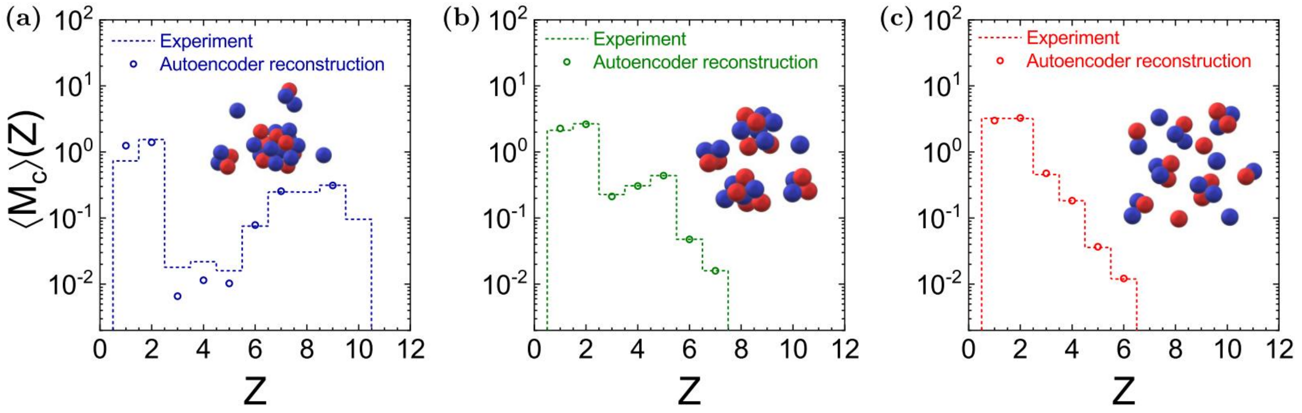
PLB 827(2022) 137001, Y.-G. Huang, L.-G. Pang, X.F. Luo and X.-N. Wang

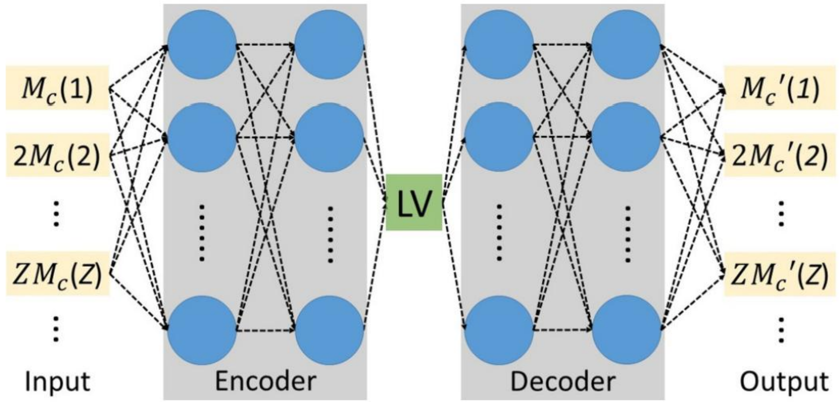
Auto Encoder for order parameter

PHYSICAL REVIEW RESEARCH **2**, 043202 (2020)

Nuclear liquid-gas phase transition with machine learning

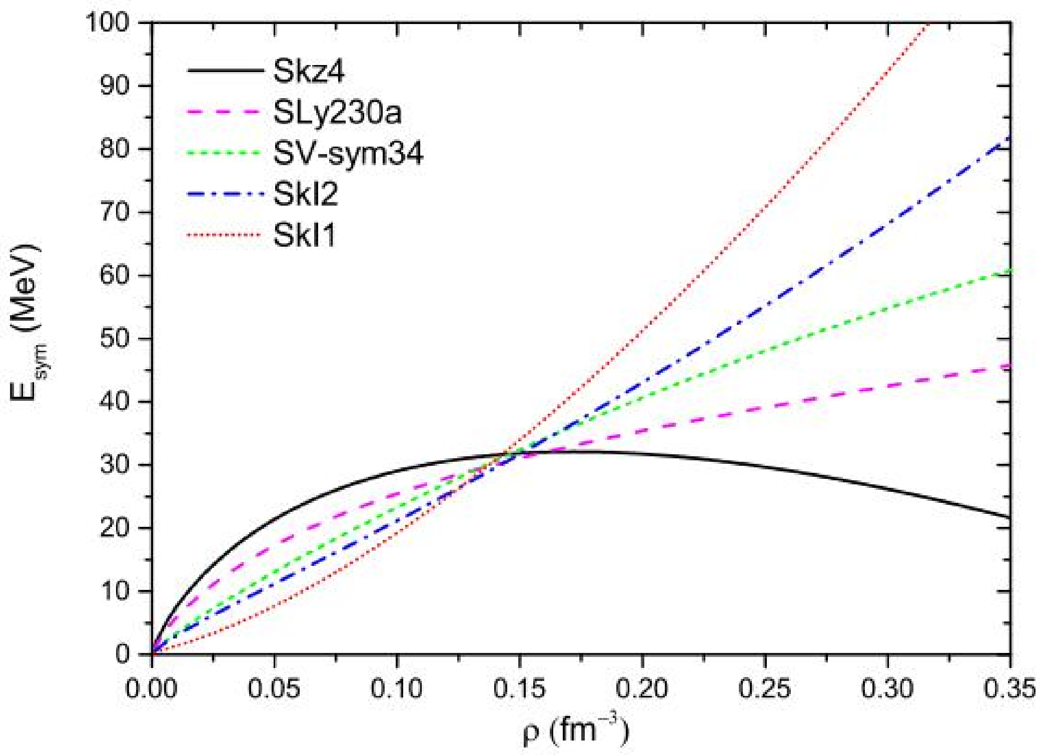
Rui Wang,[1,2,*] Yu-Gang Ma,[1,2,†] R. Wada,[3] Lie-Wen Chen,[4] Wan-Bing He,[1] Huan-Ling Liu,[2] and Kai-Jia Sun[3,5]



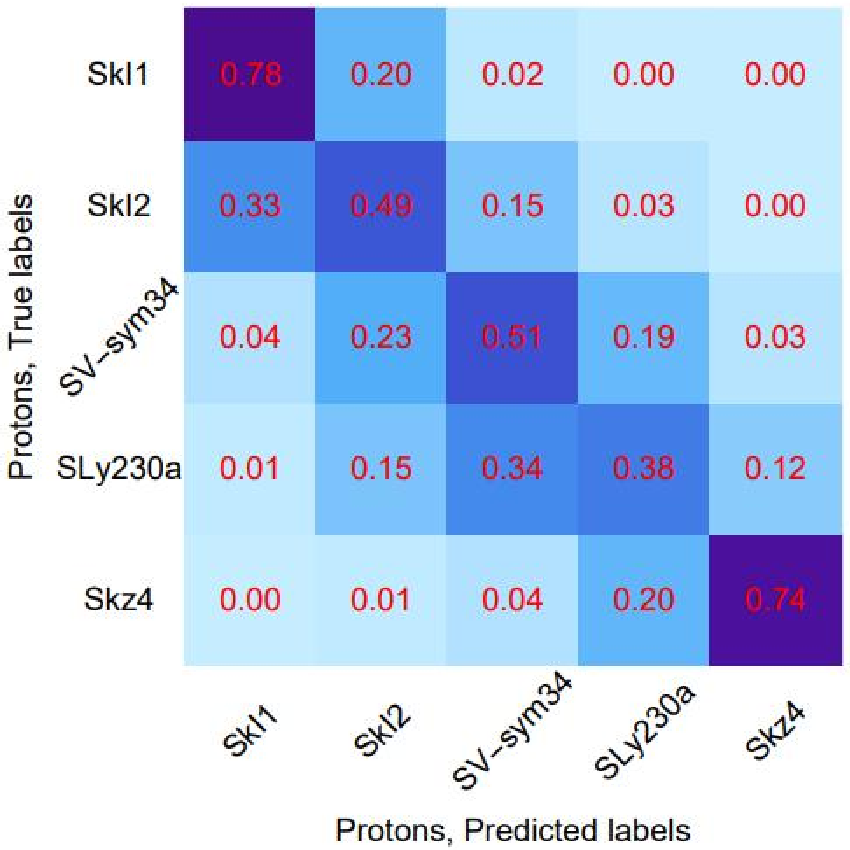


Nuclear EoS at high density region

Skyrme potential + IMQMD



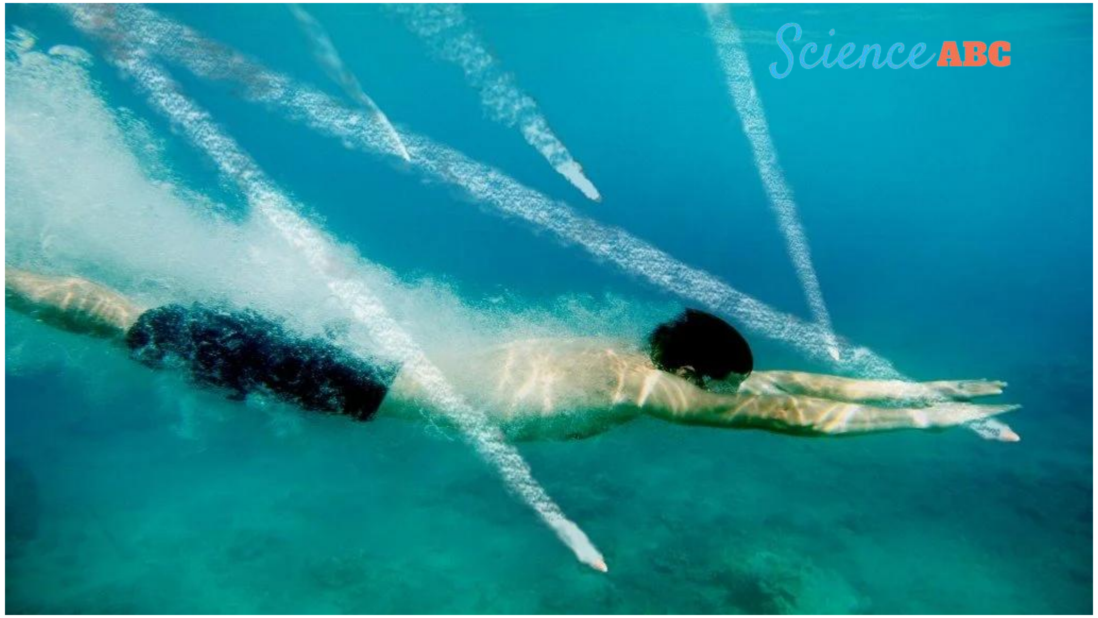
off-diagonal = misclassified



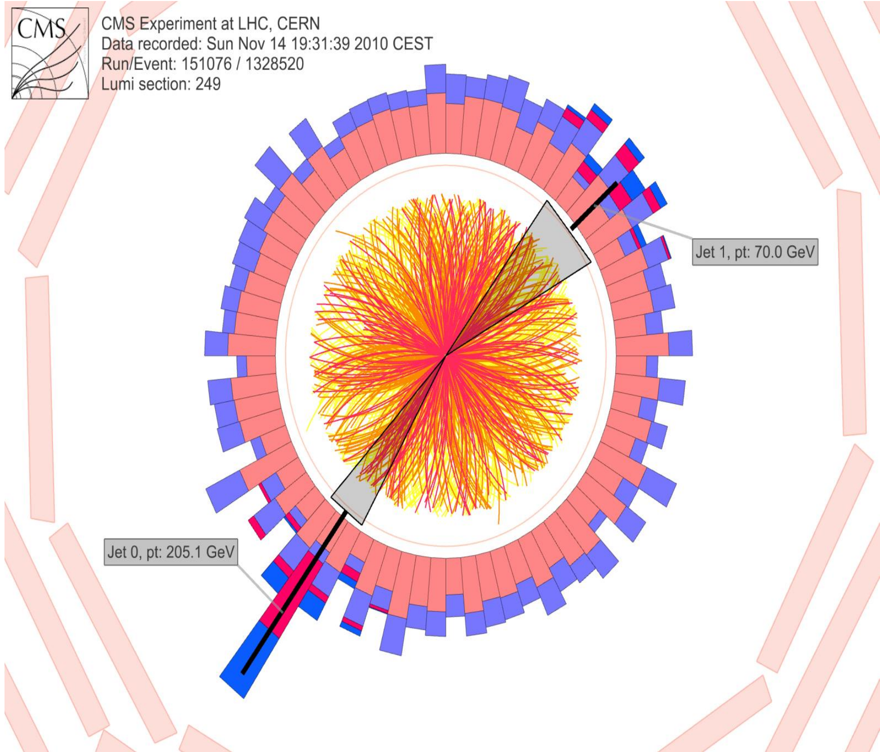
PLB 822 (2021) 136669, Y.J Wang, F.P. Li, Q.F. Li, H.L. L¨u, and K. Zhou

Jet eloss and medium response

Can Being Underwater Protect You From Bullets?

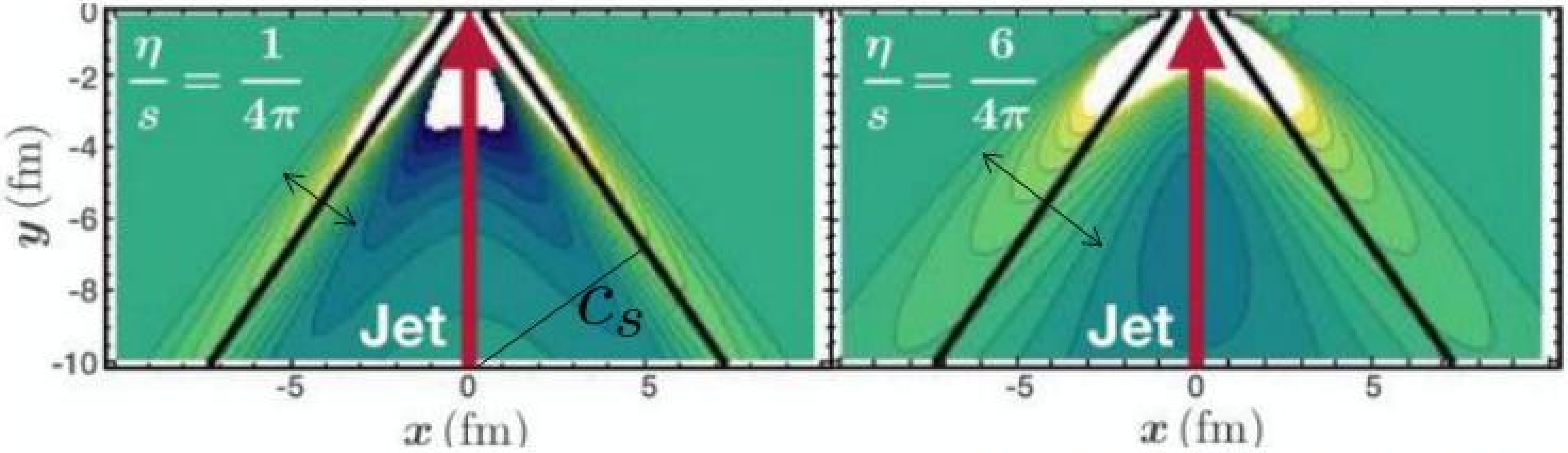


If the bullet is shot from an angle of 30 Degrees, then being underwater in the range of 3-5 feet (0.9-1.5 meters) can ensure safety from most guns.



Jet quenching in hot QGP

The nuclear EoS and Mach Cone



R.B.Neufeld. PRC79,054909(09')

Nuclear EoS： $c_s^2 = \frac{dP}{d\epsilon} = \sin^2\theta$

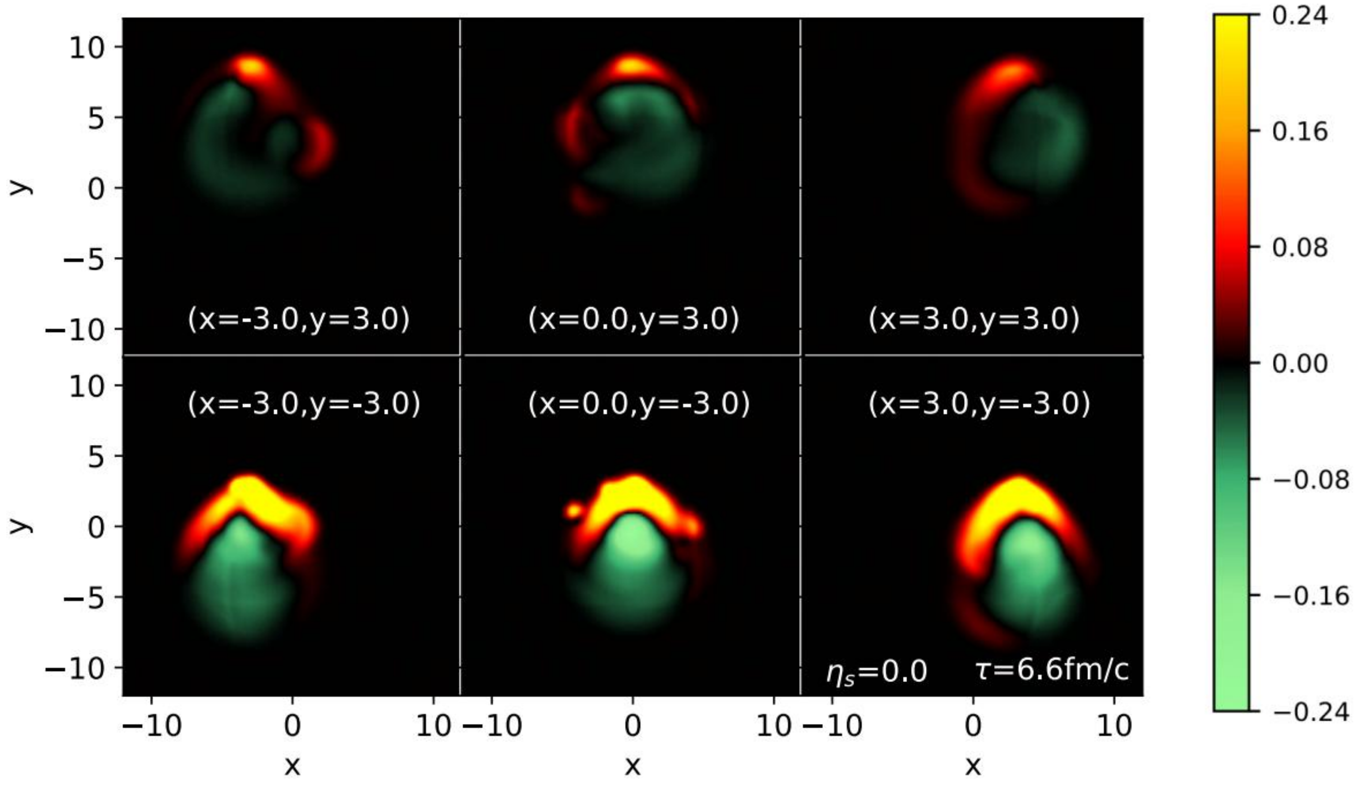
Shear Viscosity： width of the shock wave

Medium response for nuclear EoS

$$p\partial f(p) = -C(p) \quad (p\cdot u > p^0_{cut})$$

$$\partial_\mu T^{\mu\nu}(x) = j^\nu(x)$$

$$j^\nu = \sum_i p_i^\nu \delta^{(4)}(x - x_i)\theta(p^0_{cut} - p\cdot u)$$



LBT: YY He, T Luo, XN Wang, Y Zhu,
PRC 91 (2015) 054908, PRC 97 (2018) 1, 019902

CLVisc:
LG Pang, Q Wang, XN Wang, PRC 86 (2012) 024911
LG Pang, H Petersen, XN Wang, PRC 97 (2018) 6, 064918
XY Wu, GY Qin, LG Pang, XN Wang,PRC 105 (2022) 3, 034909

CoLBT:

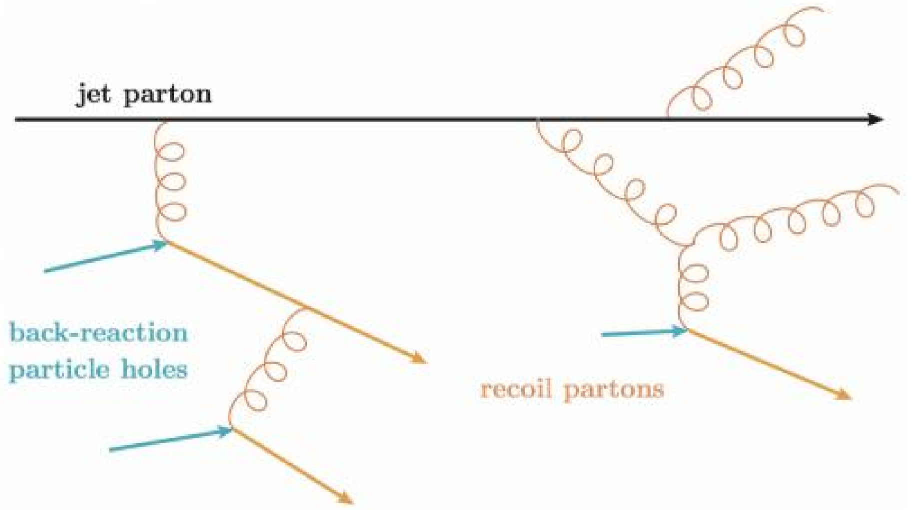
W Chen, T Luo, SS Cao, LG Pang, XN Wang,
PLB 777 (2018) 86-90

LBT: Linear Boltzmann Transport

$$p_1 \partial f_1 = -\int dp_2 dp_3 dp_4 (f_1 f_2 - f_3 f_4) \left|M_{12\to 34}\right|^2 (2\pi)^4 \delta^4 (\sum_i p^i) + inelastic$$

Medium-induced gluon(HT):

$$\frac{dN_g}{dz d^2 k_\perp dt} \approx \frac{2 C_A \alpha_s}{\pi k_\perp^4} P(z) \hat{q} (\hat{p} \cdot u) sin^2 \frac{k_\perp^2 (t - t_0)}{4z(1-z)E}$$

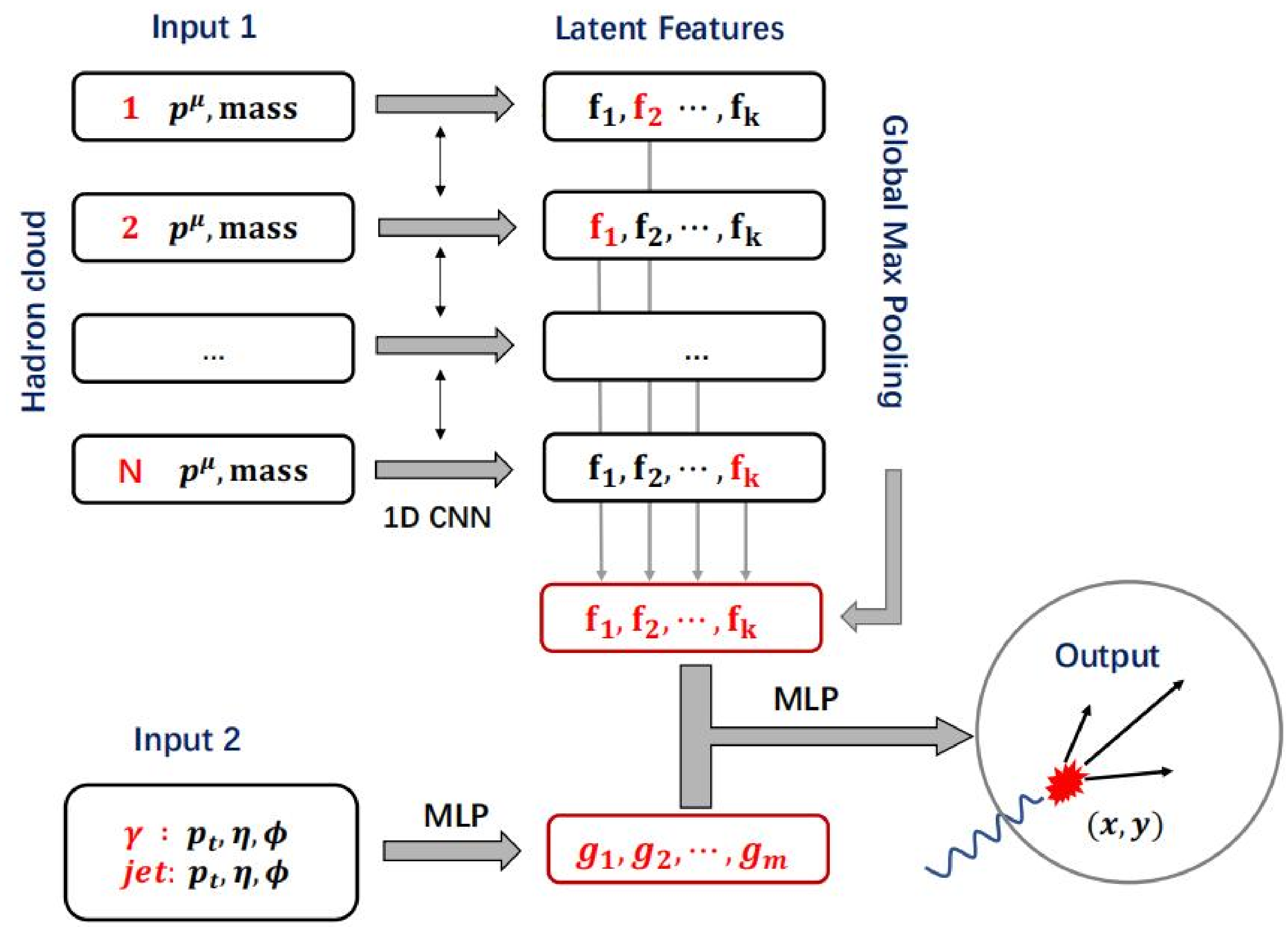


Tracked partons:

- Jet shower partons
- Thermal recoil partons
- Radiated gluons
- Negative partons(Back reaction induced by energy-momentum conservation)

YY He, T Luo, XN Wang, Y Zhu, PRC 91 (2015) 054908, PRC 97 (2018) 1, 019902

DL assisted jet tomography (gamma-jet)

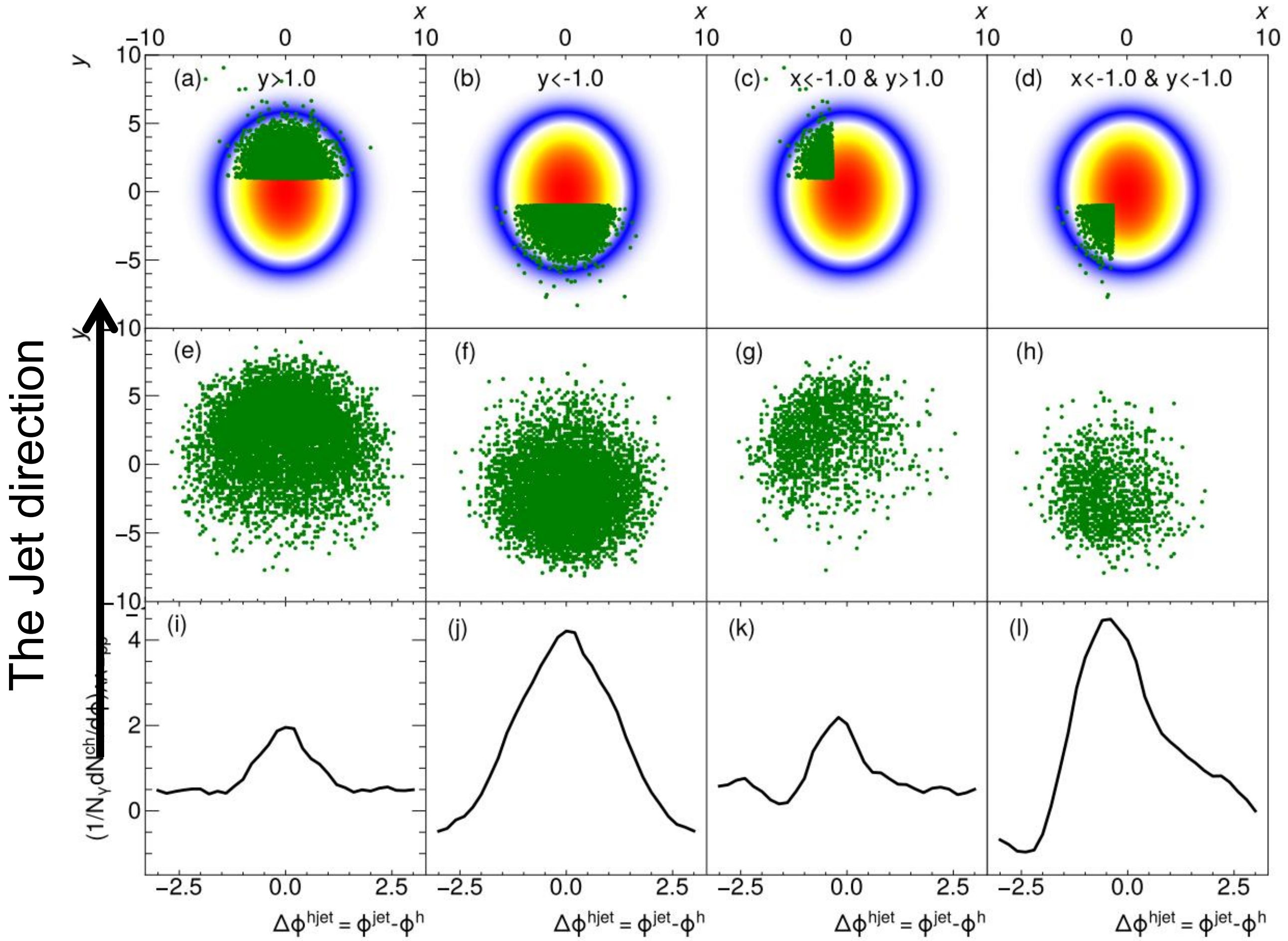


$ij \to kl$	$\lvert M\rvert^2_{ij\to kl}$	
$gg \to gg$	$\frac{9}{2}g_s^4\left(3 - \frac{ut}{s^2} - \frac{us}{t^2} - \frac{st}{u^2}\right)$	(A-1)
$gg \to q\bar{q}$	$\frac{3}{8}g_s^4\left(\frac{4}{9}\frac{t^2+u^2}{tu} - \frac{t^2+u^2}{s^2}\right)$	(A-2)
$gq \to gq$ $g\bar{q} \to g\bar{q}$	$g_s^4\left(\frac{s^2+u^2}{t^2} - \frac{4}{9}\frac{s^2+u^2}{su}\right)$	(A-3)
$q_iq_j \to q_iq_j$ $q_i\bar{q}_j \to q_i\bar{q}_j$ $\bar{q}_iq_j \to \bar{q}_iq_j$ $\bar{q}_i\bar{q}_j \to \bar{q}_i\bar{q}_j$	$\frac{4}{9}g_s^4\frac{s^2+u^2}{t^2}, \quad i \neq j$	(A-4)
$q_iq_i \to q_iq_i$ $\bar{q}_i\bar{q}_i \to \bar{q}_i\bar{q}_i$	$\frac{4}{9}g_s^4\left(\frac{s^2+u^2}{t^2} + \frac{s^2+t^2}{u^2} - \frac{2}{3}\frac{s^2}{tu}\right)$	(A-5)
$q_i\bar{q}_i \to q_j\bar{q}_j$	$\frac{4}{9}g_s^4\frac{t^2+u^2}{s^2}$	(A-6)
$q_i\bar{q}_i \to q_i\bar{q}_i$	$\frac{4}{9}g_s^4\left(\frac{s^2+u^2}{t^2} + \frac{t^2+u^2}{s^2} - \frac{2}{3}\frac{u^2}{st}\right)$	(A-7)
$q\bar{q} \to gg$	$\frac{8}{3}g_s^4\left(\frac{4}{9}\frac{t^2+u^2}{tu} - \frac{t^2+u^2}{s^2}\right)$	(A-8)

$$(x_i^{\mathrm{net}}, y_i^{\mathrm{net}}) = f(\{\vec{p}\}_i, \theta),$$

Z Yang, YY He, W Chen, WY Ke, LG Pang, XN Wang, EPJC 83 (2023) 7, 652

DL assisted jet tomography



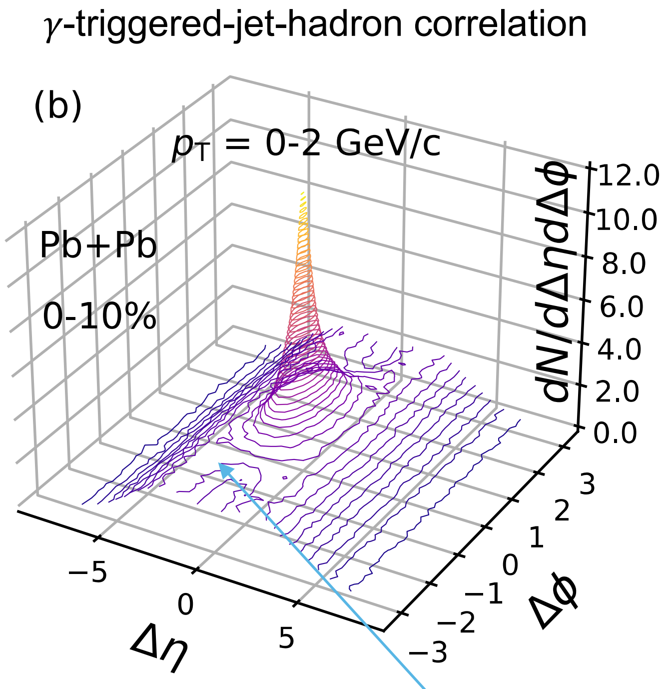
Network prediction

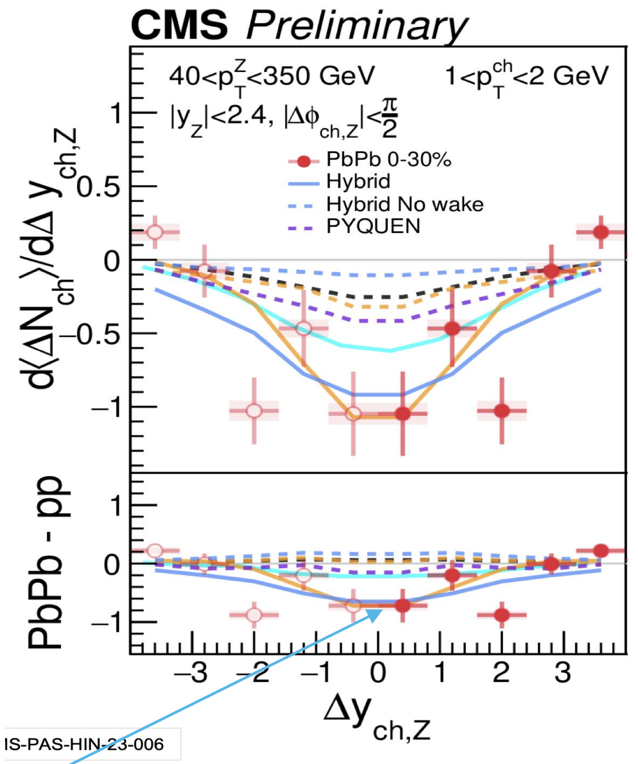
True production locations

DL assisted jet tomography:
The diffusion wake signals are enhanced significantly, allowing for more differential studies

Z Yang, YY He, W Chen, WY Ke, LG Pang, XN Wang, EPJC 83 (2023) 7, 652
Z Yang, T Luo, W Chen, LG Pang, XN Wang, PRL 130 (2023) 5, 052301

Jet induced sound wave in QGP





Diffusion Wake

2024: First Observation of the Predicted Diffusion Wake in LHC Pb+Pb Collisions

Auto Differentiation: machine precision

• Forward Mode

Introduce dual numbers: $x \to x + \dot{x}\mathbf{d}$

where $\mathbf{d}^2 = 0$

$$(x+\dot{x}\mathbf{d}) + (y+\dot{y}\mathbf{d}) = x + y + (\dot{x}+\dot{y})\mathbf{d}$$
$$(x+\dot{x}\mathbf{d}) - (y+\dot{y}\mathbf{d}) = x - y + (\dot{x}-\dot{y})\mathbf{d}$$
$$(x+\dot{x}\mathbf{d}) * (y+\dot{y}\mathbf{d}) = xy + (x\dot{y}+\dot{x}y)\mathbf{d}$$
$$\frac{1}{x+\dot{x}\mathbf{d}} = \frac{1}{x} - \frac{\dot{x}}{x^2}\mathbf{d} \qquad (x \neq 0)$$

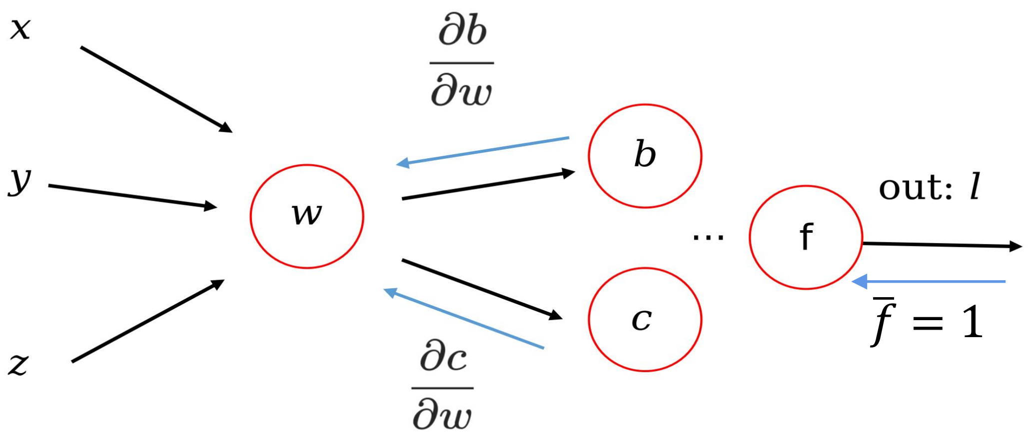
Forward mode for $R^1 \to R^n$

Reverse mode for $R^n \to R^1$

Differentiable Physics Simulation, Physics-Informed Neural Networks (PINNs)

• Reverse Mode

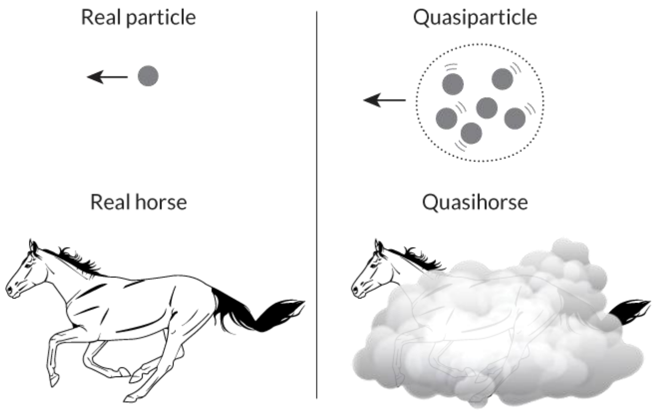
adjoint number: $\bar{w} = \frac{\partial l}{\partial w}$

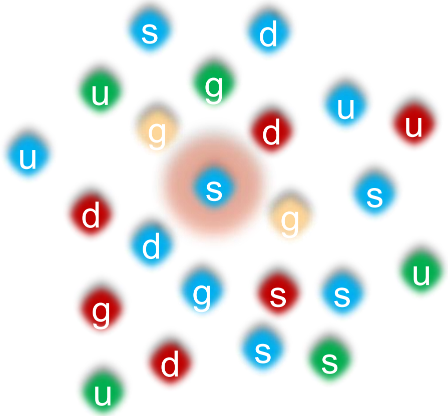


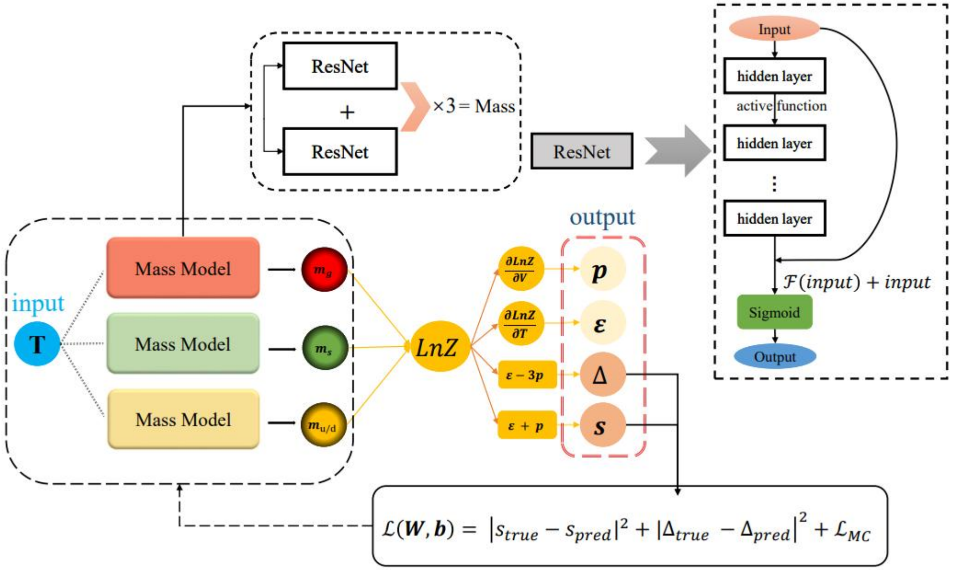
$$\text{step } 1 : \bar{w} = 0$$
$$\text{step } 2 : \bar{w} = \bar{w} + \bar{b}\frac{\partial b}{\partial w}$$
$$\text{step } 3 : \bar{w} = \bar{w} + \bar{c}\frac{\partial c}{\partial w}$$

Training and optimization

Deep Learning Quasi Parton Model







$$\ln Z(T) = \ln Z_g(T) + \ln Z_{u,d}(T) + \ln Z_s(T),$$

Fermi-Dirac distributions,

$$\ln Z_g(T) = -\frac{16V}{2\pi^2}\int_0^\infty p^2 dp \ln\left[1 - \exp\left(-\frac{1}{T}\sqrt{p^2 + m_g^2(T)}\right)\right], \quad (2)$$

$$\ln Z_{q_i}(T) = +\frac{12V}{2\pi^2}\int_0^\infty p^2 dp \ln\left[1 + \exp\left(-\frac{1}{T}\sqrt{p^2 + m_{q_i}^2(T)}\right)\right], \quad (3)$$

quarks, $m_s(T, \theta_2)$ for strange quark and $m_g(T, \theta_3)$ for gluons, where θ_1, θ_2 and θ_3 are the parameters in DNN shown in Fig. 1.

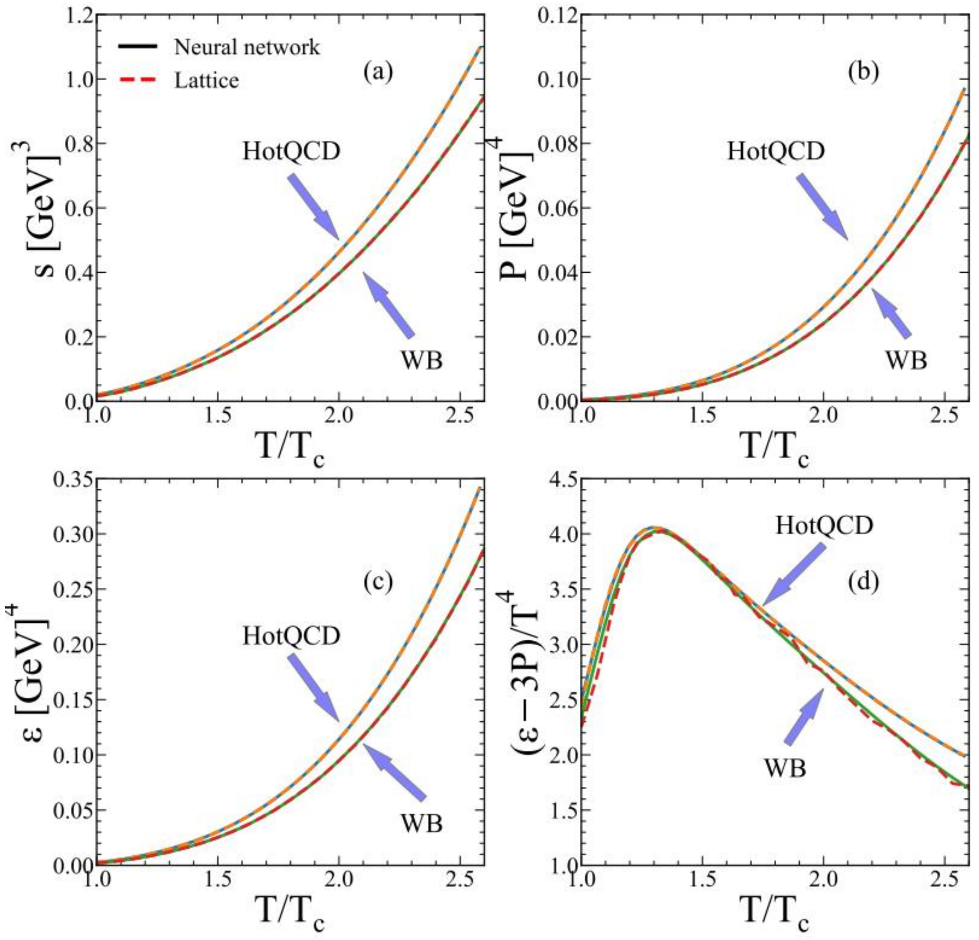
The resulting pressure and energy density are computed using the following statistical formulae,

$$P(T) = T\left(\frac{\partial \ln Z(T)}{\partial V}\right)_T, \quad (5)$$

$$\epsilon(T) = \frac{T^2}{V}\left(\frac{\partial \ln Z(T)}{\partial T}\right)_V, \quad (6)$$

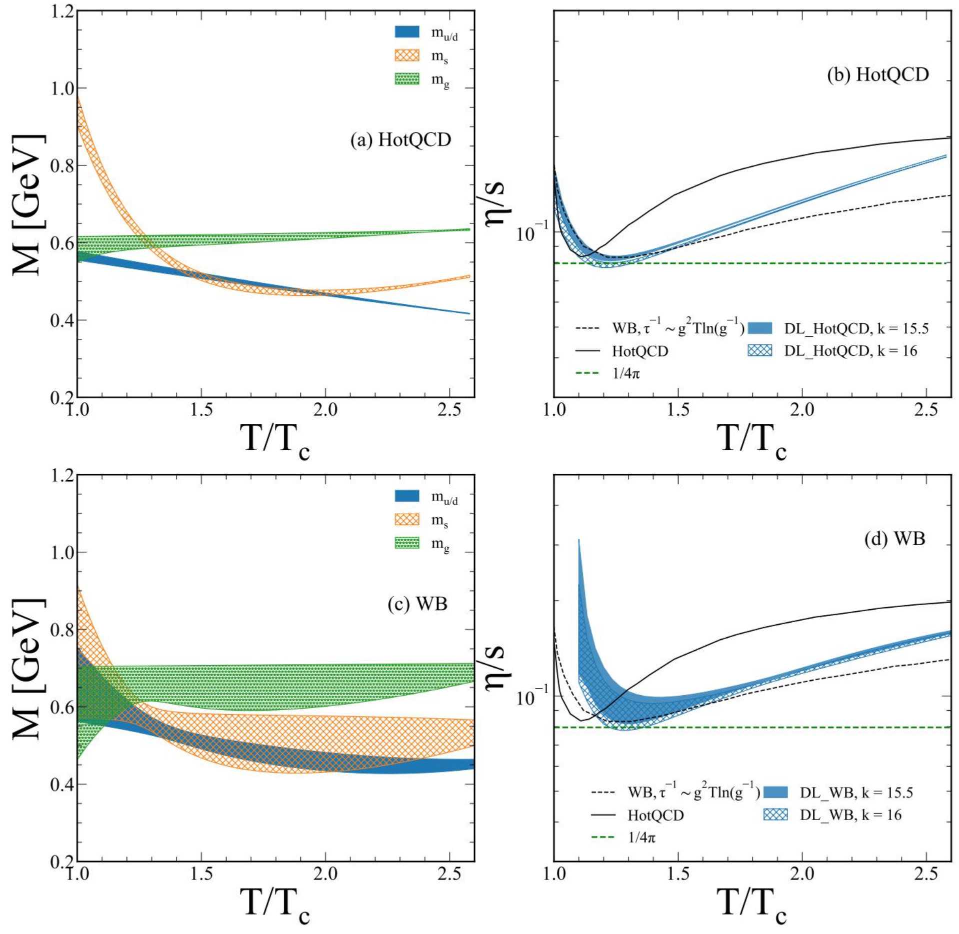
Deep Learning Quasi Parton Model

EoS vs Lattice QCD



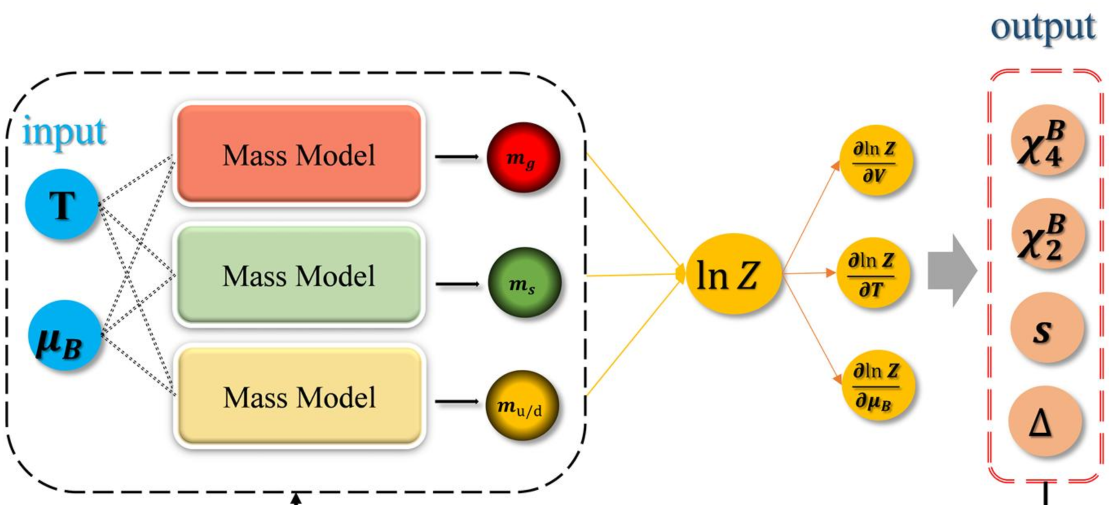
Learned Mass

η/s



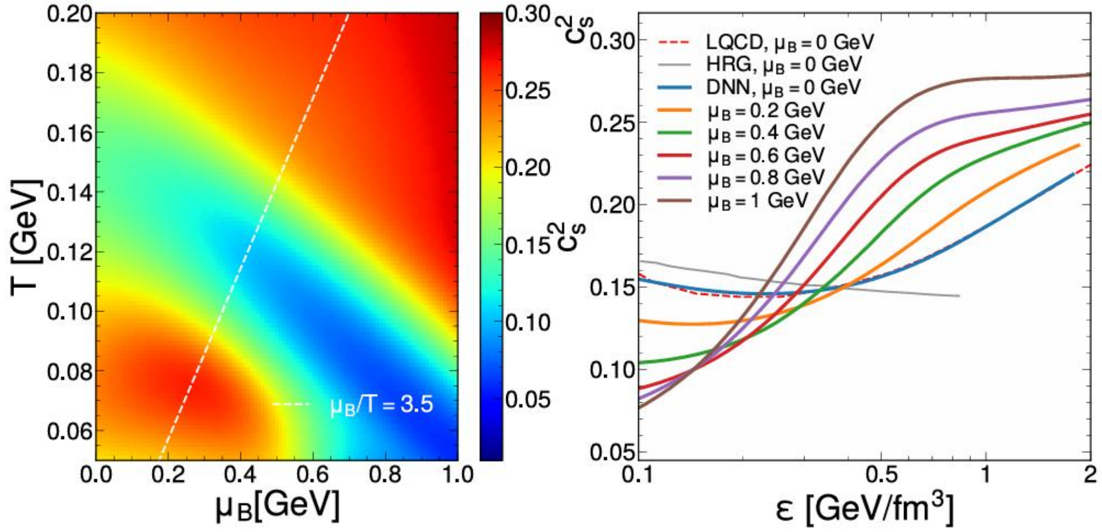
FuPeng Li, HL Lu, LG Pang, GY Qin, PLB 2023

Extended to 2 dimensions: $\boldsymbol{m(T,\mu_B)}$



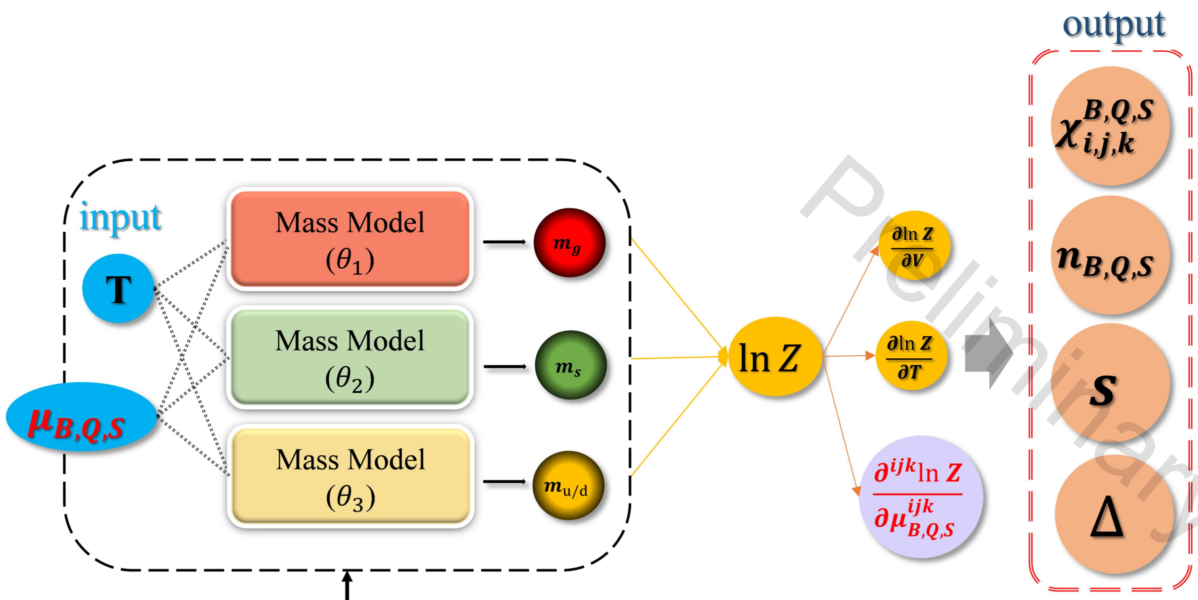
$$\mathcal{L}(\theta_1,\theta_2,\theta_3) = |s_{NN} - s_{input}| + |\frac{|\Delta_{NN} - \Delta_{input}}{T}| + |\chi^B_{2,NN} - \chi^B_{2,input}| + |\chi^B_{4,NN} - \chi^B_{4,input}| + \mathcal{L}_{MC}$$

Equation Of State



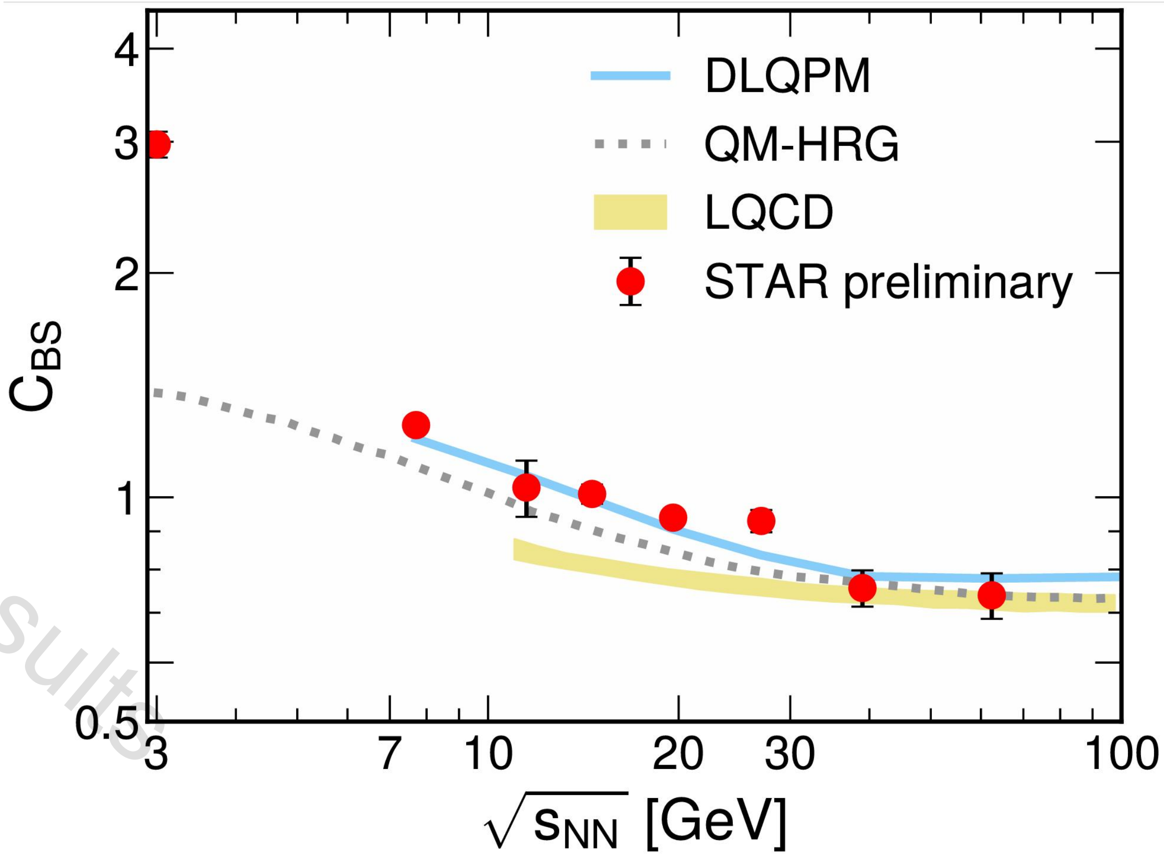
FP Li, LG Pang, GY Qin. PLB 868 (2025) 139692

To 4 dimensions: $m(T, \mu_B, \mu_Q, \mu_S)$



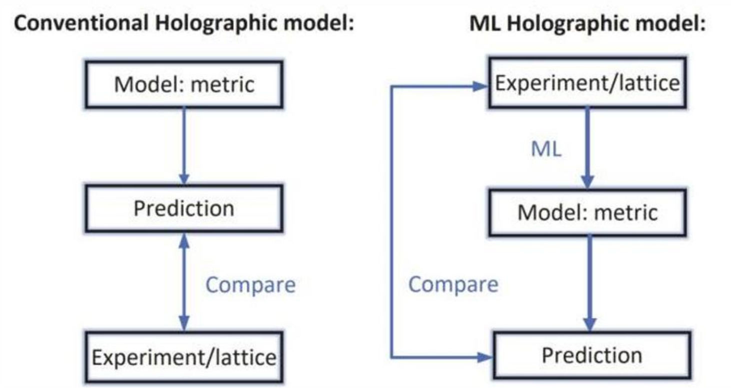
$$\mathcal{L}(\theta_1, \theta_2, \theta_3) = |s_{DNN} - s_{LQCD}| + |\frac{|\Delta_{DNN} - \Delta_{LQCD}}{T}| + |\chi^{B,Q,S}_{i,j,k,DNN} - \chi^{B,Q,S}_{i,j,k,LQCD}| + |n_{B,Q,S}| + | + \mathcal{L}_{MC}$$

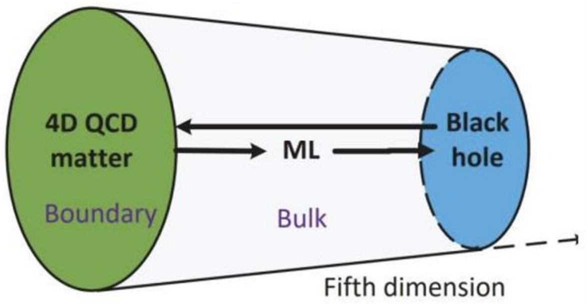
Baryon-Strangeness Correlation

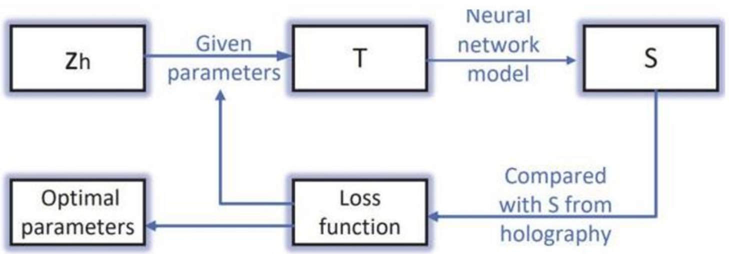


FP Li, LG Pang, GY Qin. in prepare

DL for holographic model using PINN







Einstein-Maxwell-Dilation model

O. DeWolfe, S. S. Gubser, and C. Rosen, Phys. Rev. D 83, 086005 (2011), arXiv:1012.1864.

Action: $$S_b = \frac{1}{16\pi G_5}\int d^5x\left[\sqrt{-g}R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

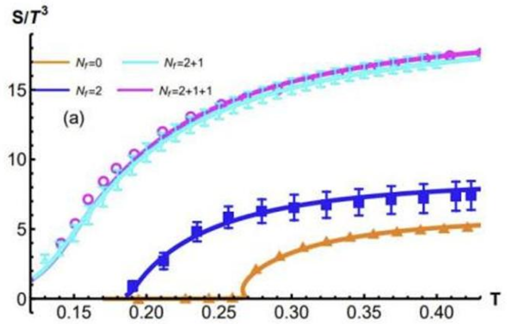
Non-conformal

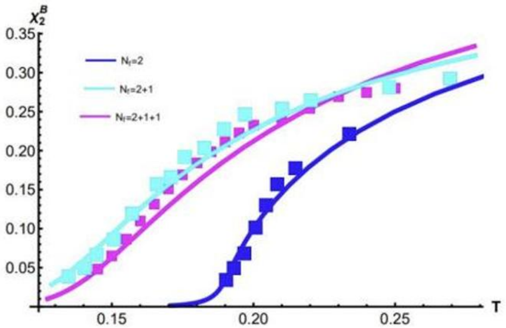
ϕ is dilaton F is the tensor of gauge field

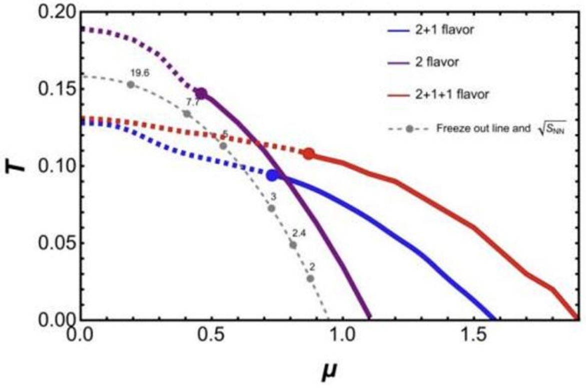
Metric ansatz: $$ds^2 = \frac{e^{2A(z)}}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + d\vec{x}^2\right]$$

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1),\ f(z) = e^{cz^2 - A(z) + k}$$

$$s = \frac{e^{3A(z_h)}}{4G_5 z_h^3}. \qquad \chi_2^B = \frac{1}{T^2}\frac{\partial\rho}{\partial\mu}.$$

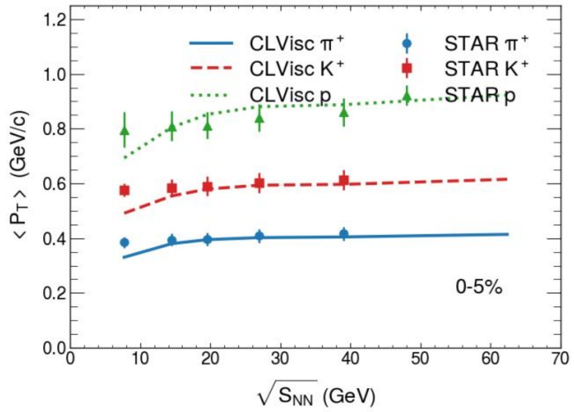


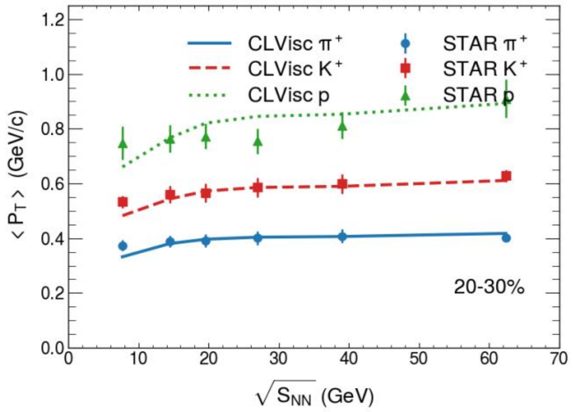


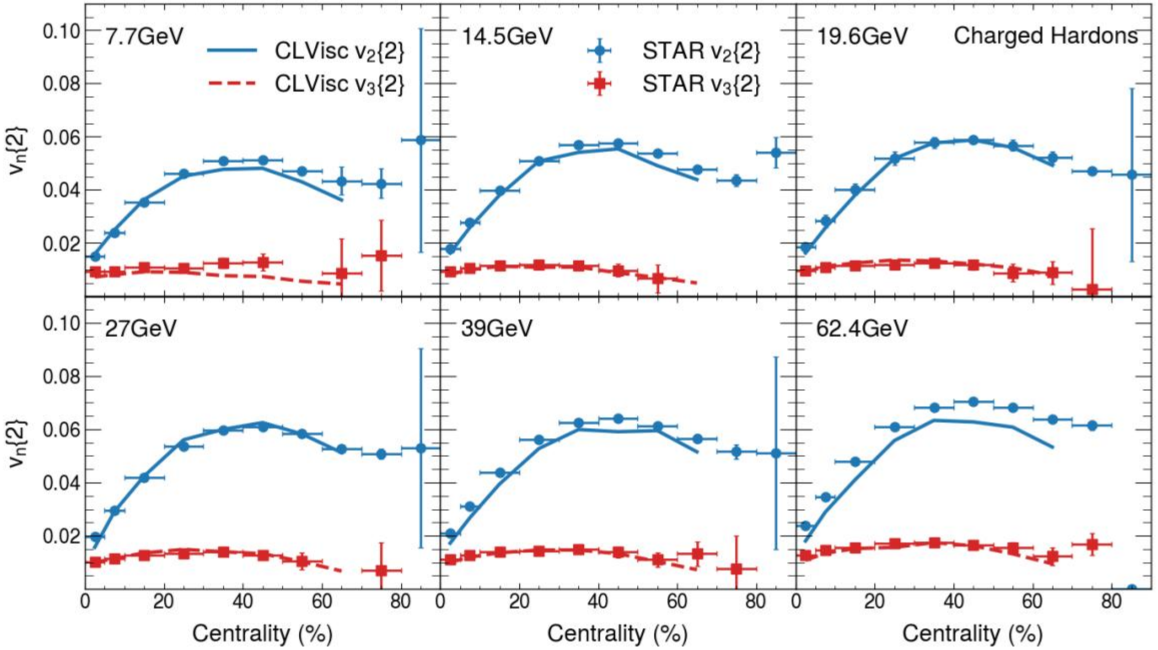


X. Chen, M. Huang, **Phys.Rev.D** 109 (2024) L051902; **JHEP**02 (2025)123

Extend CLVisc







$$\nabla_\mu T^{\mu\nu} = 0\,, \qquad T^{\mu\nu} = eU^\mu U^\nu - P\Delta^{\mu\nu} + \pi^{\mu\nu}\,,$$

$$\nabla_\mu J^\mu = 0\,, \qquad J^\mu = nU^\mu + V^\mu\,,$$

$$\Delta^{\mu\nu}_{\alpha\beta} D\pi^{\alpha\beta} = -\frac{1}{\tau_\pi}\left(\pi^{\mu\nu} - \eta_v \sigma^{\mu\nu}\right) - \frac{4}{3}\pi^{\mu\nu}\theta - \frac{5}{7}\pi^{\alpha<\mu}\sigma^{\nu>}_{\alpha} + \frac{9}{70}\frac{4}{e+P}\pi^{<\mu}_{\alpha}\pi^{\nu>\alpha}\,, \tag{10}$$

$$\Delta^{\mu\nu} DV_\nu = -\frac{1}{\tau_V}\left(V^\mu - \kappa_B \nabla^\mu \frac{\mu_B}{T}\right) - V^\mu\theta - \frac{3}{10}V_\nu\sigma^{\mu\nu}\,, \tag{11}$$

- ✓ **Net baryon conservation**
- ✓ **Relaxation equation for net baryon diffusion**
- ✓ **Works for BES energy range.**
- ➢ **Future work: 4D EoS + transport coefficients**

XY Wu, GY Qin, LG Pang, XN Wang, PRC 105 (2022) 3, 034909

Review articles

Nature Review Physics (2025)

nature reviews physics https://doi.org/10.1038/s42254-024-00798-x

Perspective

Check for updates

Physics-driven learning for inverse problems in quantum chromodynamics

Gert Aarts[1], Kenji Fukushima[2], Tetsuo Hatsuda[3], Andreas Ipp[4], Shuzhe Shi[5], Lingxiao Wang[3] & Kai Zhou[6,7]

Abstract

The integration of deep learning techniques and physics-driven designs is reforming the way we address inverse problems, in which accurate physical properties are extracted from complex observations. This is particularly relevant for quantum chromodynamics (QCD) – the theory of strong interactions – with its inherent challenges in interpreting

Sections

Introduction

Physics-driven learning

QCD physics

Conclusions and outlook

Prog. Part. Nucl. Phys. 135 (2024) 104084

Progress in Particle and Nuclear Physics 135 (2024) 104084

Contents lists available at ScienceDirect

Progress in Particle and Nuclear Physics

journal homepage: www.elsevier.com/locate/ppnp

ELSEVIER

Review

Check for updates

Exploring QCD matter in extreme conditions with Machine Learning

Kai Zhou[a,b,*], Lingxiao Wang[a,*], Long-Gang Pang[c,*], Shuzhe Shi[d,e,*]

[a] Frankfurt Institute for Advanced Studies (FIAS), D-60438 Frankfurt am Main, Germany
[b] School of Science and Engineering, The Chinese University of Hong Kong, Shenzhen 518172, China
[c] Institute of Particle Physics and Key Laboratory of Quark and Lepton Physics (MOE), Central China Normal University, Wuhan, 430079, China
[d] Department of Physics, Tsinghua University, Beijing 100084, China

Review Of Modern Physics (2022)

Colloquium: Machine learning in nuclear physics

Amber Boehnlein, Markus Diefenthaler, Nobuo Sato, Malachi Schram, Veronique Ziegler, Cristiano Morten Hjorth-Jensen, Tanja Horn, Michelle P. Kuchera, Dean Lee, Witold Nazarewicz, Peter Ostro Orginos, Alan Poon, Xin-Nian Wang, Alexander Scheinker, Michael S. Smith, and Long-Gang Pang

Rev. Mod. Phys. **94**, 031003 – Published 8 September 2022

Article | References | No Citing Articles | PDF | HTML | Export Citation

Nuclear Science and Techniques (2023) 34:88
https://doi.org/10.1007/s41365-023-01233-z

REVIEW ARTICLE

Nucl. Sci. Tech. 34 (2023) 6, 88

High-energy nuclear physics meets machine learning

Wan-Bing He[1,2] · Yu-Gang Ma[1,2] · Long-Gang Pang[3] · Hui-Chao Song[4] · Kai Zhou[5]

: 18 April 2023 / Published online: 21 June 2023

Thank you!

Science China Physics, Mechanics & Astronomy

Machine learning in nuclear physics at low and intermediate energies

Wanbing He,[1,2,*] Qingfeng Li,[3,4,†] Yugang Ma,[1,2,‡] Zhongming Niu,[5,§] Junchen Pei,[6,7,¶] and Yingxun Zhang[8,9,**]