Nuclear structure corrections for low-energy precision tests of the Standard Model via nuclear beta decays
VUD-NUCLEAR

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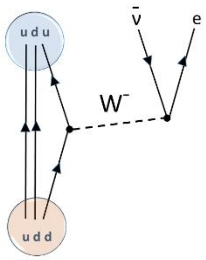


Joint Workshop on TYL/JFPPN and FKPPN 2026
Hamamatsu, Japan, May 18 – 20, 2026

Nuclear structure correction for low-energy precision tests of the Standard Model via nuclear beta decay

- Introduction: Fermi decay, CVC hypothesis and V_{ud} matrix element of the CKM matrix

- Isospin-symmetry breaking correction to Superallowed Fermi beta decay δ_C within the valence-space shell model with realistic radial wave functions and within the ab-initio no-core shell model
  - current status and results obtained by our collaborations

- Towards *further refining* calculations of isospin-symmetry breaking correction δ_C - present project VUD-NUCLEAR

- Conclusions and perspectives

Tests of the Standard Model in nuclear decays

Fermions in the Standard Model

$$\begin{pmatrix} u \\ d \end{pmatrix} \begin{pmatrix} c \\ s \end{pmatrix} \begin{pmatrix} t \\ b \end{pmatrix}$$
$$\begin{pmatrix} e \\ \nu_e \end{pmatrix} \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}$$

Cabibbo - Kobayashi - Maskawa (CKM) quark-mixing matrix

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

N. Cabibbo (1963); M. Kobayasi, T. Maskawa (1973).

Testing grounds:

- at colliders: search for direct production
- at low energies in nuclear beta decay: in precision experiments

Nuclear Matrix elements are needed !

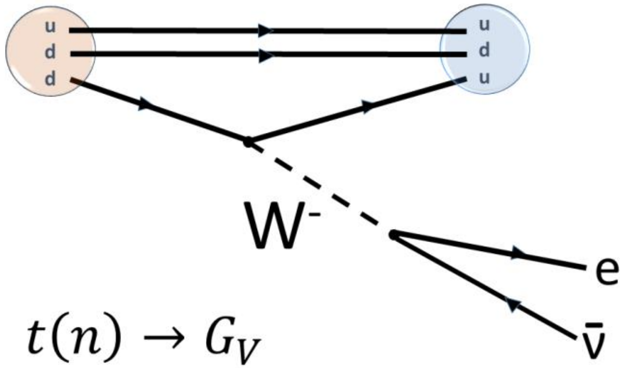
What does the Standard Model prescribe?

- V_{ik} to be extracted from experiment
- Standard Model: V is unitary

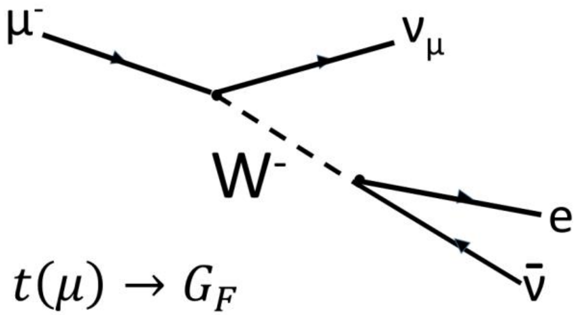
$$\sum_k |V_{ik}|^2 = \sum_k |V_{ki}|^2 = 1$$

$\Rightarrow$ constraints on new physics beyond the Standard Model

$|V_{ud}|$ from pion, nucleon and nuclear beta decay



$t(n) \rightarrow G_V$



$t(\mu) \rightarrow G_F$

$$|V_{ud}| \quad = G_V/G_F$$

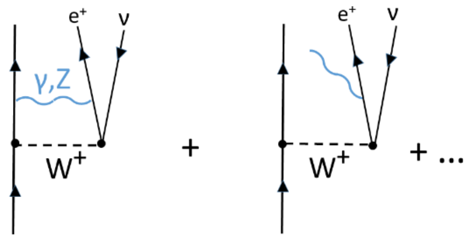
- Superallowed nuclear 0^+ -> 0^+ beta decay (nuclear structure effects)
- Mirror T=1/2 transitions (F/GT ratio, nuclear structure effects)
- Neutron decay $n \rightarrow p + e^- + \bar{\nu}_e$ (Lifetime)
- Pion decay $\pi^+ \rightarrow \pi^0 + e^+ + \nu_e$ (Branching ratio)

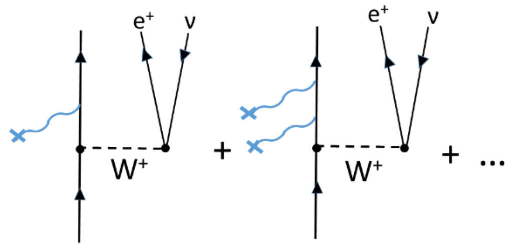
Current status:
J.C. Hardy, I. S. Towner, PRC102. 045501 (2020) ;
M. Gonzalez-Alonzo, O. Navillat-Cuncic, N. Severijns, PPNP104, 165 (2019)

Superallowed $0^+ \rightarrow 0^+$ beta decay

$$Ft^{0^+\rightarrow 0^+} \equiv ft^{0^+\rightarrow 0^+}(1+\delta'_R)(1+\delta_{NS}-\delta_C) = \frac{K}{|M_F^0|^2 G_V^2 (1+\Delta_R)}$$

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)





Radiative corrections

Much recent interest !

$$\Delta_R^V = (2.454 \pm 0.019)\%$$

$$\delta'_R \sim (1.50 \pm \sim 0.12)\%$$

$$|\delta_{NS}| \lesssim 0.4\%$$

Sirlin, Marciano, Jaus, Rasche, ..
Seng, Gorchtein, Cirigliano,…
+ talks by Misha, Emanuele, Garrett, etc

Nuclear-structure correction

$$|M_F|^2 = |M_F^0|^2(1-\delta_C)$$

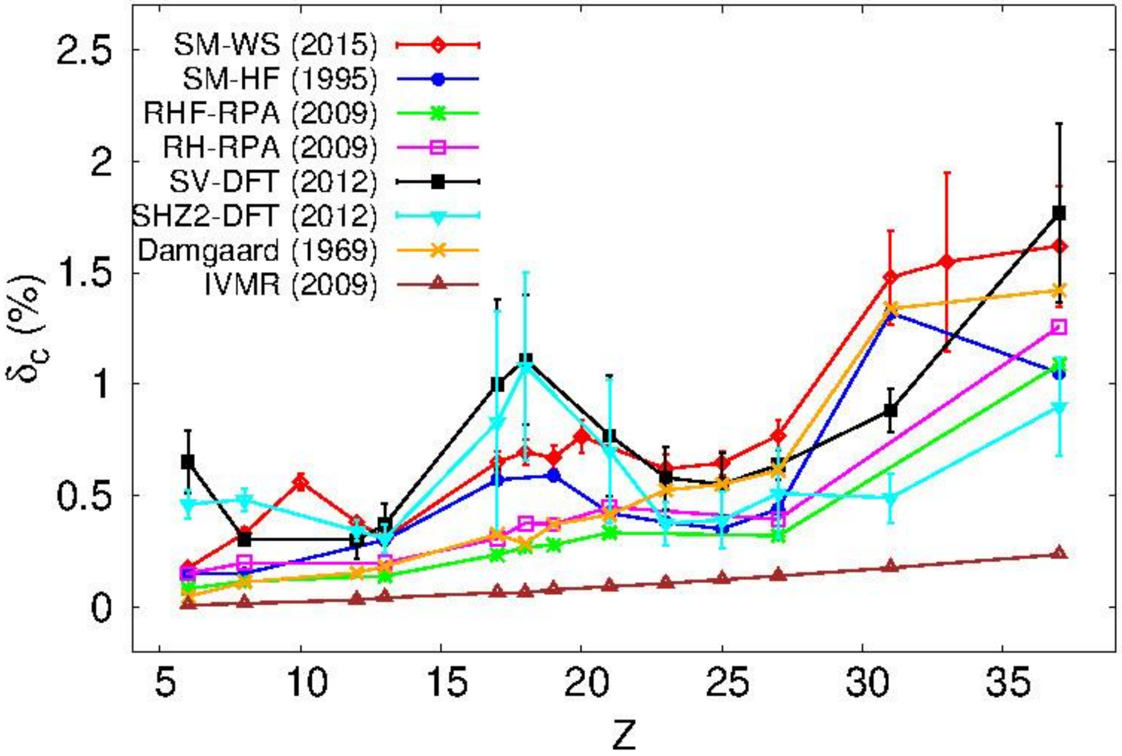
$$|M_F^0|^2 = T(T+1) - T_{zi}T_{zf}$$

$$\delta_C \approx 0.1 - 2.0\%$$

Many-body approach with charge-dependent forces

Towner, Hardy (1973 – 2020)
and many others (next slide)

Isospin-symmetry breaking correction – divergence of model predictions



Diagonalization
+ experimentally
constrained

- SM-WS : Shell-model + WS, Towner, Hardy (2015, 2020)
- SM-HF : Shell-model + HF Ormand, Brown (1989, 1995)

Variational
+ global EDF

- SV/SHZ2-DFT : J- , T-projected DFT, Satula et al (2012)
- RHF-RPA : Relativistic RPA, Liang, Giai, Meng (2009)

"gross"

- Damgaard : Harmonic-oscillator model, Damgaard (1969), Towner (1977)
- IVMR : Isovector Monopole Resonance, Auerbach (2009)

→ *CVC test*
Towner, Hardy
PRC82 (2010)

Superallowed $0^+ \rightarrow 0^+$ beta decay: status by 2020

$$\mathcal{F}t = (1+\delta'_R)(1+\delta_{NS}-\delta_C)ft = \frac{K}{M_0^2 G_F{}^2 |V_{ud}|^2 (1+\Delta_R)}$$

$$\mathcal{F}t = 3072.24 \pm 0.57\ sec$$

$\chi 2/\nu = 0.47$

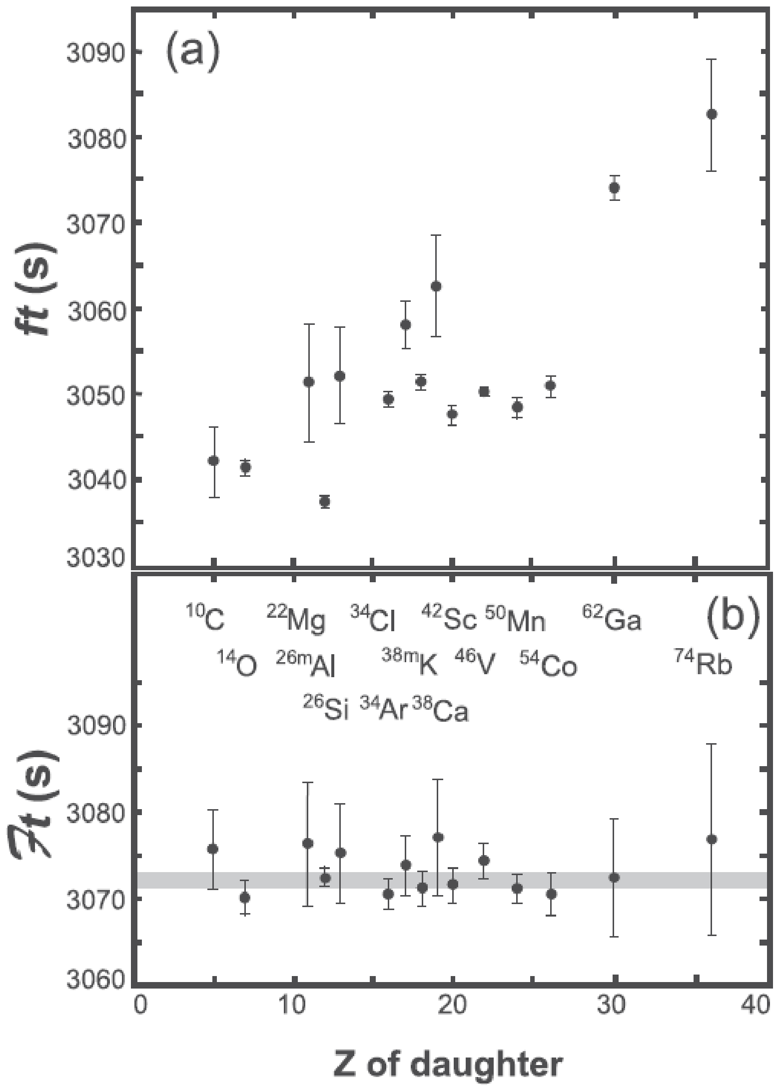
$$|V_{ud}| = 0.97373(31)$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(5)$$

Further revisions of radiative corrections are ongoing since 2018: for review see Gorchtein, Seng, ARNPS 74 (2024)

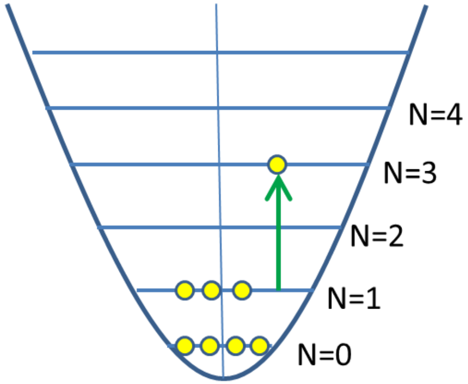
The purpose of our work: to revise the shell-model computation δ_C of in large-scaled calculations With realistic WS or HF wave functions

Hardy, Towner (2020)



No-Core Shell model - configuration-interaction approach

$$H = T - T_{CM} + V_{NN}\ (+V_{NNN})$$



$$H|\Psi_n\rangle = E_n|\Psi_n\rangle$$

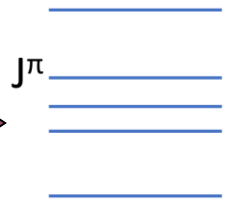
Harmonic Oscillator Basis

$$|\Psi_n\rangle = \sum_k c_{kn}\,|\Phi_k\rangle$$

$$\langle\Phi_k|\Phi_l\rangle = \delta_{kl}$$

$$\sum_{k=1}^{d}\langle\Phi_l|H|\Phi_k\rangle\, c_{kn} = E_n c_{ln}$$

$$\begin{pmatrix} H_{11} & H_{12} & K & H_{1d} \\ H_{21} & H_{22} & K & H_{2d} \\ M & & O & \\ H_{d1} & H_{d2} & K & H_{dd} \end{pmatrix}$$

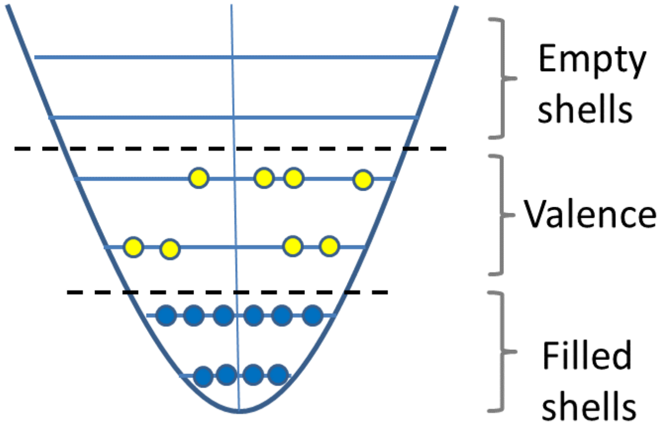


Ab-initio **No-Core Shell Model : sufficiently large model space so that the results for A nucleons do not depend on the basis parameters (hw and Nmax)**

Advantages : Conservation of symmetries of the Hamiltonian, use of bare NN+3N forces, exact separation of the COM, detailed information on low-energy states and transitions

Valence-space shell model for heavier nuclei

Restricted model space: valence space beyond an inert core



Effective operators

$$H|\Psi_n\rangle = E_n|\Psi_n\rangle \quad \Rightarrow \quad H_{eff}|\Psi_n^M\rangle = (E_n - E_C)|\Psi_n^M\rangle$$

Core energy

$$H_{eff} = \sum_\alpha \varepsilon_\alpha a_\alpha^\dagger a_\alpha + \frac{1}{4}\sum_{\alpha\beta\gamma\delta} \langle\alpha\beta|V_{res}|\delta\gamma\rangle a_\alpha^\dagger a_\beta^\dagger a_\gamma a_\delta$$

Empirical

Microscopic derived from NN potential via MBPT, IMSRG, …

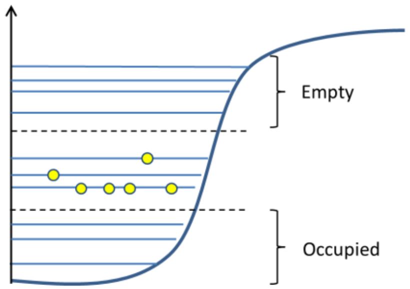
Phenomenological (constrained by the data)

- **Current status :**

- Excellent description with empirical (phenomenological) interactions
- Microscopic interactions -> recent progress and challenges

Fermi matrix element within the shell model

Fermi β-decay matrix element



$$H|\Psi\rangle = E|\Psi\rangle \quad \Rightarrow \quad \begin{array}{l} |\Psi_i\rangle = \sum c_{ki}|\Phi_k\rangle \\ |\Psi_f\rangle = \sum c_{kf}|\Phi_k\rangle \end{array}$$

Single-particle matrix element (radial overlap) δ_{C2}

$$M_F = \langle\Psi_f|T_+|\Psi_i\rangle = \sum_\alpha \langle\Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle\langle\alpha_n|t_+|\alpha_p\rangle$$

$$\langle\Psi_f|a^\dagger_{\alpha_n} a_{\alpha_p}|\Psi_i\rangle \equiv \rho_\alpha \quad (\neq \rho^T_\alpha)$$

One-body transition Density evaluated between initial and final nuclear states δ_{C1}

$$\langle\alpha_n|t_+|\alpha_p\rangle = \int_0^\infty R_{\alpha_n}(r)R_{\alpha_p}(r)r^2dr \equiv \Omega_\alpha \quad (\neq 1)$$

$$\alpha = (n_\alpha, l_\alpha, j_\alpha, m_\alpha)$$

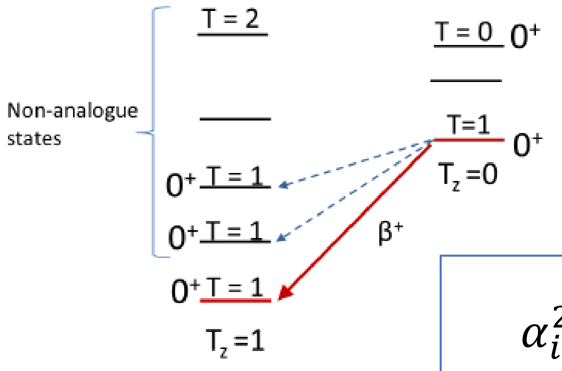
Miller, Schwenk PRC78,80 (2009,2010): Radial excitations !

- Large-scale calculations
- Global parameterization of INC forces
- Revisit WS procedure
- Implicate HF wave functions

I.S. Towner, J.C. Hardy; (1973 – 2020)
W.E. Ormand, B.A. Brown (1985 – 1995)

Shell-model evaluation of δ_{C1}

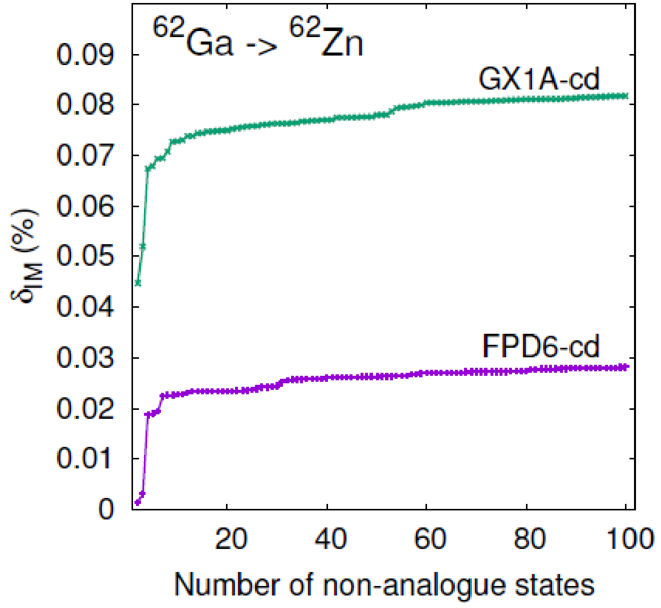
Towner and Hardy:
- Local parametrization of the CD term
- A few (?) non-analogue states considered

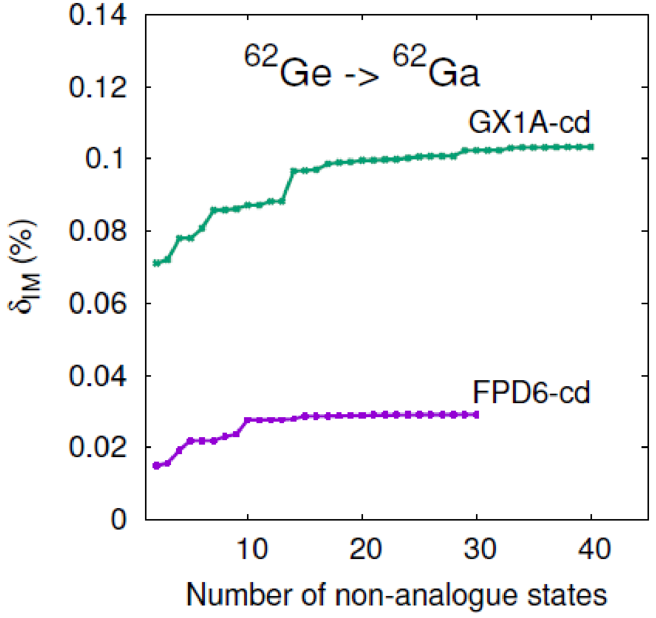


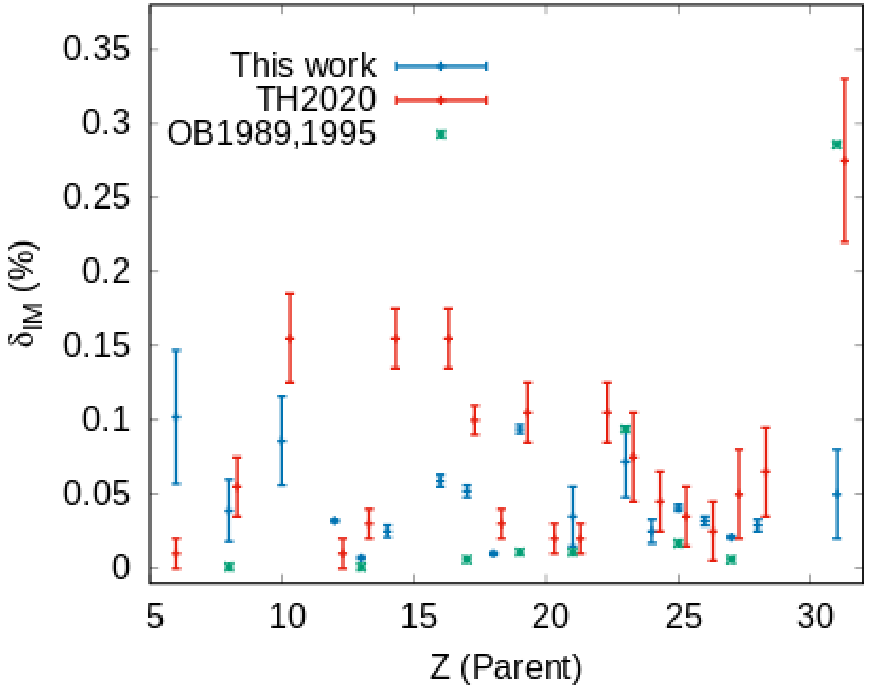
We sum over the Fermi strengths to non-analogue states!
⇒ experimental information on non-analogue strength is highly desirable!

$$\alpha_i^2 \sim \frac{\langle V_{CD}\rangle^2}{\Delta E^2}$$

$$M_{Fi}^2 \sim \frac{\Delta E_{th}^2}{\Delta E_{exp}^2}$$







Xayavong, Smirnova, Nowacki, PRC112 (2025)

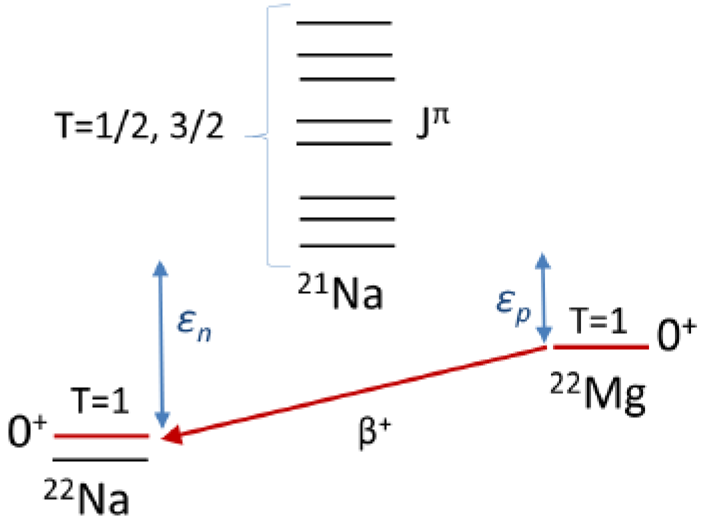
Shell-model evaluation of δ_{C2} beyond the closure approximation

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha} \langle \Psi_f | a^{\dagger}_{\alpha_n} a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega_{\alpha})$$

$$\Downarrow$$

Experimental information on Relevant spectroscopic Factors is highly desirable !

$$\delta_{C2} = \frac{2}{M_F^0} \sum_{\alpha,\pi} \langle \Psi_f | a^{\dagger}_{\alpha_n} | \pi \rangle^T \langle \pi | a_{\alpha_p} | \Psi_i \rangle^T (1 - \Omega_{\alpha}^{\pi})$$



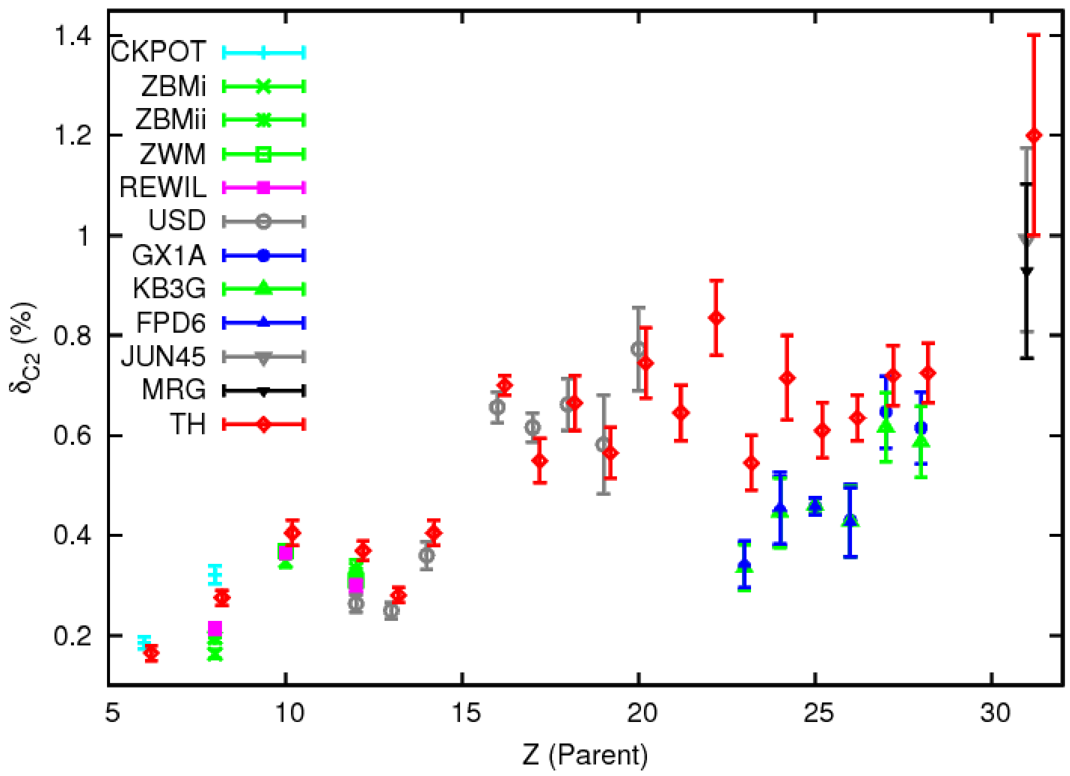
- *Spectroscopic amplitudes* (from the shell-model):

$$\langle \Psi_f | a^{\dagger}_{\alpha_n} | \pi \rangle = \frac{\langle \Psi_f || a^{\dagger}_{\alpha_n} || \pi \rangle}{\sqrt{2J_f + 1}}$$

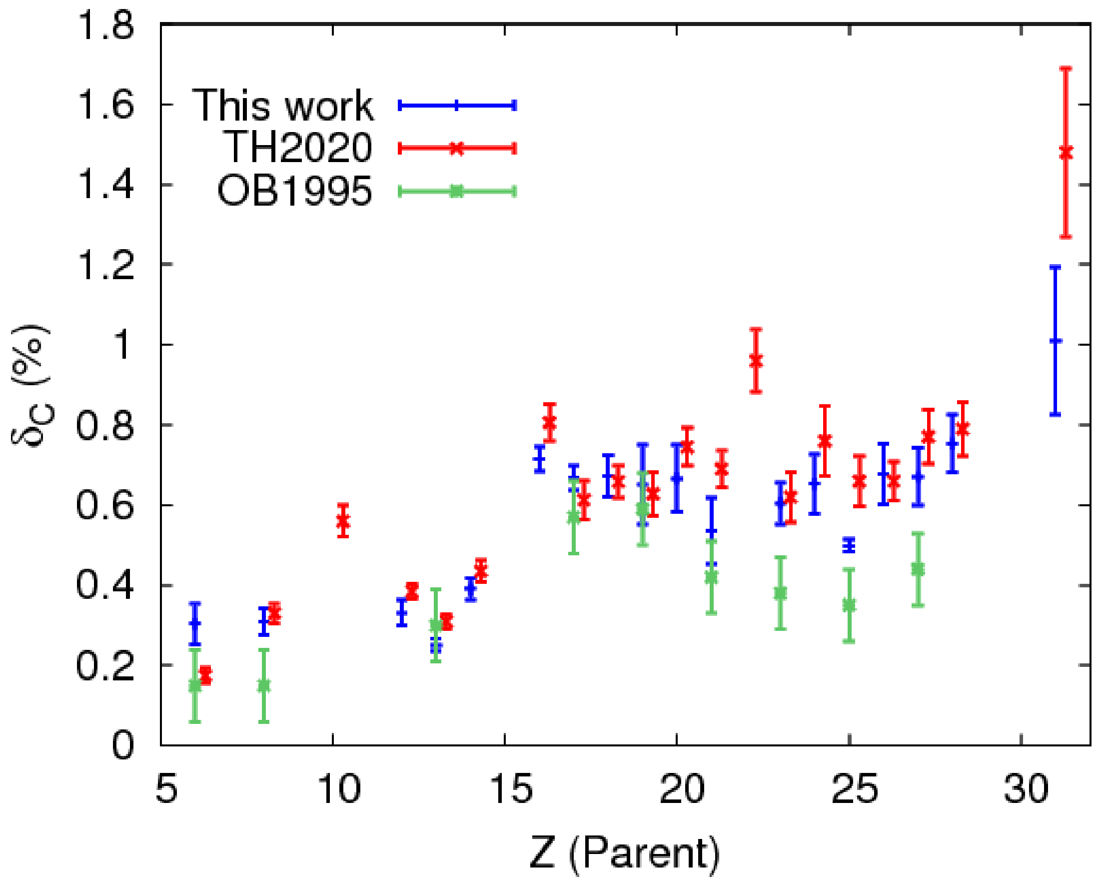
- *Radial-overlap integrals* (from a realistic single-particle potential)

$$\Omega_{\alpha}^{\pi} = \int_0^{\infty} R^{\pi}_{\alpha_n}(r) R^{\pi}_{\alpha_p}(r) r^2 dr$$

Shell-model + Woods-Saxon evaluation of δ_{C2} and δ_C



Preliminary, since no well adjusted INC interaction for psd and ZBM2 model spaces !



Xayavong, Smirnova, Nowacki, PRC112, 055503 (2025)

Results obtained by our collaboration

- Isospin-symmetry breaking correction to Superallowed Fermi beta decay δ_C within shell model with WS or Hartree-Fock wave functions

  L. Xayavong, N. Smirnova, PRC97, 024324 (2018) ⇨ included in the evaluation by Towner and Hardy, 2020

  L. Xayavong, N. Smirnova, PRC105, 044308 (2022)

  L. Xayavong, N. Smirnova, F. Nowacki, PRC112, 055503 (2025)

- Higher-order terms in δ_C

  L. Xayavong, N. Smirnova, PRC109, 014317 (2024)

- Isospin symmetry breaking in mirror Gamow-Teller transitions

  L. Xayavong, Y. Lim, PRC111, 015501 (2025)

  L. Xayavong, Y. Lim, PRC112, 014313 (2025)

- Towards *ab-initio* no-core shell-model calculation of isospin-symmetry breaking correction δ_C

  L. Xayavong, Y. Lim, N. Smirnova, C. Johnson PRC113, 034317 (2026)

FKPPN Project 2026 – goal (1)

Construction of precise isospin-nonconserving interactions for *psd, ZBM2* and *pfg* model spaces for more precision calculation of the δ_{C1} correction within the large-scale shell model

^{10}C, ^{14}O, ^{18}Ne, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{42}Ti, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{66}As

Model spaces and effective interactions (+ charge-dependence)

- p-shell: CKPOT *(Cohen-Kurath, 1965)*
- $(p_{1/2}sd_{5/2})$-shell: ZBM's *(Zuker et al, 1969)*, REWIL *(Reehal, Wildenthal, 1973)*
- sd-shell: USD *(Wildenthal, 1984)*, USDA/USDB *(Brown, Richter, 2006)*
- $(sd_{3/2}f_{7/2}p_{3/2})$-shell: ZBM2 *(Nowacki et al, 2014)*
- pf-shell: KB3G *(Poves et al, 2004)*, GXPF1A *(Honma et al, 2004)*
- $pf_{5/2}g_{9/2}$: JUN45 *(Honma et al, 2009)*, MRG *(Nowacki et al, 1996)*

Only Coulomb Is currently Used !

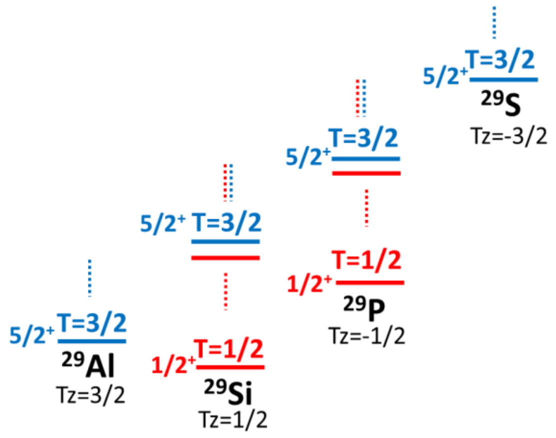
NuShellX@MSU shell-model code *(B.A. Brown, W.D.M. Rae, Nucl. Data Sheets 120, (2014).*

Valence-space isospin non-conserving (INC) Shell Model

Include Coulomb + empirical effective charge-dependent term perturbatively

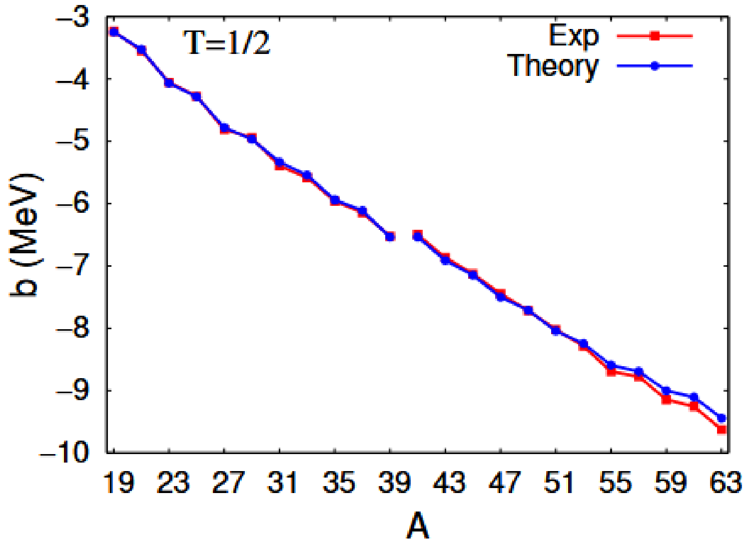
$$H_{INC} = H_0^{pn} + V_{res} + V_{CD}$$ with

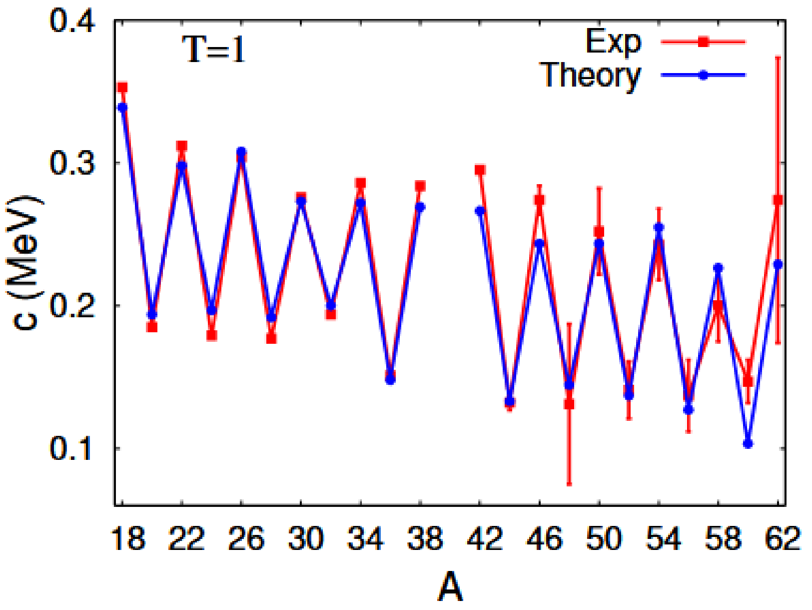
$$V_{CD} = V_C + V_{CSB}^{(1)} + V_{CIB}^{(2)}$$



$$\langle \eta T T_z | H_{INC} | \eta T T_z \rangle = a^{th}(\eta, T) + b^{th}(\eta, T) T_z + c^{th}(\eta, T) T_z^2$$

$$\mathcal{M}(\eta, T, T_z) = a(\eta, T) + b(\eta, T) T_z + c(\eta, T) T_z^2 ,$$





RMS errors (theory & experiment) :

	b (keV)	c (keV)	N data (b)	N data (c)
sd shell	32	11	81 (all T)	51 (T=1,3/2,2)
pf shell	50	20	g.s. (T=1/2,1)	g.s. (T=1)

FKKPN Project 2026 – goal (2)

Refining Woods-Saxon potential parameters from global systematics for more precise calculation of δ_{C2} correction within the large-scale shell model

Parameterization

$$V_{WS}(r) = -\frac{V_0}{1+\exp\left(\frac{r-R}{a}\right)} - V_{ls}(r)\vec{l}\cdot\vec{\sigma} + V_C(r) \quad + Vas + Vsurf$$

- A. Bohr, B.R. Mottelson modified (BM_m) from *Nuclear Structure, Vol. I.*
- N. Schwierz, I. Wiedenhöver, A. Volya (SWV) from *nucl-th:0709.3525*

Under stringent experimental constraints:

- (V_0,r_0) are adjusted simultaneously to reproduce experimental **nucleon separation energies** and **charge radii**

$$\psi(r) \to \exp\left(-\frac{\sqrt{2m|\epsilon|}r}{\hbar}\right)$$

- A new approach to nuclear charge radii beyond the closure approximation: $\langle r^2\rangle_{ch} = \frac{1}{Z}\sum_{\pi\alpha}\langle\alpha|r^2|\alpha\rangle^{\pi}|\langle\Psi_i||a_\alpha^\dagger||\pi\rangle|^2$
- Thorough **propagation of uncertainties**.

Conclusions and Prospects

Conclusions :

- Superallowed 0+ -> 0+ beta decay provides the most precise way to test the CVC hypothesis and to derive the |Vud| matrix element of the CKM matrix for its unitarity tests => numerous experimental programs at RIB facilities throughout the world

- Phenomenological shell model provides a robust framework in the description of isospin symmetry breaking and currently assures the most precise value of the nuclear structure correction

- Our projects VUD-NUCLEAR aims at further improving of the shell-model + WS precision : construction of new effective charge-dependent interactions and constraining further the WS potential

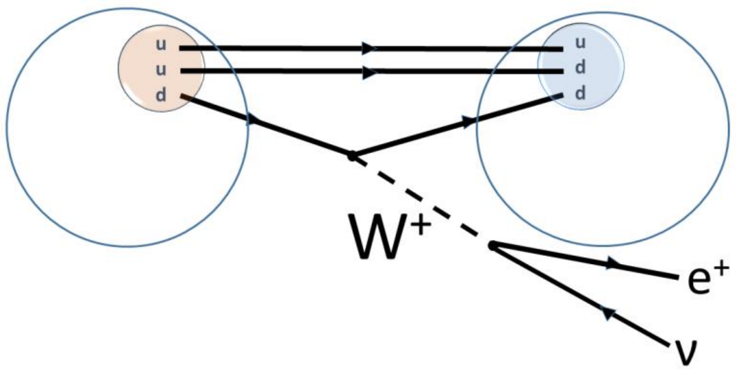
Perspectives and further plans :

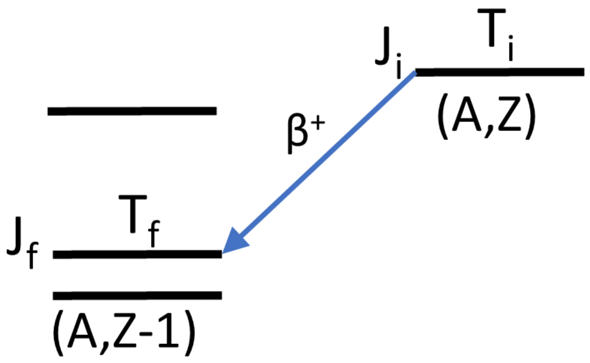
- The issue of the exact Fermi operator
- Mirror transitions (between T=1/2 states)
- High-dimensions and heavier nuclei (up to A=100)

GOAL : Towards high precision tests of the Standard Model and stringent constraints on the possible physics beyond it

BACK-UP SLIDES

Nuclear beta decay





Total decay rate:

$$\Gamma = \frac{\ln 2}{t} = \frac{G_V^2}{K}\left[g_V^2 f_V |M_F|^2 + g_A^2 f_A |M_{GT}|^2\right]$$

$$M_F = \langle \Psi_f | \sum_k t_\pm | \Psi_i \rangle\,, \quad M_{GT} = \langle \Psi_f | \sum_k \sigma t_\pm | \Psi_i \rangle$$

$$K = \frac{2\pi^3 \ln 2 \hbar^7}{m_e^5 c^4}$$

$g_V=1$, $g_A \approx 1.27$

$$f = \int_1^{W_0} (W - W_0)^2 p W F(Z,W) S(Z,W) \mathrm{d}W$$

$$W_0 = Q_{EC}/(m_e c^2) - 1$$

For 0+ -> 0+ beta decay:

$$ft^{0^+ \to 0^+} = \frac{K}{|M_F|^2 G_V^2}\,.$$

Should be constant (for fixed T) – a consequence of the Conserved Vector Current (CVC) hypothesis

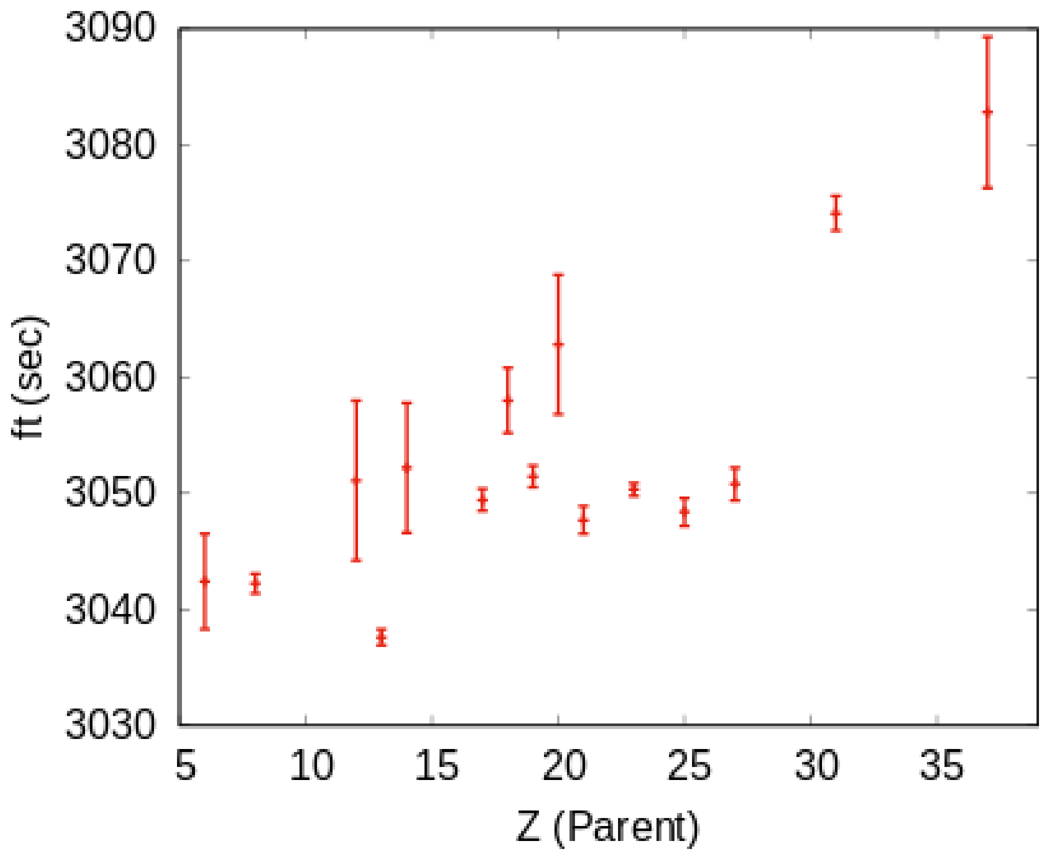
Superallowed $0^+ \rightarrow 0^+$ beta decay

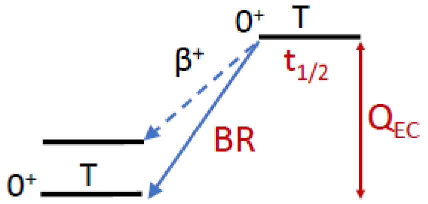
15 best known $T = 1$ emitters ($ft^{0^+ \rightarrow 0^+}$-value known with a precision $\lesssim 0.4\%$):

^{10}C, ^{14}O, ^{22}Mg, ^{26m}Al, ^{26}Si, ^{34}Cl, ^{34}Ar, ^{38m}K, ^{38}Ca, ^{42}Sc, ^{46}V, ^{50}Mn, ^{54}Co, ^{62}Ga, ^{74}Rb

J.C. Hardy, I.S. Towner, PRC102, 045501 (2020)

Experimental data on $t_{1/2}$, BR, Q_{EC}





Statistical rate function:

$$f = f(Z, Q_{EC})$$

Partial halflife:

$$t = \frac{t_{1/2}}{BR}$$

$$ft^{0^+ \rightarrow 0^+} = \frac{K}{|M_F|^2 G_V^2} \, .$$

Many-body approaches to isospin-symmetry breaking

- Shell model (from 60's . . .)

  W.E. Ormand, B.A. Brown et al 1985 –; S.Nakamura, K.Muto, T.Oda, 1994; A.P. Zuker, S.M. Lenzi, M.A. Bentley et al, 2001 – 2018; K. Kaneko et al, 2010 – 2018; Y.H. Lam, N. Smirnova, E. Caurier, 2013.

- Gamow shell-model, SMEC

  N. Michel, W. Nazarewicz, M. Ploszajczak, PRC82 (2010), . . .

- HF + Tamm-Dankoff, RPA, HTDA

  I. Hamamoto, H. Sagawa, N. V. Giai, J. Dobaczewski, T. Suzuki, G. Colo, J. Le Bloas, N. Auerbach et al, 1993 – 2018

- Relativistic RPA

  H. Liang, N. V. Giai, J. Meng, 2009 –

- J-projected and T-projected DFT with ISB terms

  W. Satula, J. Dobaczewski et al, 2009 – 2018, P. Baczyk et al, PLB778, 178 (2018)

- VAP technique on the HFB basis

  A. Petrovici et al, 2008 – 2018

- Isovector giant monopole resonance

  G. Colo et al (1993); N. Auerbach, Phys. Rep. 98, 273 (1983); PRC (2009)

Diagonalization + experimentally constrained

Variational + global energy-density functional

“gross”

Theoretical description of isospin-symmetry breaking

Sources of isospin symmetry breaking (nucleon level):

- Coulomb interaction between protons
- Charge-dependent (CD) nuclear forces and Mp and Mn differences

- Ab-initio calculations: use CD NN potentials V (CD-Bonn, chiral EFT N^nLO potentials)
- Phenomenological approach: constrain effective CD forces from experiment to add to V

$$\hat{V}_{INC} = \hat{V}_{Coul} + \hat{V}_{CD} = \sum_{\lambda=0,1,2} \hat{V}^{(\lambda)}_{INC}, \quad \text{where} \quad \begin{cases} \hat{V}^{(0)}_{INC} = (v_{pp} + v_{nn} + v^{T=1}_{np})/3 \\ \hat{V}^{(1)}_{INC} = v_{pp} - v_{nn} \\ \hat{V}^{(2)}_{INC} = (v_{pp} + v_{nn})/2 - v^{T=1}_{np} \end{cases}$$

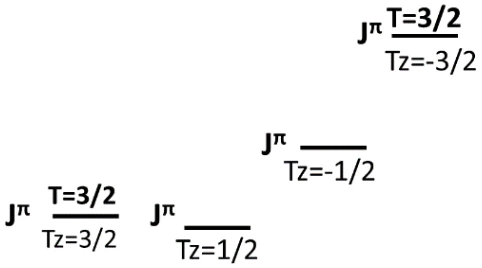
$$\langle \Psi_{TT_z} | \hat{V}_{INC} | \Psi_{TT_z} \rangle = E^{(0)}(\eta, T) + E^{(1)}(\eta, T) T_z + E^{(2)}(\eta, T) \left[3T_z^2 - T(T+1) \right]$$

Fit to experimental b and c coefficients through the valence space !

Nuclei along N=Z line: isospin-symmetry breaking

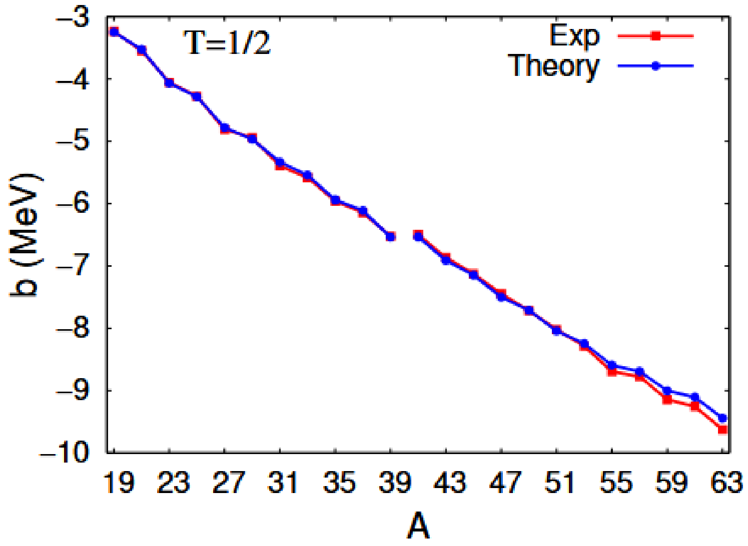
Isospin non-conserving Hamiltonian
(Coulomb + effective charge-dependent components)

$$H_{INC} = H_0 + V_{res} + V_{INC}$$



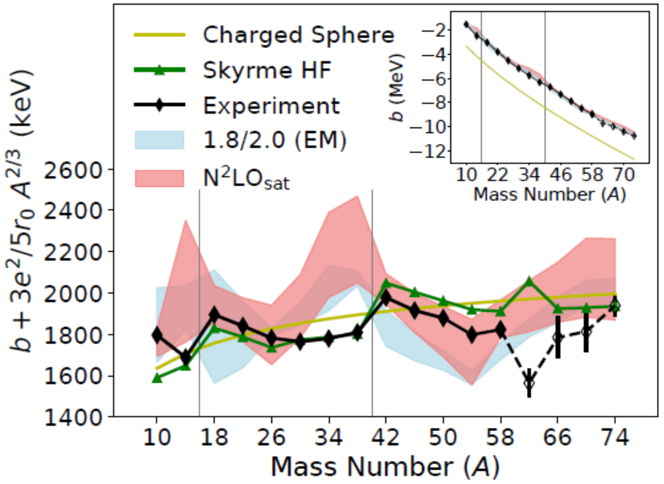
$$M(\eta, T, M_T) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$

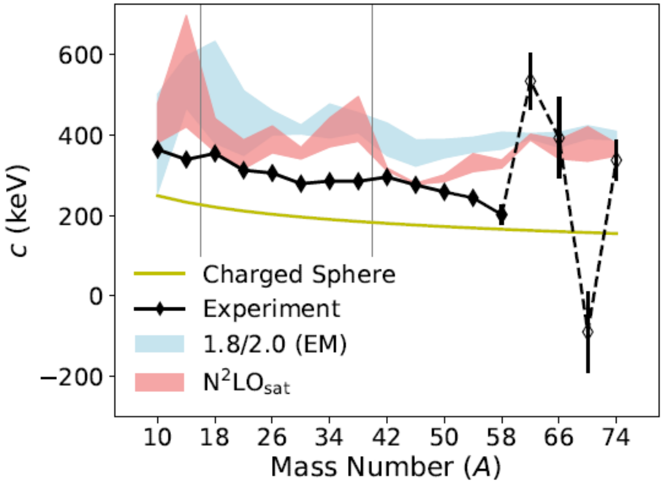
sd shell : rms(b) = 30 keV, rms(c) = 10 keV

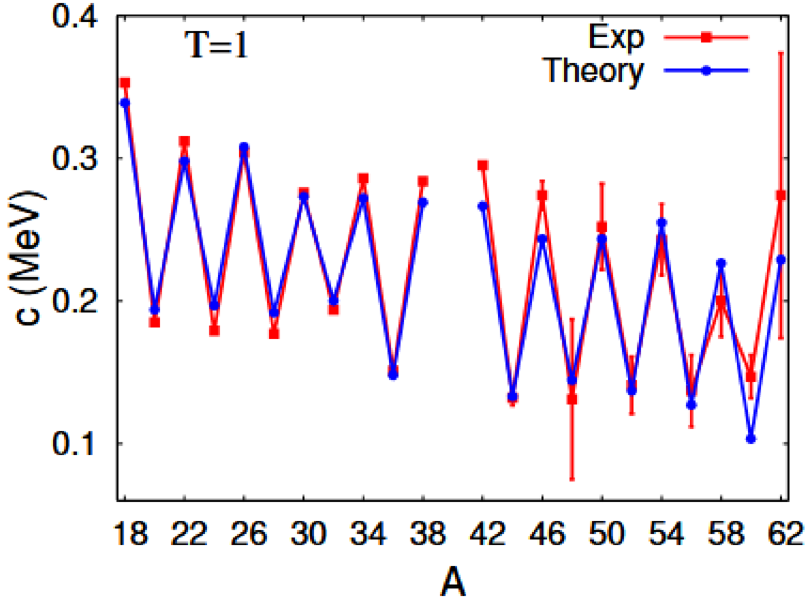


Ormand, Brown (1989), Lam, Smirnova, Caurier (2013); Magilligan, Brown (2020)

IMSRG: Martin et al, PRC104, 014324 (2021) – A=10,14,18,22,…,74

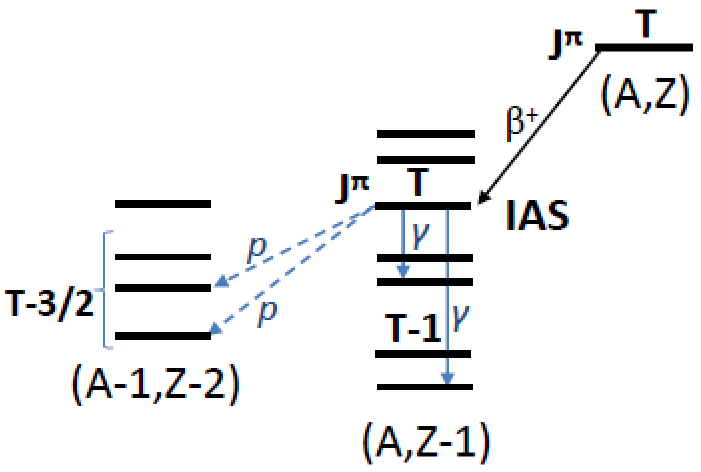


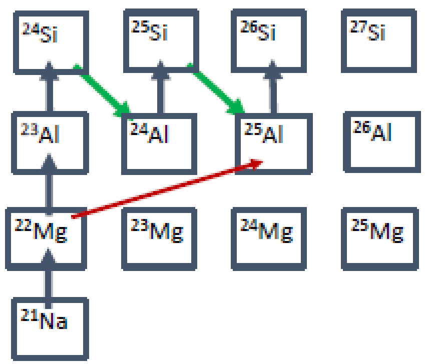


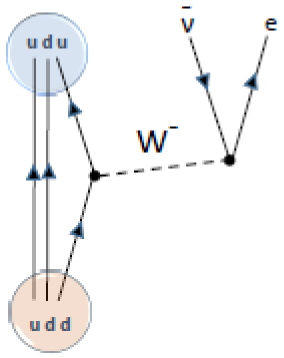


Applications of isospin-symmetry breaking

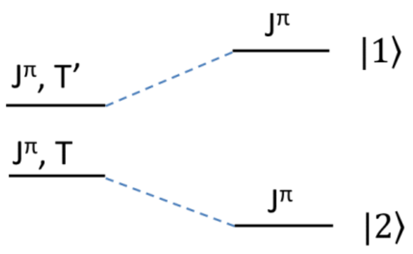
- Structure and decay modes of proton-rich nuclei and nuclei along $N = Z$
- Applications for nuclear astrophysics (masses, (p,γ), (α,p), $\ldots$ reaction rates)
- Corrections to weak-interaction processes in nuclei for the tests of the Standard Model (CVC hypothesis, CKM matrix, . . .)







Isospin-forbidden decays and isospin mixing



$$\alpha^2 \sim \frac{\langle V_{CD} \rangle^2}{\Delta E^2}$$

Current status :

- Reliable prediction of isospin-forbidden decay rates and of isospin mixing on theoretical grounds is challenging, because of poorly determined ΔE
- Safe predictions of Coulomb mixing matrix elements
- Efficient way: combination of theory and experiment !
- Lack of precision for the search of higher-order terms in IMME in quartets and quintets

$$M(\eta,T,T_z) = a(\eta,T) + b(\eta,T)T_z + c(\eta,T)T_z^2 + d(\eta,T)T_z^3 + e(\eta,T)T_z^4$$

Isospin-symmetry breaking: NN interaction and light nuclei

Sources of isospin-symmetry breaking : $m_u \neq m_d$ and electromagnetic interactions between quarks

- Coulomb interaction between protons
- $M_p - M_n \approx 1.3\ MeV$ and charge-dependent forces of nuclear origin

Experimental evidence on charge-dependent components of NN forces

1) NN scattering length in the $^1S_0$ channel:

$$a_{nn} - a_{pp} = 1.65 \pm 0.60\ fm$$ → *Charge-symmetry breaking (CSB)* $$\boldsymbol{V_{pp} - V_{nn} \neq 0}$$

$$\frac{1}{2}(a_{nn} + a_{pp}) - a_{np} = 5.6 \pm 0.6\ fm$$ → *Charge-independence breaking (CIB)* $$\boldsymbol{V_{pn}^{T=1} - \frac{1}{2}(V_{pp} + V_{nn}) \neq 0}$$

2) Nolen-Schiffer anomaly (1969): mirror binding energy differences (^3H – ^3He , …)

Henley, Miller, 1979

Class I : $1, \boldsymbol{t}(j) \cdot \boldsymbol{t}(k)$
Class II : $t_z(j)t_z(k)$
Class III : $t_z(j) + t_z(k)$
Class IV : $t_z(j) - t_z(k)$, $[\boldsymbol{t}(j) \times \boldsymbol{t}(k)]_z$

Ab-initio approaches →

Light nuclei in *ab-initio* approaches :
A = 3,7,8: GFMC with CD interactions
Wiringa, Pastore, Pieper, Miller, PRC88 (2013)

G.A. Miller et al PR194, 1 (1990); χEFT : U. van Kolck, J.L. Friar, E.Epelbaum, …

$$\boldsymbol{V_I > V_{II} > V_{III} > V_{IV}}$$

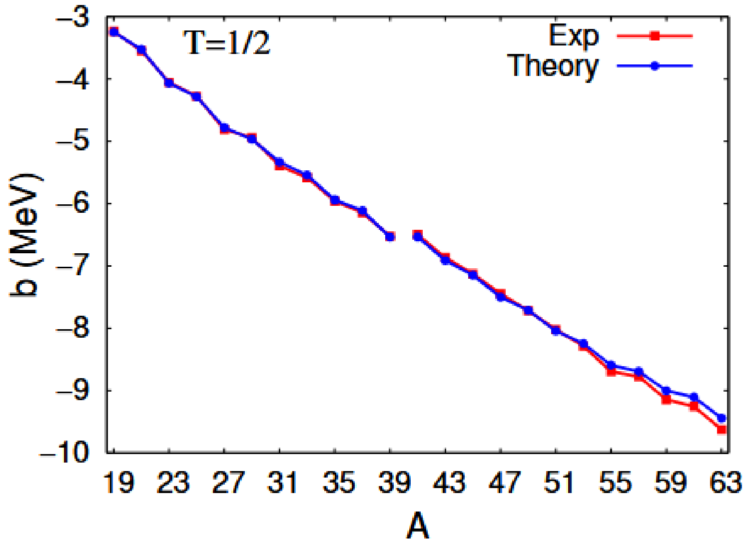
$$\hat{H}_{V-A} = \frac{G_V}{\sqrt{2}} \hat{J}_\mu^\dagger \hat{j}^\mu + h.c.$$

$$\begin{aligned}
\hat{J}_\mu^\dagger &= \hat{V}_\mu + \hat{A}_\mu \\
\hat{V}_\mu &= i\overline{\psi}_p \left(g_V(k^2)\gamma_\mu + \frac{g_W(k^2)}{2m_N}\sigma_{\mu\nu}k_\nu + ig_S(k^2)k_\mu \right)\psi_n \\
\hat{A}_\mu &= i\overline{\psi}_p \left(g_A(k^2)\gamma_\mu + \frac{g_T(k^2)}{2m_N}\sigma_{\mu\nu}k_\nu + ig_P(k^2)k_\mu \right)\gamma_5\psi_n
\end{aligned}$$

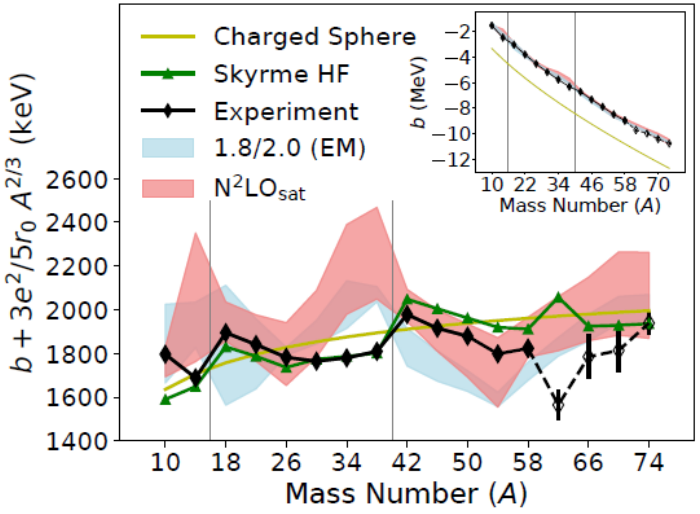
Nuclei along N=Z line: isospin-symmetry breaking

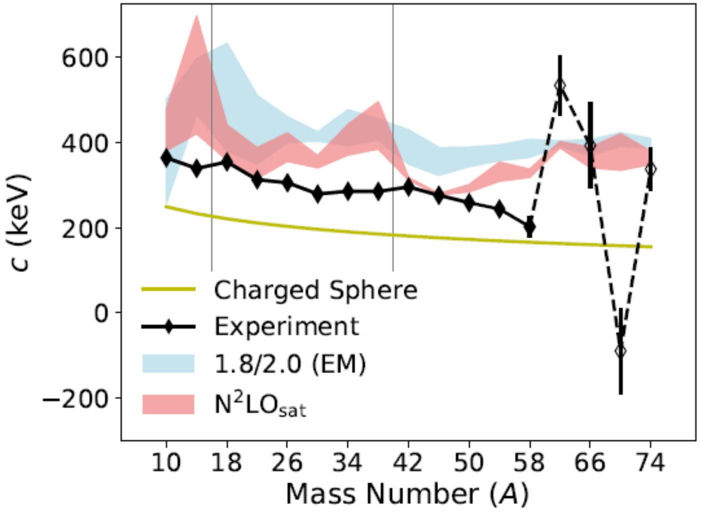
$$M(\eta, T, M_T) = a(\eta, T) + b(\eta, T)T_z + c(\eta, T)T_z^2$$

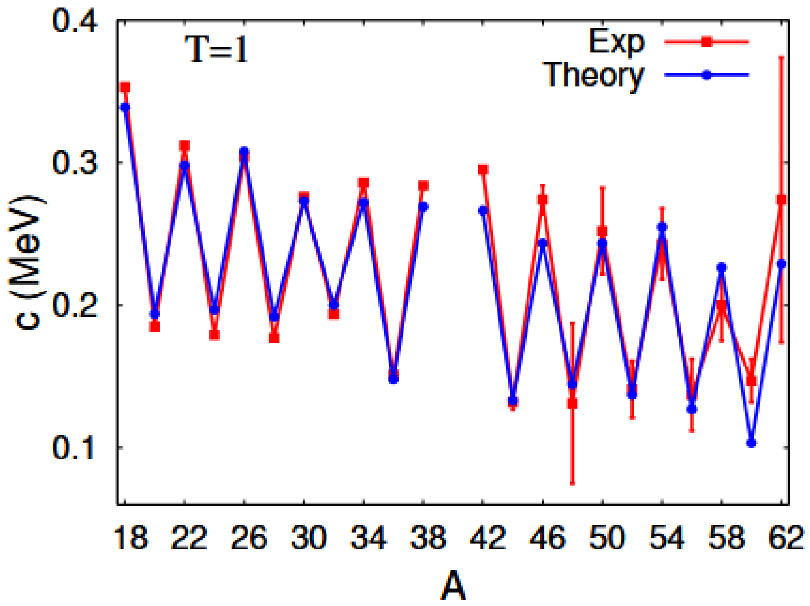
	b (keV)	c (keV)	N data (b)	N data (c)
sd shell	32	11	81 (all T)	51 (T=1,3/2,2)
pf shell	50	20	g.s. (T=1/2,1)	g.s. (T=1)



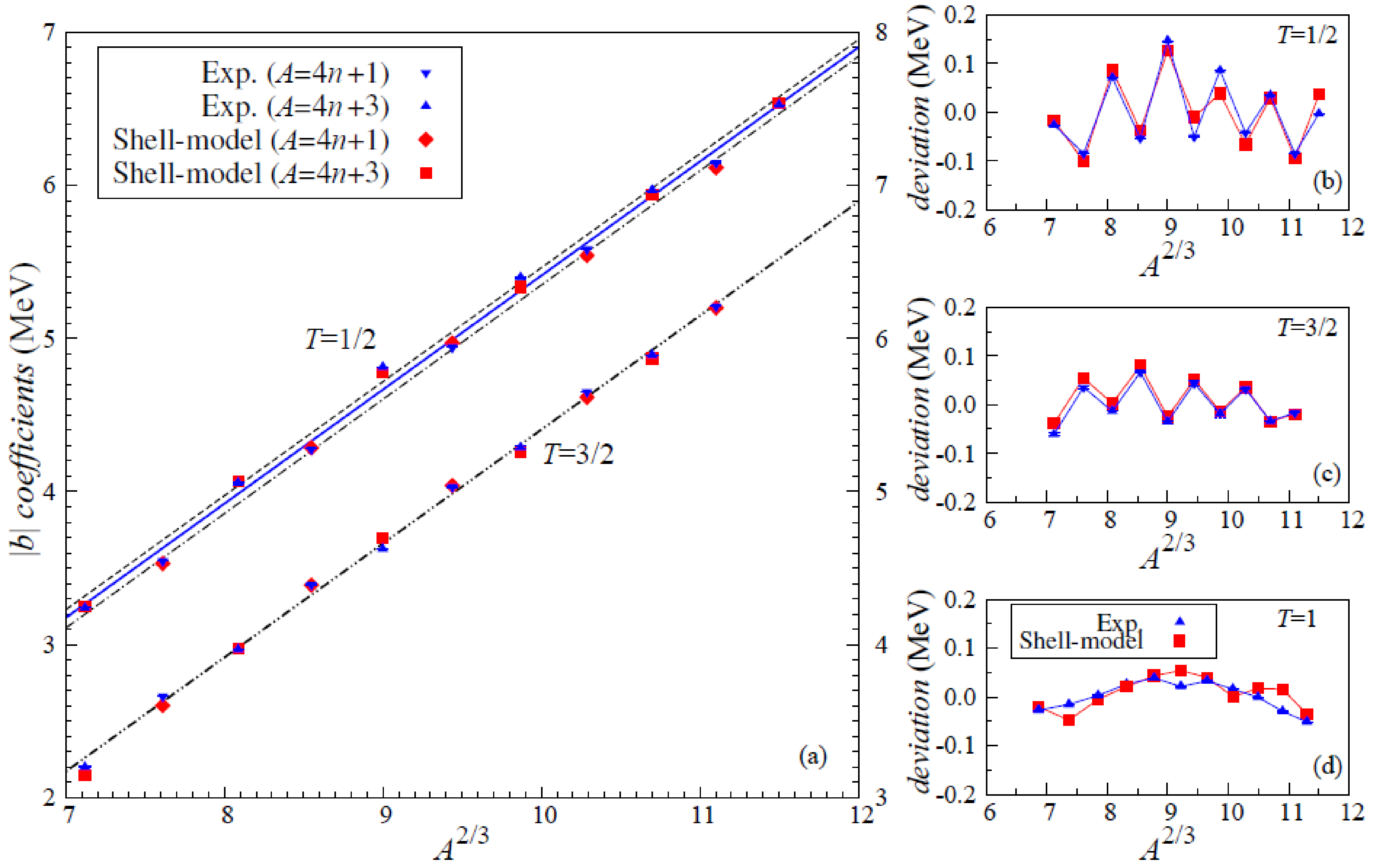
IMSRG: Martin et al, PRC104, 014324 (2021) – A=10,14,18,22,…,74







Theoretical description of isospin-symmetry breaking



Lam, Smirnova, Caurier (2013)

IMME b and c coefficients of lowest and excited multiplets

Importance for mass and excitation energies predictions

$$M_{T_z} = a + bT_z + cT_z^2$$

T=1

$$\begin{aligned} M_1 &= a + b + c \\ M_0 &= a \\ M_{-1} &= a - b + c \end{aligned}$$

- Prediction of masses and excited levels in proton-rich nuclei, e.g. if *Tz*>0 :

$$M_{-T_z} = M_{T_z}^{exp} + 2b^{th}T_z$$

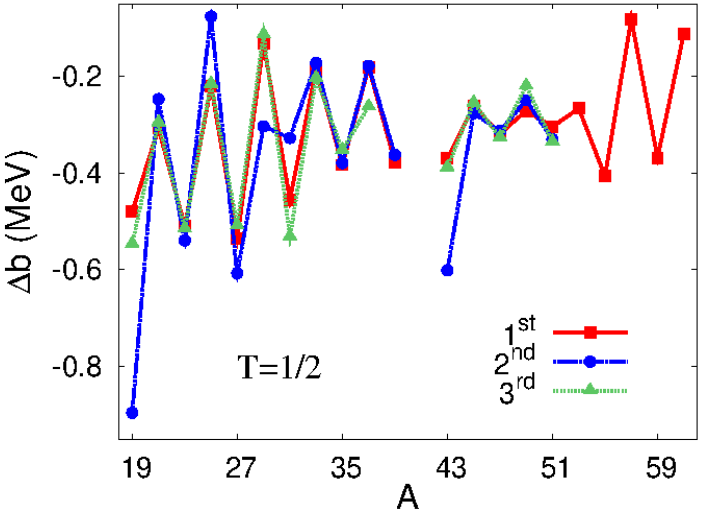
- Particular case of triplets :

$$M_{-1} = 2M_0^{exp} - M_1^{exp} + 2c^{th}$$

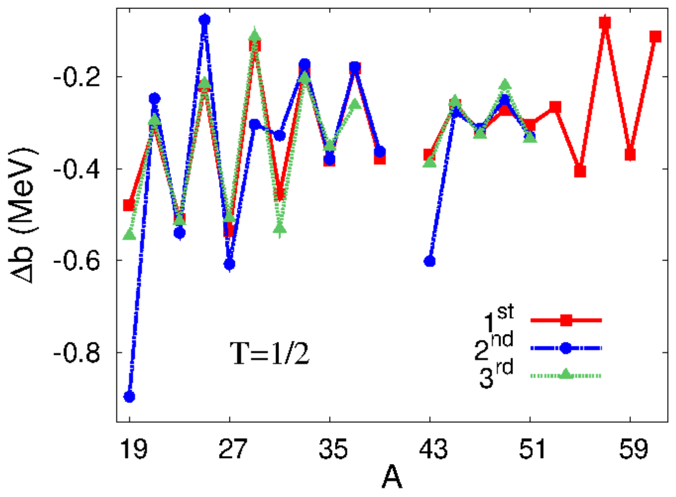
Important for nuclear astrophysics applications!

Brown, Richter (from 2011); Kaneko et al (2013)

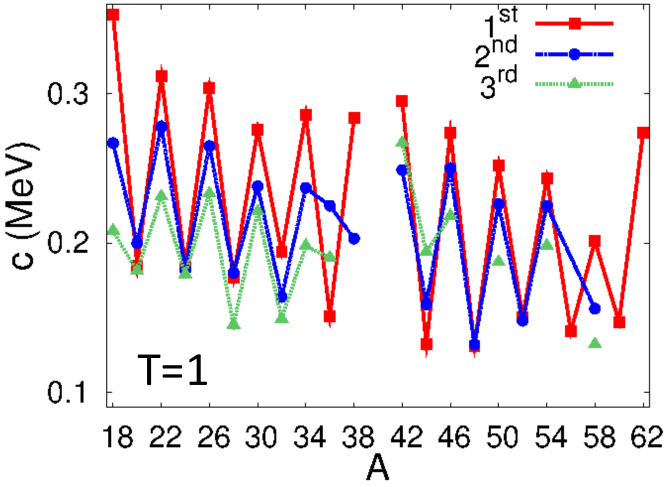
Experiment



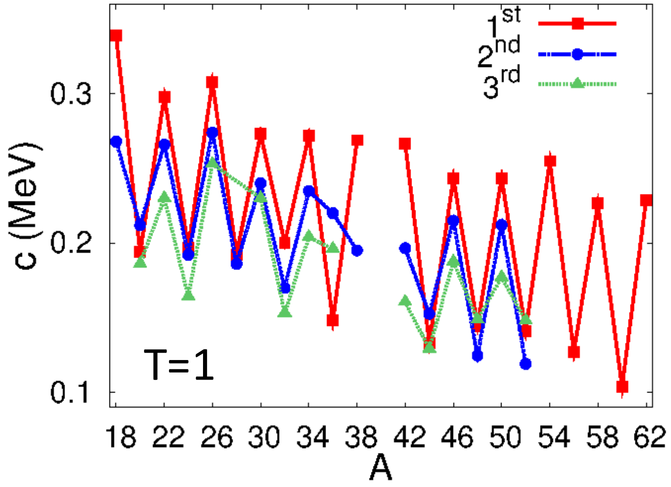
Theory



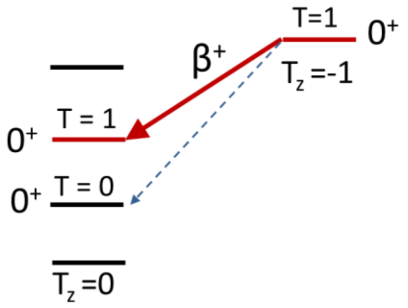
Experiment



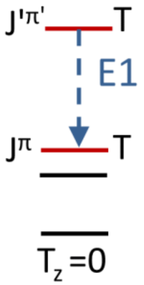
Theory



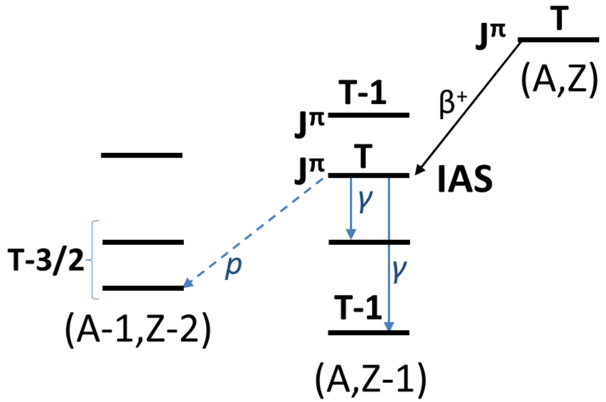
Isospin-forbidden decays and isospin mixing (?)



Isospin-forbidden Fermi beta decay



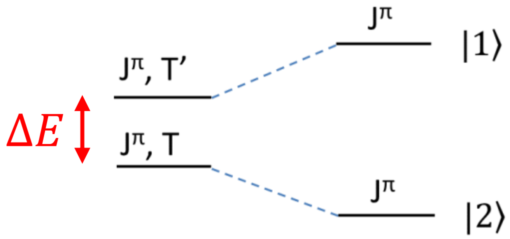
Isospin-forbidden E-M transitions



Isospin-forbidden proton emission

$$H_{INC} = H_0 + V + V_{CD}$$ ⇨ ***Isospin mixing !***

How reliable can these phenomena be described theoretically?



$$|1\rangle = \sqrt{1-\alpha^2}|T\rangle + \alpha|T'\rangle$$

$$|2\rangle = \alpha|T\rangle + \sqrt{1-\alpha^2}|T'\rangle$$

$$\alpha^2 \sim \frac{\langle V_{CD}\rangle^2}{\Delta E^2}$$

$\langle V_{CD}\rangle^2$ ← Reliable (at low energies)

ΔE^2 ← ***Challenging...***

$$\langle V_{CD}\rangle = \langle T|V_{CD}|T'\rangle$$

p-shell	< 150(40) keV
sd-shell	< 100(40) keV
pf-shell	< 40(20) keV

Fermi matrix element within the shell model

Isospin-symmetry limit

$$M_F^0 = \sum_\alpha \rho_\alpha^T \Omega_\alpha^T = \pm\sqrt{T(T+1) - T_{zi}T_{zf}}\,, \qquad \Omega_\alpha^T = 1$$

Realistic model (Coulomb and charge-symmetry breaking effective nuclear forces)

$$M_F = \sum_\alpha \rho_\alpha \Omega_\alpha$$

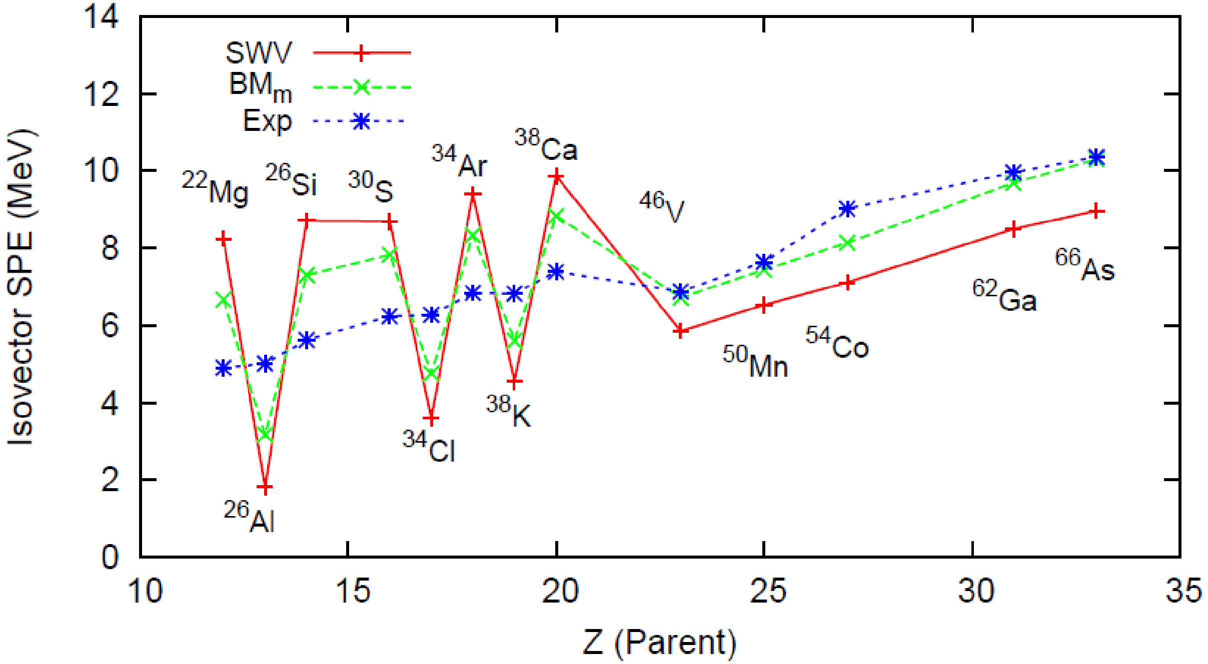
In first order perturbation theory:

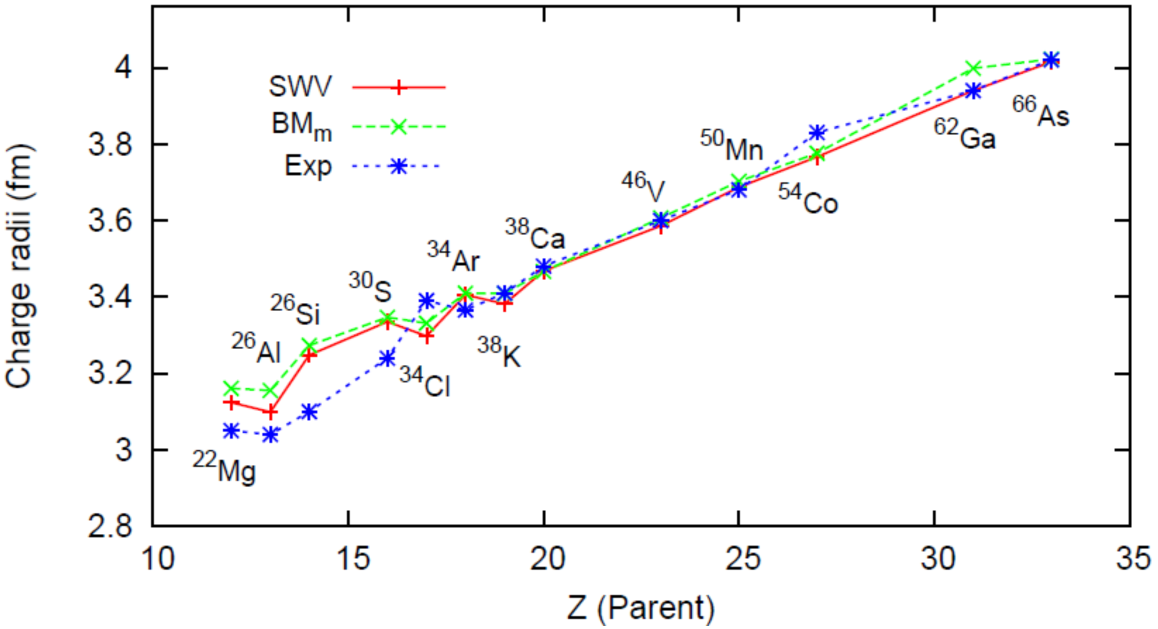
$$|M_F|^2 \approx |M_F^0|^2 \Big[1 - \underbrace{\frac{2}{M_F^0} \sum_\alpha \left(\rho_\alpha^T - \rho_\alpha \right)}_{\delta_{C1}} - \underbrace{\frac{2}{M_F^0} \sum_\alpha \rho_\alpha^T (1 - \Omega_\alpha)}_{\delta_{C2}} \Big],$$

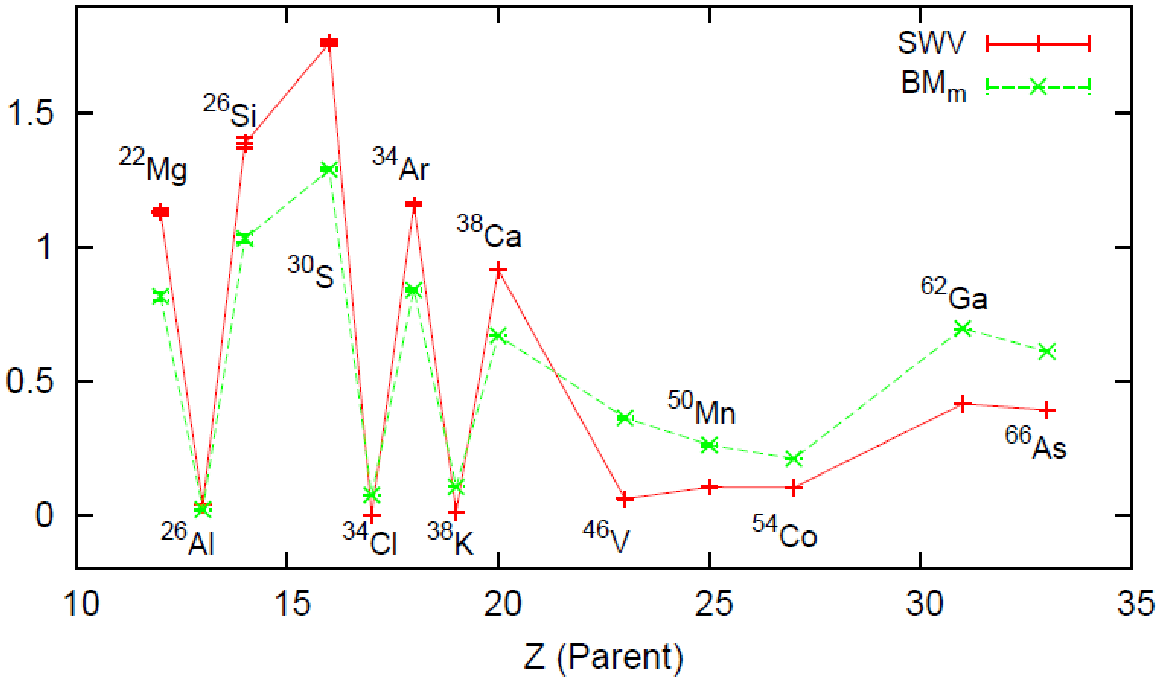
$$\delta_C \approx \delta_{C1} + \delta_{C2}$$

- δ_{C1} is the *isospin-mixing* part
- δ_{C2} is the *radial-overlap* part

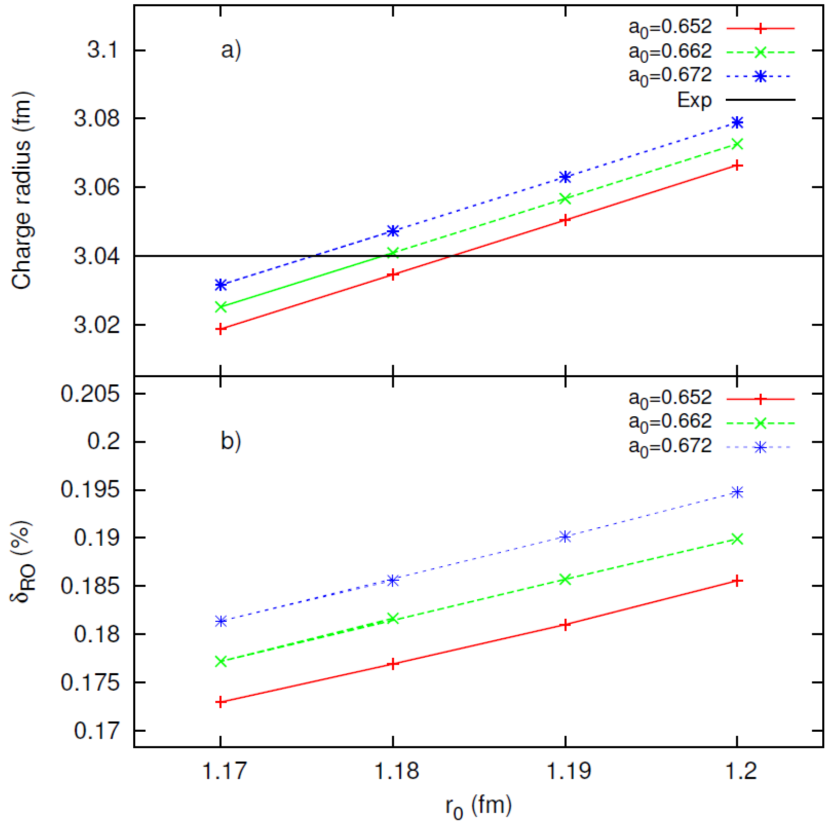
WS Non-adjusted !

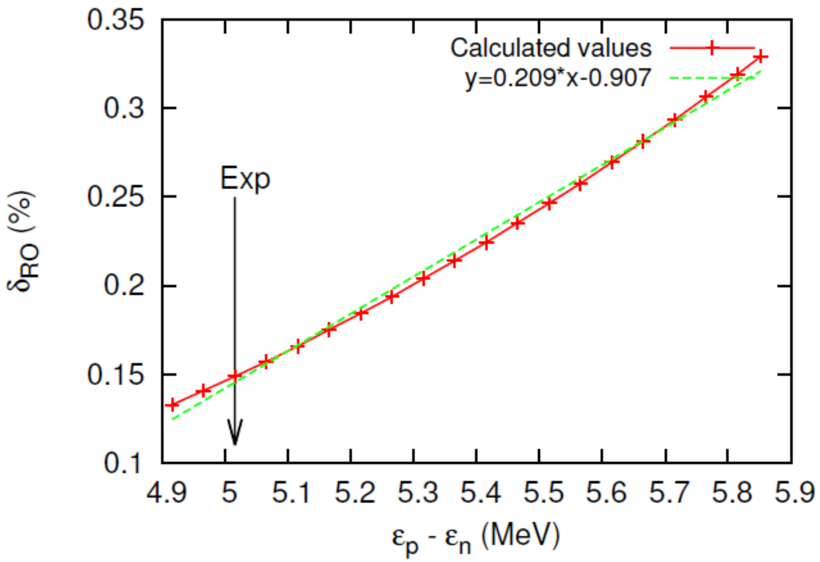


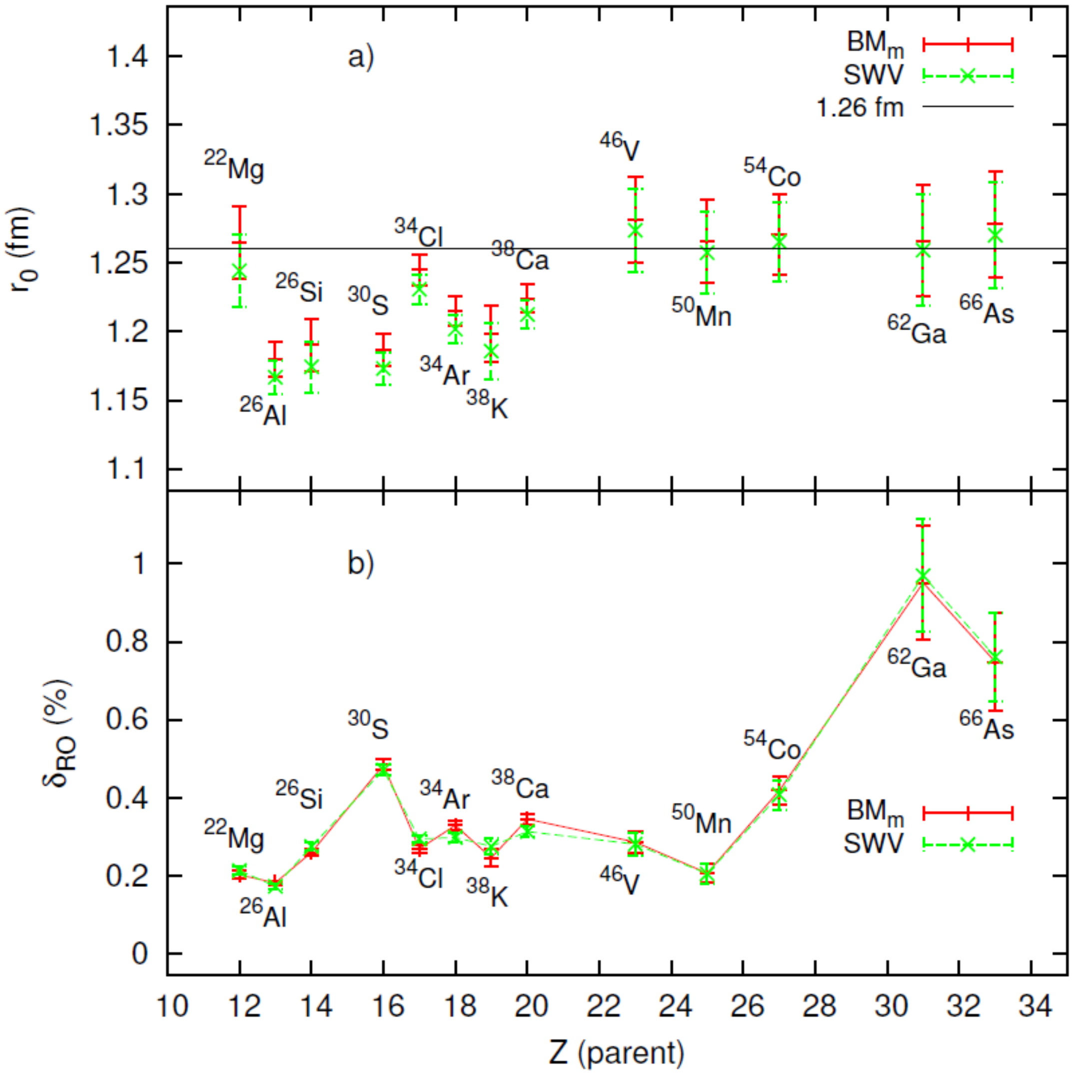




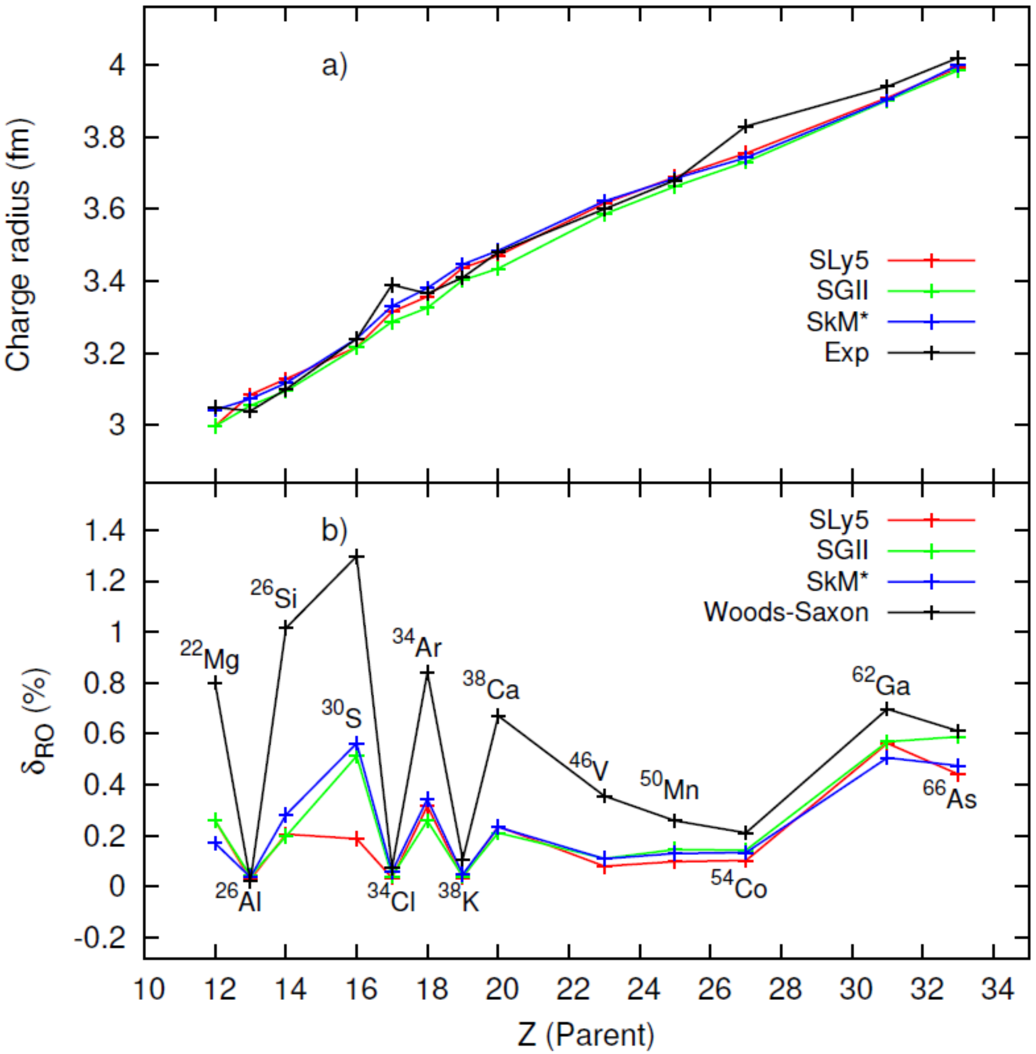
WS Adjusted !


a)
b)
Charge radius (fm)
δ_{RO} (%)
r_0 (fm)
a_0=0.652
a_0=0.662
a_0=0.672
Exp


Calculated values
y=0.209*x-0.907
Exp
δ_{RO} (%)
$\varepsilon_p - \varepsilon_n$ (MeV)


a)
b)
BM_m
SWV
1.26 fm
r_0 (fm)
δ_{RO} (%)
Z (parent)
^{22}Mg
^{26}Al
^{26}Si
^{30}S
^{34}Cl
^{34}Ar
^{38}K
^{38}Ca
^{46}V
^{50}Mn
^{54}Co
^{62}Ga
^{66}As

WS and HF Non-adjusted !



WS and HF Adjusted !