RELATIONSHIP BETWEEN $0\nu\beta\beta$ AND LBL MEASUREMENTS

From parameter determination to natural flavor predictions

NPN2026 (The Hirosawa City Center, Mito)

May/26/2026

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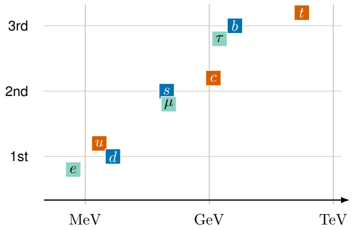
Flavor Structures in the SM + Neutrino Masses

✓ **Fermion masses : hierarchical pattern across generations**

quark/charged lepton masses



neutrino masses

✓ $\Delta m^2_{21} \simeq 7.5 \times 10^{-5}\,\mathrm{eV}^2$

$(m_2 > m_1$ with $0 < \theta_{12} < \pi/4)$

✓ $|\Delta m^2_{31}| \simeq 2.5 \times 10^{-3}\,\mathrm{eV}^2$

✓ mass ordering : not yet determined

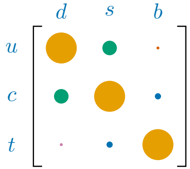
✓ absolute mass scale :

$\Sigma m_\nu < \mathcal{O}(0.1)\mathrm{eV}$(cosmology)

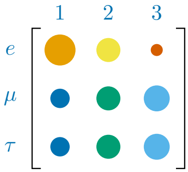
$\Sigma m_\nu < \mathcal{O}(1)\mathrm{eV}$($\beta$-decay)

✓ **Fermions with different flavors mix with each other**

CKM: small mixing



PMNS: large mixing



Flavor Puzzle?

The flavor puzzle is the question of the origin of these structures

Many flavor models have been proposed:

flavor symmetries, texture zeros, zero minors ...

However, the observed flavor structure can simply be **fitted by parameters**

$$\mathcal{L}_{\text{flavor}} = -\, y_i^u\, \delta_{ij}\, H Q_i \bar{u}_j - y_i^d\, V_{ij}^{\text{CKM}}\, H^\dagger Q_i \bar{d}_j \\ -\, y_i^\ell\, \delta_{ij}\, H^\dagger \bar{e}_i L_j - \frac{1}{2 v_{\text{EW}}^2} \left(U^* m_\nu^{\text{diag}} U^\dagger\right)_{ij} (H L_i)(H L_j) + \text{h.c.}$$

[All fermions are written as left-handed Weyl fermions]

In frequentist statistics:

✓ parameters = fixed constants

$\Rightarrow$ **flavor structure just tells us that the world is built that way.**

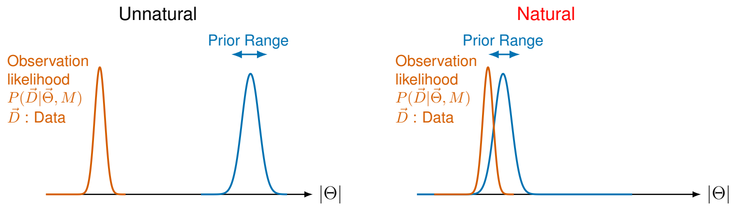
Naturalness as Typicality

To go beyond fitting, we need a notion of naturalness.

A flavor model M with $\mathcal{O}(1)$ parameters $\vec{\Theta}$

In Bayesian statistics :

- ✓ parameters = random variables
  drawn from the prior distribution $\pi(\vec{\Theta}|M)$
- ✓ Naturalness = whether the observed data lie in a typical region
  predicted by the prior



Naturalness is quantified as the overlap between likelihood and prior distribution.

$\mathcal{O}(1)$ Yukawa Coupling Constants

The prior distribution of the complex-valued 3×3 Yukawa couplings,

$$P(Y^F) = \left(\frac{1}{\sqrt{2\pi}\sigma}\right)^{18} \exp\left[-\frac{\mathrm{tr}[Y^\dagger Y]}{2\sigma^2}\right], \quad \sigma \sim 1$$

[integration measure $d\,\mathrm{Re}(Y^F_{ij})d\,\mathrm{Im}(Y^F_{ij})$]

Why Gaussian?

✓ Zero mean and common variance: no preferred flavor entry.

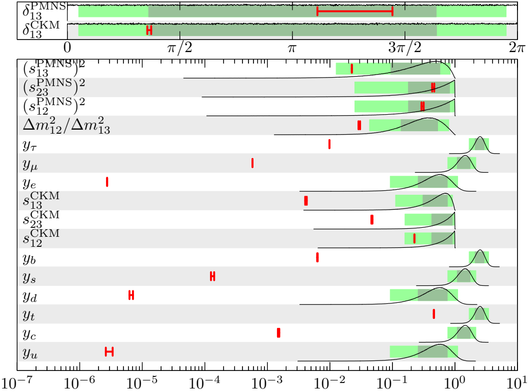
✓ Maximum entropy ($S[p] = \int dx p(x) \log p(x)$) for fixed mean and variance: least biased choice.

✓ The width $\sigma \sim 1$ implements generic $\mathcal{O}(1)$ couplings.

$\mathcal{O}(1)$ Yukawa Coupling Constants

What happens without flavor structure?

Distributions of dimensionless observables (normal ordering)



Red bars : Observed Likelihood

Curves and green shaded region :

Posterior distribution (1σ and 2σ ranges)

Without flavor structure, fitting is quite unnatural ...

Example : Froggatt–Nielsen Mechanism

In the FN mechanism, hierarchies arise from powers of a small parameter:

$$Y_{ij} \sim c_{ij}\, \epsilon^{|f_i + f_j|}, \qquad c_{ij} = \mathcal{O}(1)\ .$$

- ✓ f's : U(1) charge of quarks/leptons
- ✓ ϵ : order parameter of U(1) symmetry
- ✓ c_{ij} random $\mathcal{O}(1)$ variables

Ex. $f_Q = (3,2,0)\ , \quad f_{\bar{u}} = (5,2,0)\ , \quad f_{\bar{d}} = (4,4,3)\ ,$

$$f_L = (4,3,3)\ , \quad f_{\bar{e}} = (4,1,0)\ ,$$

$$y_u : y_c : y_t = \epsilon^8 : \epsilon^4 : \epsilon^0, \quad y_d : y_s : y_b = \epsilon^7 : \epsilon^6 : \epsilon^3,$$

$$y_e : y_\mu : y_\tau = \epsilon^8 : \epsilon^4 : \epsilon^3,$$

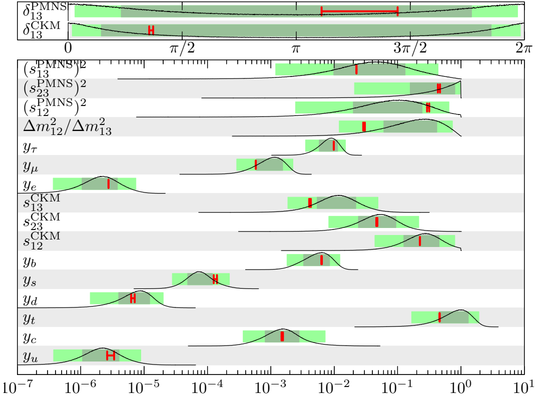
$$|V_{\rm CKM}| \sim \begin{pmatrix} \epsilon^0 & \epsilon^1 & \epsilon^3 \\ \epsilon^1 & \epsilon^0 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & \epsilon^0 \end{pmatrix}, \quad |U_{\rm PMNS}| \sim \begin{pmatrix} \epsilon^0 & \epsilon^1 & \epsilon^1 \\ \epsilon^1 & \epsilon^0 & \epsilon^0 \\ \epsilon^1 & \epsilon^0 & \epsilon^0 \end{pmatrix}\ .$$

$\Rightarrow \epsilon \sim 0.2$ (Cabbibo angle)

Example : Froggatt-Nielsen Mechanism

Distributions of dimensionless observables (normal ordering)

$\epsilon = 0.185,\ f_{\bar{u}} = (5,2,0),\ f_{\bar{d}} = (4,4,3),\ f_Q = (3,2,0),\ f_{\bar{e}} = (4,1,0),\ f_L = (4,3,3)$



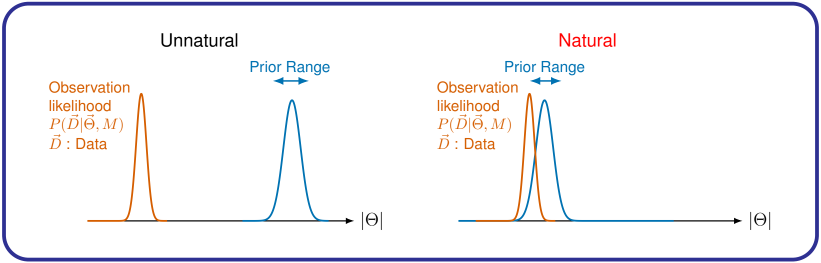
Red bars : Observed Likelihood

Curves and green shaded region :

Posterior distribution (1σ and 2σ ranges)

Flavor structure is naturally explained !

Bayes Factor as Relative Naturalness



✓ Evidence = overlap between $P(\vec{D}|\vec{\Theta}, M)$ and $\pi(\vec{\Theta}|M)$:

$$P(\vec{D}|M_i) = \int d\Theta P(\vec{D}|\vec{\Theta}, M)\pi(\vec{\Theta}|M_i)$$

✓ Model comparison is quantified by the Bayes factor:

$$B_{ij} = P(\vec{D}|M_i)/P(\vec{D}|M_j)$$

The larger the Bayes factor, the more “natural” the model is

Bayes Factor as Relative Naturalness

✓ Model comparison is quantified by the Bayes factor:

$$B_{ij} = P(\vec{D}|M_i)/P(\vec{D}|M_j).$$

✓ Jeffreys' scale provides a conventional interpretation of B_{ij}.

Jeffreys' scale

$\log_{10} B_{ij}$	B_{ij}	Strength of evidence
$0-1/2$	$1-3.2$	Barely worth mentioning
$1/2-1$	$3.2-10$	Substantial
$1-2$	$10-100$	Strong
>2	>100	Decisive

✓ Example: FN mechanism with an appropriate charge assignment

$$B = Z_{\rm FN}/Z_{\rm no\ FN} > 10^{50}.$$

[JHEP 03 (2025) 150, M.I., S. Shirai and K. Watanabe]

The observed flavor structure is overwhelmingly more typical with FN.

What is a Good Flavor Model?

Flavor model building $=$ flavor origin $\times$ typicality (reasonable prior)

✓ **Origin:** the model should explain why flavor structures appear:

mass hierarchy, small CKM mixing, large PMNS mixing.

✓ **Typicality:** the observed data should lie in a typical region of the model prediction:

$$P(\vec{D}|M) \text{ large.}$$

✓ **Caveat:** The prior distributions are not unique.
They should be chosen and interpreted carefully.

A good flavor model should not only fit the data, but should predict them naturally.

Long-Base-Line Experiments

- ✓ Hyper-K/DUNE ($\nu_\mu/\bar{\nu}_\mu \to \nu_{e,\mu}/\bar{\nu}_{e,\mu}$)
  $\Rightarrow$ sensitive to $\delta_{\rm CP}$, θ_{23}, $|\Delta m^2_{31}|$ and mass ordering

- ✓ JUNO (reactor $\bar{\nu_e} \to \bar{\nu_e}$)
  $\Rightarrow$ sensitive to θ_{12}, Δm^2_{21}, $|\Delta m^2_{31}|$ and mass ordering

- ✓ Combination with atmospheric-neutrino data and other oscillation experiments
  $\Rightarrow$ important for reducing degeneracies and improving precision

These measurements will provide new insight into flavor models !

Long-Baseline Experiments: Inverted Ordering?

- ✓ Inverted ordering (IO) is difficult to realize as a typical prediction.

$$m_3 < m_1 \simeq m_2, \qquad \Delta m_{21}^2 \ll |\Delta m_{31}^2|.$$

$\Rightarrow$ two heavier masses need to be nearly degenerate.

- ✓ Without a special structure, one expects

$$|m_1 - m_2| = \mathcal{O}(m_{1,2}),$$

rather than a pseudo-degenerate pair.

Fine-tuning lowers the evidence through prior-volume suppression.

If inverted ordering is established...

- ✓ IO would point to a special structure, e.g. a pseudo-Dirac pair:

$$m_\nu \sim \frac{v_{\rm EW}^2}{M_R} \begin{pmatrix} \epsilon & 1 & 1 \\ 1 & \epsilon & \epsilon \\ 1 & \epsilon & \epsilon \end{pmatrix} \circ c_{ij} \quad \Rightarrow \quad \begin{array}{l} m_1 \simeq m_2 \gg m_3, \\ \Delta m_{21}^2/|\Delta m_{31}^2| = \mathcal{O}(\epsilon). \end{array}$$

[see e.g. King and Singh, Nucl. Phys. B596, 81 (2001)]

- ✓ Can it reproduce sizable θ_{13} and small $\Delta m_{21}^2/|\Delta m_{31}^2|$ naturally?

IO would be a serious challenge for many flavor models.

Long-Baseline Experiments: Special vs Typical?

✓ Suppose future data find

$$\theta_{23}^{\text{obs}} = \frac{\pi}{4} \pm \sigma_{23}, \qquad \sigma_{23} \ll 1.$$

✓ **Sharp model** M_{sharp}: $\theta_{23} = \pi/4$ is predicted by a symmetry.

$$\Rightarrow \quad P(D|M_{\text{sharp}}) \simeq \frac{1}{\sqrt{2\pi}\sigma_{23}}.$$

✓ **Broad model** M_{broad}: θ_{23} is broadly distributed (e.g., FN model).

$$\Rightarrow \quad P(D|M_{\text{broad}}) \simeq \pi\left(\frac{\pi}{4}\middle| M_{\text{broad}}\right).$$

If $\theta_{23} = \pi/4$ is confirmed with high precision,
$\Rightarrow$ a sharp prediction can beat many broad predictions!

The same argument applies to other special values, such as

$$\delta_{\text{CP}} = 0, \ \pm\pi/2.$$

Future observations will teach us about the “structure” part of flavor models!

Absolute Neutrino Mass

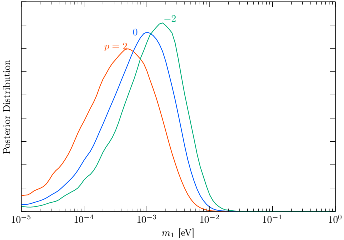
✓ Neutrino masses in the seesaw mechanism

$$(m_\nu)_{ij} = -v_{\rm EW}^2 \, (y_D)_{i\alpha} (M_R^{-1})_{\alpha\beta} (y_D^T)_{\beta j},$$
$$(y_D)_{i\alpha} = \epsilon^{|f_{L_i} + f_{\bar{N}_\alpha}|} \circ c^D_{i\alpha},$$
$$(M_R)_{\alpha\beta} = M_0 \, \epsilon^{|f_{\bar{N}_\alpha} + f_{\bar{N}_\beta}|} \circ c^R_{\alpha\beta}.$$

$\Rightarrow$ The prediction for m_ν depends on the prior for M_0.

✓ For M_0, the choice of prior distribution is nontrivial.
We adopt a power-law prior: $\pi(M_0) \propto M_0^p, \quad -2 \le p \le 2.$

✓ Posterior distribution in the FN model of $m_1 \Rightarrow m_1 = \mathcal{O}(1)$ meV.



A blue-tilted prior $(p = 2)$ tends to predict smaller m_1.

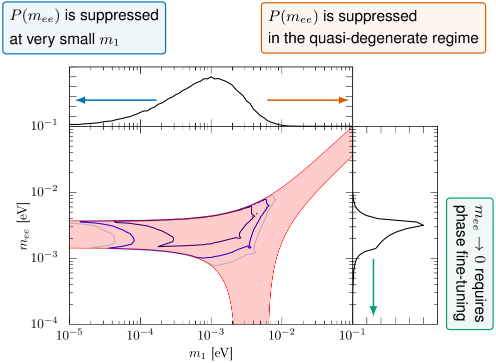
Absolute Neutrino Mass

✓ Effective Majorana mass of ν_e relevant for $0\nu\beta\beta$ (normal ordering)

$$m_{ee} := \left|\sum_i U_{ei}^2 m_i\right| = \left|m_1 c_{12}^2 c_{13}^2 + m_2 s_{12}^2 c_{13}^2 e^{2i(\eta_2-\eta_1)} + m_3 s_{13}^2 e^{-2i(\delta_{\rm CP}+\eta_1)}\right| .$$

$\eta_{1,2}$: Majorana phases in the PMNS matrix.

✓ Posterior distribution of m_{ee} in the FN model ($p = 0$)



⇒ Typicality favors $m_{ee} \gtrsim 1\,\mathrm{meV}$!

Summary

✓ Typicality provides a quantitative notion of naturalness.

Naturalness $\Longleftrightarrow$ large evidence $P(\vec{D}|M)$.

✓ **Flavor model = structure $\times$ typicality**

Ex.) FN mechanism gives an overwhelmingly natural fit

✓ Future oscillation data will sharpen the test.

- ✓ IO is challenging for typicality
- ✓ Special values such as $\theta_{23} = \pi/4$ may also challenge typicality

$\Rightarrow$ **new insight on the "structure" part of flavor model**

✓ Typicality favors the effective majorana mass for $0\nu\beta\beta$ decay

$$m_{ee} \gtrsim 1\,\mathrm{meV}.$$

Future oscillation and $0\nu\beta\beta$ data will provide crucial tests of flavor models.