Exact “Two-Zero Minor texture” Analyses in Neutrino Physics Are Incomplete Impact on the Neutrino Mass Sum

6th New Physics Opportunities at Neutrino Facilities Workshop

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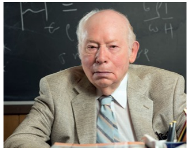
The Big problem: Flavor puzzle

Who ordered this observed pattern?

		Normal Ordering ($\Delta\chi^2 = 0.6$)		Inverted Ordering (best fit)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC19 without SK atmospheric data	$\sin^2\theta_{12}$	$0.307^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.561^{+0.012}_{-0.015}$	$0.430 \to 0.596$	$0.562^{+0.012}_{-0.015}$	$0.437 \to 0.597$
	$\theta_{23}/^\circ$	$48.5^{+0.7}_{-0.9}$	$41.0 \to 50.5$	$48.6^{+0.7}_{-0.9}$	$41.4 \to 50.6$
	$\sin^2\theta_{13}$	$0.02195^{+0.00054}_{-0.00058}$	$0.02023 \to 0.02376$	$0.02224^{+0.00056}_{-0.00057}$	$0.02053 \to 0.02397$
	$\theta_{13}/^\circ$	$8.52^{+0.11}_{-0.11}$	$8.18 \to 8.87$	$8.58^{+0.11}_{-0.11}$	$8.24 \to 8.91$
	$\delta_{CP}/^\circ$	177^{+19}_{-20}	$96 \to 422$	285^{+25}_{-28}	$201 \to 348$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.534^{+0.025}_{-0.023}$	$+2.463 \to +2.606$	$-2.510^{+0.024}_{-0.025}$	$-2.584 \to -2.438$
		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 6.1$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
IC24 with SK atmospheric data	$\sin^2\theta_{12}$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$	$0.308^{+0.012}_{-0.011}$	$0.275 \to 0.345$
	$\theta_{12}/^\circ$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$	$33.68^{+0.73}_{-0.70}$	$31.63 \to 35.95$
	$\sin^2\theta_{23}$	$0.470^{+0.017}_{-0.013}$	$0.435 \to 0.585$	$0.550^{+0.012}_{-0.015}$	$0.440 \to 0.584$
	$\theta_{23}/^\circ$	$43.3^{+1.0}_{-0.8}$	$41.3 \to 49.9$	$47.9^{+0.7}_{-0.9}$	$41.5 \to 49.8$
	$\sin^2\theta_{13}$	$0.02215^{+0.00056}_{-0.00058}$	$0.02030 \to 0.02388$	$0.02231^{+0.00056}_{-0.00056}$	$0.02060 \to 0.02409$
	$\theta_{13}/^\circ$	$8.56^{+0.11}_{-0.11}$	$8.19 \to 8.89$	$8.59^{+0.11}_{-0.11}$	$8.25 \to 8.93$
	$\delta_{CP}/^\circ$	212^{+26}_{-41}	$124 \to 364$	274^{+22}_{-25}	$201 \to 335$
	$\frac{\Delta m^2_{21}}{10^{-5}\ \mathrm{eV}^2}$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$	$7.49^{+0.19}_{-0.19}$	$6.92 \to 8.05$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ \mathrm{eV}^2}$	$+2.513^{+0.021}_{-0.019}$	$+2.451 \to +2.578$	$-2.484^{+0.020}_{-0.020}$	$-2.547 \to -2.421$

Nufit 6.1



Weinberg once identified the flavor pattern as the mystery he most wanted to see solved.

(cern courier volume57 number9)

(Focus here: the neutrino sector)

A solution: Zeros in m_ν

Who ordered this observed pattern?

Assuming …

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

Zero minor

$$m_\nu = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

Zero texture

$*$: Non-Zero element

Zeros lead to correlations

$$\theta_{12}, \theta_{23}, \theta_{13}, \delta_{\rm CP}, \eta_1, \eta_2, m_1, m_2, m_3$$

If the predicted correlations naturally reproduce the **observed flavor** pattern, they may reveal an **underlying mechanism** behind the flavor puzzle.

(Other special structures are possible)

Flavor symmetry as the Origins of Zeros

What is the underlying mechanism?

Flavor symmetry

Example: $\mathrm{U(1)}_{\mathrm{L}_\mu-\mathrm{L}_\tau}$

	Field			$U(1)_{L_\mu-L_\tau}$
	L_e	$\bar{e}_e$	$\bar{N}_e$	0
Leptons	L_μ	$\bar{e}_\tau$	$\bar{N}_\tau$	$+1$
	L_τ	$\bar{e}_\mu$	$\bar{N}_\mu$	-1
Scalar		ϕ		$+1$

$$m_D = \begin{pmatrix} \lambda_e v_{\mathrm{EW}} & 0 & 0 \\ 0 & \lambda_\mu v_{\mathrm{EW}} & 0 \\ 0 & 0 & \lambda_\tau v_{\mathrm{EW}} \end{pmatrix} \quad M = \begin{pmatrix} M_{ee} & \kappa_{e\mu} v_\phi & \kappa_{e\tau} v_\phi \\ \kappa_{e\mu} v_\phi & 0 & M_{\mu\tau} \\ \kappa_{e\tau} v_\phi & M_{\mu\tau} & 0 \end{pmatrix}$$

Dirac mass   Majorana mass

Flavor <u>symmetry breaking</u> $\langle\phi\rangle = v_\phi$

$m_\nu = -m_D M^{-1} m_D^T$ (Type-I seesaw)

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

"Two-Zero Minor"

Outline

Who ordered this observed pattern?

Flavor symmetry

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix} \qquad m_\nu = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

Zero minor   Zero texture

1. Zero minors/textures are constrained (review)
2. Exact zeros are not expected from UV-complete model perspective
3. Soft-breaking effects relax the cosmological constraints

Most are excluded by oscillation data

$m_\nu =$

$A_1 : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$, ~~$A_2 : \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$~~, $B_1 : \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$,

$B_2 : \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}$, $B_3 : \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}$, $B_4 : \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix}$,

$C_1 : \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$, ~~$D_1 : \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$~~, ~~$D_2 : \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}$~~,

~~$E_1 : \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}$~~, ~~$E_2 : \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}$~~, ~~$E_3 : \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$~~,

~~$F_1 : \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$~~, ~~$F_2 : \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$~~, ~~$F_3 : \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$~~.

$m_\nu^{-1} =$

$A_1 : \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}$, $A_2 : \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}$, $B_3 : \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix}$,

$B_4 : \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}$, $B_5 : \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}$, $B_6 : \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix}$,

$D : \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$, ~~$S_1 : \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}$~~, ~~$S_2 : \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$~~,

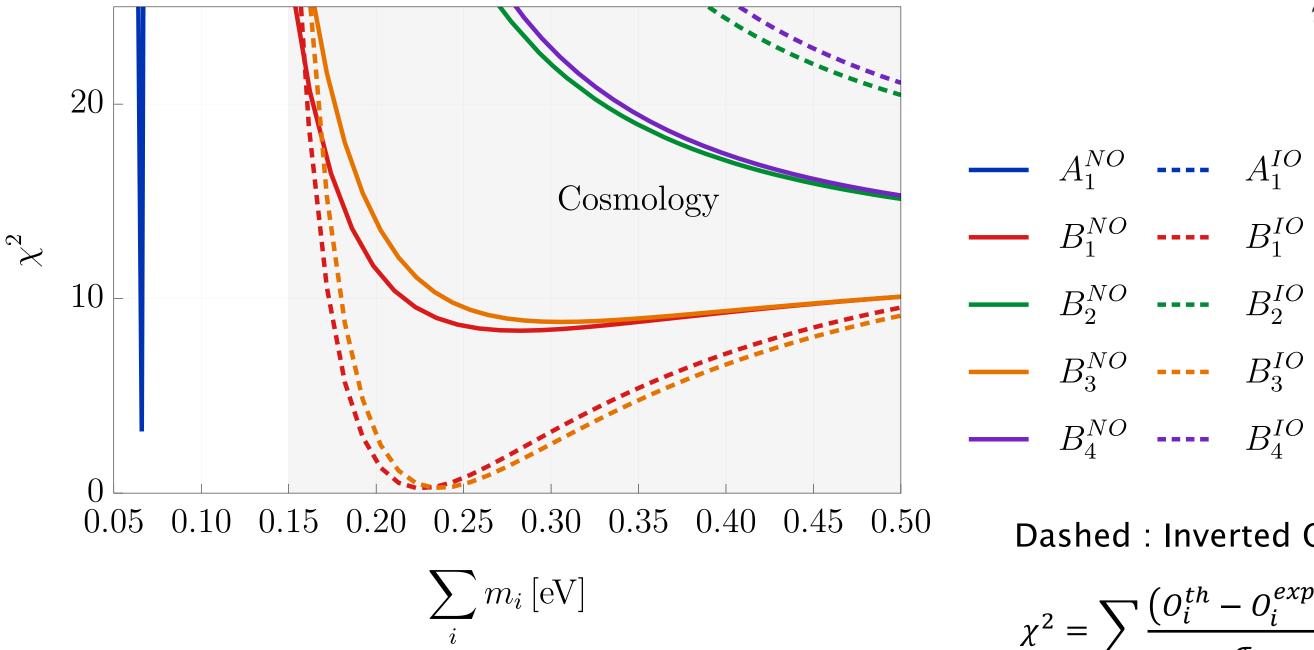
~~$S_3 : \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}$~~, ~~$F_1 : \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}$~~, ~~$F_2 : \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}$~~,

~~$F_3 : \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}$~~, ~~$F_4 : \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}$~~, ~~$F_5 : \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}$~~.

Most structures are inconsistent with oscillation data

(**notation** 2004.05622 and 0708.2423)

Cosmology strongly constrains the remaining cases



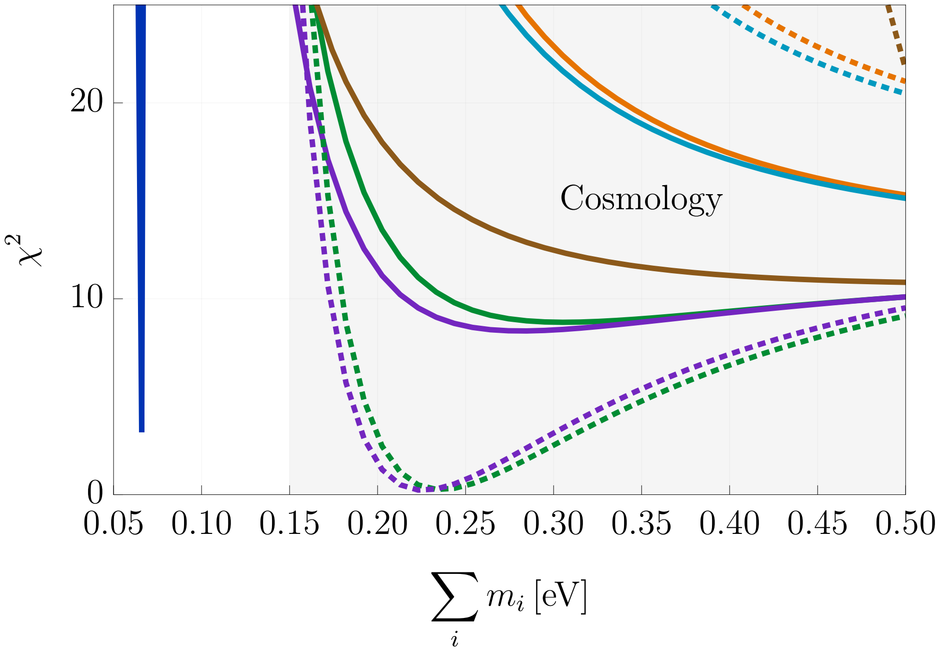
$$m_\nu = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

Zero texture

Dashed : Inverted Ordering

$$\chi^2 = \sum_i \frac{\left(O_i^{th} - O_i^{exp}\right)^2}{\sigma_i}$$

Cosmology strongly constrains the remaining cases



A_1^{NO} A_1^{IO}
A_2^{NO} A_2^{IO}
B_3^{NO} B_3^{IO}
B_4^{NO} B_4^{IO}
B_5^{NO} B_5^{IO}
B_6^{NO} B_6^{IO}
D^{NO} D^{IO}

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

Zero minor

Dashed : Inverted Ordering

$$\chi^2 = \sum_i \frac{\left(O_i^{th} - O_i^{exp}\right)^2}{\sigma_i}$$

Zero textures/minors are strongly constrained

$$m_\nu = \quad A_1: \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \quad A_2: \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad B_1: \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

$$B_2: \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}, \quad B_3: \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \quad B_4: \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$C_1: \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}, \quad D_1: \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \quad D_2: \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix},$$

$$E_1: \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}, \quad E_2: \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}, \quad E_3: \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix},$$

$$F_1: \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \quad F_2: \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}, \quad F_3: \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}.$$

$$m_\nu^{-1} = \quad A_1: \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}, \quad A_2: \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \quad B_3: \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$B_4: \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \quad B_5: \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}, \quad B_6: \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

$$D: \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}, \quad S_1: \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad S_2: \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix},$$

$$S_3: \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \quad F_1: \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}, \quad F_2: \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix},$$

$$F_3: \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}, \quad F_4: \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad F_5: \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}.$$

Most structure are inconsistent with oscillation data

and

Most surviving structures are severely constrained by cosmology

Outline

Flavor symmetry

as the origin of flavor pattern

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix} \qquad m_\nu = \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}$$

Zero minor   Zero texture

Flavor symmetry cannot be the origin of flavor pattern?

1. Zero minors/textures are constrained (review)
2. Exact zeros are not expected from UV-complete model perspective
3. Soft-breaking effects relax the cosmological constraints

Revisiting "Zeros"

Are texture zeros and minor zeros radiatively stable?

Example: $\mathrm{U(1)}_{\mathrm{L}_\mu-\mathrm{L}_\tau}$

$$m_D = \begin{pmatrix} \lambda_e v_{\mathrm{EW}} & 0 & 0 \\ 0 & \lambda_\mu v_{\mathrm{EW}} & 0 \\ 0 & 0 & \lambda_\tau v_{\mathrm{EW}} \end{pmatrix} \qquad M = \begin{pmatrix} M_{ee} & \kappa_{e\mu} v_\phi & \kappa_{e\tau} v_\phi \\ \kappa_{e\mu} v_\phi & 0 & M_{\mu\tau} \\ \kappa_{e\tau} v_\phi & M_{\mu\tau} & 0 \end{pmatrix}$$

Dirac mass   Majorana mass

Flavor symmetry breaking $\langle \phi \rangle = v_\phi$

Radiative corrections turns 0 into small nonzero entries, ϵ

Example UV Model

SM + 3 RH Neutrino + SSB scalar + $\mathrm{U(1)_{L_\mu - L_\tau}}$

$$\mathcal{L} = \mathcal{L}_{\rm SM} + \mathcal{L}_{\phi,Z'} + \sum_{\ell=e,\mu,\tau} i\bar{N}_\ell^\dagger \bar{\sigma}^\mu D_\mu \bar{N}_\ell$$
$$- \sum_{\ell=e,\mu,\tau} \lambda_\ell L_\ell \bar{N}_\ell H + \text{h.c.}$$
$$- \frac{1}{2} M_{ee}\bar{N}_e\bar{N}_e - M_{\mu\tau}\bar{N}_\mu\bar{N}_\tau - \kappa_{e\mu}\bar{N}_e\bar{N}_\mu\phi - \kappa_{e\tau}\bar{N}_e\bar{N}_\tau\phi^\dagger + \text{h.c.}$$

	Field			$U(1)_{L_\mu - L_\tau}$
Leptons	L_e	$\bar{e}_e$	$\bar{N}_e$	0
	L_μ	$\bar{e}_\tau$	$\bar{N}_\tau$	+1
	L_τ	$\bar{e}_\mu$	$\bar{N}_\mu$	−1
Scalar		ϕ		+1

$$m_\nu = -m_D M_R^{-1} m_D^T$$

$$m_D = \begin{pmatrix} \lambda_e v_{\rm EW} & 0 & 0 \\ 0 & \lambda_\mu v_{\rm EW} & 0 \\ 0 & 0 & \lambda_\tau v_{\rm EW} \end{pmatrix}$$

Dirac mass

$$M = \begin{pmatrix} M_{ee} & \kappa_{e\mu} v_\phi & \kappa_{e\tau} v_\phi \\ \kappa_{e\mu} v_\phi & 0 & M_{\mu\tau} \\ \kappa_{e\tau} v_\phi & M_{\mu\tau} & 0 \end{pmatrix}$$

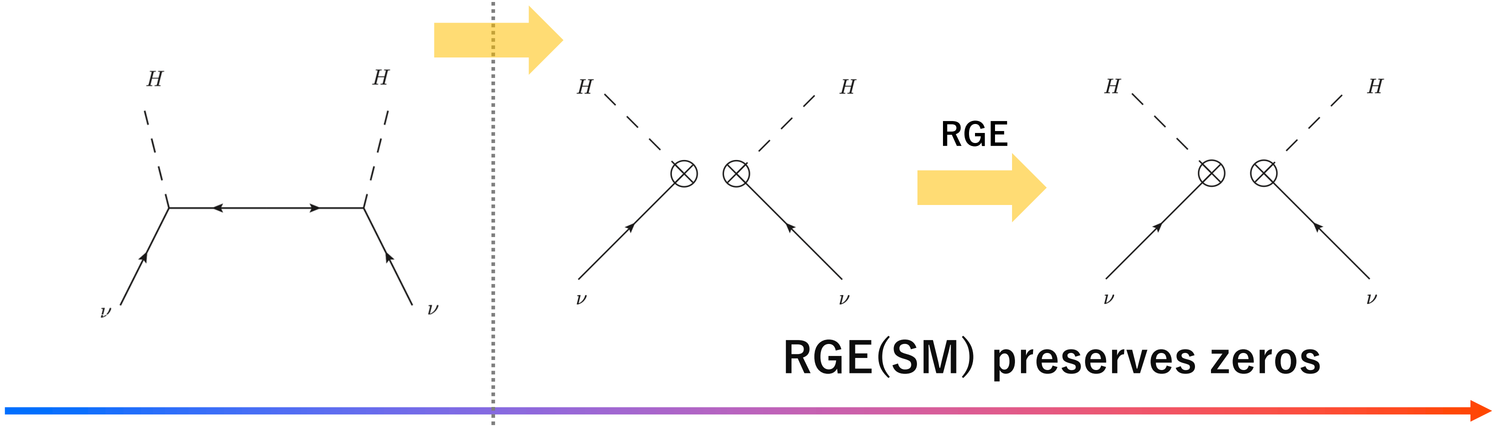
Majorana mass

$$m_\nu^{-1} = \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}$$

Type-D Zero Minor

RGE and Threshold Effects

What about Threshold Effect ?

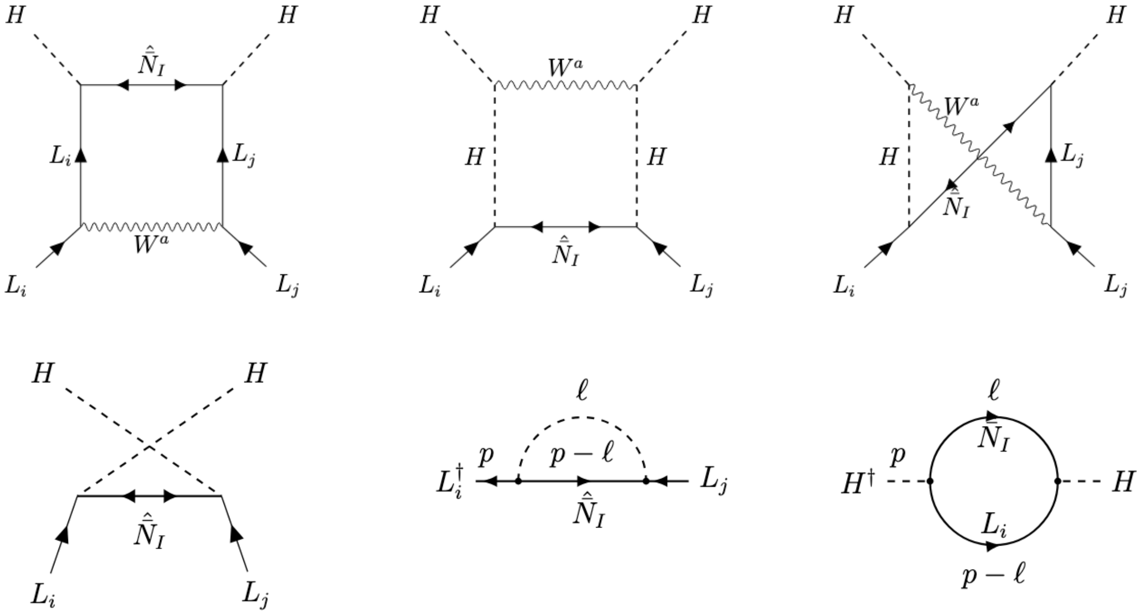


$\frac{C_{ll'}}{\Lambda} L_l H \, L_{l'} H$

$\frac{C_{ll'}}{\Lambda} L_l H \, L_{l'} H$

$\sim 10^{14}\text{GeV}$

Relevant diagrams



and charged yukawa interaction

Outline

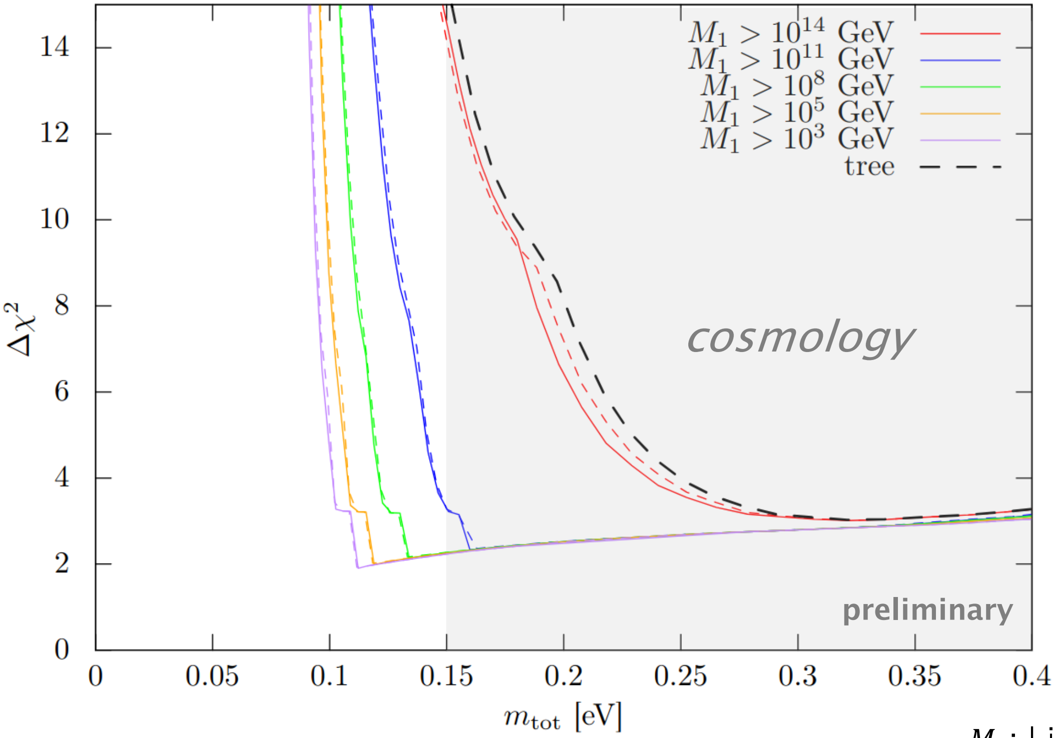
Flavor symmetry
as the origin of flavor pattern

$$m_\nu^{-1} = \underbrace{\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}}_{\text{tree}} + \underbrace{\begin{pmatrix} * & * & * \\ * & \epsilon & * \\ * & * & \epsilon' \end{pmatrix}}_{\text{radiative}}$$

1. Zero minors/textures are constrained (review)
2. Exact zeros are not expected from UV-complete model perspective
3. Soft breaking effects relax the cosmological constraints

(SM + 3 RH Neutrino + SSB scalar + $\mathrm{U}(1)_{\mathrm{L}_\mu - \mathrm{L}_\tau}$ **)**

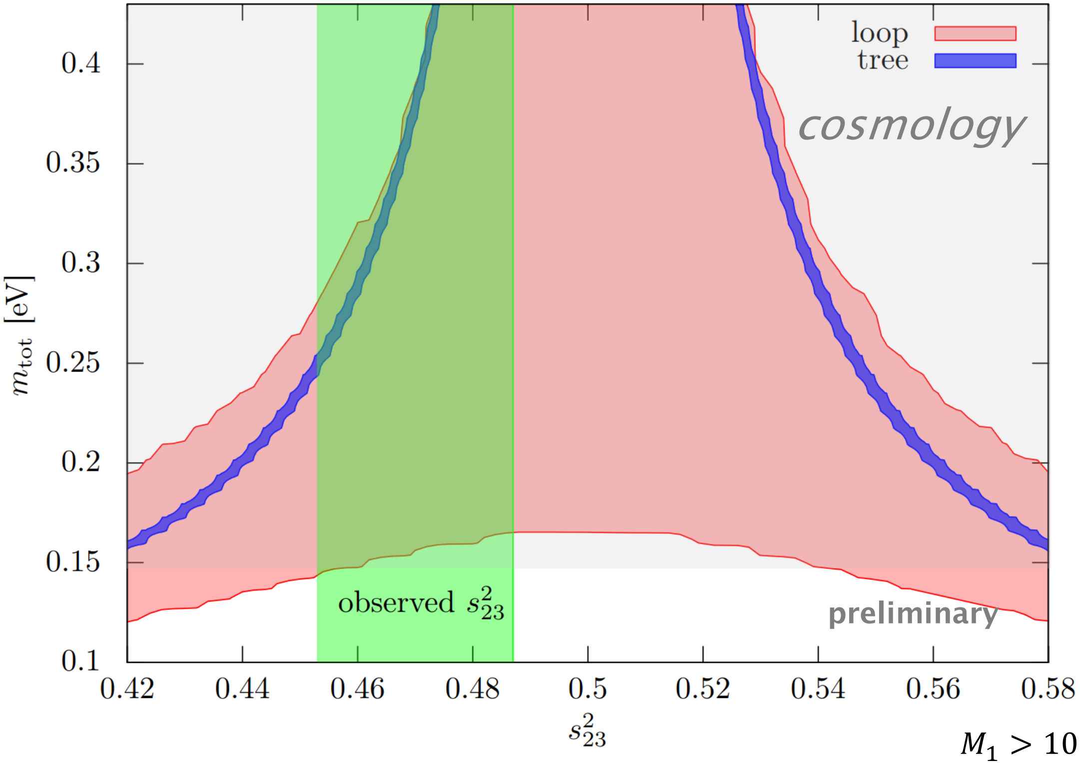
Impact on $\sum_i m_i$



Δ: sym – (nosym)

M_1: Lightest RHN mass

s_{23}^2 vs $\sum_i m_i$

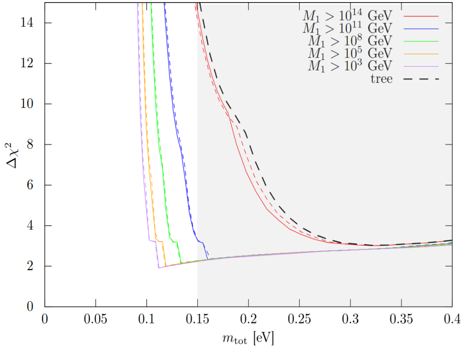


$M_1 > 10^{11} GeV$

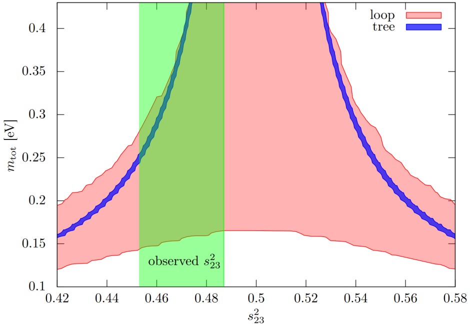
Conclusions

Flavor symmetry

Can be the origin of flavor pattern



$$m_\nu^{-1} = \underbrace{\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}}_{\text{tree}} + \underbrace{\begin{pmatrix} * & * & * \\ * & \epsilon & * \\ * & * & \epsilon' \end{pmatrix}}_{\text{radiative}}$$



$*$: Non-Zero element

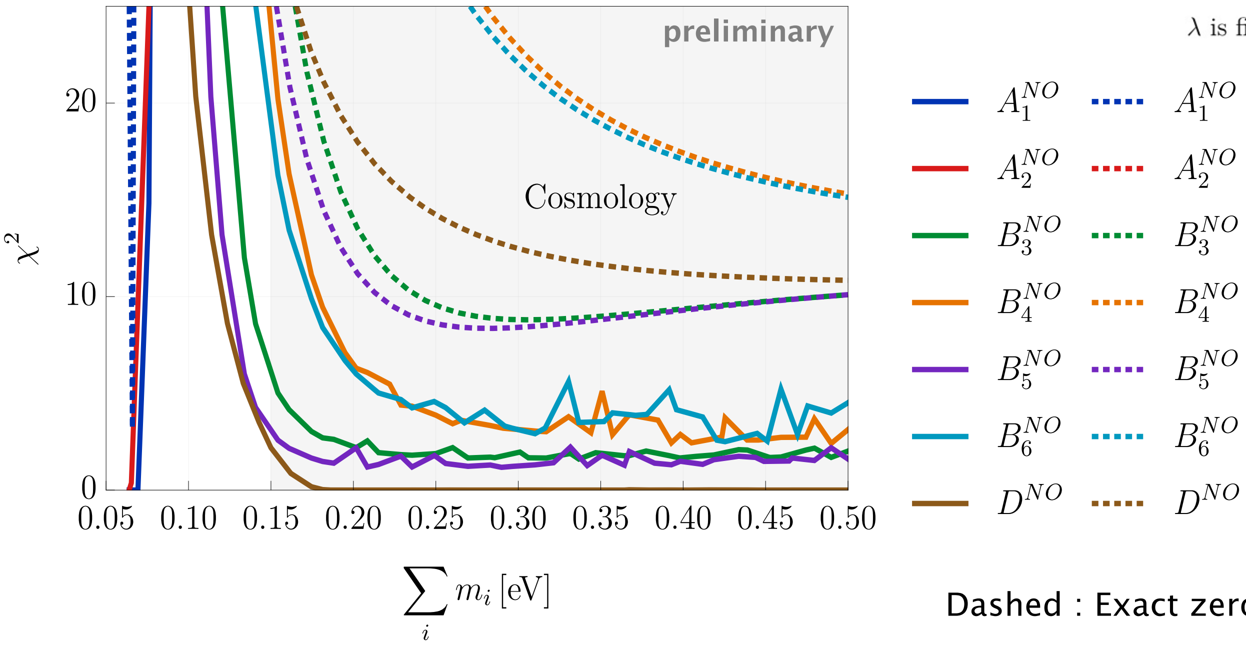
1. Exact zeros are broken radiatively
2. Soft breaking effect modify the prediction
3. Neutrino physics provides a probe of flavor symmetry

Bottom up analysis

minor breaking is introduced in $K = M_\nu^{-1}$:

$$K^{(0)} = \left(M_\nu^{(0)}\right)^{-1}, \quad K = K^{(0)} + \lambda \Delta K, \quad M_\nu = K^{-1},$$

$$\lambda \text{ is fixed by } \frac{\left\| M_\nu - M_\nu^{(0)} \right\|_F}{\left\| M_\nu^{(0)} \right\|_F} = 0.1$$



$$\chi^2 = \sum_i \frac{\left(O_i^{th} - O_i^{exp}\right)^2}{\sigma_i}$$

Dashed : Exact zero (tree level)

Which observables are important?

$$m_\nu = \quad A_1: \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix}, \quad A_2: \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad B_1: \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

$$B_2: \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}, \quad B_3: \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \quad B_4: \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$C_1: \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}, \quad D_1: \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \quad D_2: \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix},$$

$$E_1: \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix}, \quad E_2: \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}, \quad E_3: \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix},$$

$$F_1: \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \quad F_2: \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}, \quad F_3: \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}.$$

$$m_\nu^{-1} = \quad A_1: \begin{pmatrix} * & * & * \\ * & * & 0 \\ * & 0 & 0 \end{pmatrix}, \quad A_2: \begin{pmatrix} * & * & * \\ * & 0 & 0 \\ * & 0 & * \end{pmatrix}, \quad B_3: \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & 0 \end{pmatrix},$$

$$B_4: \begin{pmatrix} * & 0 & * \\ 0 & 0 & * \\ * & * & * \end{pmatrix}, \quad B_5: \begin{pmatrix} * & 0 & * \\ 0 & * & * \\ * & * & 0 \end{pmatrix}, \quad B_6: \begin{pmatrix} * & * & 0 \\ * & 0 & * \\ 0 & * & * \end{pmatrix},$$

$$D: \begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}, \quad S_1: \begin{pmatrix} 0 & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix}, \quad S_2: \begin{pmatrix} 0 & 0 & * \\ 0 & * & * \\ * & * & * \end{pmatrix},$$

$$S_3: \begin{pmatrix} * & 0 & 0 \\ 0 & * & * \\ 0 & * & * \end{pmatrix}, \quad F_1: \begin{pmatrix} 0 & * & * \\ * & * & * \\ * & * & 0 \end{pmatrix}, \quad F_2: \begin{pmatrix} 0 & * & * \\ * & 0 & * \\ * & * & * \end{pmatrix},$$

$$F_3: \begin{pmatrix} 0 & * & * \\ * & * & 0 \\ * & 0 & * \end{pmatrix}, \quad F_4: \begin{pmatrix} * & * & 0 \\ * & * & 0 \\ 0 & 0 & * \end{pmatrix}, \quad F_5: \begin{pmatrix} * & 0 & * \\ 0 & * & 0 \\ * & 0 & * \end{pmatrix}.$$

Some structure can not explain $\boldsymbol{\theta_{23}} > 45$ or < 45

$\boldsymbol{\delta_{CP}}$ determination is important because of correlations of model prediction

Of course, ordering is most important

Two-loop?

$$m_\nu^{-1} = \underbrace{\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}}_{\text{tree}} + \underbrace{\begin{pmatrix} * & * & * \\ * & \epsilon & * \\ * & * & \epsilon' \end{pmatrix}}_{\text{radiative}}$$

Leading order is One-loop

0 and ϵ are very different

Two-loop is Next leading order, which is negligible since it is smaller than LO (one-loop)

Large-log?

$$m_\nu^{-1} = \underbrace{\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}}_{\text{tree}} + \underbrace{\begin{pmatrix} * & * & * \\ * & \epsilon & * \\ * & * & \epsilon' \end{pmatrix}}_{\text{radiative}}$$

$\log\left(\frac{M_2^2}{M_1^2}\right)$ are appear in the ϵ

The coupling is small, but the effect is logarithmically enhanced

Is RGE improvement needed?

ϵ has an O(1) % uncertainty when $\alpha log \sim 0.1$

ϵ is Leading order, so that our result is not badly affected by RGE

$0\nu\beta\beta$

$$m_\nu^{-1} = \underbrace{\begin{pmatrix} * & * & * \\ * & 0 & * \\ * & * & 0 \end{pmatrix}}_{\text{tree}} + \underbrace{\begin{pmatrix} * & * & * \\ * & \epsilon & * \\ * & * & \epsilon' \end{pmatrix}}_{\text{radiative}}$$

some zero texture/minors are constrained by $0\nu\beta\beta$

But ϵ is One-loop level, so that its evaluation is model dependent.

Flavor violating decay process are also important (e.g. 2511.08679)