Probing new intra-atomic force with isotope shifts

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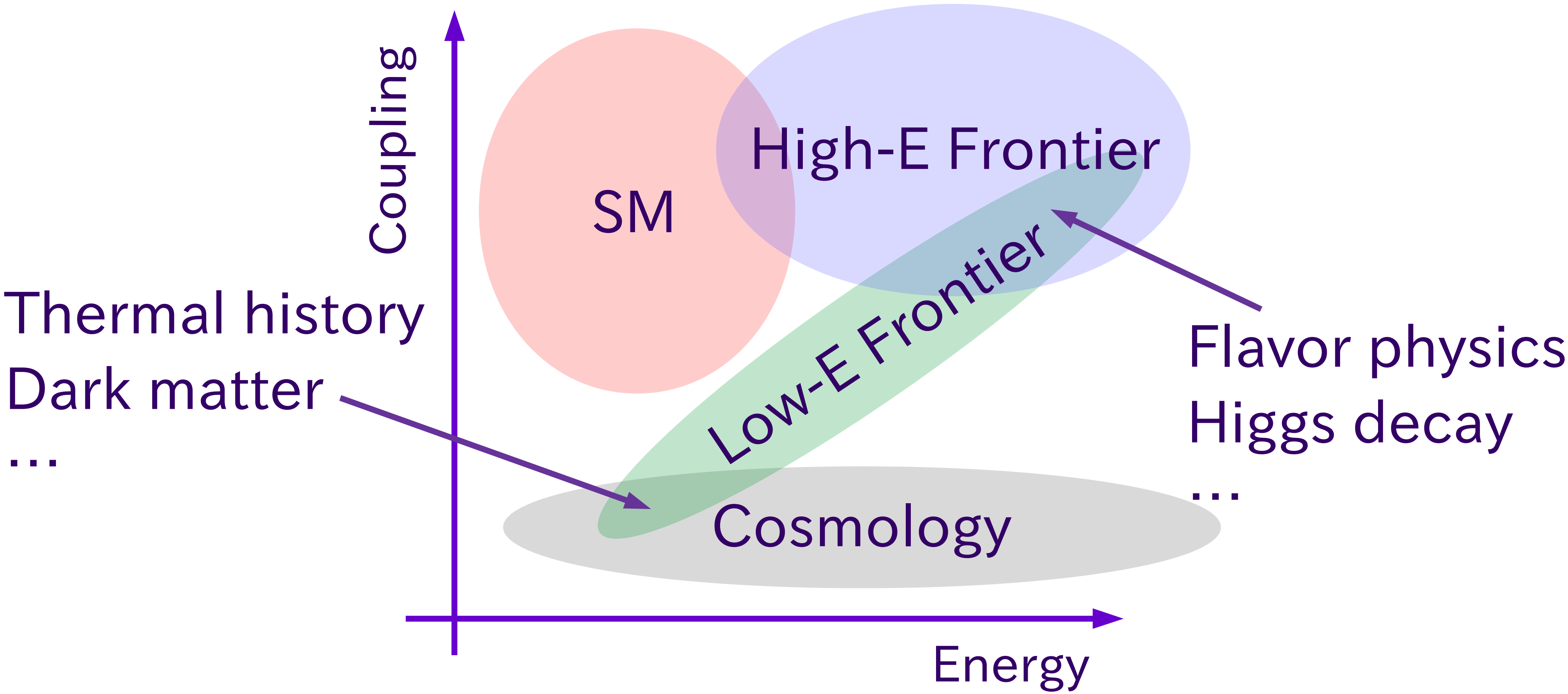
Based on 1710.11443
with K. Mikami & M. Tanaka (Osaka U)

Physics of light new particles

- Interaction strength $\sim \dfrac{g}{M} : \dfrac{1}{1\ \mathrm{TeV}} = \dfrac{10^{-4}}{100\ \mathrm{MeV}}$



- Many talks and posters have told us the importance.

Precision measurements

- Error of the electron g-2 is $O(10^{-10})$.

$$\frac{g_e - 2}{2} = \begin{cases} -0.001\,159\,652\,180\,73(28)_{\mathrm{EX}} \\ -0.001\,159\,652\,181\,64(76)_{\mathrm{TH}} \end{cases}$$

- Error of the atomic clocks $O(10^{-15}$–$10^{-18})$.

$$^{87}\mathrm{Sr}: \ \ 429\ 228\ 004\ 229\ 873.4\ \mathrm{Hz}$$

(From Wikipedia:atomic clock)

  - The calculation of the spectrum is too difficult.

- Reduce the uncertainty.
  - The new constraint on **the light new boson**.

Plan

- Introduction
- The linearity and its violation
- The field shift and its higher order
- The particle shift
- Numerical results and other constraints
- Conclusion

Isotope shift and the linearity

- Isotope shifts follow a linearity.

$$\delta H_{A'A} = \delta K_{A'A} + \delta V_{A'A}$$

Isotope dependence.

$$\delta\nu = G\,\delta\mu + F\,\delta\langle r^2\rangle$$

Wave function dependence.

- Linearity for isotope pairs.

1963: W. H. King

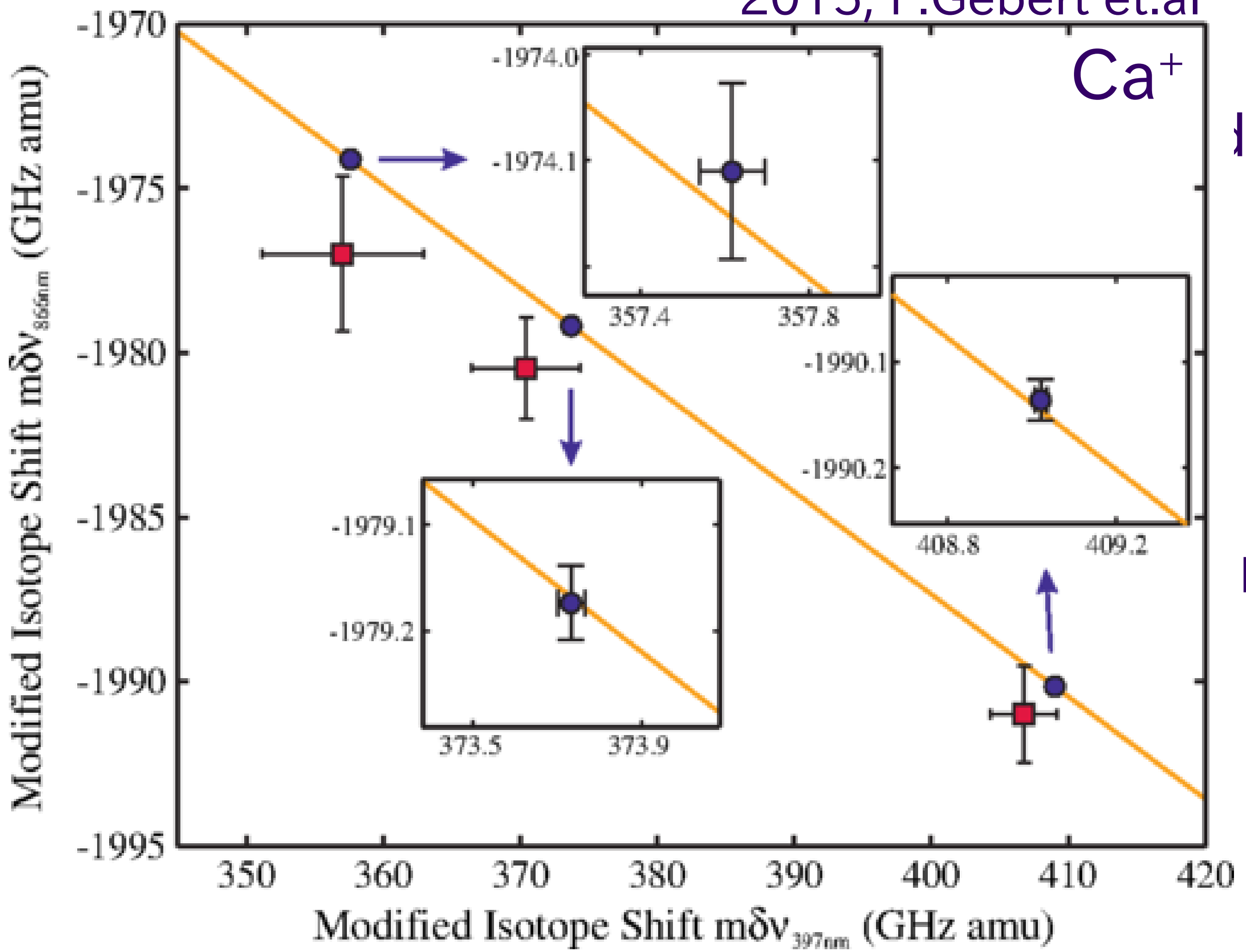
$$\frac{\delta\nu_2}{\delta\mu} = \boxed{\frac{F_2}{F_1}}\frac{\delta\nu_1}{\delta\mu} + \boxed{G_2 - \frac{F_2}{F_1}G_1}$$

Constant for isotope pairs.

Isotope shift and the linearity

- Isotope shifts follow a linearity.

2015, F.Gebert et.al



lence.

H. King

Isotope shift and the linearity

non

does not

- Isotope shifts follow a linearity.

$$\delta H_{A'A} = \delta K_{A'A} + \delta V_{A'A}$$

Isotope dependence.

$$\delta\nu = G\,\delta\mu + F\,\delta\langle r^2\rangle + X$$

NLO corrections
Yukawa potential

Wave function dependence.

Non

- Linearity for isotope pairs.

2016, C. Delaunay et. al

$$\frac{\delta\nu_2}{\delta\mu} = \frac{F_2}{F_1}\frac{\delta\nu_1}{\delta\mu} + G_2 - \frac{F_2}{F_1}G_1 + \left(X_2 - \frac{F_2}{F_1}X_1\right)/\delta\mu$$

Constant for isotope pairs.

Field shift

Def: $\int d\vec{r}\ \left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right)\ \delta V(\vec{r})$

$$-Z\alpha \int d\vec{r'}\ \frac{\delta\rho(\vec{r'})}{|\vec{r}-\vec{r'}|}$$

Expand

$$\propto \int_0^\infty dr' \int_0^{r'} dr\, r^2 \sum_k \xi_k r^k \left(r' - \frac{r'^2}{r}\right) \delta\rho(r')$$

$$\delta\langle r^k\rangle = \int d\vec{r}\, r^k \delta\rho(r)$$

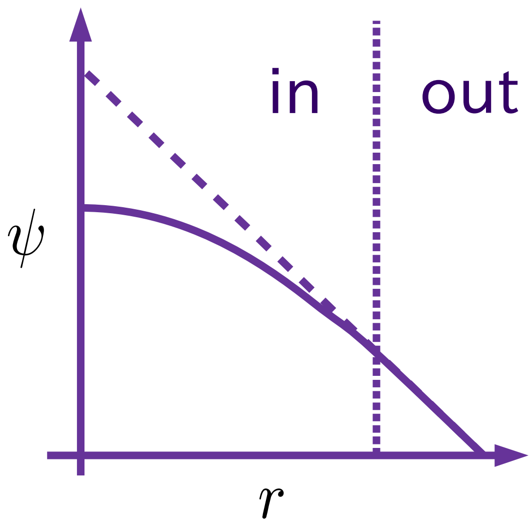
$$= Z\alpha \sum_k \frac{\xi_k}{(k+3)(k+2)} \delta\langle r^{k+2}\rangle$$

1969, E. C. Seltzer

- NLO field shift

$$\delta\nu = G\delta\mu + F\delta\langle r^2\rangle + \tilde{F}\delta\langle r^4\rangle + \cdots$$

$$\psi \sim \chi_0 + \chi_2 r^2 + \cdots$$



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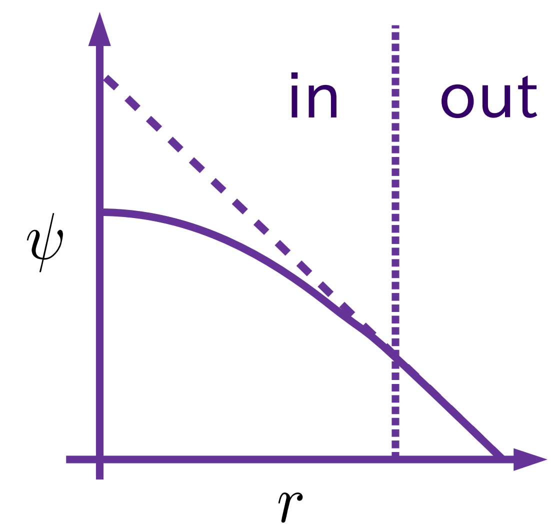

Field shift

Def: $\int d\vec{r}\, \left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right) \delta V(\vec{r})$

$$\delta V(\vec{r}) = -Z\alpha \int d\vec{r'}\, \frac{\delta\rho(\vec{r'})}{|\vec{r} - \vec{r'}|}$$

Expand

$$\propto \int_0^\infty dr' \int_0^{r'} dr\, r^2 \sum_k \xi_k r^k \left(r' - \frac{r'^2}{r}\right) \delta\rho(r')$$

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▶ NLO field shift

$$\delta\nu = G\delta\mu + F\delta\langle r^2\rangle + \tilde{F}\delta\langle r^4\rangle + \cdots$$

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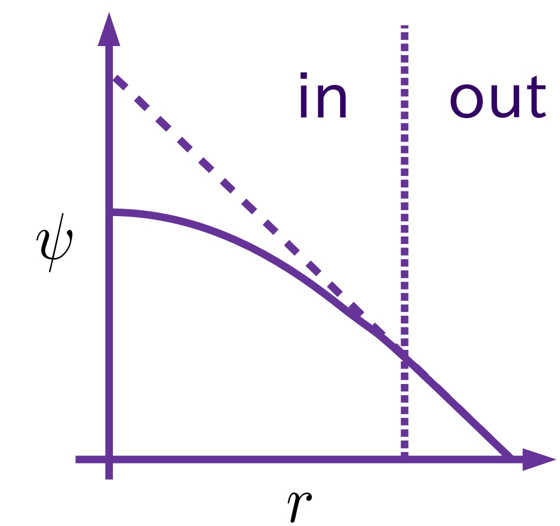
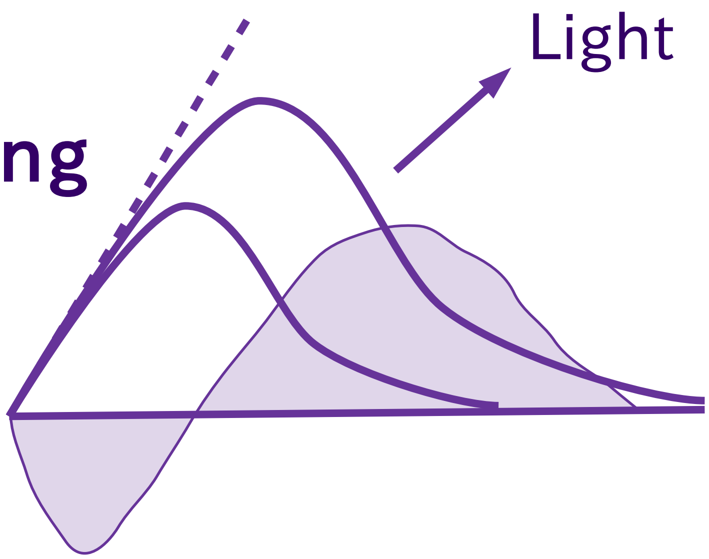


Particle shift

Def: $$\int d\vec{r}\,\left(|\psi_j(\vec{r})|^2 - |\psi_i(\vec{r})|^2\right)(A' - A)\frac{g_n g_e}{4\pi}\frac{e^{-mr}}{r}$$

- Similar to the field shift.
- Sensitive to the **e-n coupling**



- For heavy mediator

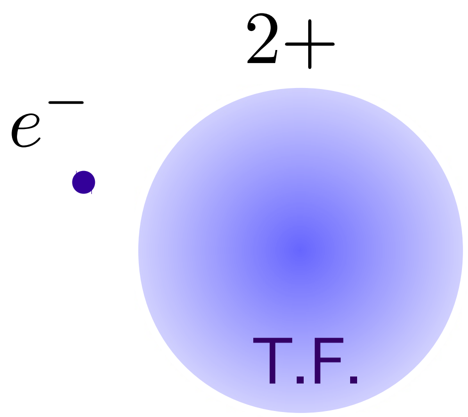
$$= (A' - A)\frac{g_n g_e}{4\pi}\sum_k \frac{k!}{m^{k+2}}\xi_k$$

- $$\delta\nu = G\delta\mu + F\left(\delta\langle r^2\rangle + c_0/m^2\right) + \tilde{F}\left(\delta\langle r^4\rangle + c_2/m^4\right) + \cdots$$

Keep the linearity

Non-linearity

Wave functions of ions

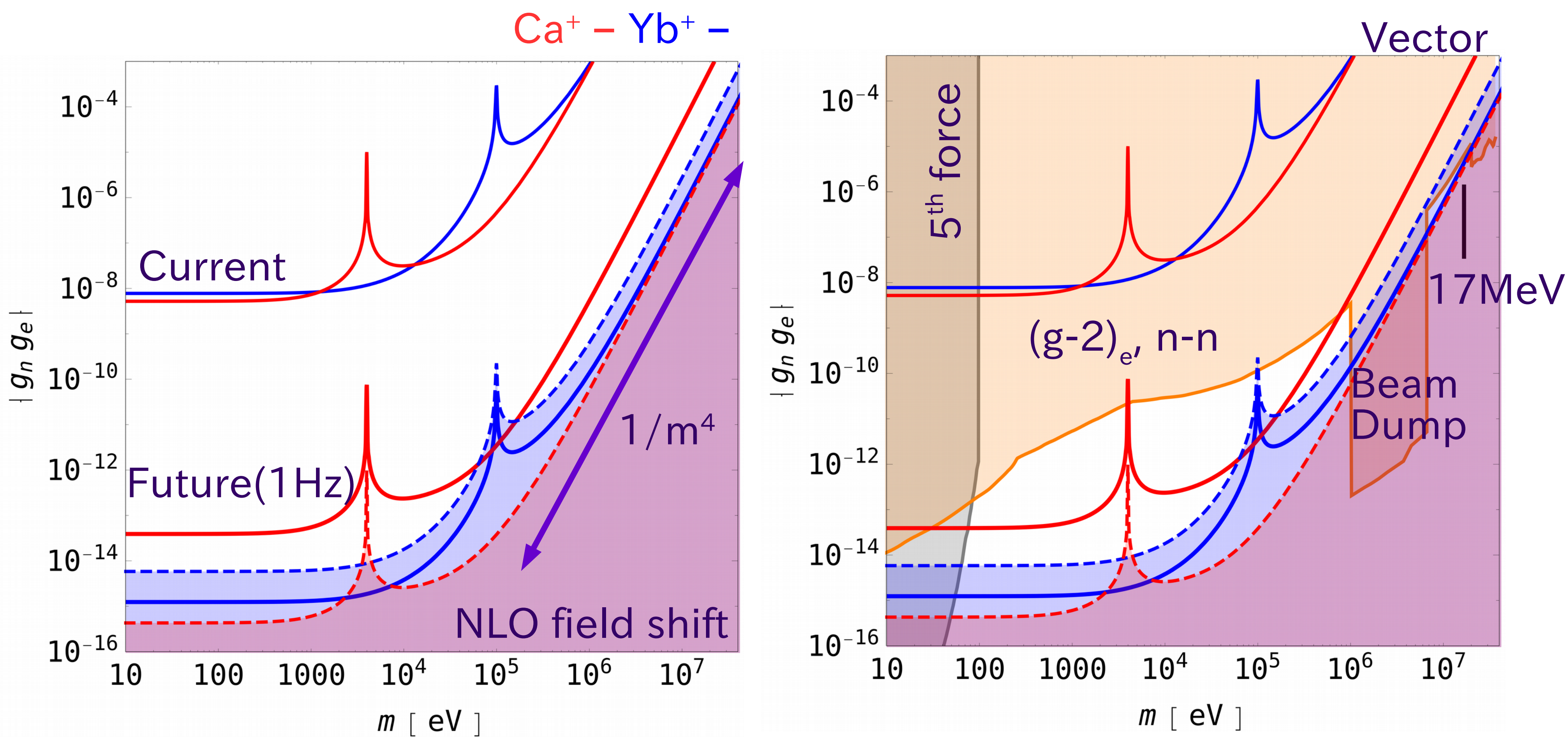


Single electron
+
The **Thomas-Fermi** potential
(Semi-classical free electron gas.)

Ca^+	$^2S_{1/2} \rightarrow {}^2P_{1/2}$ (323meV) $4s \rightarrow 4p$ (386meV)	$^2D_{3/2} \rightarrow {}^2P_{1/2}$ (704meV) $3d \rightarrow 4p$ (-1309meV)
Yb^+	$^2S_{1/2} \rightarrow {}^2P_{1/2}$ (301meV) $6s \rightarrow 6p$ (309meV)	$^2D_{3/2} \rightarrow {}^2D[3/2]_{1/2}$ (760meV) $4f \rightarrow 6s$ (39.5meV)

- s- & p-states are ☺, d- & f-states are ☹ .
- Numerically, good agreement with other results.

Sensitivity and constraints



- NLO field shift limits the future sensitivity.
- 100 eV – 1 MeV is the main target.

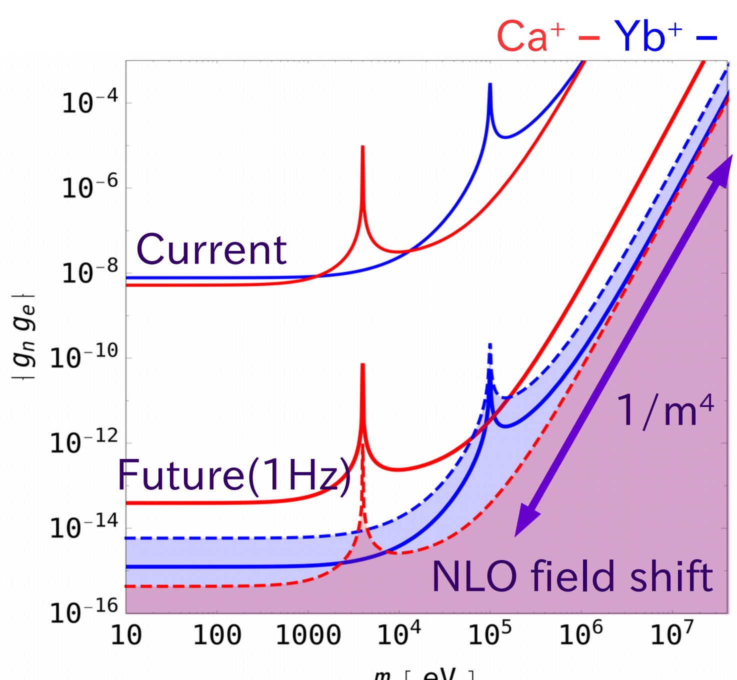
Conclusion

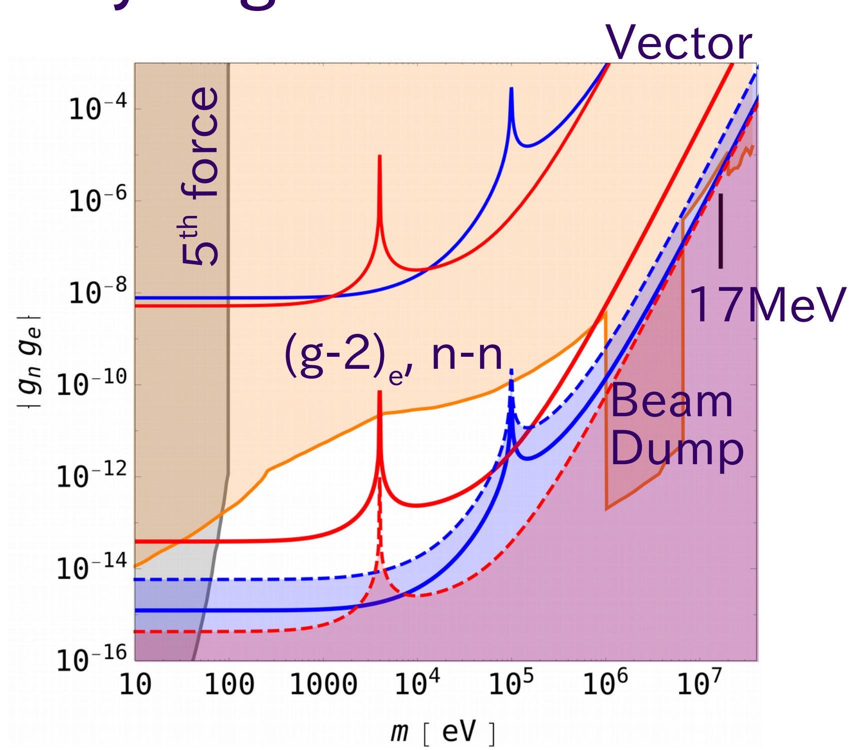
Precision spectroscopy + King´s linearity

▼

New physics as the non-linearity

- SM background of NLO field shift.
- The scaling law at the heavy region.





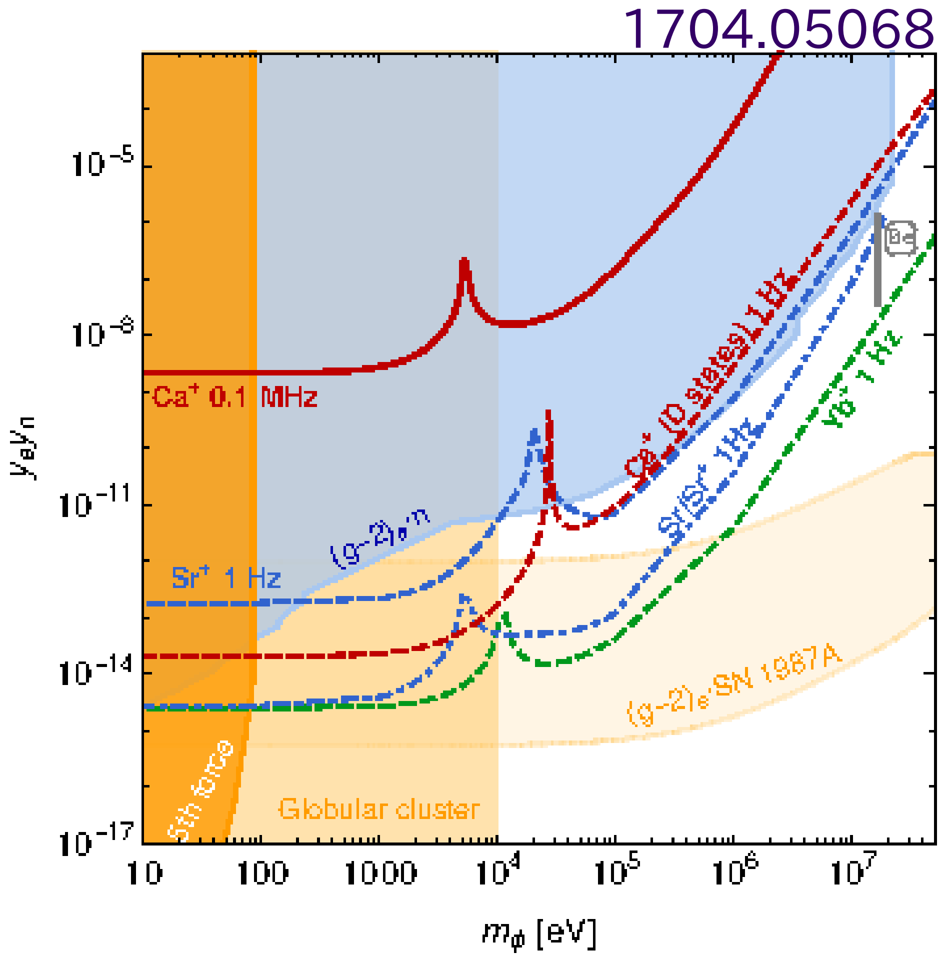
Other issues

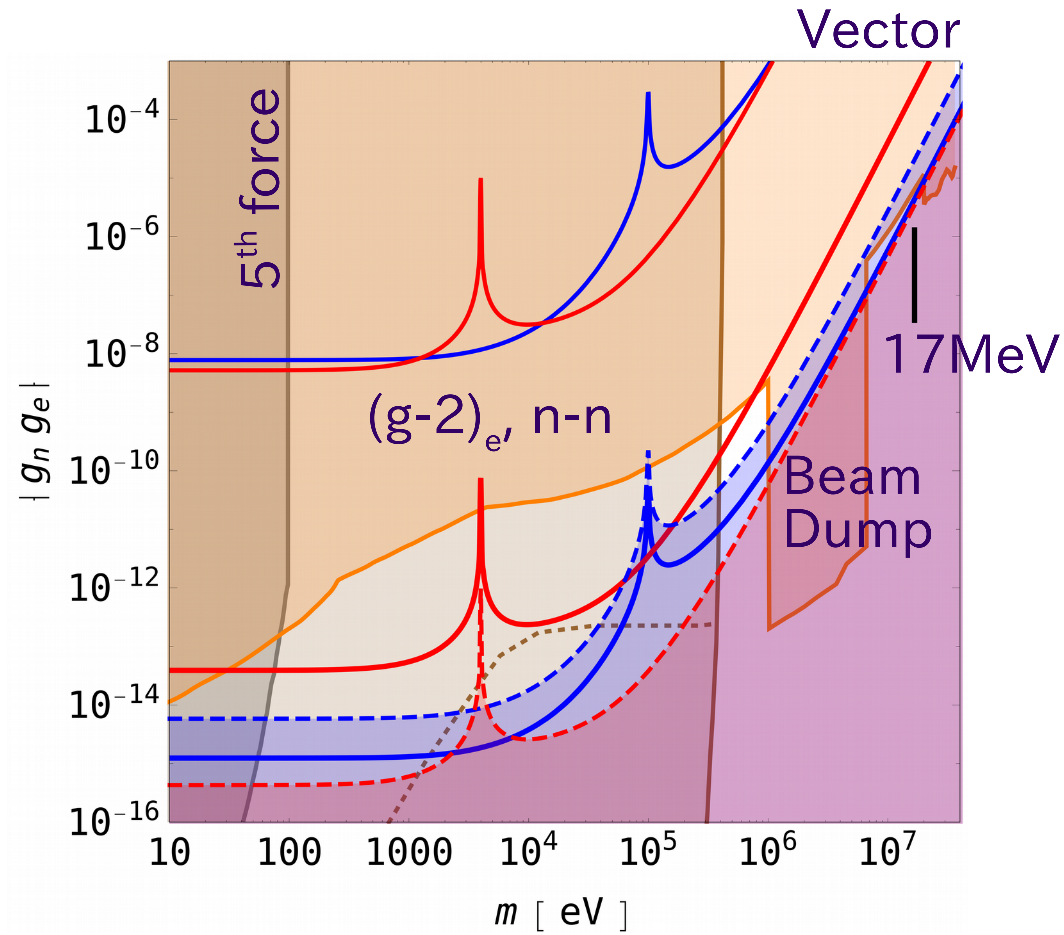
- **Relativistic effects** on wave function and potential.
  - The inner behavior can be modified.
  - Possibly important for light elements.
- **Isotope shift corrections** to wave functions.
  - Can be large for heavy elements.
- **Details of nuclear density** distributions.
  - Angular distributions.

And others...

- **Suggest appropriate atoms to experiments.**

Some comments





- The stellar cooling has large uncertainty.
- Our result is smooth because of the analytic study.