中国科学院大学
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AI for Holographic QCD

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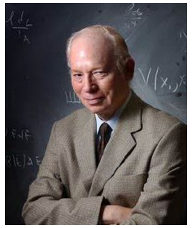
The 2nd "AI+HEP in East Asia" workshop, Jan.19-23,2026

Outline

1, A brief introduction on hQCD
2, Quantitative era of hQCD under the help of AI
3, Summary and outlook

Challenges of QCD: from UV to IR



Holographic QCD might help! How does it work?

Lagrangian of Quarks & Gluons at UV

QM, NJL, SM, HLS,

CHPT,
NRQCD……

color flux tube

Dual superconductor …

Emergent world:

Oberservables (IR)

Hadron physics(spectra, form factors, parton distribution functions…), QCD phase transitions(HICs, Cosmology), EOS (Neutron stars)

Strong QCD:

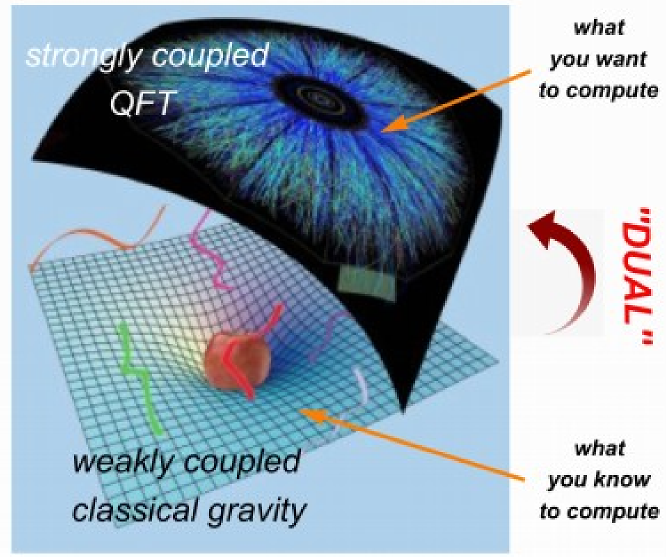
UV:
$SU(N_f)_L \times SU(N_f)_R$

SU(Nc=3)

LQCD, DSE/fRG

IR: chiral symmetry breaking & confinement

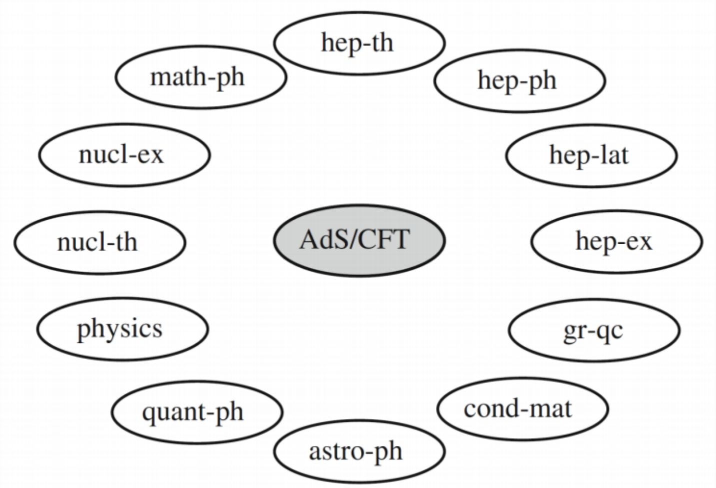
One of the biggest breakthroughs in theory: AdS/CFT



Matteo Baggioli 1908.02667v2 [hep-th]

Juan Martin Maldacena. Adv. Theor. Math. Phys., 2:231–252, 1998.
Edward Witten. Adv. Theor. Math. Phys., 2:253–291, 1998.

- The AdS/CFT duality spans all physics arXivs (Lect.Notes Phys. 903 (2015) pp.1-294)



Holographic Duality: (d+1)-Gravity/ (d)-QFT

27 years later, is it possible to use AdS/CFT describing the QCD world? How far we are from the gravity duality of QCD?

From UV to IR: RG flow and holography

AdS/CFT as a RG flow

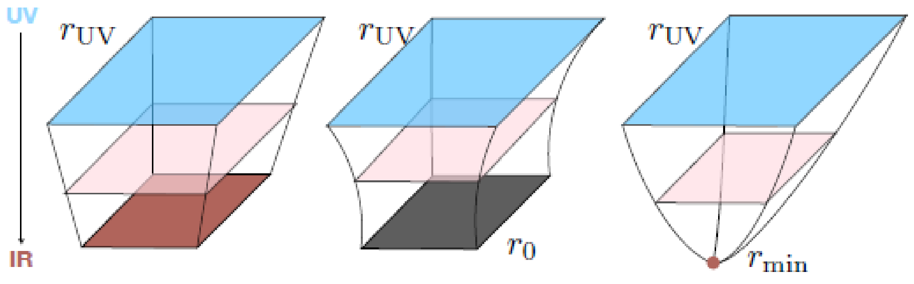


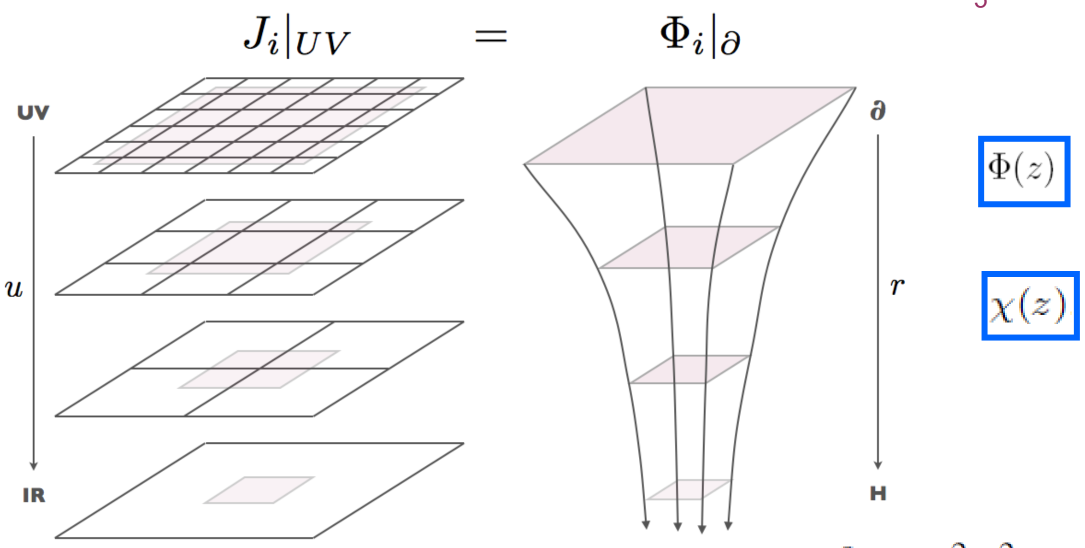
FIG. 1. AdS/CFT as an RG flow. The left panel represents an RG fixed point, so that the entire geometry is scale-invariant (empty AdS). The middle panel shows a thermal state, where the IR geometry is instead a black hole with horizon at r_0. The third panel represents an RG flow where the UV fixed point flows to gapped theory in the IR, ending smoothly at a minimum radius $r_{\min}$. Only the first geometry is fully scale invariant.

Hong Liu,Julian Sonner, 1810.02367,
Rept.Prog.Phys. 83 (2019) 1, 016001

Holographic Duality: (d+1)-Gravity/ (d)-QFT

5D field theory or bulk field theory

A AdS_5



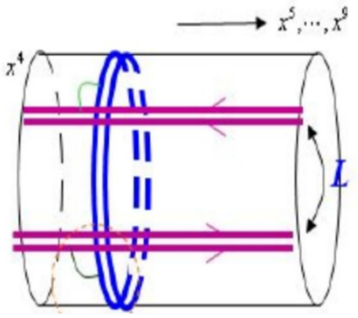
From UV to IR

$\mathrm{Tr}\langle G^2\rangle$ $\langle g^2A^2\rangle$ $\langle\bar{q}q\rangle$

Deformed AdS_5

Dynamical holographic QCD !
QCD Dynamics at IR induces deformation of AdS5 metric

“Entanglement” of gluodynamics and quark dynamics



Top-down	D4/D8,D3/D7	Dp brane: D4, D3	Dq brane: D8, D7
4D EFT	PNJL	Polyakov-loop potential	NJL model
bottom-up	DhQCD	Dilaton-Background	Flavor probe

Dynamical holographic QCD framework

$$S = S_G + \lambda S_f$$

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

$$S_f = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr\left(|DX|^2 + V_X + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right)$$

Danning Li, M.H., JHEP2013, arXiv:1303.6929

"Entanglement" of gluodynamics and quark dynamics

Gluonic background + matter field
(Confinement & chiral symmetry breaking)

Improved hQCD model:

U. Gursoy, E. Kiritsis, F. Nitti

$$S_{\rm V-QCD} = S_g + S_f$$

$$S_g = M^3 N_c^2 \int d^5x \sqrt{-g} \left(R - \frac{4}{3} \frac{(\partial\lambda)^2}{\lambda^2} + V_g(\lambda) \right)$$

$$S_f = -M^3 N_f N_c \int d^5x \, V_f(\lambda,\tau) \sqrt{-\det\left(g_{\mu\nu} + \kappa(\lambda)\partial_\mu\tau\partial_\nu\tau + w(\lambda)F_{\mu\nu}\right)}$$

$$V_g(\lambda) = 12 \left[1 + V_1\lambda + \frac{V_2\lambda^2}{1+\lambda/\lambda_0} + V_{\rm IR} e^{-\lambda_0/\lambda}(\lambda/\lambda_0)^{4/3}\sqrt{\log(1+\lambda/\lambda_0)} \right].$$

DeWolfe-Gubser-Rosen model:

Phys.Rev.D 83 (2011) 086005

$$V_\phi(\phi) = -12\cosh(c_1\phi) + (6c_1^2 - \frac{3}{2})\phi^2 + c_2\phi^6,$$

Dual fields: $\phi \leftrightarrow G_{\mu\nu}G^{\mu\nu}$, $\mathfrak{a} \leftrightarrow G_{\mu\nu}\widetilde{G}^{\mu\nu}$, $(A_\mu^{L/R})^{ij} \leftrightarrow \bar{\psi}^i(1\pm\gamma_5)\gamma_\mu\psi^j$

$$\mathcal{S}_{\mathrm{V-QCD}} = S_g + S_{\mathrm{DBI}} + S_a + S_{\mathrm{CS}}$$

$$\begin{aligned}
S_g &= M^3N_c^2\int d^5x\,\sqrt{-g}\left[R - \frac{4}{3}(\partial\phi)^2 + V_g(\phi)\right]\\
S_{\mathrm{DBI}} &= -M^3N_c\int d^5x\; V_f(\phi)\;\mathrm{Tr}\left[\sqrt{-\det(g_{\mu\nu} + w(\phi)F^{(L)}_{\mu\nu})} + (L\leftrightarrow R)\right]\\
S_a &= -\frac{M^3N_c^2}{2}\int d^5x\sqrt{-g}\;Z(\phi)\left[\partial_\mu\mathfrak{a} - \mathrm{Tr}\left(A_\mu^L - A_\mu^R\right)/N_c\right]^2\\
S_{\mathrm{CS}} &= \frac{iN_c}{24\pi^2}\int \mathrm{Tr}\Big[-iA_L\wedge F_L\wedge F_L + \frac{1}{2}A_L\wedge A_L\wedge A_L\wedge F_L +\\
&\quad + \frac{i}{10}A_L\wedge A_L\wedge A_L\wedge A_L\wedge A_L + (L\leftrightarrow R)\Big]
\end{aligned}$$

"Entanglement" of gluodynamics and quark dynamics

$$S = S_G + \lambda S_f$$

Confinement &
chiral symmetry breaking

DhQCD model:
(simplest version)
Can add any probe action

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

$$S_f = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr\left(|DX|^2 + V_X + \frac{1}{4g_5^2}(F_L^2 + F_R^2)\right)$$

Bulk field theory or 5D field theory

5D(Bulk) field theory, has operator/bulk field correspondence,

flexible to extend to pure gluon sector, 2-flavor, 3-flavor, 4-flavor

finite temperature, chemical potential, magnetic field, rotation

Hadron Physics and QCD matter, Hydrodynamics……

How does it work?

Boundary QFT **Bulk Gravity**

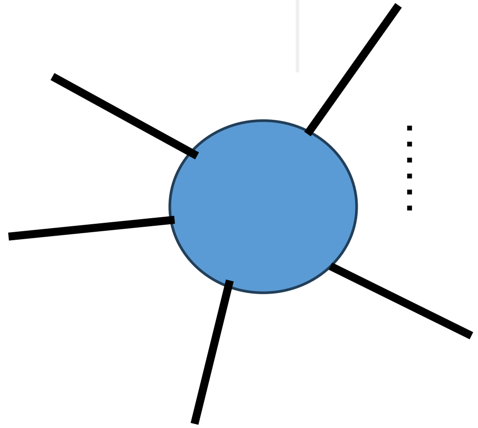
Local operator $\mathcal{O}_i(x)$ $\Delta(d-\Delta) = m^2 L^2$ **Bulk field** $\Phi_i(x,r)$

- Operator/Field correspondence:

4D boundary operator $\mathcal{O}(x)$
local, gauge invariant, scaling dim. Δ $\Longleftrightarrow$ 5D bulk field
$\phi(x, z \to 0) \to z^{4-\Delta}\phi_0(x) + z^{\Delta} < \mathcal{O}(x) >$

$$\left\langle e^{i\int d^4x \phi_0(x)\mathcal{O}(x)} \right\rangle_{CFT} = e^{iS_{5D}[\phi(x,z)]}|_{\phi(x,z\to 0)\to\phi_0(x)}$$

$$\boxed{Z_{\mathrm{QFT}}[J_i] \;=\; Z_{\mathrm{QG}}[\Phi[J_i]]} \qquad Z_{\mathrm{QFT}}[J] \;\simeq\; e^{-I_{\mathrm{GR}}[\Phi[J]]}$$



$$\langle \mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle = \frac{\delta^n I_{\mathrm{GR}}[\Phi[J_i]]}{\delta J_1(x_1)\ldots\delta J_n(x_n)}\Big|_{J_i=0}$$

Correlators:

Using bulk field to calculate observables: two-point correlation gives mass spectra, three-point correlation gives form factor, and so on.

Hadron spectra and form factors:

glueball, light flavor, heavy flavor

Pure gluon sector

$$\mathscr{L}_G = -\frac{1}{4} G^a_{\mu\nu}(x) G^{\mu\nu,a}(x),$$

Danning Li, M.H., JHEP2013, arXiv:1303.6929

IR: Gluon condensate D=4 $\mathrm{Tr}\langle G^2 \rangle$

Effective gluon mass D=2 $\langle g^2 A^2 \rangle$ ⇨ String tension, linear confinement

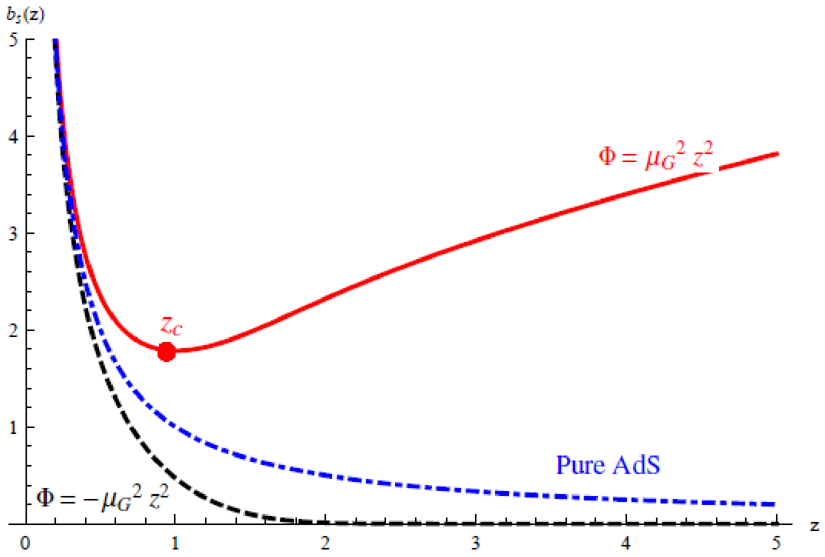
$$S_b^s = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g^s} e^{-2\Phi} \Big[R^s + 4 g^{sMN} \partial_M \Phi \partial_N \Phi - V^s(\Phi) - \frac{h(\Phi)}{4} e^{\frac{4\Phi}{3}} g^{sM\widetilde{M}} g^{sN\widetilde{N}} F_{MN} F_{\widetilde{M}\widetilde{N}} \Big],$$

Dimension-2 dilaton field

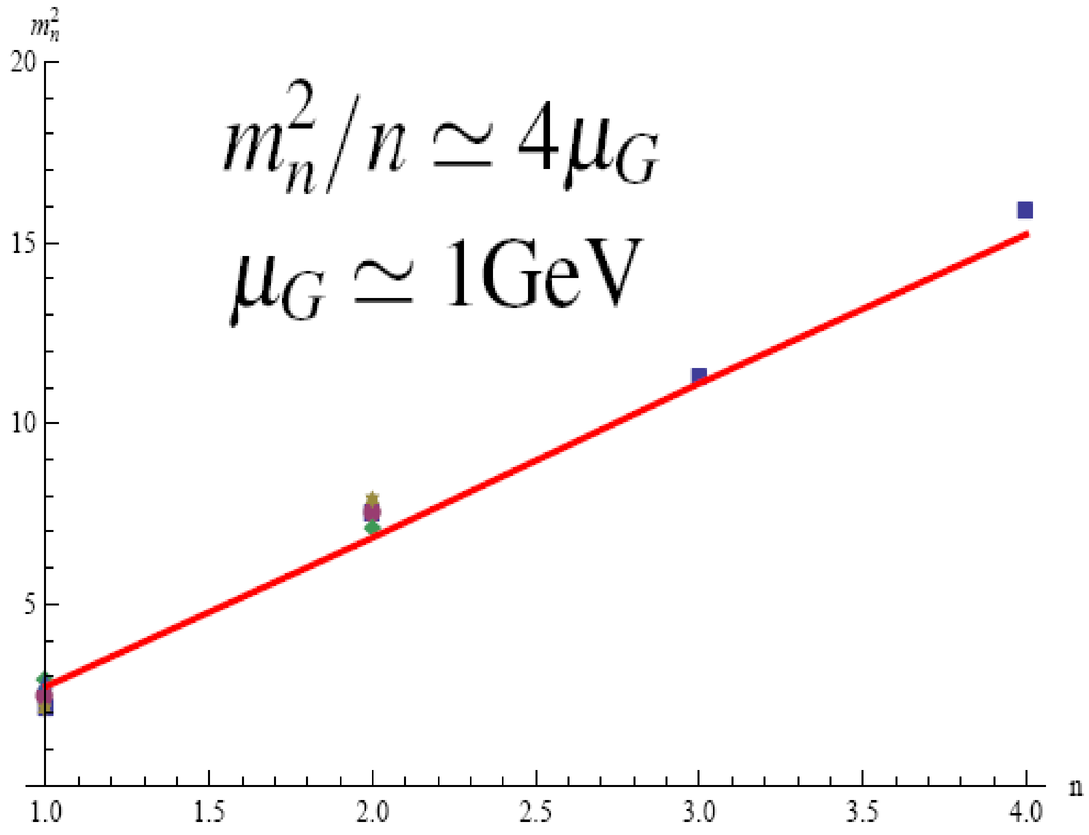
$$\phi(z) = bz^2,$$

$$V_\phi(\phi) = \frac{1}{L^2} 2 \times 2^{\frac{3}{4}} \times 3^{\frac{1}{4}} \phi^{\frac{3}{2}} \left[\Gamma\left(\frac{5}{4}\right)\right]^2 \left\{ \left[I_{\frac{1}{4}}\left(\frac{\phi}{\sqrt{6}}\right)\right]^2 - 4\left[I_{-\frac{3}{4}}\left(\frac{\phi}{\sqrt{6}}\right)\right]^2 \right\},$$

$$A_E(z) = -\ln\left[\frac{2^{\frac{3}{8}} \times 3^{\frac{1}{8}} \Gamma\left(\frac{5}{4}\right) I_{\frac{1}{4}}\left(\frac{bz^2}{\sqrt{6}}\right)}{b^{\frac{1}{4}} \sqrt{z}}\right],$$



Scalar glueball   **D.N. Li, M.H., JHEP2013, arXiv:1303.6929**



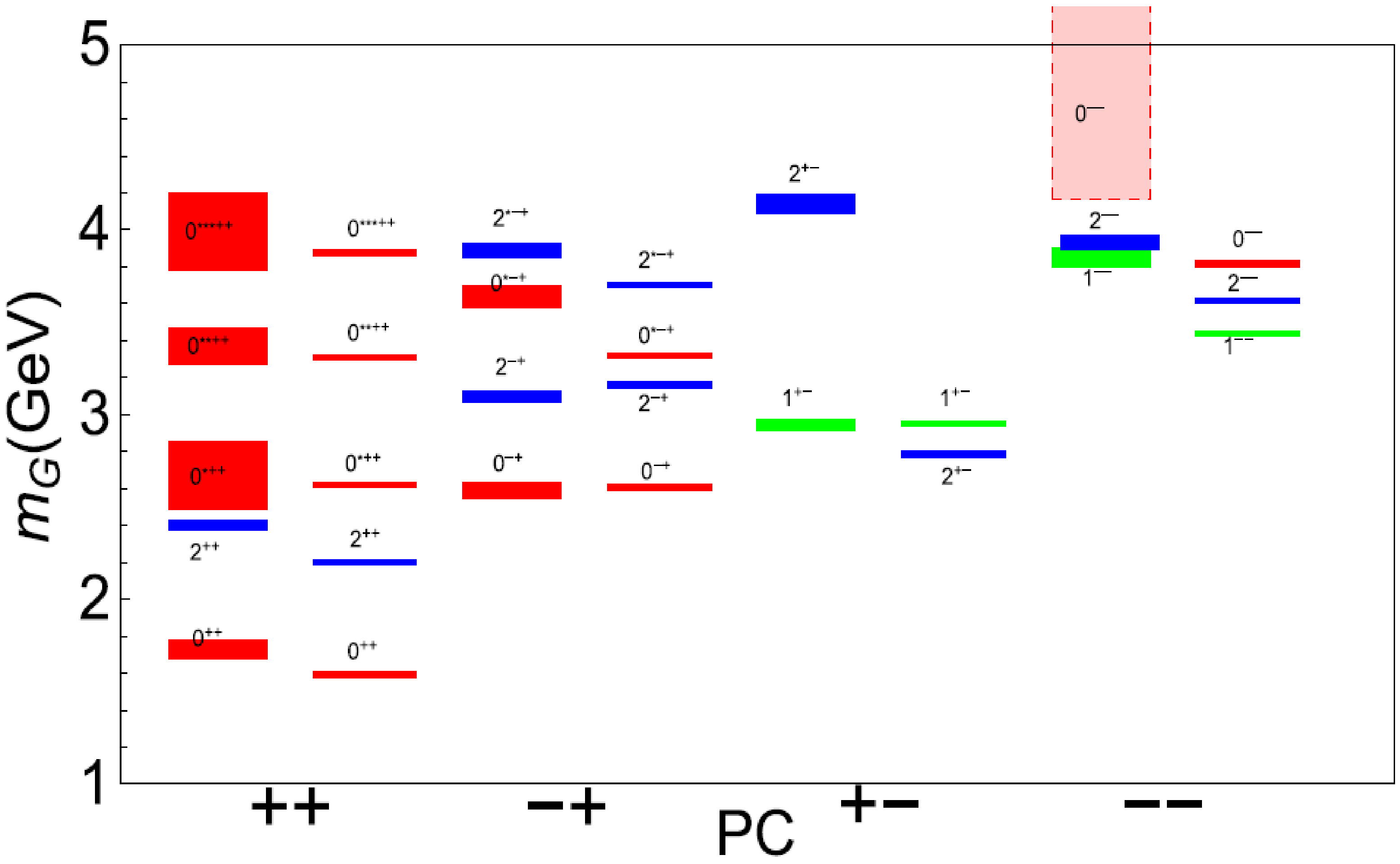
hep-lat/0508002.

[hep-lat/0510074].

[hep-lat/0103027].

[hep-lat/9901004]

Glueball spectra: **Yidian Chen, M.H., 1511.07018**



Agree well with lattice result except 0^{--} and 2^{+-} but …

J^{PC}	4-dimensional operator: $\mathscr{O}(x)$	Δ	p	M_5^2
0^{++}	$Tr(G^2) = \vec{E}^a \cdot \vec{E}^a - \vec{B}^a \cdot \vec{B}^a$	4	0	0
0^{-+}	$Tr(G\tilde{G}) = \vec{E}^a \cdot \vec{B}^a$	4	0	0
0^{+-}	$\mathrm{Tr}\left(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}(D_\mu G_{\rho\alpha})\right)$	9	0	45
0^{--}	$\mathrm{Tr}\left(\{(D_\tau G_{\mu\nu}), (D_\tau G_{\rho\nu})\}\left(D_\mu \tilde{G}_{\rho\alpha}\right)\right)$	9	0	45
1^{-+}	$f^{abc}\partial_\mu \left[G^a_{\mu\nu}\right]\left[G^b_{\upsilon\rho}\right]\left[G^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu \left[G^a_{\mu\nu}\right]\left[\tilde{G}^b_{\upsilon\rho}\right]\left[\tilde{G}^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu \left[\tilde{G}^a_{\mu\nu}\right]\left[G^b_{\upsilon\rho}\right]\left[\tilde{G}^c_{\rho\alpha}\right]$, $f^{abc}\partial_\mu \left[\tilde{G}^a_{\mu\nu}\right]\left[\tilde{G}^b_{\upsilon\rho}\right]\left[G^c_{\rho\alpha}\right]$	7	1	24
1^{+-}	$d^{abc}\left(\vec{E}_a \cdot \vec{E}_b\right)\vec{B}_c$	6	1	15
1^{--}	$d^{abc}\left(\vec{E}_a \cdot \vec{E}_b\right)\vec{E}_c$	6	1	15
2^{++}	$E^a_i E^a_j - B^a_i B^a_j - trace$	4	2	4
2^{-+}	$E^a_i B^a_j + B^a_i E^a_j - trace$	4	2	4
2^{+-}	$d^{abc}\mathcal{S}\left[E^i_a\left(\vec{E}_b \times \vec{B}_c\right)^j\right]$	6	2	16
2^{--}	$d^{abc}\mathcal{S}\left[B^i_a\left(\vec{E}_b \times \vec{B}_c\right)^j\right]$	6	2	16
3^{+-}	$d^{abc}\mathcal{S}\left[B^i_a B^j_b B^k_c\right]$	6	3	15
3^{--}	$d^{abc}\mathcal{S}\left[E^i_a E^j_b E^k_c\right]$	6	3	15

Lin Zhang, Chutian Chen, Yidian Chen, M.H.
Phys.Rev.D 105 (2022) 2, 026020

$$S_{\mathscr{G}} = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\partial_M\mathscr{G}\partial^M\mathscr{G} + M^2_{\mathscr{G},5}(z)\mathscr{G}^2)$$

$$S_V = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\frac{1}{2}F^{MN}F_{MN} + M^2_{\mathscr{V},5}(z)\mathscr{V}^2),$$

$$S_T = -\frac{1}{2}\int d^5x\sqrt{g_s}e^{-p\Phi}(\nabla_L h_{MN}\nabla^L h^{MN} - 2\nabla_L h^{LM}\nabla^N h_{NM} + 2\nabla_M h^{MN}\nabla_N h - \nabla_M h\nabla^M h + M^2_{h,5}(z)(h^{MN}h_{MN} - h^2)),$$

$$-\mathscr{G}_n^{''} + V_{\mathscr{G}}\mathscr{G}_n = m^2_{\mathscr{G},n}\mathscr{G}_n,$$

$$V_{\mathscr{G}} = \frac{3A_s^{''} + \frac{3}{z^2} - p\Phi^{''}}{2} + \frac{\left[3A_s^{'} - \frac{3}{z} - p\Phi^{'}\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{\mathrm{r.m.}}\Phi}M^2_{\mathscr{G},5}.$$

$$-\mathscr{V}_n^{''} + V_{\mathscr{V}}\mathscr{V}_n = m^2_{\mathscr{V},n}\mathscr{V}_n,$$

$$V_{\mathscr{V}} = \frac{A_s^{''} + \frac{1}{z^2} - p\Phi^{''}}{2} + \frac{\left[A_s^{'} - \frac{1}{z} - p\Phi^{'}\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{\mathrm{r.m.}}\Phi}M^2_{\mathscr{V},5}.$$

$$-\mathscr{T}_n^{''} + V_{\mathscr{T}}\mathscr{T}_n = m^2_{\mathscr{T},n}\mathscr{T}_n,$$

$$V_{\mathscr{T}} = \frac{3A_s^{''} + \frac{3}{z^2} - p\Phi^{''}}{2} + \frac{\left[3A_s^{'} - \frac{3}{z} - p\Phi^{'}\right]^2}{4} + \frac{1}{z^2}e^{2A_s}e^{-c_{\mathrm{r.m.}}\Phi}M^2_{\mathscr{T},5}.$$

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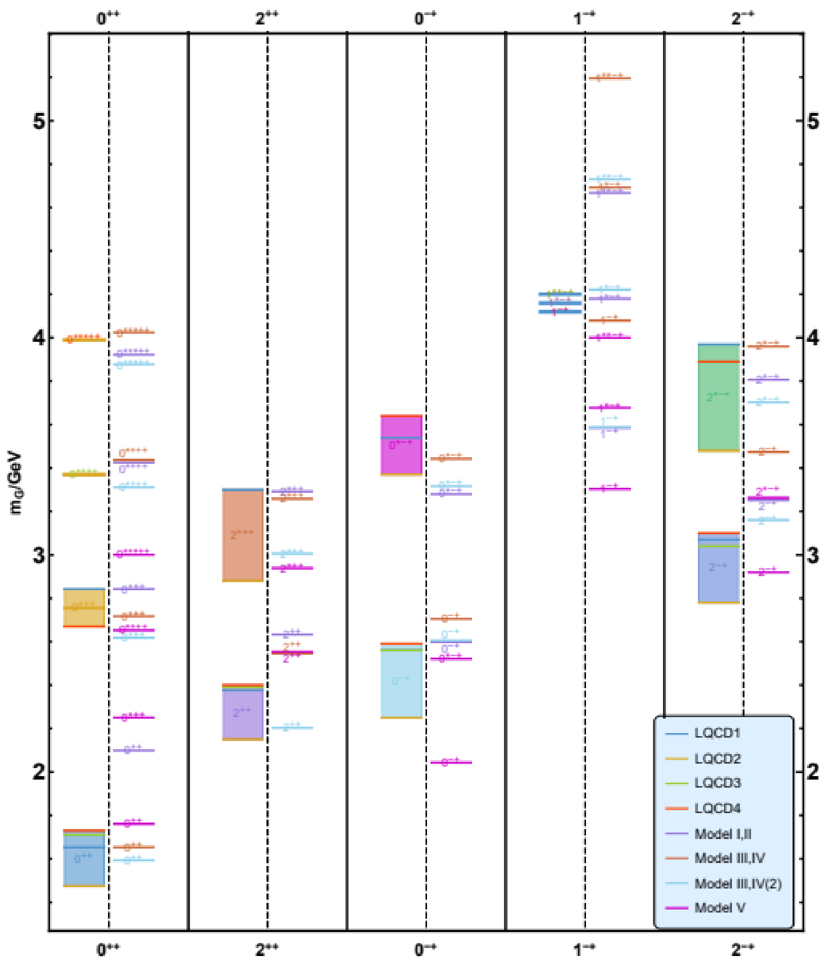
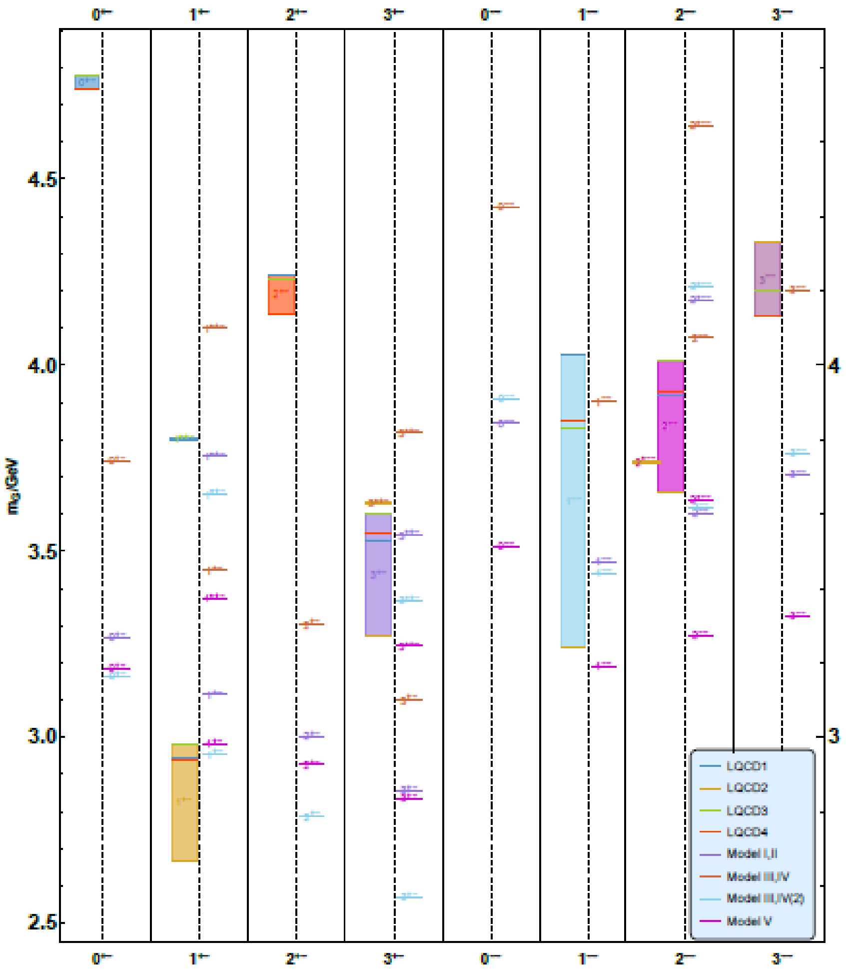


Glueball



Odd ball

Lin Zhang, Chutian Chen, Yidian Chen, M.H.
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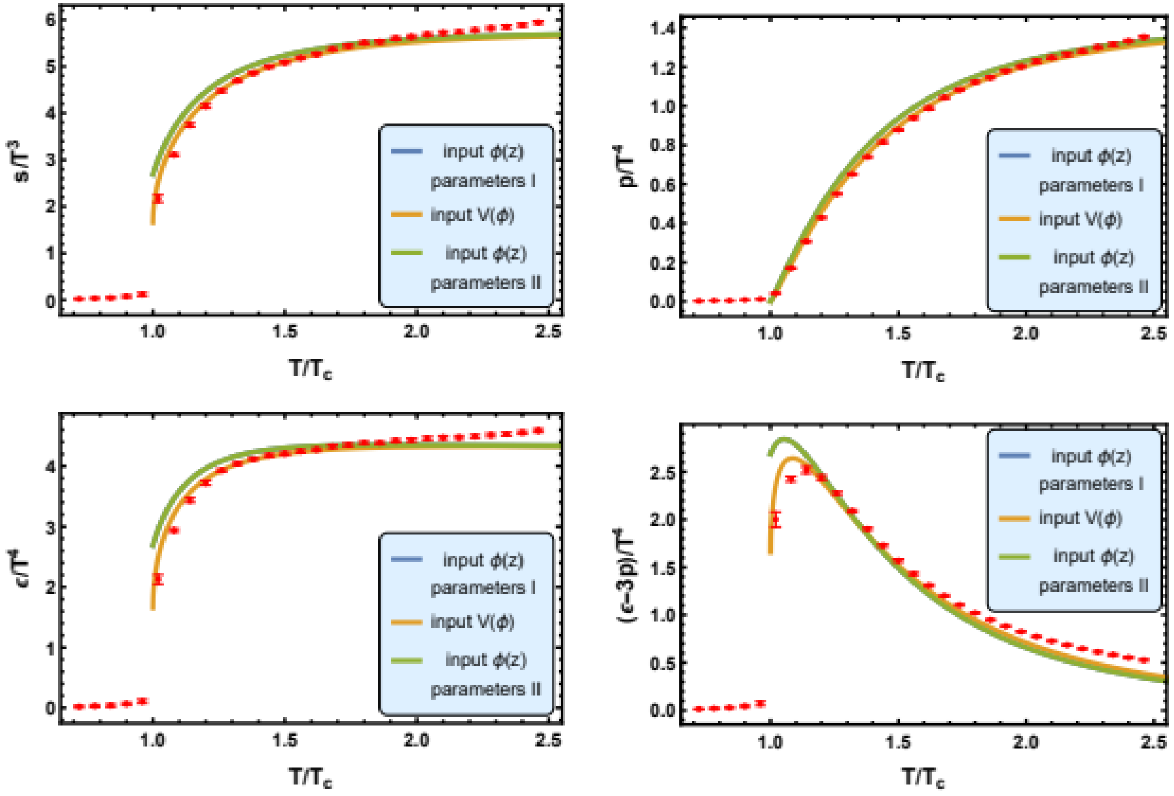
gluon background

$$\phi(z) = c_1 z^2,$$

Agree well with lattice results on EOS for pure gluon system



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Quardratic dilaton field describes pure gluon system reasonably well.

Graviton-Dilaton-Scalar system for 2-flavor system

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Action for pure gluon system: Graviton-dilaton coupling

$$S_G = \frac{1}{16\pi G_5}\int d^5x\sqrt{g_s}e^{-2\Phi}\left(R + 4\partial_M\Phi\partial^M\Phi - V_G(\Phi)\right)$$

Gluonic background

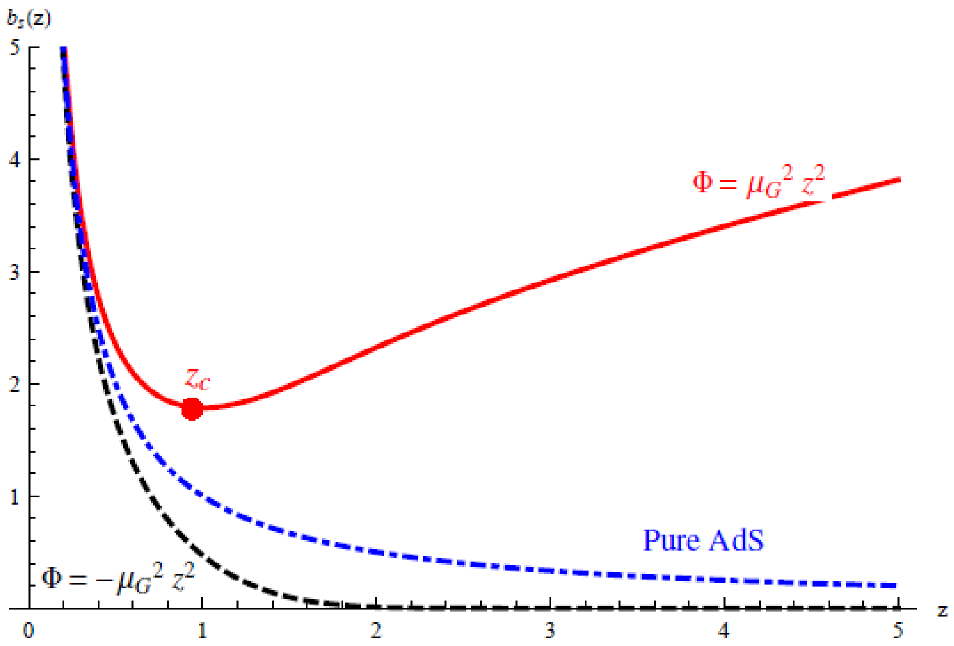
Action for light hadrons: KKSS model (promote dilaton field to a dynamical field)

$$S_{KKSS} = -\int d^5x\sqrt{g_s}e^{-\Phi}Tr(|DX|^2 + V_X(X^+X,\Phi) + \frac{1}{4g_5^2}(F_L^2 + F_R^2)).$$

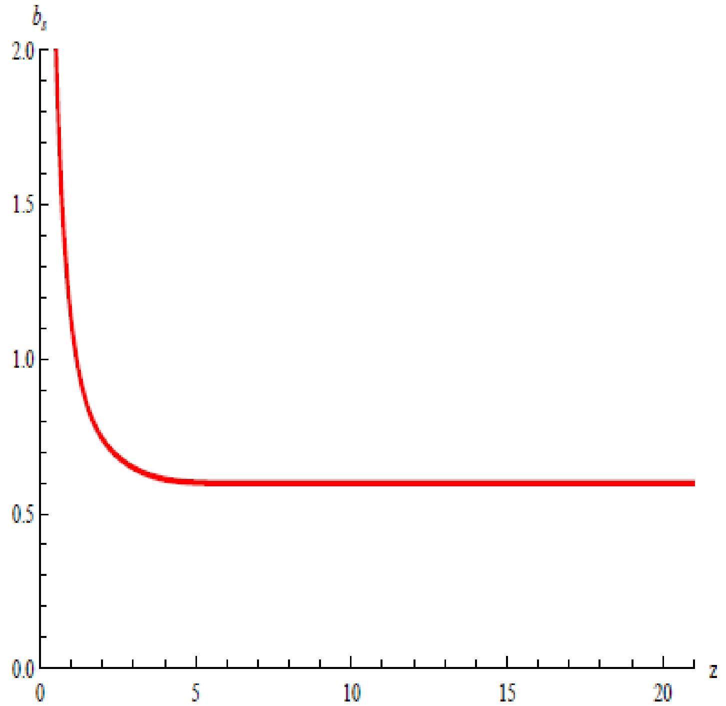
5D linear sigma model

Total action: $$S = S_G + \frac{N_f}{N_c}S_{KKSS}.$$

Quenched background



Unquenched background



$$-A_s'' + A_s'^2 + \frac{2}{3}\Phi'' - \frac{4}{3}A_s'\Phi' - \frac{\lambda}{6}e^{\Phi}\chi'^2 = 0,$$

$$\Phi'' + (3A_s' - 2\Phi')\Phi' - \frac{3\lambda}{16}e^{\Phi}\chi'^2 - \frac{3}{8}e^{2A_s - \frac{4}{3}\Phi}\partial_\Phi\left(V_G(\Phi) + \lambda e^{\frac{7}{3}\Phi}V_C(\chi,\Phi)\right) = 0,$$

$$\chi'' + (3A_s' - \Phi')\chi' - e^{2A_s}V_{C,\chi}(\chi,\Phi) = 0.$$

D.N. Li, M.H., JHEP2013, arXiv:1303.6929

Two-point correlator gives the spectra:

$$-s_n'' + V_s(z) s_n = m_n^2 s_n,$$
$$-\pi_n'' + V_{\pi,\varphi}\pi_n = m_n^2(\pi_n - e^{A_s}\chi\varphi_n),$$
$$-\varphi_n'' + V_\varphi \varphi_n = g_5^2 e^{A_s}\chi(\pi_n - e^{A_s}\chi\varphi_n),$$
$$-v_n'' + V_v(z) v_n = m_{n,v}^2 v_n,$$
$$-a_n'' + V_a a_n = m_n^2 a_n,$$

$$V_s = \frac{3A_s'' - \phi''}{2} + \frac{(3A_s' - \phi')^2}{4} + e^{2A_s} V_{C,\chi\chi},$$
$$V_{\pi,\varphi} = \frac{3A_s'' - \phi'' + 2\chi''/\chi - 2\chi'^2/\chi^2}{2} + \frac{(3A_s' - \phi' + 2\chi'/\chi)^2}{4},$$
$$V_\varphi = \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4},$$
$$V_v = \frac{A_s'' - \phi''}{2} + \frac{(A_s' - \phi')^2}{4},$$
$$V_a = \frac{A_s' - \phi'}{2} + \frac{(A_s' - \phi')^2}{4} + g_5^2 e^{2A_s}\chi^2.$$

Three-point correlator gives form factor:

$$f_\pi^2 F_\pi(Q^2) = \frac{N_f}{g_5^2 N_c}\int dz e^{A_s - \Phi} V(q^2, z)\{(\partial_z\varphi)^2 + g_5^2\chi^2 e^{2A_s}(\pi - \varphi)^2\},$$

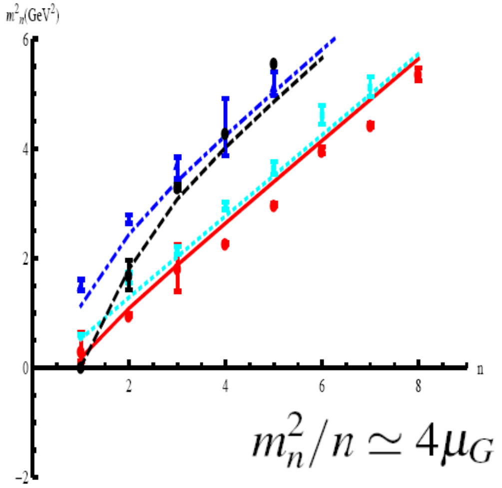
$$Dilaton\ in\ Mod\ I: \qquad \Phi(z) = \mu_G^2 z^2$$
$$Dilaton\ in\ Mod\ II: \qquad \Phi(z) = \mu_G^2 z^2 \tanh(\mu_{G^2}^4 z^2/\mu_G^2)$$

	Mod IA	Mod IB	Mod IIA	Mod IIB
G_5/L^3	0.75	0.75	0.75	0.75
m_q (MeV)	5.8	5.0	8.4	6.2
$\sigma^{1/3}$ (MeV)	180	240	165	226
μ_G	0.43	0.43	0.43	0.43
μ_{G^2}	-	-	0.43	0.43

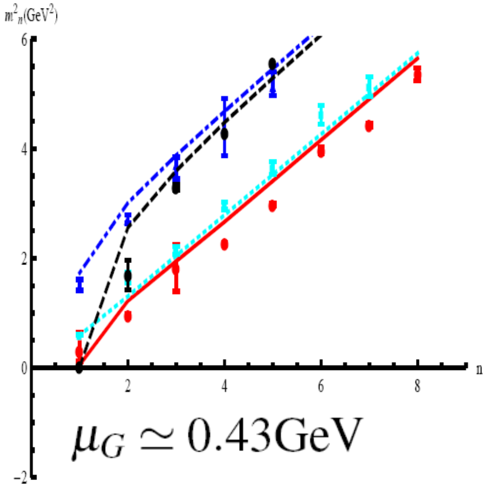
Table 7. Two sets of parameters.

Produced hadron spectra and pion form factor comparing with data

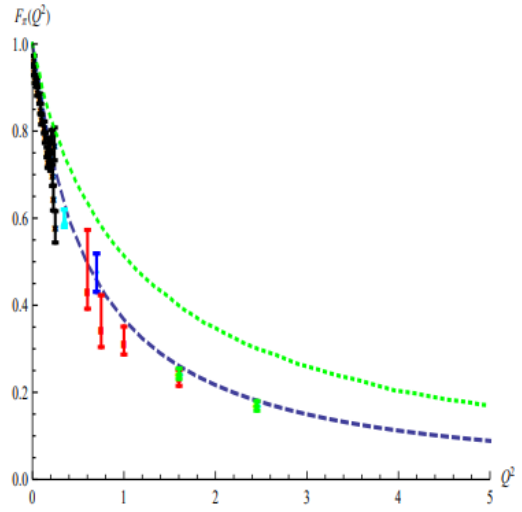
D.N. Li, M.H., JHEP2013, arXiv:1303.6929



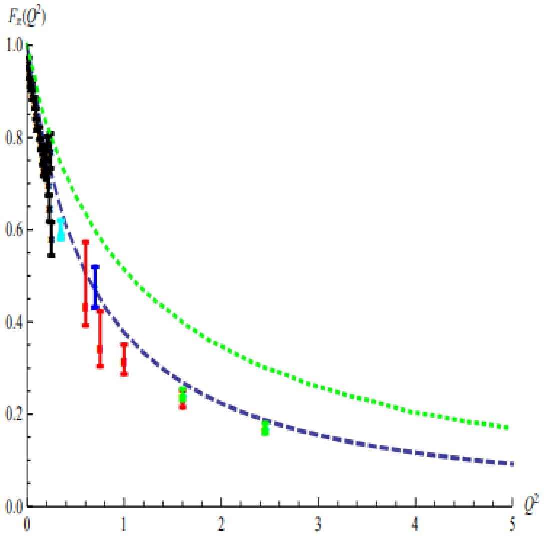
(Mod IA)



(Mod IB)



(Mod IA)



(Mod IIA)

Ground states: chiral symmetry breaking
Excitation states: linear confinement

Spectra and pion form factor cannot be simultaneously produced!

Graviton-Dilaton-Scalar system for 4-flavor system

$$S_M = -\int_\epsilon^{z_m} d^5x\sqrt{-g}e^{-\phi}\,\mathrm{Tr}\left\{(D^M X)^\dagger (D_M X) + M_5^2|X|^2 + \frac{1}{4g_5^2}\left(L^{MN}L_{MN} + R^{MN}R_{MN}\right) + (D^M H)^\dagger (D_M H) + M_5^2|H|^2\right\},$$

$$V = V^a t^a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega' + \frac{\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3}{\sqrt{12}}\psi \end{pmatrix},$$

$$A = A^a t^a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & a_1^+ & K_1^+ & \bar{D}_1^0 \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & K_1^0 & D_1^- \\ K_1^- & \bar{K}_1^0 & -\sqrt{\frac{2}{3}}(f_1) + \frac{\chi_{c1}}{\sqrt{12}} & D_{s1}^- \\ D_1^0 & D_1^+ & D_{s1}^+ & -\frac{3}{\sqrt{12}}\chi_{c1} \end{pmatrix},$$

$$\pi = \pi^a t^a = \frac{1}{\sqrt{2}}\begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3}{\sqrt{12}}\eta_c \end{pmatrix}.$$

extended Linear Sigma Model

W. I. Eshraim, F. Giacosa and D. H. Rischke,
Eur. Phys. J. A 51, no.9, 112 (2015)

Solving background with chiral condensate:

$$S^{(0)} = -\frac{1}{4}\int_\epsilon^{z_m} d^5x\left\{\frac{e^{-\phi(z)}}{z^3}\left(2v_l'(z)v_l'(z) + v_s'(z)v_s'(z) + v_c'(z)v_c'(z)\right) - \frac{e^{-\phi(z)}}{z^5}\left(3\left(2v_l(z)^2 + v_s(z)^2 + v_c(z)^2\right) - \frac{\kappa}{4}\left(2v_l(z)^4 + v_s(z)^4 + v_c(z)^4\right)\right) + \frac{e^{-\phi(z)}}{z^3}\left(h_c'(z)h_c'(z)\right) - \frac{3e^{-\phi(z)}}{z^5}h_c(z)^2\right\}$$

Y.D. Chen, M.H.arXiv: 2110.08215,
Phys.Rev.D 105 (2022) 2, 026021
H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975,
Phys.Rev.D 108 (2023) 8, 086034) arXiv:2309.06156

Two-point correlation function gives mass

$$S^{(2)} = -\int d^5x \left\{ \eta^{MN} \frac{e^{-\phi(z)}}{z^3} \left(\left(\partial_M \pi^a - A_M^a\right) \left(\partial_N \pi^b - A_N^b\right) M_A^{ab} - V_M^a V_N^b M_V^{ab} + V_{HM}^a V_{HN}^b M_{V_H}^{ab} \right) + \frac{e^{-\phi(z)}}{4 g_5^2 z} \eta^{MP} \eta^{NQ} \left(V_{MN}^a V_{PQ}^b + A_{MN}^a A_{PQ}^b \right) \right\}$$

$$\langle J_\mu^{V,a} J_\nu^{V,b} \rangle \simeq \delta^{ab} (q_\mu q_\nu - q^2 g_{\mu\nu}) \times \frac{F_\rho^2}{q^2 - m_\rho^2},$$

Vector field

$$V = V^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & \rho^+ & K^{*+} & \bar{D}^{*0} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega'}{\sqrt{6}} + \frac{\psi}{\sqrt{12}} & K^{*0} & D^{*-} \\ K^{*-} & \bar{K}^{*0} & -\sqrt{\frac{2}{3}}\omega' + \frac{\psi}{\sqrt{12}} & D_s^{*-} \\ D^{*0} & D^{*+} & D_s^{*+} & -\frac{3}{\sqrt{12}}\psi \end{pmatrix},$$

Axial vector field

$$A = A^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & a_1^+ & K_1^+ & \bar{D}_1^0 \\ a_1^- & -\frac{a_1^0}{\sqrt{2}} + \frac{f_1}{\sqrt{6}} + \frac{\chi_{c1}}{\sqrt{12}} & K_1^0 & D_1^- \\ K_1^- & \bar{K}_1^0 & -\sqrt{\frac{2}{3}} f_1 + \frac{\chi_{c1}}{\sqrt{12}} & D_{s1}^- \\ D_1^0 & D_1^+ & D_{s1}^+ & -\frac{3}{\sqrt{12}}\chi_{c1} \end{pmatrix},$$

$$a = 1, 2, ..., 15$$

Pseudoscalar field

$$\pi = \pi^a t^a = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & \pi^+ & K^+ & \bar{D}^0 \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} + \frac{\eta_c}{\sqrt{12}} & K^0 & D^- \\ K^- & \bar{K}^0 & -\sqrt{\frac{2}{3}}\eta + \frac{\eta_c}{\sqrt{12}} & D_s^- \\ D^0 & D^+ & D_s^+ & -\frac{3}{\sqrt{12}}\eta_c \end{pmatrix}.$$

EOM for hadrons: eigenvalues give hadron spectra

$$-s_n'' + V_s(z)s_n = m_n^2 s_n,$$
$$-\pi_n'' + V_{\pi,\varphi}\pi_n = m_n^2(\pi_n - e^{A_s}\chi\varphi_n),$$
$$-\varphi_n'' + V_\varphi\varphi_n = g_5^2 e^{A_s}\chi(\pi_n - e^{A_s}\chi\varphi_n),$$
$$-v_n'' + V_v(z)v_n = m_{n,v}^2 v_n,$$
$$-a_n'' + V_a a_n = m_n^2 a_n,$$

$$\langle J_\mu^{V,a} J_\nu^{V,b}\rangle \simeq \delta^{ab}(q_\mu q_\nu - q^2 g_{\mu\nu}) \times \frac{F_\rho^2}{q^2 - m_\rho^2},$$

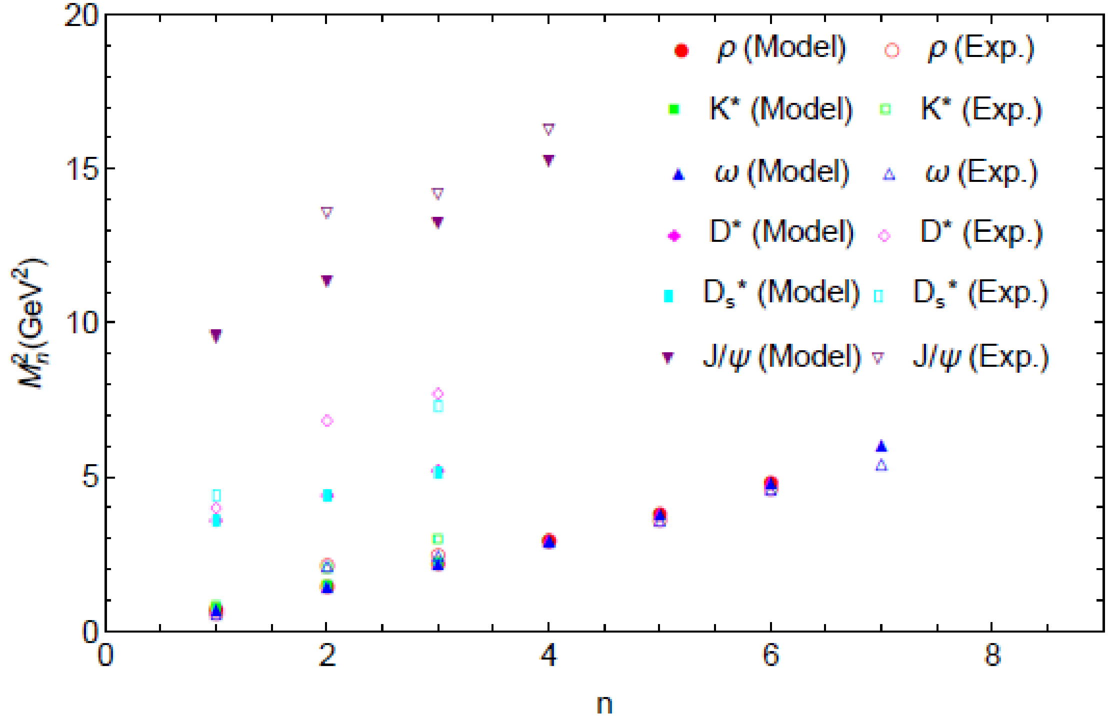
$$V_s = \frac{3A_s'' - \Phi''}{2} + \frac{(3A_s' - \Phi')^2}{4} + e^{2A_s}V_{C,\chi\chi},$$
$$V_{\pi,\varphi} = \frac{3A_s'' - \Phi'' + 2\chi''/\chi - 2\chi'^2/\chi^2}{2} + \frac{(3A_s' - \Phi' + 2\chi'/\chi)^2}{4},$$
$$V_\varphi = \frac{A_s'' - \Phi''}{2} + \frac{(A_s' - \Phi')^2}{4},$$
$$V_v = \frac{A_s'' - \Phi''}{2} + \frac{(A_s' - \Phi')^2}{4},$$
$$V_a = \frac{A_s' - \Phi'}{2} + \frac{(A_s' - \Phi')^2}{4} + g_5^2 e^{2A_s}\chi^2.$$

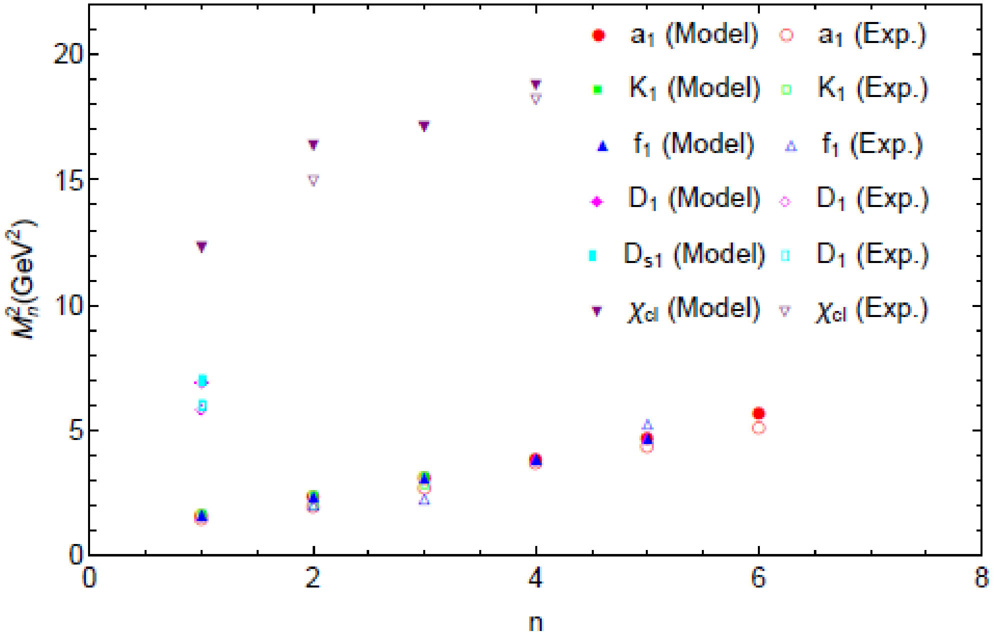
4-flavor hadron spectra: ground state and excitation states

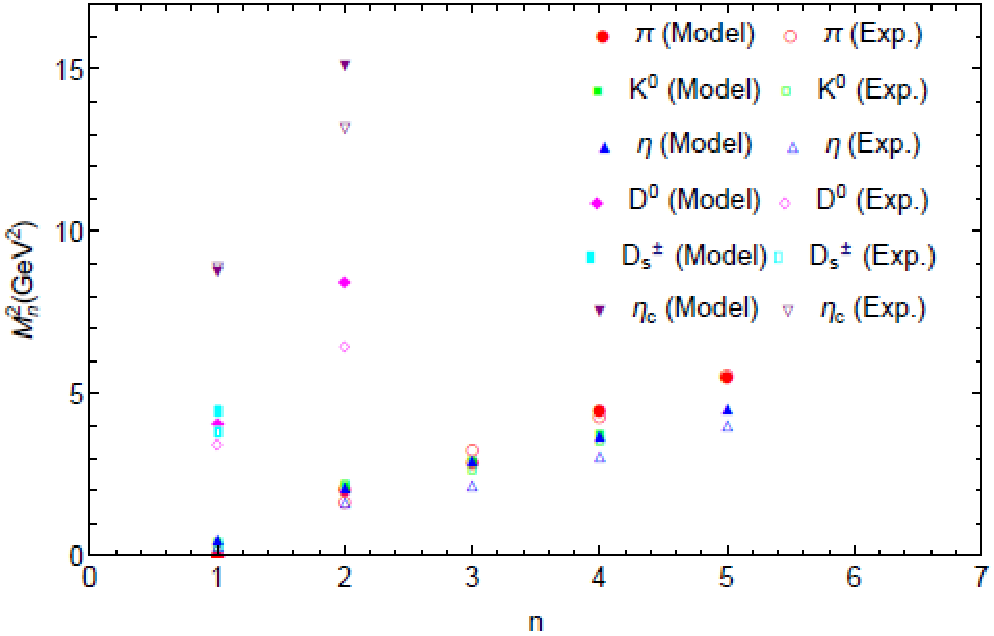
H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975 *Phys.Rev.D 108 (2023) 8, 086034)*, arXiv:2309.06156

$\mu = 0.43$	$\sigma_u = (0.2962)^3$
$m_u = 0.0032$	$\sigma_s = (0.2598)^3$
$m_s = 0.1423$	$\sigma_c = (0.302)^3$
$m_c = 1.5971$	$z_m = 10$
$m_{hc} = 1.985$	$\kappa = 30$

The values of the free parameters with the unit of GeV







Three-point correlation function gives EM and semi-leptonic form factors

$$S^{(3)} = -\int d^5x \left\{ \eta^{MN} \frac{e^{-\phi(z)}}{z^3} \left(2\left(A_M^a - \partial_M \pi^a\right) V_N^b \pi^c g^{abc} + V_M^a \left(\partial_N\left(\pi^b \pi^c\right) - 2A_M^b \pi^c\right) h^{abc} \right.\right.$$
$$\left.\left. - V_M^a V_N^b \pi^c k^{abc}\right) + \frac{e^{-\phi(z)}}{2g_5^2 z} \eta^{MP} \eta^{NQ} (V_{MN}^a V_P^b V_Q^c + V_{MN}^a A_P^b A_Q^c + A_{MN}^a V_P^b A_Q^c + A_{MN}^a A_P^b V_Q^c) f^{bca} \right\}$$

$$\left\langle 0 \left| \mathcal{T}\left\{ J_{A\parallel}^{\alpha a}(x) J_{V\perp}^{\mu b}(y) J_{A\parallel}^{\beta c}(w) \right\} \right| 0 \right\rangle = -\frac{\delta^3 \, \mathrm{S}(V\pi\pi)}{\delta A_{\parallel\alpha}^{0a}(x) \delta V_{\perp\mu}^{0b}(y) \delta A_{\parallel\beta}^{0c}(w)},$$

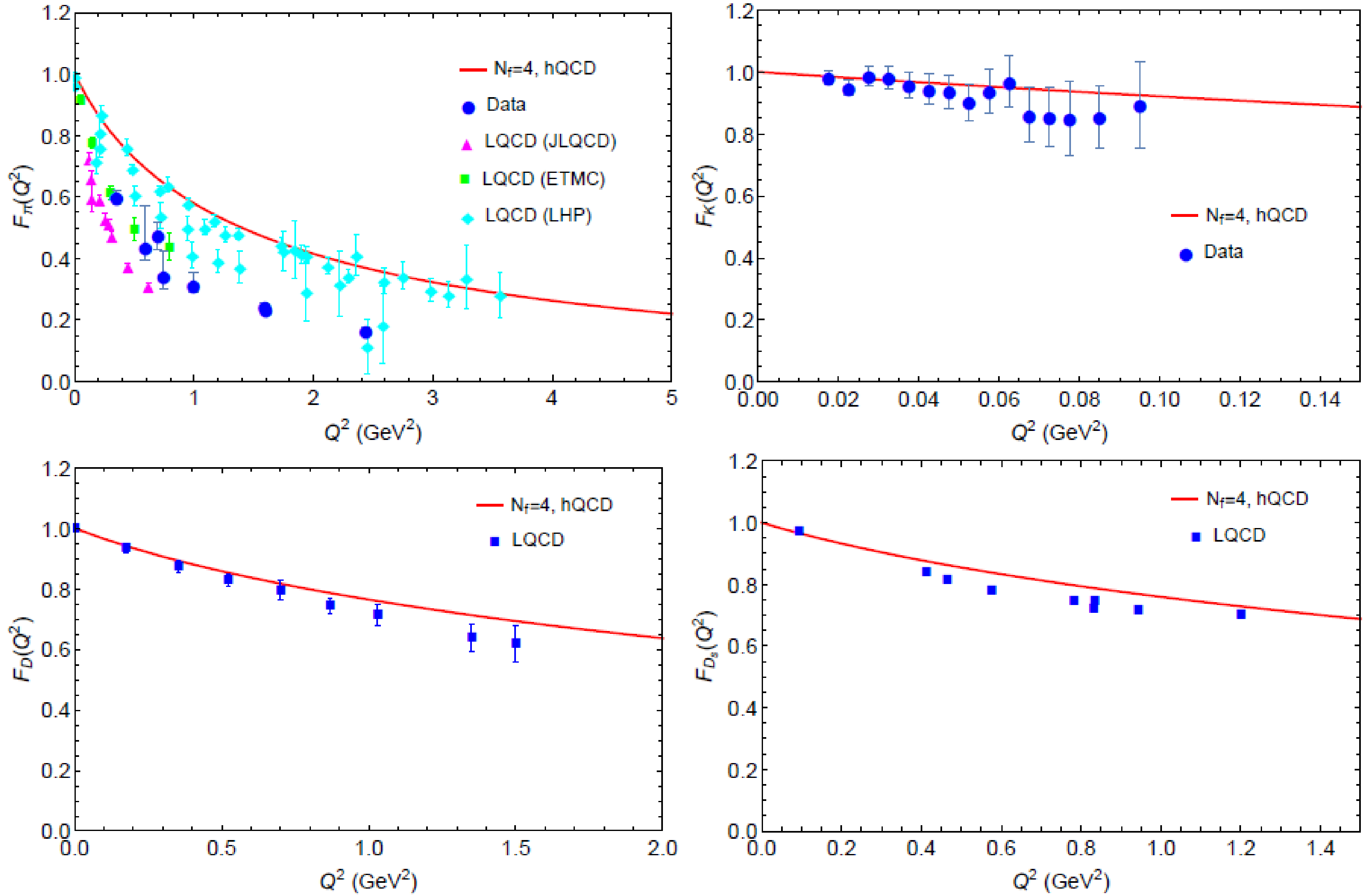
$$\left\langle 0 \left| \mathcal{T}\left\{ J_{V\perp}^{\mu a}(x) J_{V\perp}^{\nu b}(y) J_{V\perp}^{\alpha c}(w) \right\} \right| 0 \right\rangle = -\frac{\delta^3 \, \mathrm{S}(VVV)}{\delta V_{\perp\mu}^{0a}(x) \delta V_{\perp\nu}^{0b}(y) \delta V_{\perp\alpha}^{0c}(w)},$$

$$\left\langle 0 \left| \mathcal{T}\left\{ J_{A\perp}^{\alpha a}(x) J_{V\perp}^{\mu b}(y) J_{A\perp}^{\beta c}(w) \right\} \right| 0 \right\rangle = -\frac{\delta^3 \, \mathrm{S}(VAA)}{\delta A_{\perp\alpha}^{0a}(x) \delta V_{\perp\mu}^{0b}(y) \delta A_{\perp\beta}^{0c}(w)},$$

H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975 *Phys.Rev.D 108 (2023) 8, 086034)*, arXiv:2309.06156

Psudoscalar mesons

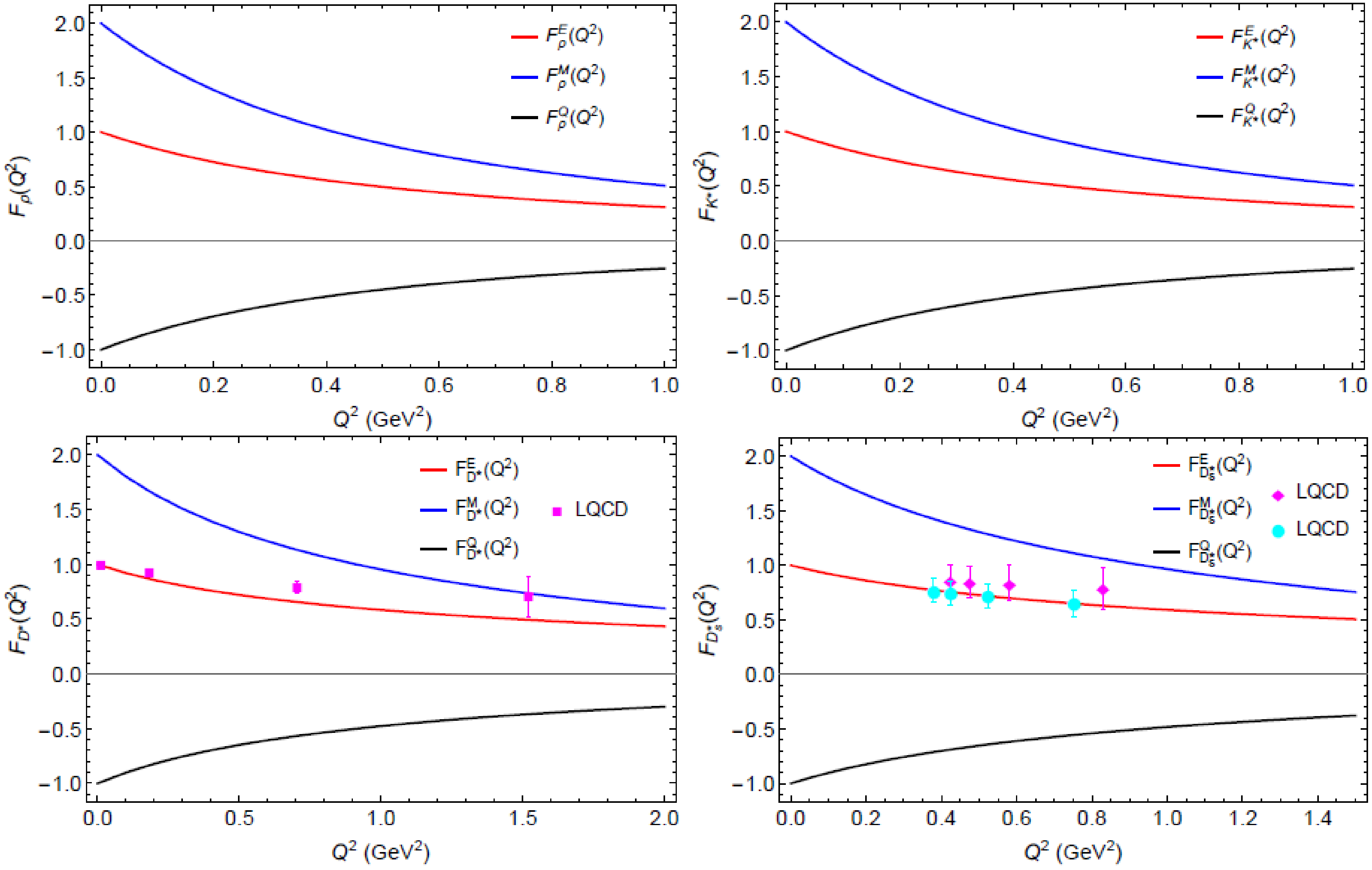
H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975 *Phys.Rev.D 108 (2023) 8, 086034)*, arXiv:2309.06156



Data(π)(G. M. Huber et al., prc(2008)), LQCD(JLQCD)(T. Kaneko et al., pos (2008)), LQCD(ETMC)(R. Frezzotti et al., prd(2009)), LQCD(LHP)(F. D. R. Bonnet et al., prd(2005)) Data(K)(S. R. Amendolia et al., plb(1986)), LQCD(D)(K. U. Can et al., PLB(2013)), LQCD(D_s)(N. Li, Y. J. Wu, EPJA(2017))

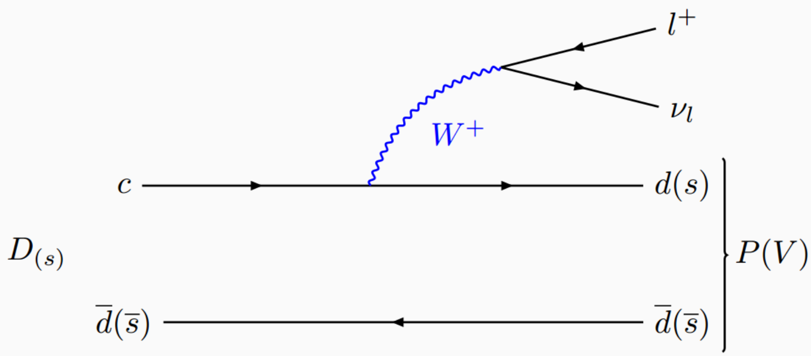
Vector mesons

H.Ameld, Y.D. Chen, M.H.arXiv:2308.14975, arXiv:2309.06156



Semileptonic decays

- Semileptonic decays involve the transition of a heavy meson (such as B or D) to a lighter meson via the exchange of a W boson.
- Understanding the form factors governing these transitions is essential for precision measurements of CKM matrix elements and testing the Standard Model.



- The matrix elements within the SM is defined by (M. A. Ivanov et al., Front. Phys.(2019))

$$\mathcal{M}\left(D_{(s)} \to (P,V) l^+ \nu_l\right) = \frac{G_F}{\sqrt{2}} V_{cq}^* \left\langle (P,V) | \bar{q} \gamma^\mu (1-\gamma_5) c | D_{(s)} \right\rangle \bar{\nu}_l \gamma^\mu (1-\gamma_5) l$$

- The transition form factors are defined by (M. Wirbel et al., Z. Phys. C(1985))

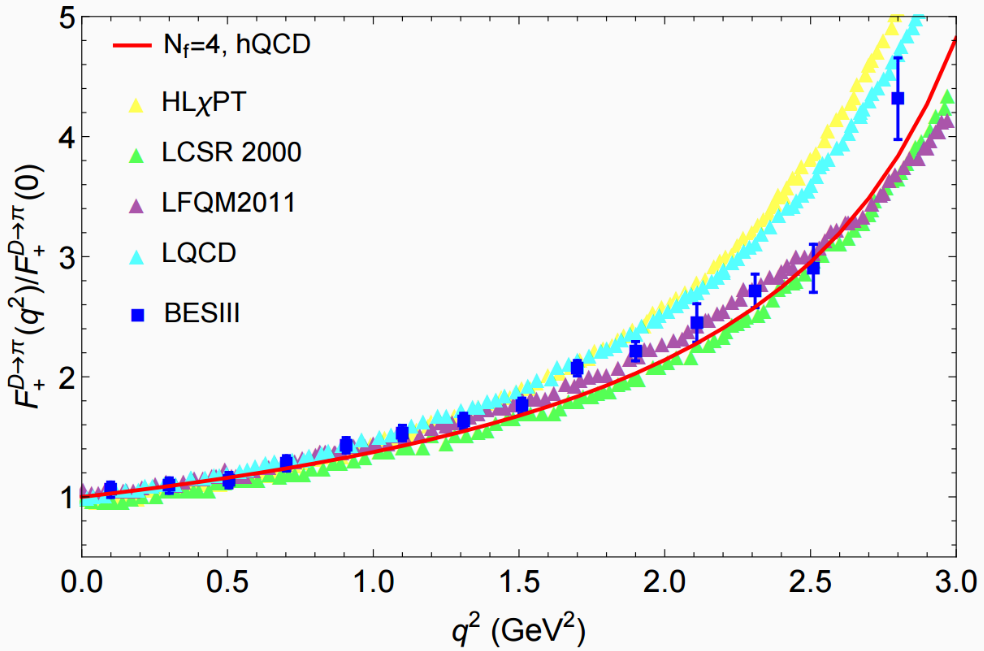
$$\left\langle P\left(p_2\right) \left| V^\mu \right| D_{(s)}\left(p_1\right)\right\rangle = F_+\left(q^2\right)\left[P^\mu - \frac{M_1^2 - M_2^2}{q^2} q^\mu\right] + F_0\left(q^2\right) \frac{M_1^2 - M_2^2}{q^2} q^\mu$$

$$\left\langle V\left(p_2, \epsilon_2\right) \left| V^\mu - A^\mu \right| D_{(s)}\left(p_1\right)\right\rangle = -\left(M_1 + M_2\right) \epsilon_2^{*\mu} A_1\left(q^2\right) + \frac{\epsilon_2^* \cdot q}{M_1 + M_2} P^\mu A_2\left(q^2\right)$$
$$+ 2M_2 \frac{\epsilon_2^* \cdot q}{q^2} q^\mu \left[A_3\left(q^2\right) - A_0\left(q^2\right)\right] + \frac{2i\varepsilon_{\mu\nu\rho\sigma} \epsilon_2^{*\nu} p_1^\rho p_2^\sigma}{M_1 + M_2} V\left(q^2\right)$$

$D \to \pi l^+ \nu_l$

$$F_+(q^2) = \int dz \frac{e^{-\phi(z)}}{z} \left(f^{abc} \partial_z \phi^a \mathcal{V}^b(q^2, z) \partial_z \phi^c - \frac{2g_5^2}{z^2} (\pi^a - \phi^a) \mathcal{V}^b(q^2, z)(\pi^c - \phi^c)(g^{abc} - h^{bac}) \right)$$

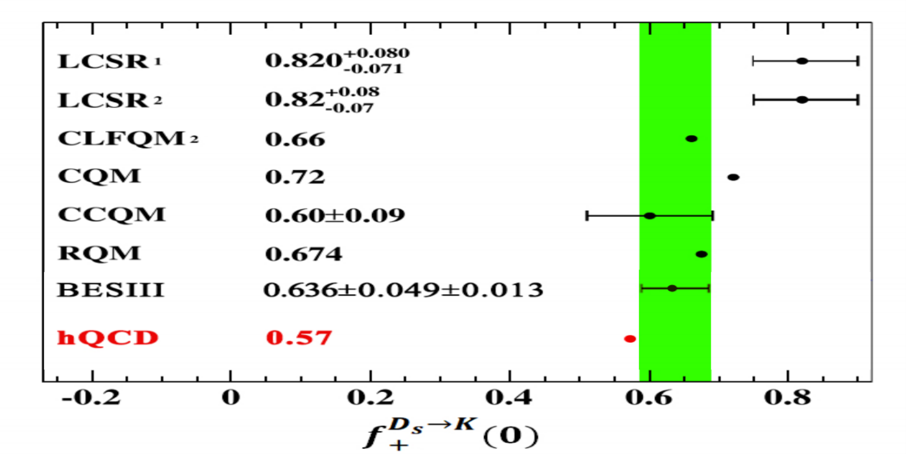
FFs	hQCD	LCSR2000	HLχPT	LFQM2011	LQCD	Exp.
$f_+^{D\to\pi}(0)$	0.58	0.65	0.61	0.66	0.64	0.6372$\pm$0.008

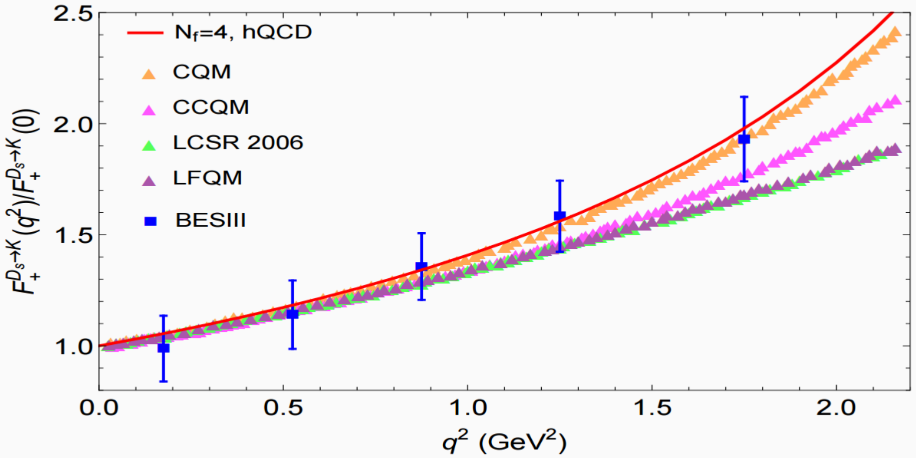


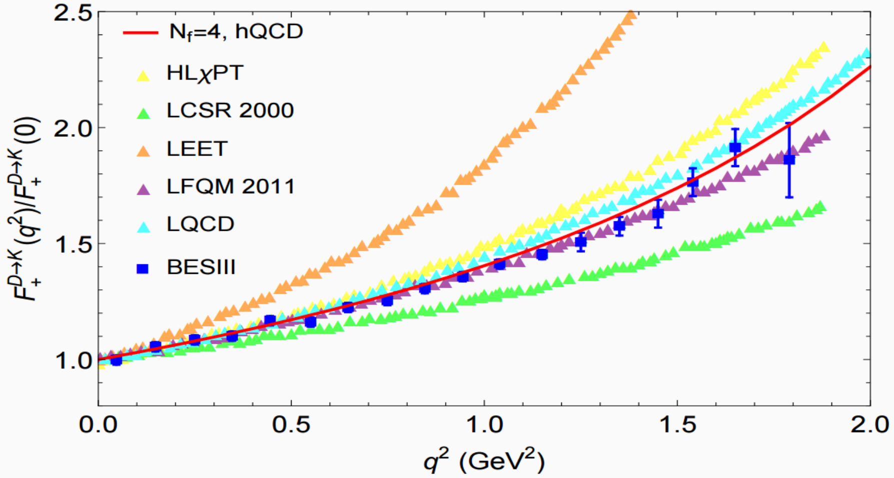
- HLχPT: S. Fajfer and J. F. Kamenik, prd(2005)
- LCSR: A. Khodjamirian et al., prd(2000)
- LFQM: R. C. Verma, J. Phys. G (2012)
- LQCD: C. Aubin et al, prl(2005)
- BESIII: M. Ablikim et al., prd(2015)

$D_{(s)} \to K l^+ \nu_l$

$$F_+(q^2) = \int dz \frac{e^{-\phi(z)}}{z} \left(f^{abc} \partial_z \phi^a \mathcal{V}^b(q^2, z) \partial_z \phi^c - \frac{2g_5^2}{z^2} (\pi^a - \phi^a) \mathcal{V}^b(q^2, z)(\pi^c - \phi^c)(g^{abc} - h^{bac}) \right)$$







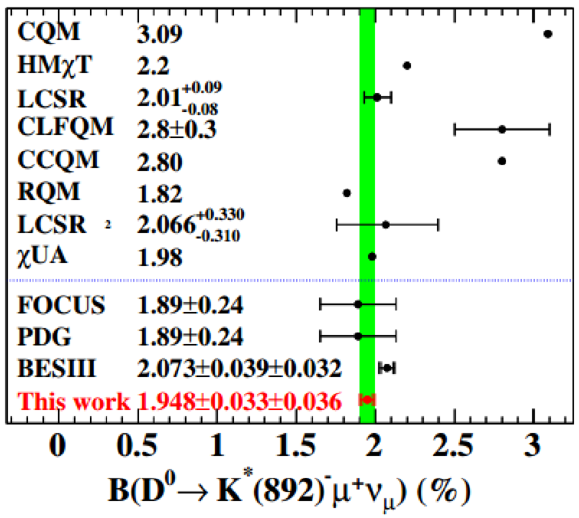
HLχPT: S. Fajfer and J. F. Kamenik, prd(2005) ; LCSR: A. Khodjamirian et al., prd(2000) ; LEET: J. Charles et al., prd(1999) ; LFQM: R. C. Verma, J. Phys. G (2012) ; LQCD: C. Aubin et al, prl(2005) ; BESIII: M. Ablikim et al., prd(2015) ; CQM: D. Melikhov and B. Stech, prd(2000) ; CCQM: N. R. Son et al., prd(2018) ; LCSR: Y. L. Wu, et al., IJMPA(2006) ; BESIII: M. Ablikim et al., prl(2019)

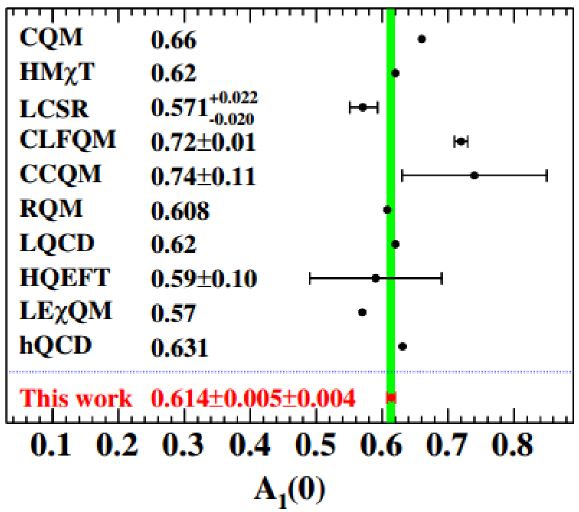
$D \to K^*$

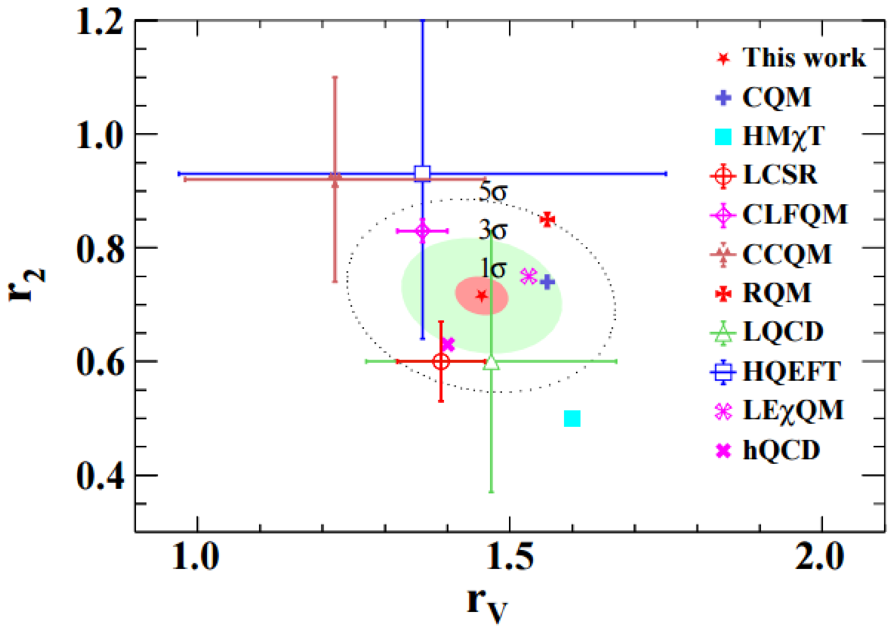
$$V(q^2) = \frac{(M_1+M_2)g_5^2}{2}\int dz \frac{e^{-\phi(z)}}{z^3} k^{abc} V^a(z)\mathcal{V}^b(q^2,z)\pi^c(z)$$

$$A_1(q^2) = \int dz \frac{e^{-\phi(z)}}{z}\left(\frac{M_1^2+M_2^2-q^2}{2(M_1+M_2)}\right) f^{bac}\mathcal{A}^a(q^2,z)V^b(z)\phi^c(z)$$
$$-\int dz \frac{e^{-\phi(z)}}{z^3}\frac{2g_5^2}{(M_1+M_2)}\mathcal{A}^a(q^2,z)V^b(z)\pi^c(z)(g^{abc}-h^{bac})$$

$$r_V = \frac{V(q^2=0)}{A_1(q^2=0)}$$
$$r_2 = \frac{A_2(q^2=0)}{A_1(q^2=0)}$$







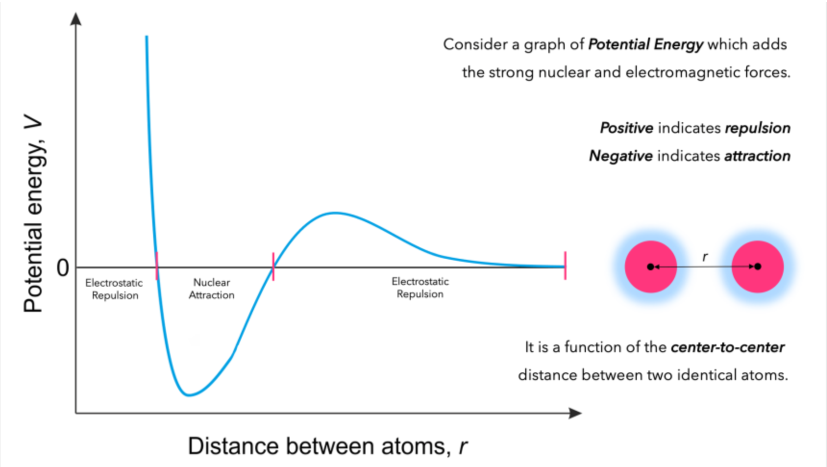
BESIII Collaboration, Phys.Rev.Lett. 135 (2025) 11, 111803

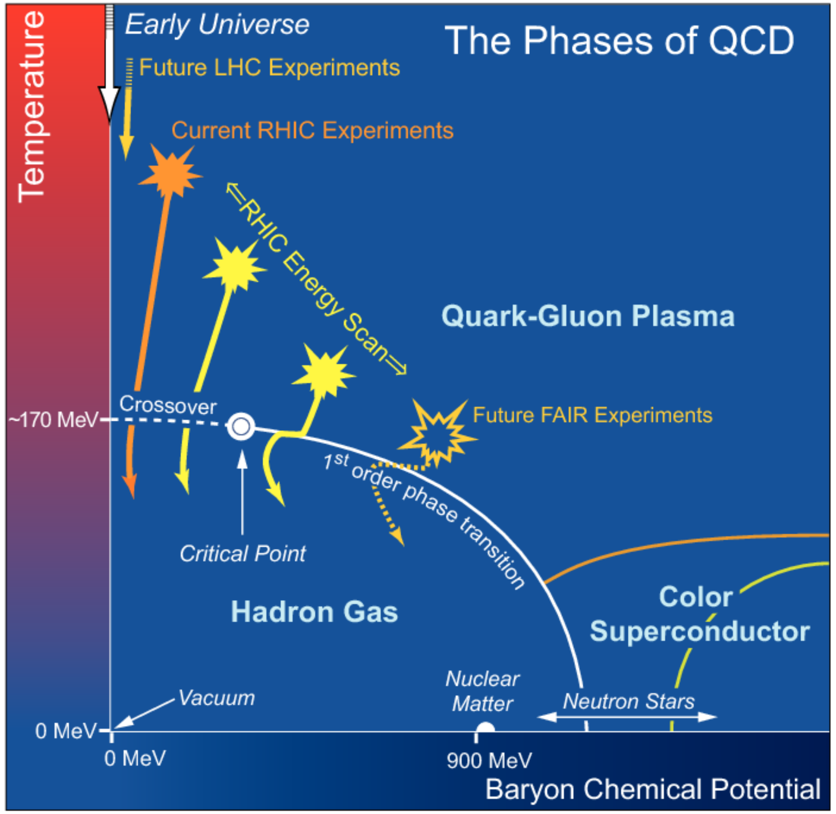
II. Quantitative era of hQCD under the help of AI

1, Status of CEP from theory;
2, QCD phase transitions under rotation

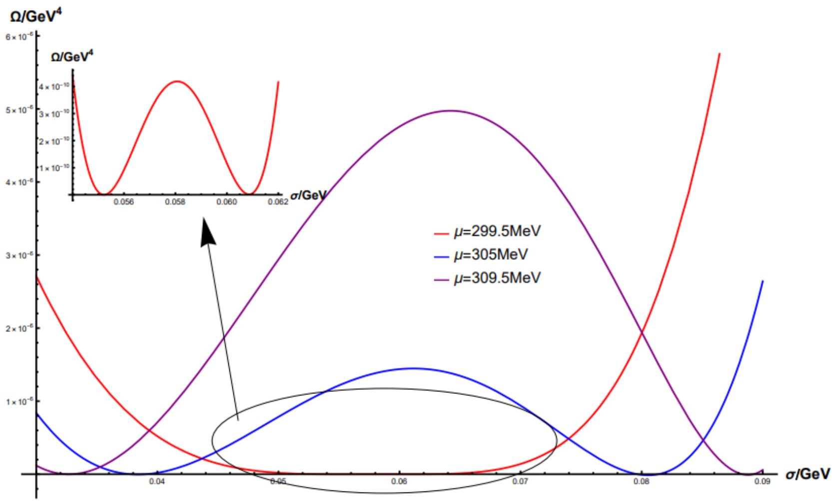
Existence of CEP at high baryon chemical potential

potential barrier is needed for 1st-order phase transition

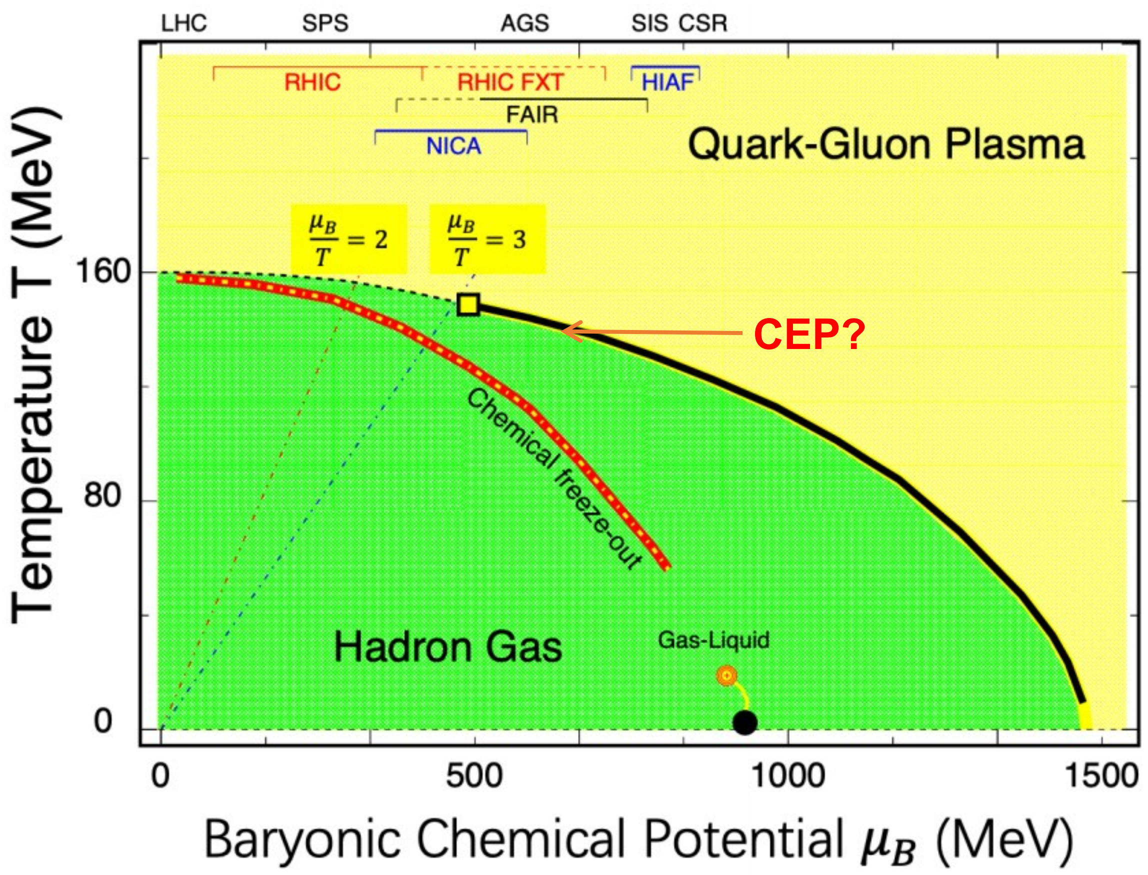


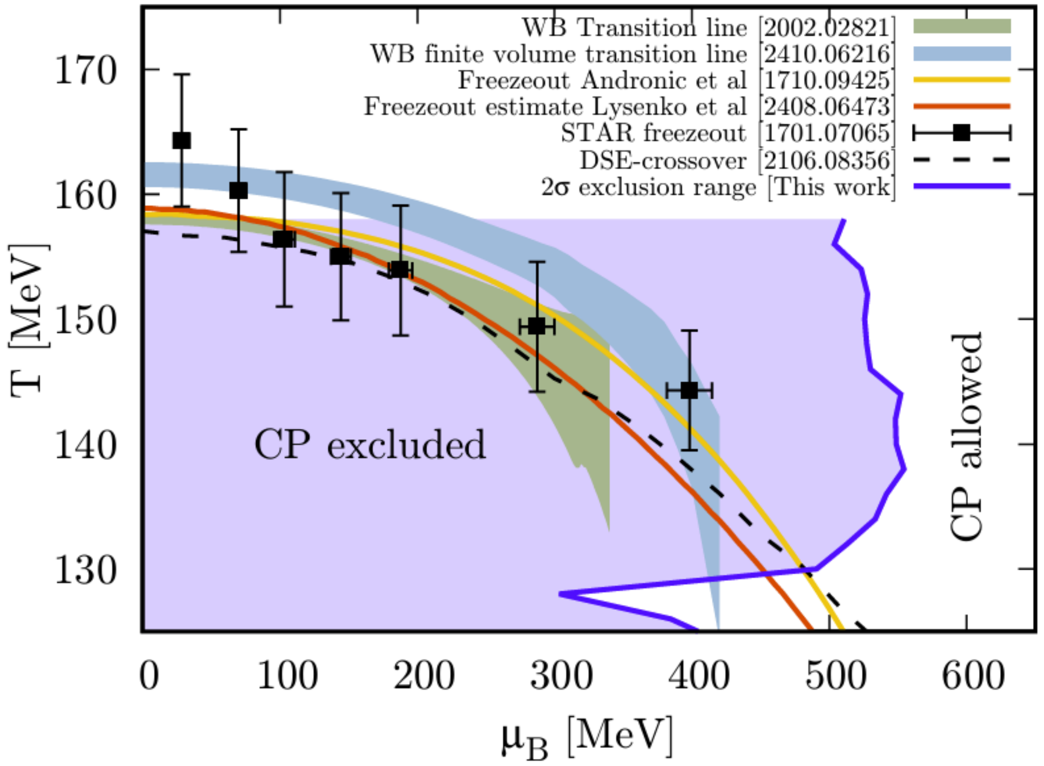


mu_B plays the role of repulsive vector interaction, potential barrier develops when mu_B increases, indicating a 1st-order phase transition.



Latest lattice constraints on CEP





Lattice QCD constraints on the critical point,
Szabolcs Bors´anyi,1 Zolt´an Fodor, et.al., …arXiv: 2502.10267

EOS for cold QCD matter

The Einstein-Maxwell-dilaton system at finite baryon density

$$S_E = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$ds^2 = \frac{L^2 e^{2A(z)}}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + d\vec{x}^2\right]$$

the components of the vector field AM(z) are zero except the t component At(z). $A_t(0) = \mu + \rho' z^2 + \cdots$

$$\phi'(z) = \sqrt{-6\left(A'' - A'^2 + \frac{2}{z}A'\right)},$$

$$A_t(z) = \mu \frac{\int_{z_H}^{z} \frac{y}{e^A f} dy}{\int_{z_H}^{0} \frac{y}{e^A f} dy},$$

$$g(z) = 1 + \frac{1}{\int_0^{z_H} y^3 e^{-3A} dy}\left[-\int_0^z y^3 e^{-3A} dy + \left(\frac{\mu}{\int_0^{z_H} \frac{y}{e^A f} dy}\right)^2 \hat{G}\right],$$

$$V(z) = -3z^2 g e^{-2A}\left[A'' + 3A'^2 + \left(\frac{3g'}{2g} - \frac{6}{z}\right)A' - \frac{1}{z}\left(\frac{3g'}{2g} - \frac{4}{z}\right) + \frac{g''}{6g}\right],$$

$$T = \frac{1}{4\pi}\frac{z_H^3 e^{-3A(z_H)}}{\int_0^{z_H} y^3 e^{-3A(z)} dy}\left[1 - \left(\frac{\mu}{\int_0^{z_H} \frac{y}{e^A f} dy}\right)^2 \left(\int_0^{z_H} y^3 e^{-3A} dy \cdot \int_0^{z_H} \frac{x}{e^A f} dx - \int_0^{z_H} y^3 e^{-3A} dy \int_0^{y} \frac{x}{e^A f} dx\right)\right],$$

$$s = \frac{e^{3A(z_H)}}{4G_5 z_H^3},$$

$$\rho = \frac{1}{16\pi G_5}\frac{\mu}{\int_0^{z_H} \frac{y}{e^A f} dy}.$$

$$F = -\int s dT - \rho d\mu.$$

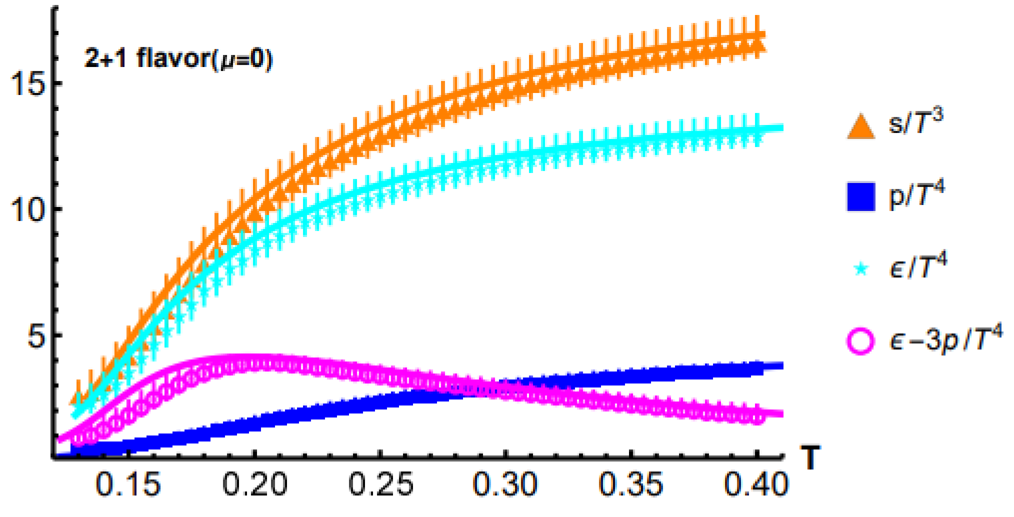
$$\epsilon = -p + sT + \mu\rho. \qquad \chi_2^B = \frac{1}{T^2}\frac{\partial\rho}{\partial\mu}.$$

Theoretical predictions for the location of CEP

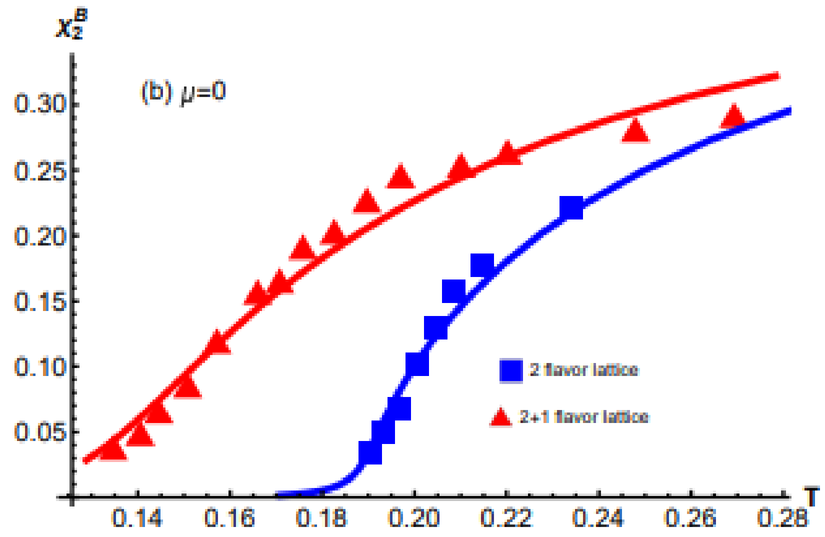
Nonperturbative theoretical calculations (rPNJL model, DSE-fRG,fRG,holographic QCD):

Strategy of model calculations:

1, Fit model paraters with Lattice QCD EOS and baryon number susceptibility at zero chemical potential;
2,Predictions at finite baryon number chemical potential.



Determine Tc at mu=0



Determine mu dependent Tc

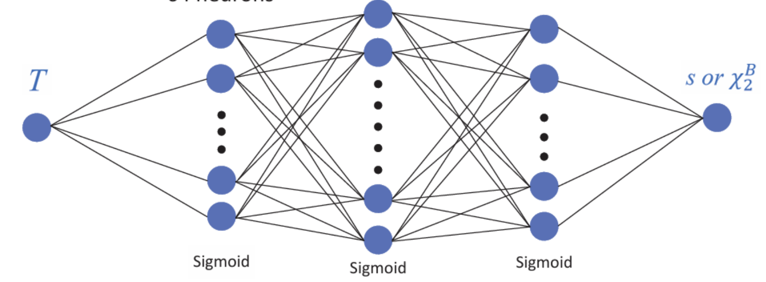
ML+hQCD:Xun Chen, MH, Phys.Rev.D 109 (2024) 5, L051902, e-Print:2401.06417

Either input A(z) or V(z)

$$S_E = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$A(z) = d\ln(az^2+1) + d\ln(bz^4+1)$$ Input A(z) ansatz

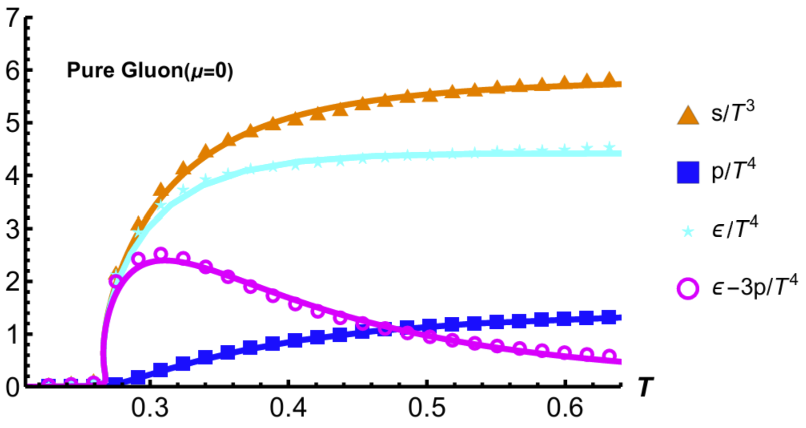
$$f(z) = e^{cz^2 - A(z) + k}.$$

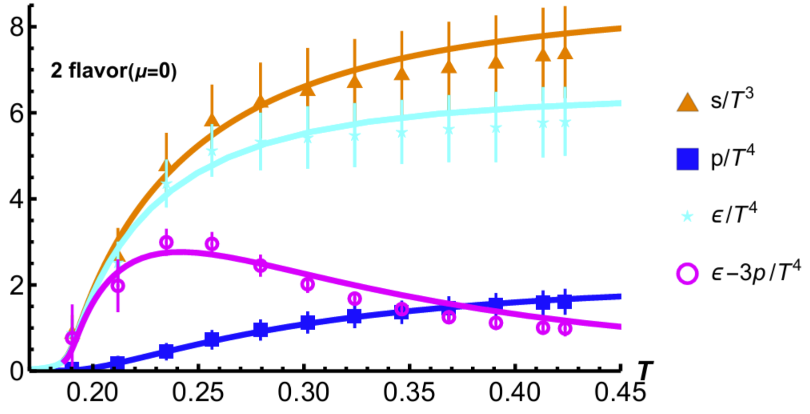


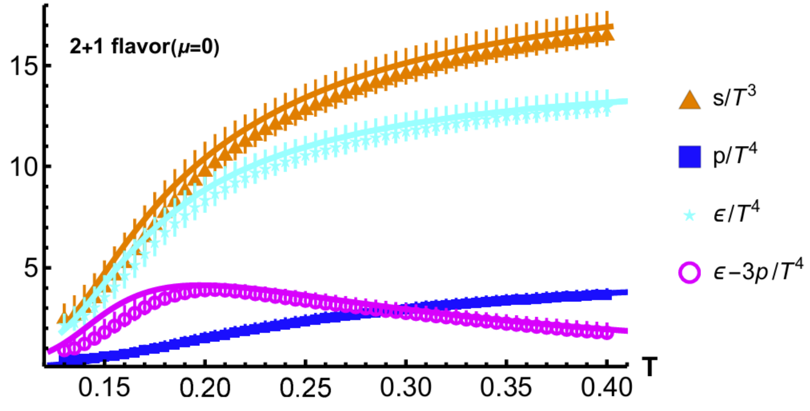
The neural network here is used to determine the model parameters from lattice EOS and baryon number susceptibility

	a	b	c	d	k	G_5	T_c
$N_f=0$	0	0.072	0	-0.584	0	1.326	0.265
$N_f=2$	0.067	0.023	-0.377	-0.382	0	0.885	0.189
$N_f=2+1$	0.204	0.013	-0.264	-0.173	-0.824	0.400	0.128

ML+hQCD:Xun Chen, MH, Phys.Rev.D 109 (2024) 5, L051902, e-Print:2401.06417 [hep-ph];e-Print: 2405.06179 [hep-ph]

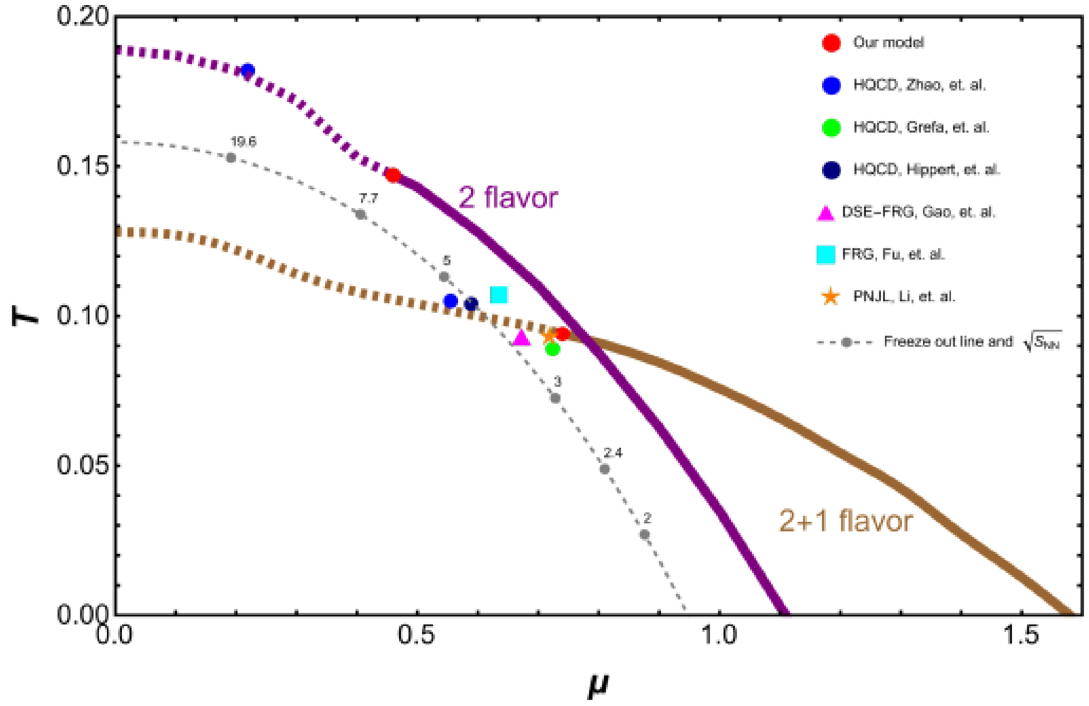


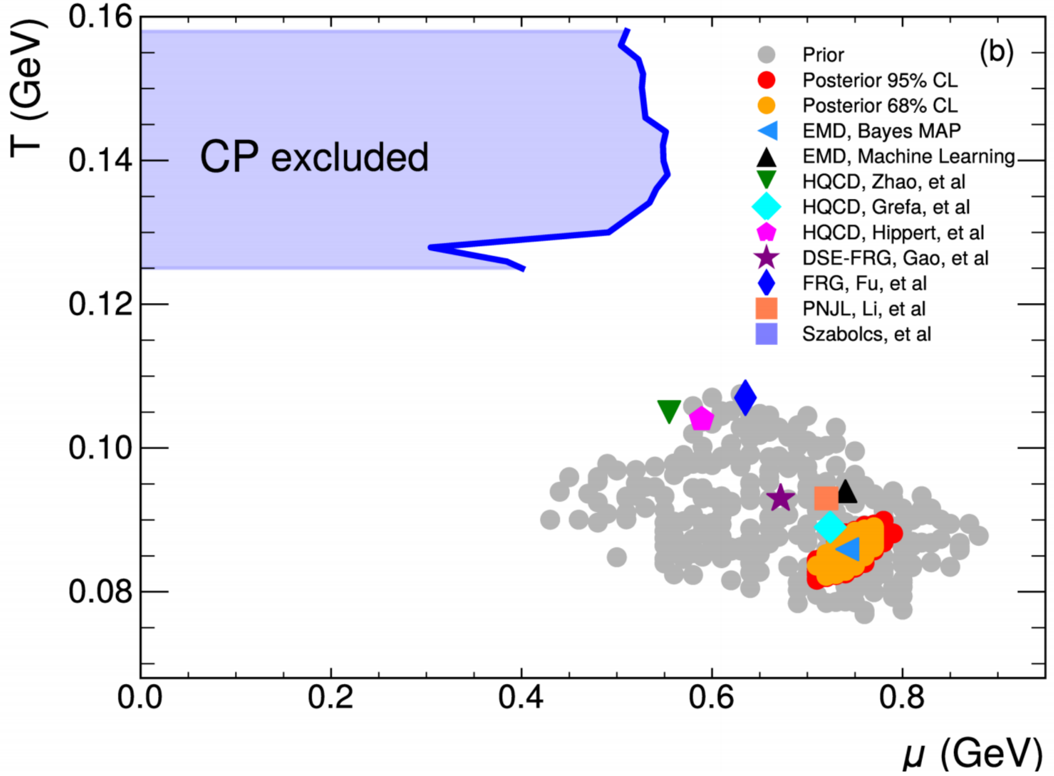




Equilibrium state

Locations of CEP from rPNJL model, holographic QCD models, DSE-fRG,fRG converge at around (Tc~100MeV, mu_B^c~700 MeV)





ML+hQCD:Xun Chen, MH, Phys.Rev.D 109 (2024) 5, L051902, e-Print:2401.06417 [hep-ph];e-Print: 2405.06179 [hep-ph]

J. Grefa, J. Noronha, J. Noronha-Hostler, I. Portillo, C. Ratti, and R. Rougemont, Phys. Rev. D 104, 034002 (2021), arXiv:2102.12042 [nucl-th]; M. Hippert, J. Grefa, T. A. Manning, J. Noronha, J. Noronha-Hostler, I. Portillo Vazquez, C. Ratti, R. Rougemont, and M. Trujillo (2023) arXiv:2309.00579 [nucl-th], Y.-Q. Zhao, S. He, D. Hou, L. Li, and Z. Li, JHEP 04, 115 (2023), arXiv:2212.14662 [hep-ph]

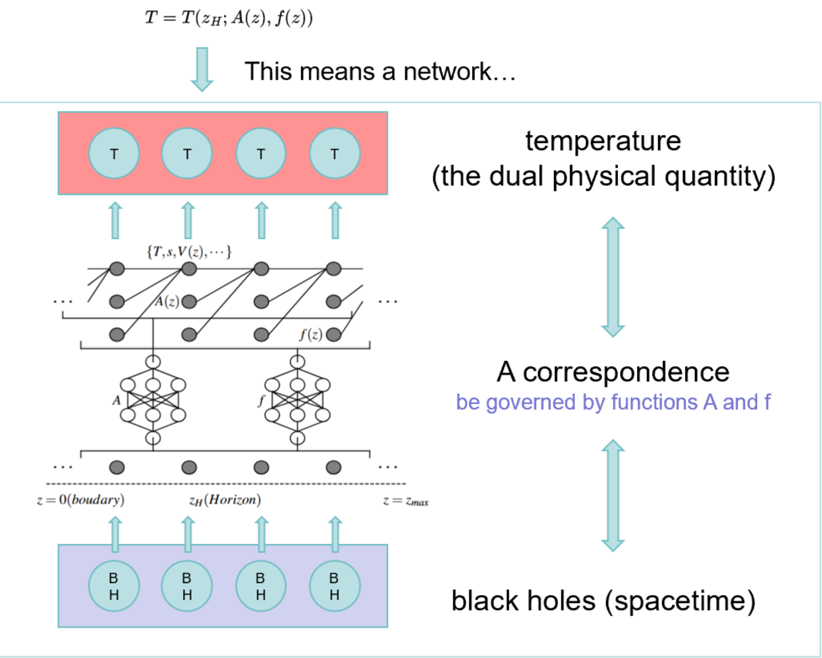
rPNJL:Zhibin Li, Kun Xu, Xinyang Wang and MH, arXiv:1801.09215

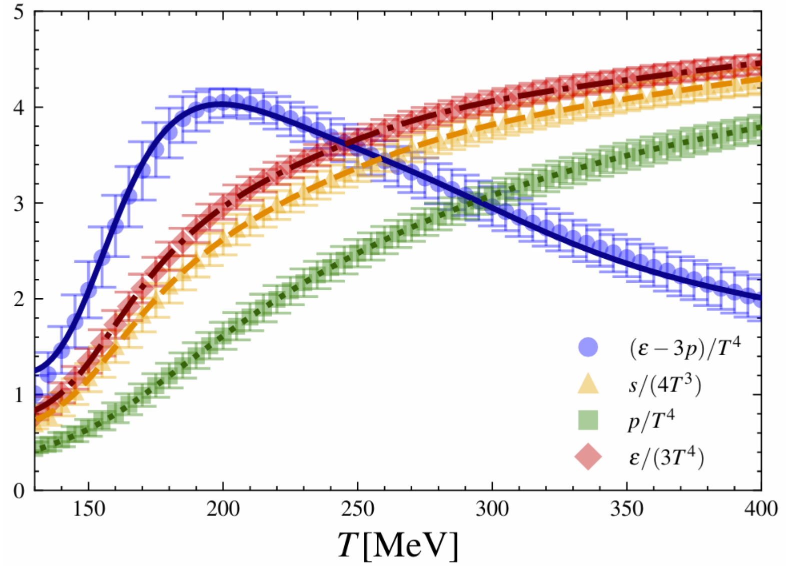
DSE-fRG: *F. Gao and J. M. Pawlowski, Phys. Rev. D 102, 034027 (2020), arXiv:2002.07500 [hep-ph].*

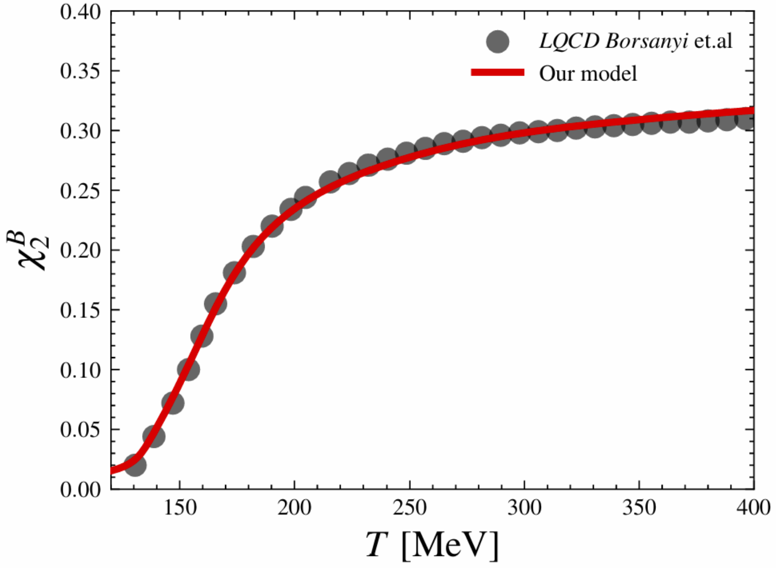
fRG:W.-j. Fu, J. M. Pawlowski, and F. Rennecke, Phys. Rev. D 101, 054032 (2020), arXiv:1909.02991 [hep-ph].

Bayesian+hQCD: Liqiang Zhu, Xun Chen, Kai Zhou, Hanzhong Zhang, M.H., e-Print: 2501.17763

Supervised learning, without input ansatz, ML lattice data and output A(z) and V(z)







$$S_E = \frac{1}{16\pi G_5}\int d^5x\sqrt{-g}\left[R - \frac{f(\phi)}{4}F^2 - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

$$ds^2 = \frac{L^2 e^{2A(z)}}{z^2}\left[-g(z)dt^2 + \frac{dz^2}{g(z)} + d\vec{x}^2\right]$$

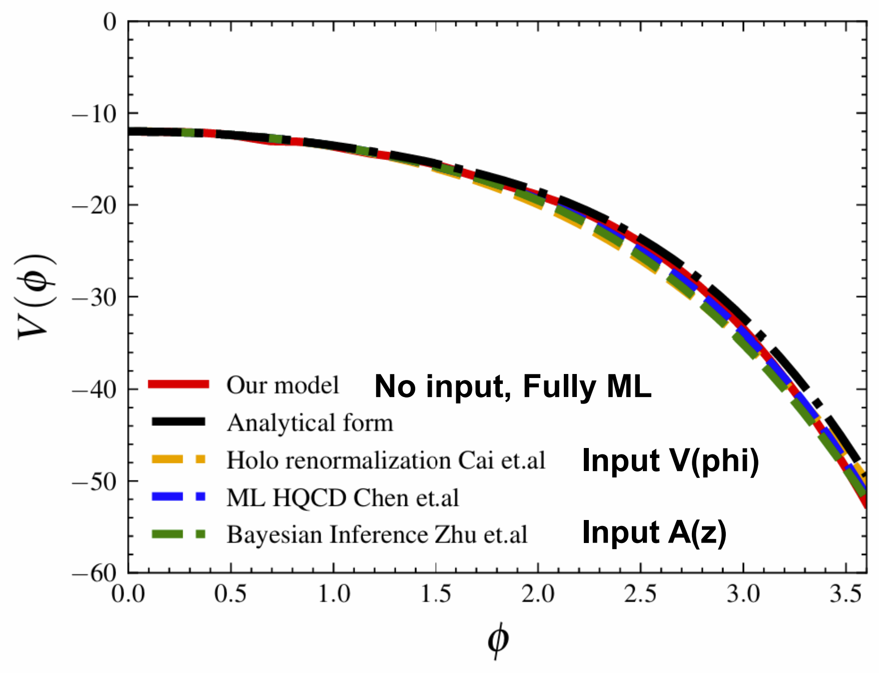
Hongan Zeng, Lingxiao Wang, MH, e-Print: 2512.06044

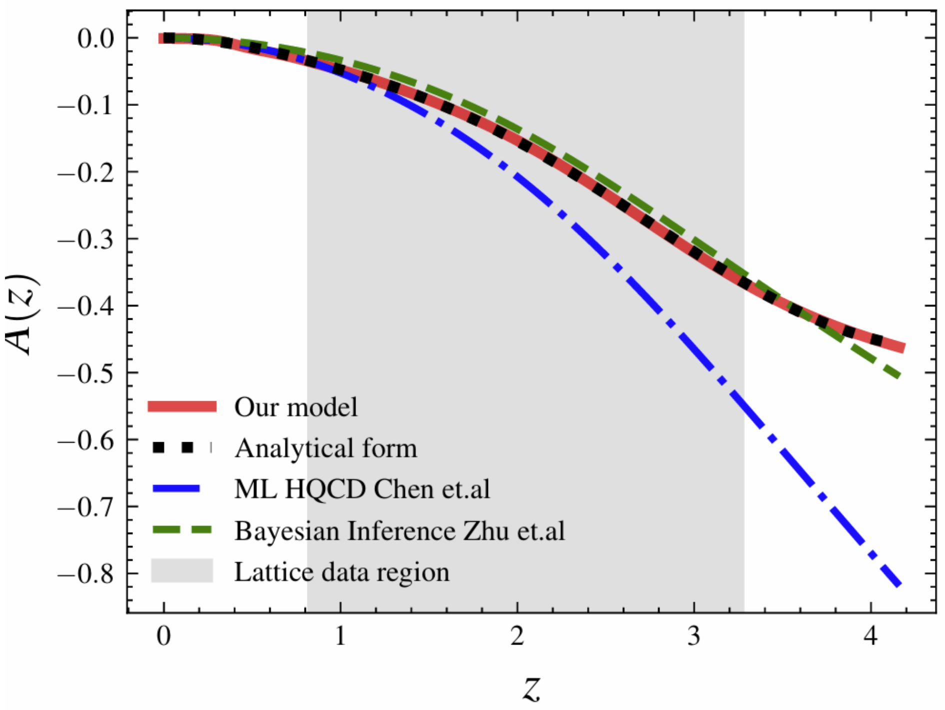
Equivalenve of machine learning A(z) and V(\phi)

$$A(z) = a_1 z^4 - a_2 \log\left(1+z^2\right) - a_3 \log\left(1+a_4 z^4\right)$$

$$f(z) = f_1 + \frac{f_2}{1+f_3(z+f_4)^2} + f_5\,\mathrm{sech}(f_6 + f_7(z+f_8)^3).$$

The corresponding constants are $a_1 = 0.00037885$, $a_2 = 0.062182$, $a_3 = 0.21002$, $a_4 = 0.020314$, $f_1 = 0.31197$, $f_2 = 0.079030$, $f_3 = 0.34070$.





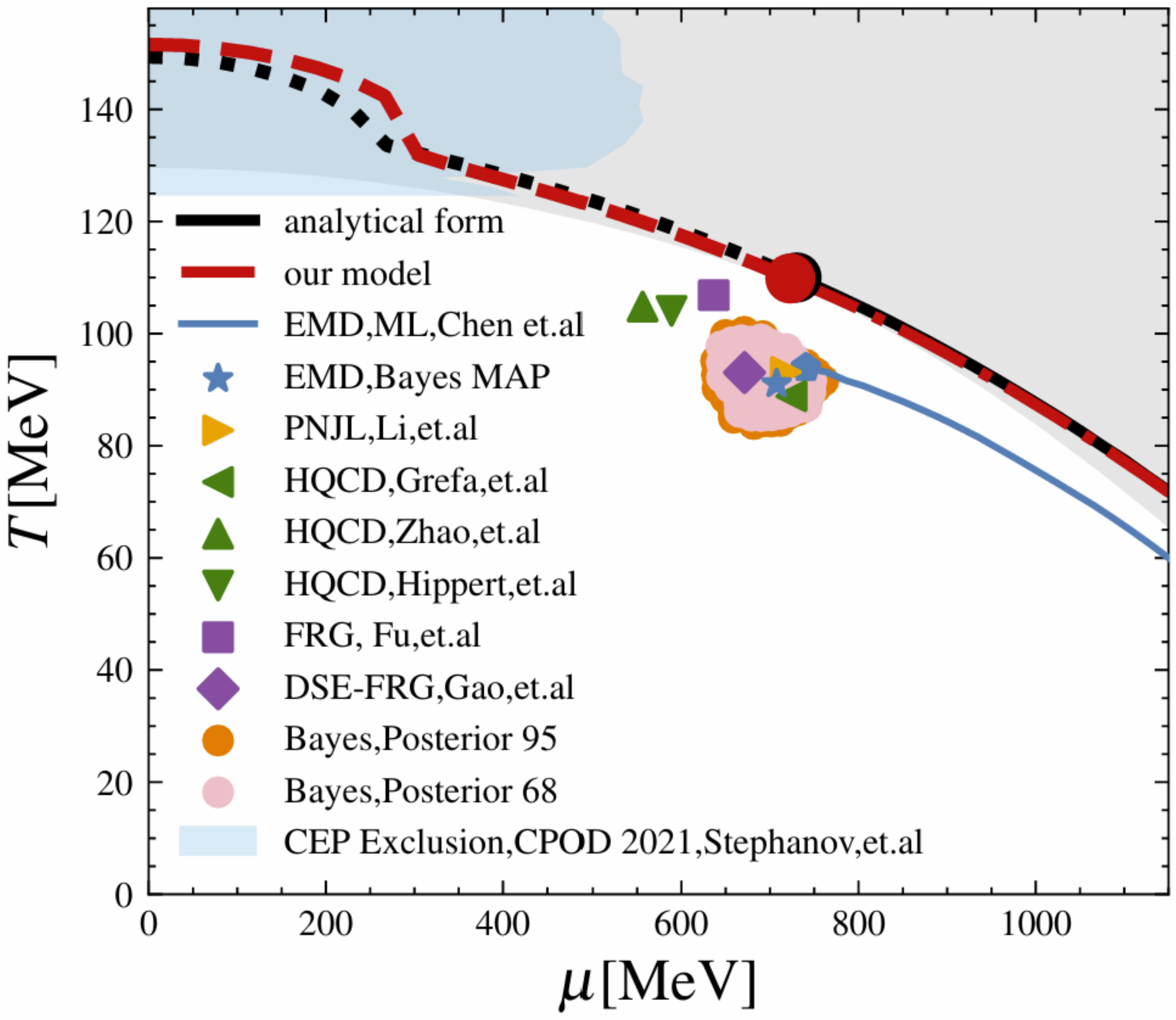
$$V(\phi) = -12\ \cosh(c_1\phi) + c_2\phi^2,$$

$$f(\phi) = c_3\ \mathrm{sech}(c_4(\phi + c_5)^3),$$

where $c_1 = 0.69048$, $c_2 = 1.2777$, $c_3 = 0.31562$, $c_4 = 0.061371$, $c_5 = 0.35482$.

Hongan Zeng, Lingxiao Wang, MH, e-Print: 2512.06044

Location of CEP



Hongan Zeng, Lingxiao Wang, MH, e-Print: 2512.06044

Conclusion and outlook

5D DhQCD offers a systematic framework to describe the emergent real world from QCD theory, including Linear confinement and chiral symmetry breaking; hadron spectra (glueball spectra, light-flavor and heavy flavor spectra) and form factors, QCD phase transitions, thermodynamical and transport propertities, and

More: 1)Nonequilibrium evolution, GW,

2) Inhomogeneous system

3) Hadron structure： PDF, ……

Future: AI will definitely accelerate building hQCD framework in quantitatively describing the nonperturbative QCD world!


中国科学院大学
University of Chinese Academy of Sciences
感谢！
Many Thanks