Model independent evaluation of the Wilson coefficient of the Weinberg operator in QCD

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in collaboration with

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based on arXiv:1712.09503

We want new physics!

Two approaches for discovering new physics

- ⋆ direct search (LHC etc.)
- ⋆ indirect search (precision measurements ⋯)

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Observables

- ⋆ g-2, flavor physics (B, K, ⋯), ⋯
- ⋆ EDM is one good observables

EDM (= Electric Dipole Moment)

Definition

$$H \supset -d_f \frac{\vec{s}}{|\vec{s}|} \cdot \vec{E}$$

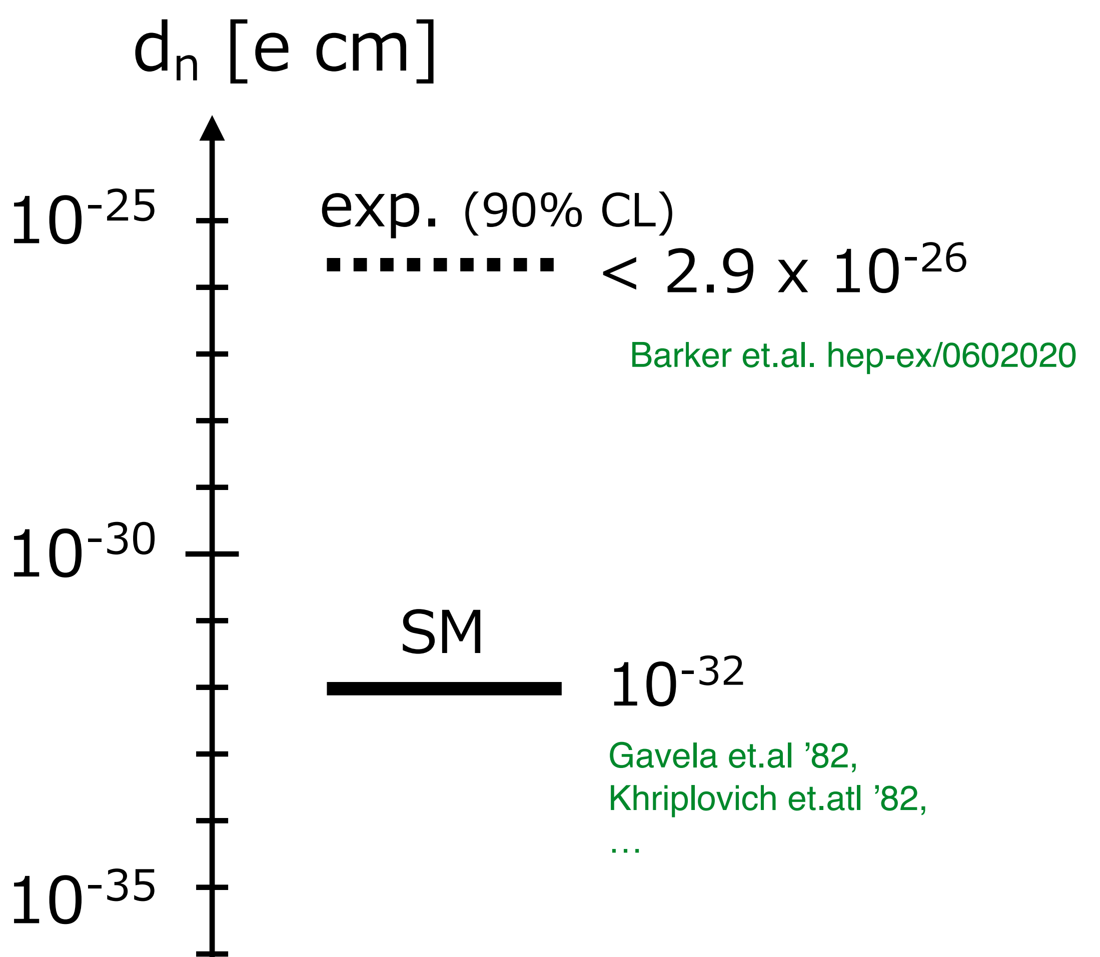
violate T (=CP)

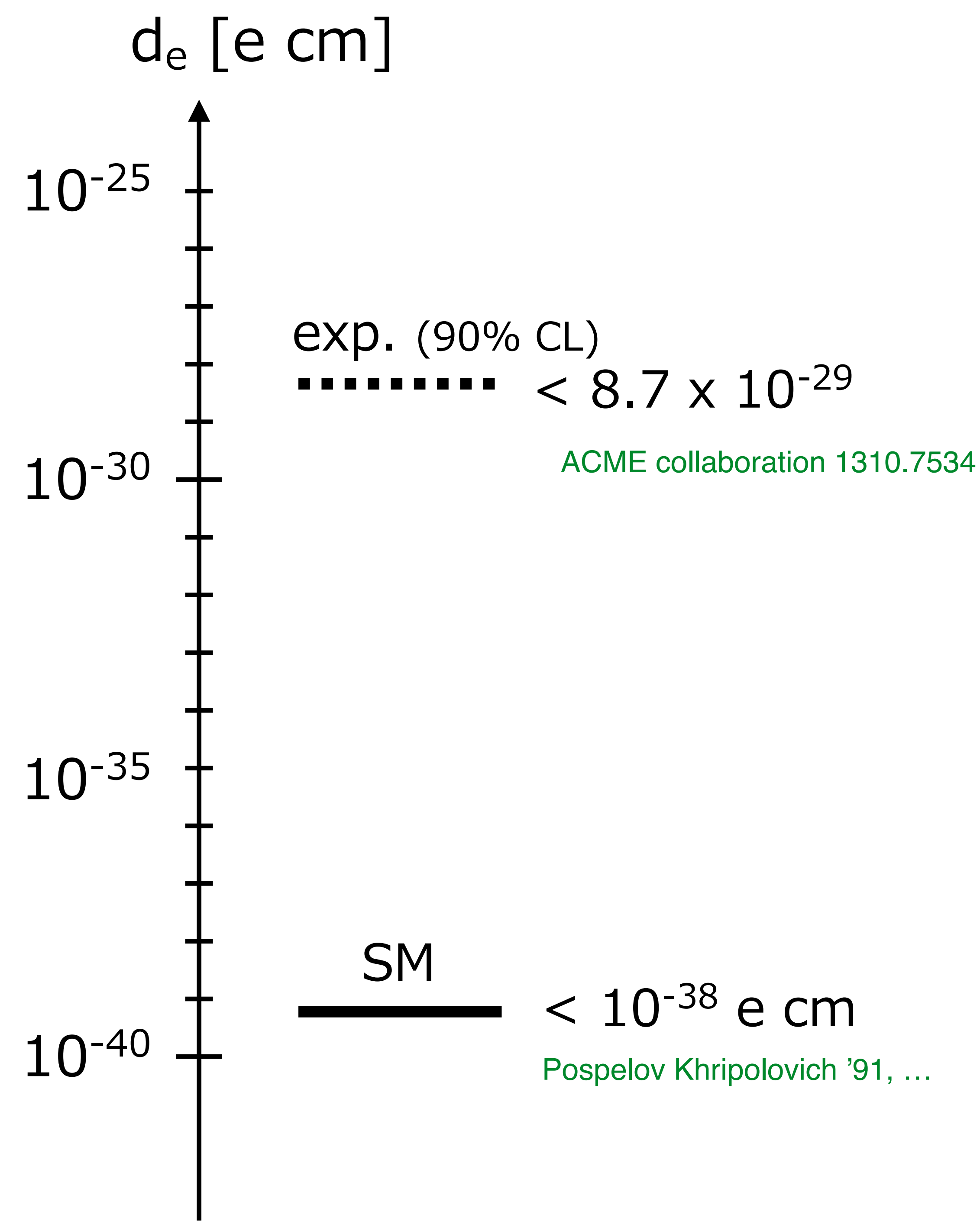
$$\vec{s} \to -\vec{s},\ \vec{E} \to \vec{E}$$

Sensitive to CP violation

⋆ BSM predict new source of CPV in many cases

current status of EDM

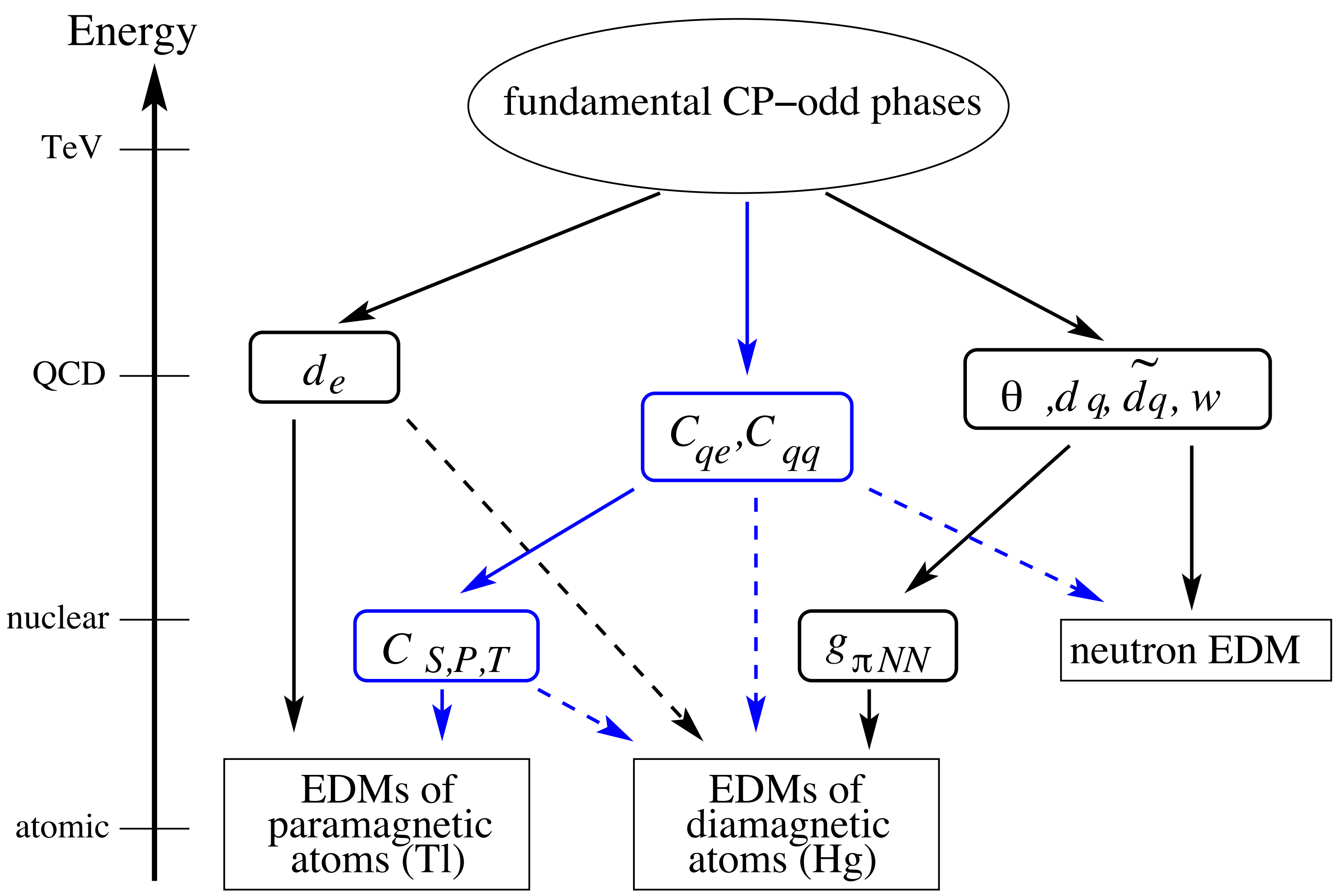




current status

- ⋆ not observed yet
- ⋆ upper bound in much far away from the SM prediction
- ⋆ observation means the discovery of new physics

neutron EDM



[Pospelov Ritz, hep-ph/0504231]

θ term

$$\frac{g_s^2}{32\pi^2}\theta G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

EDM and chromo EDM

$$-\frac{d_q}{2}\left(i\bar{q}\sigma^{\mu\nu}\gamma_5 q F_{\mu\nu}\right)$$

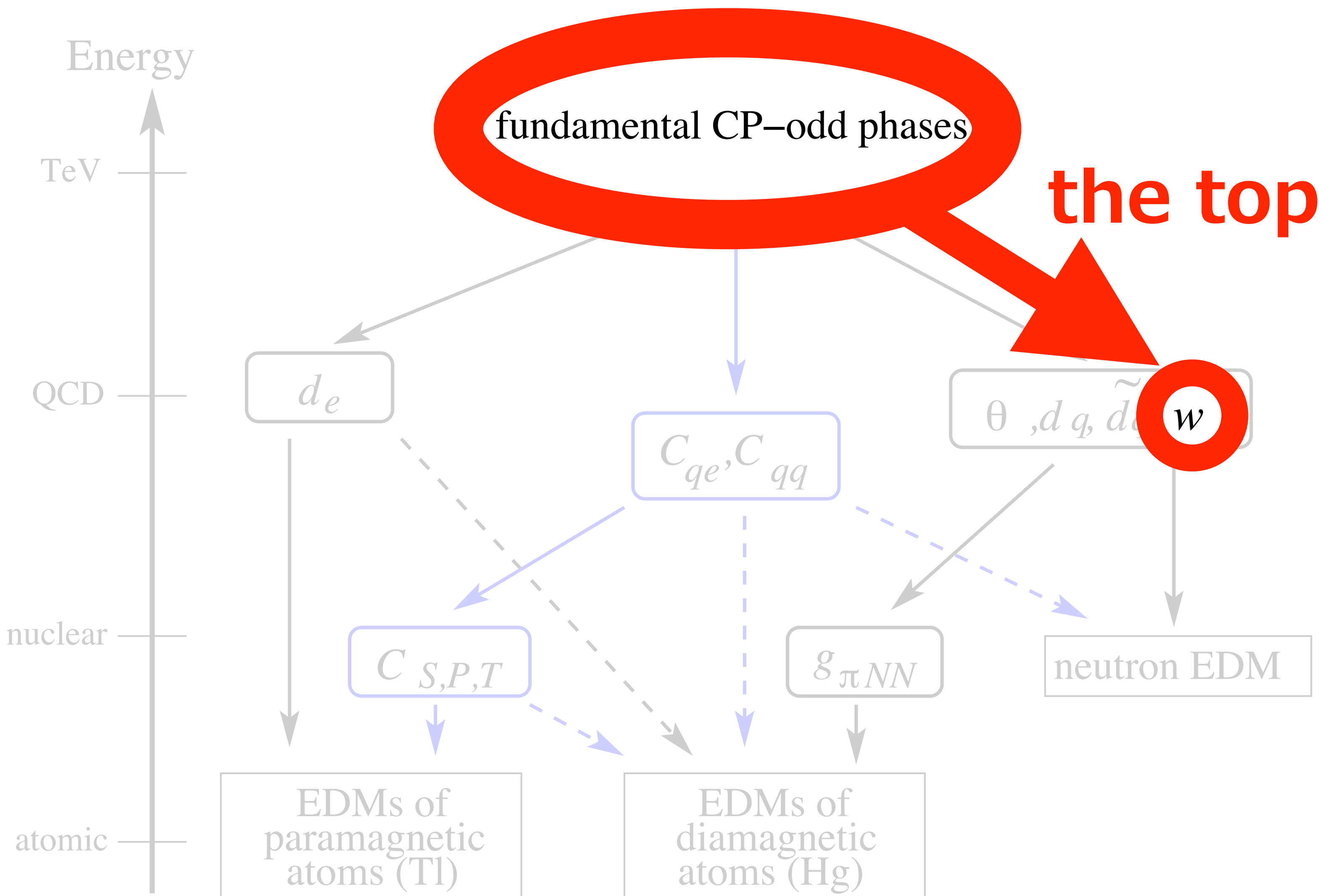
$$-\frac{\tilde{d}_q}{2}\left(i\bar{q}\sigma^{\mu\nu}\gamma_5 q G_{\mu\nu}\right)$$

the Weinberg operator

$$-\frac{w}{3}f^{abc}G^a_{\mu\nu}G^{b\nu}_{\rho}\tilde{G}^{c\rho\mu}$$

[Weinberg PRL63.2333(1989)]

neutron EDM



[Pospelov Ritz, hep-ph/

the topic in this talk

θ

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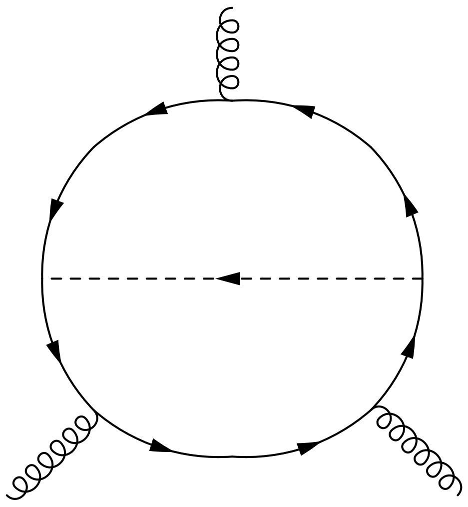
GGG~

generated at the two-loop level

- ⋆ need colored particles
- ⋆ need a source of CPV



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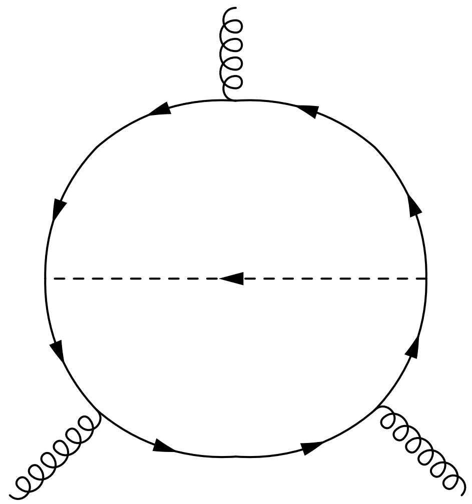

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- ⋆ need colored particles
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calculation is straightforward but tough

- ⋆ calculation was done only in a few models
  - ✳ THDM [Weinberg ’89; Dicus ’90]
  - ✳ MSSM (quark-squark-gluino) [Dai et.al ’90]
  - ✳ LR model [Chang et.al ’90; Rothstein ‘90]
- ⋆ In other models, you need to calculate 2-loop diagrams
- ⋆ contribution is not negligible

In this work

We have derived model independent formula

$$
\boxed{\begin{aligned}
w = &-\frac{g_s^3}{(4\pi)^4} 6\mathrm{Im}(sa^*) m_A m_B \\
&\times \Big\{ \left(X T_A T_A X^\dagger\right) f_1(m_A^2, m_B^2, m_S^2) + \left(X X^\dagger T_B T_B\right) f_1(m_B^2, m_A^2, m_S^2) \\
&\quad + \left(X T_A X^\dagger T_B\right) \left[f_2(m_A^2, m_B^2, m_S^2) + f_2(m_B^2, m_A^2, m_S^2) \right] \Big\}
\end{aligned}}
$$

- ⋆ we do not specify models
- ⋆ we specify Feynman rules with QCD gauge invariance
- ⋆ you can apply our result to your models immediately

Strategy

Setup

$$\boxed{\mathcal{L} \supset -\bar{\psi}_B g_{\bar{B}AS} \psi_A S - \bar{\psi}_A g_{\bar{A}B\bar{S}} \psi_B S^*}$$

⋆ two fermions (*A* and *B*)

⋆ one scalar (*S*)

⋆ QCD invariance is imposed

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- ⋆ two fermions (*A* and *B*)
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coupling

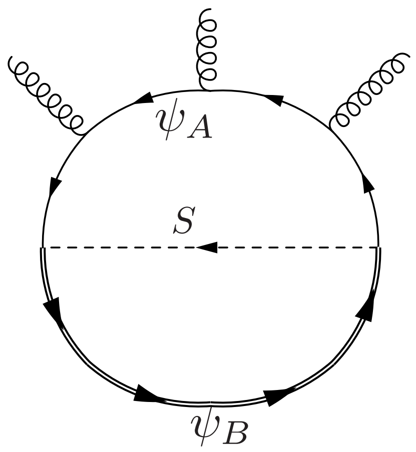
$$\boxed{\begin{aligned} g_{\bar{B}AS} =& X_{\bar{B}AS}(s + \gamma_5 a), \\ g_{\bar{A}B\bar{S}} =& X^\dagger_{\bar{A}B\bar{S}}(s^* - \gamma_5 a^*) \end{aligned}}$$

- ⋆ "*a*" and "*s*" are complex numbers
- ⋆ X contains group theory factor (examples)

(A, B, S)	$\psi_A \ \psi_B \ S$	$X_{\bar{B}AS}$
$(3, 3, 1)$	$(\psi_A)^a \ (\psi_B)^b \ S$	δ^b_a
$(3, \bar{3}, 3)$	$(\psi_A)^i \ (\psi_B)_j \ S^k$	ϵ_{ijk}
$(\bar{3}, 3, 6)$	$(\psi_A)_a \ (\psi_B)^b \ S^{ij}$	$\frac{\delta^b_i \delta^a_j + \delta^b_j \delta^a_i}{2}$

group theory factor : XTTTX

(ex.) a diagram with 3 fields for GGG~



$$g_{\bar{B}AS} = X_{\bar{B}AS}(s + \gamma_5 a),$$

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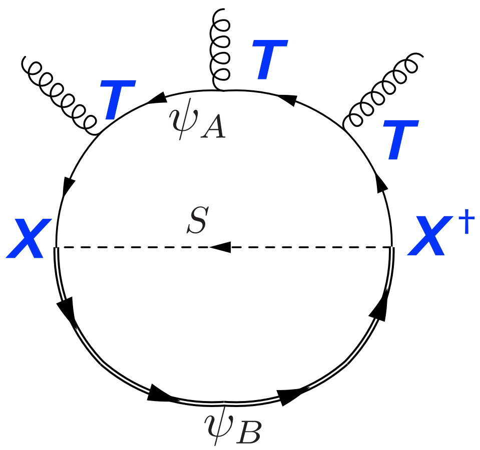

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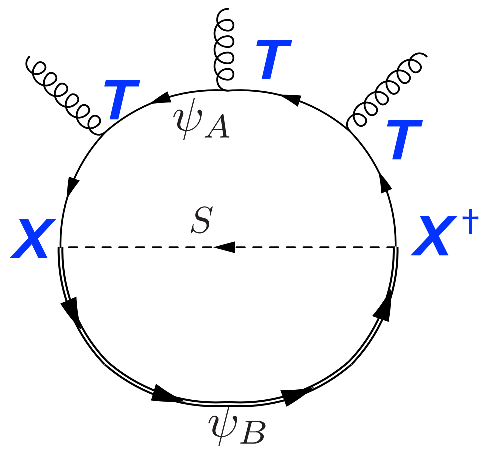

$$\propto \left(X_{\bar{B}AS} (T^a T^b T^c)_{AA'} X^\dagger_{\bar{A}'B\bar{S}} \right) \left(G^a_{\mu\nu} G^{b\nu}{}_{\rho} G^c_{\alpha\beta} \epsilon^{\rho\mu\alpha\beta} \right)$$

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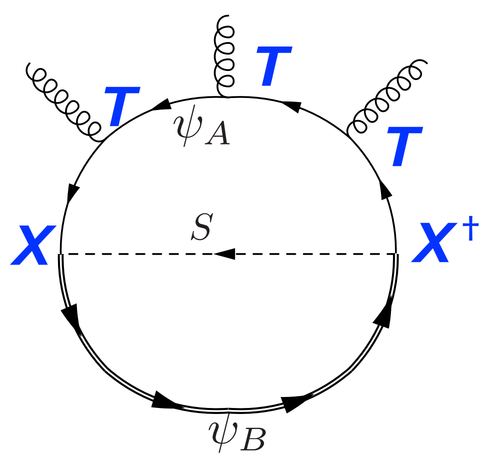

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$G^a{}_{\mu\nu}\, G^{b\nu}{}_\rho\, \varepsilon^{\rho\mu\alpha\beta}$ is anti-symmetric under exchange of a and b

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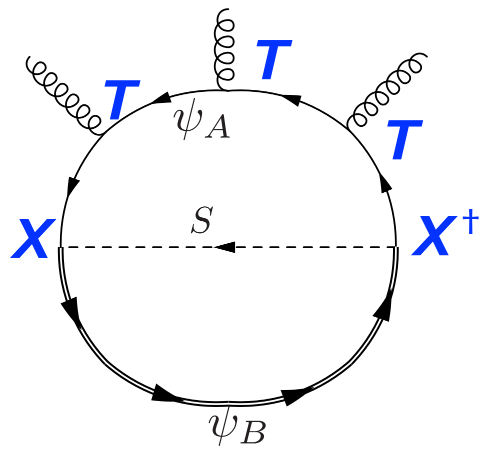

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$$=\frac{i}{2}f^{abd}\left(X_{\bar{B}AS}(T^dT^c)_{AA'}X^\dagger_{\bar{A}'B\bar{S}}\right)\left(G^a_{\mu\nu}G^{b\nu}{}_\rho G^c_{\alpha\beta}\epsilon^{\rho\mu\alpha\beta}\right).$$

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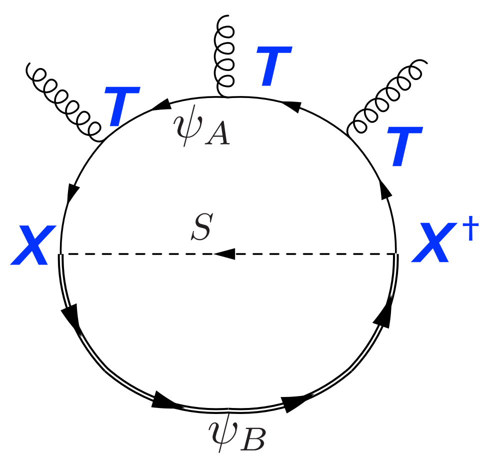

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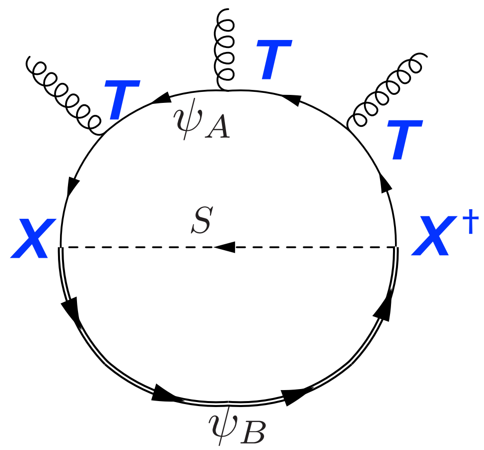

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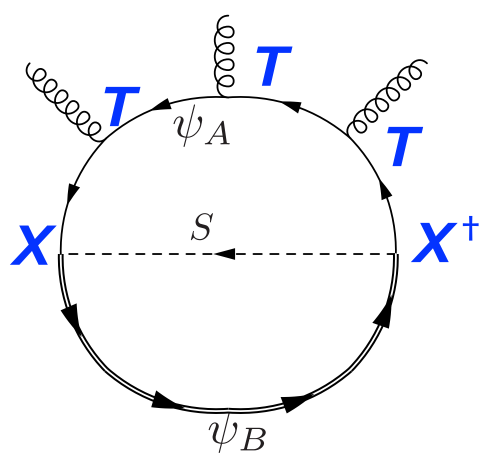

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&=\frac{i}{2}f^{abd}\delta^{dc}\left(XT_AT_AX^\dagger\right)\left(G^a_{\mu\nu}G^{b\nu}{}_\rho G^c_{\alpha\beta}\epsilon^{\rho\mu\alpha\beta}\right)\\
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\end{aligned}$$

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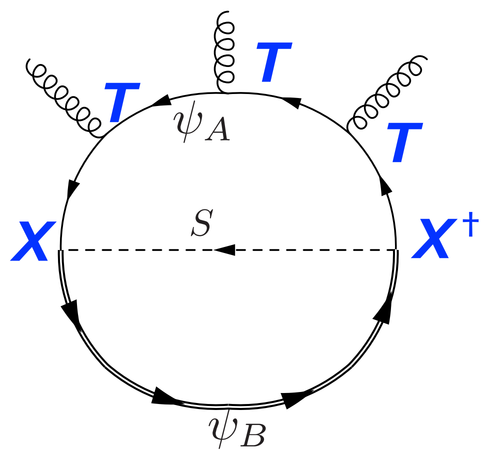

$$g_{\bar{B}AS} = X_{\bar{B}AS}(s+\gamma_5 a),$$
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$$\left(X_{\bar{B}AS}(T^dT^c)_{AA'}X^\dagger_{\bar{A}'B\bar{S}}\right)$$
$$\equiv \delta^{dc}\left(XT_AT_AX^\dagger\right)$$

(example)

$$\left(XT_AT_AX^\dagger\right) = \frac{1}{2} \text{ for } (A,B,S)\sim(3,3,1)$$
$$\mathcal{L} \supset (\bar{\psi}_B)_j\delta^j_i(s+\gamma_5 a)(\psi_A)^i S$$
$$X = \delta^j_i$$
$$X_{\bar{B}AS}(T^a)_{AA'}(T^b)_{A'A''}X^\dagger_{\bar{A}''B\bar{S}} = \mathrm{tr}(T^aT^b) = \frac{1}{2}\delta^{ab}$$

group theory factor

All the group factors we need to calculate are

- $XT_AT_AX^\dagger$, $XT_AX^\dagger T_B$, and $XX^\dagger T_BT_B$
- All the others are expressed by these 3 factors

(A,B,S)	$\psi_A\ \psi_B\ S$	$X_{\bar{B}AS}$	$XT_AT_AX^\dagger$	$XT_AX^\dagger T_B$	$XX^\dagger T_BT_B$
$(3,3,1)$	$(\psi_A)^a\ (\psi_B)^b\ S$	δ_a^b	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$
$(3,1,\bar{3})$	$(\psi_A)^a\ (\psi_B)\ S_i$	δ_a^i	$\frac{1}{2}$	0	0
$(1,3,3)$	$(\psi_A)\ (\psi_B)^b\ S^i$	δ_i^b	0	0	$\frac{1}{2}$
$(\bar{6},1,6)$	$(\psi_A)_{ab}\ (\psi_B)\ S^{ij}$	$\frac{\delta_i^a\delta_j^b+\delta_j^a\delta_i^b}{2}$	$\frac{5}{2}$	0	0
$(6,6,1)$	$(\psi_A)^{ij}\ (\psi_B)^{kl}\ S$	$\frac{\delta_i^k\delta_j^l+\delta_j^k\delta_i^l}{2}$	$\frac{5}{2}$	$\frac{5}{2}$	$\frac{5}{2}$
$(3,\bar{3},3)$	$(\psi_A)^i\ (\psi_B)_j\ S^k$	ϵ_{ijk}	1	$\frac{1}{2}$	1
$(\bar{3},3,6)$	$(\psi_A)_a\ (\psi_B)^b\ S^{ij}$	$\frac{\delta_i^b\delta_j^a+\delta_j^b\delta_i^a}{2}$	1	$-\frac{1}{4}$	1
$(3,3,8)$	$(\psi_A)^i\ (\psi_B)^j\ (S^aT^a)$	$(T^a)^j{}_i$	$\frac{2}{3}$	$-\frac{1}{12}$	$\frac{2}{3}$

Result

$$\boxed{\begin{aligned} w =& -\frac{g_s^3}{(4\pi)^4} 6\mathrm{Im}(sa^*) m_A m_B \\ & \times \Big\{ \left(X T_A T_A X^\dagger\right) f_1(m_A^2, m_B^2, m_S^2) + \left(X X^\dagger T_B T_B\right) f_1(m_B^2, m_A^2, m_S^2) \\ & \quad + \left(X T_A X^\dagger T_B\right) \left[f_2(m_A^2, m_B^2, m_S^2) + f_2(m_B^2, m_A^2, m_S^2) \right] \Big\} \end{aligned}}$$

$$\mathcal{L} \supset -\frac{w}{3} f^{abc} G^a_{\mu\nu} G^{b\nu}_{\ \ \rho} \tilde{G}^{c\rho\mu}$$

$$g_{\bar{B}AS} = X_{\bar{B}AS}(s + \gamma_5 a),$$

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$$f_1(m_A^2, m_B^2, m_S^2) \equiv \int_0^\infty d\ell_E^2 \int_0^1 dz \frac{-\ell_E^4 z(1-z)}{\left(m_S^2 z + m_B^2(1-z) + \ell_E^2 z(1-z)\right)\left(\ell_E^2 + m_A^2\right)^4}$$

$$f_2(m_A^2, m_B^2, m_S^2) \equiv \int_0^\infty d\ell_E^2 \int_0^1 dz \frac{\ell_E^4 (1-z)}{\left(m_S^2 z + m_B^2(1-z) + \ell_E^2 z(1-z)\right)\left(\ell_E^2 + m_A^2\right)^4}$$

Applications

nEDM from the Weinberg operator

$$d_N(w) = \pm e\, \Lambda_{\rm nEDM}\, w(1\ {\rm GeV}), \qquad (N = n, p),$$

$$\Lambda_{\rm nEDM} = 10 - 30\ {\rm MeV}$$

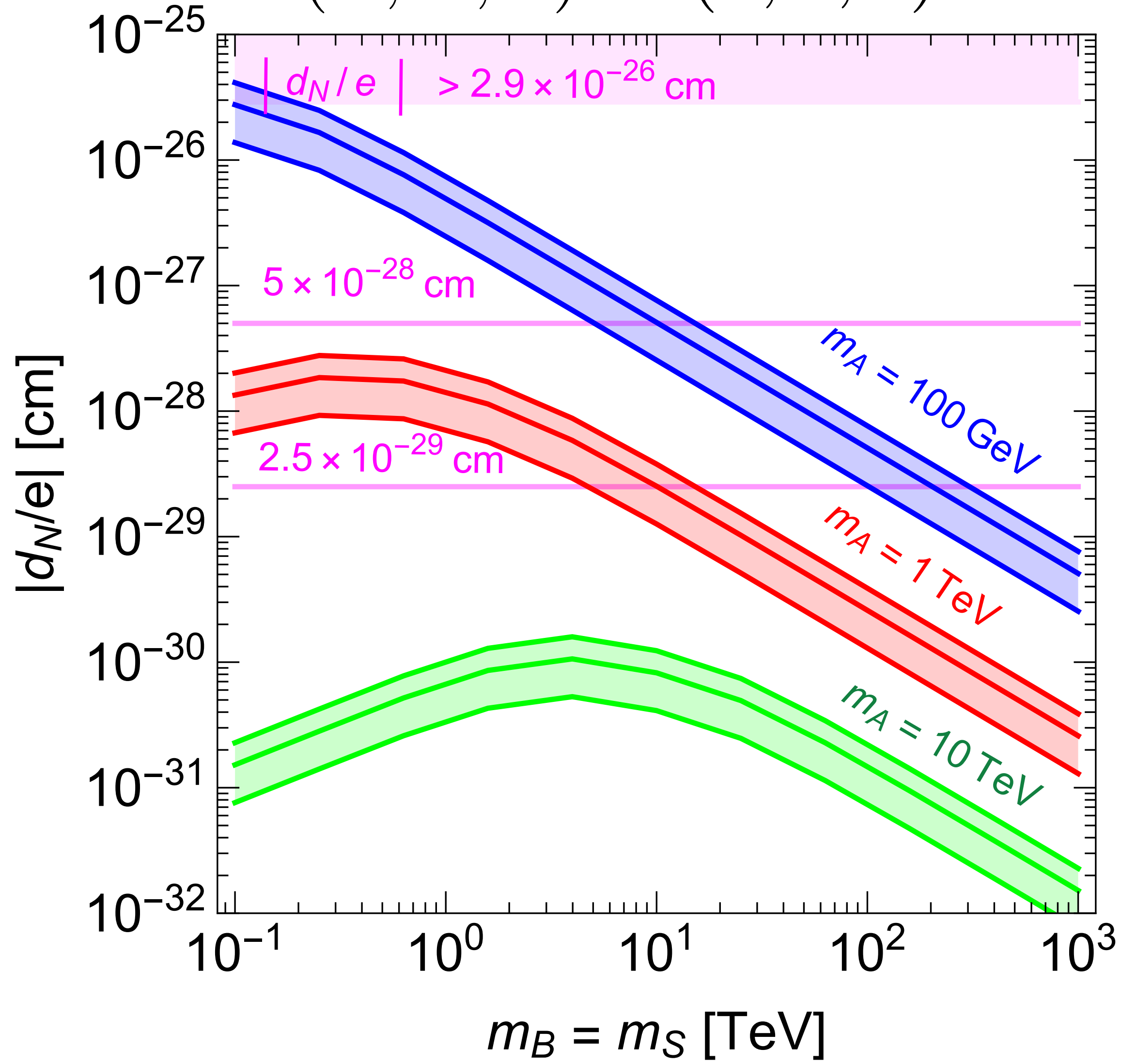
[Demir-Pospelov-Ritz '03, ⋯]

RGE for *w* [Degrassi-Franco-Marchetti-Silvestrini '05]

$$\frac{d}{d\ln\mu} w(\mu) = \frac{g_s^2(\mu)}{16\pi^2}(N_C + 2N_f) w(\mu)$$

Im(sa*) = 0.25

$(A, B, S) \sim (3, 1, \overline{3})$



current bound [Barker et.al. hep-ex/0602020]

prospect 1 [Altarev et al. Nucl. Instrum. Meth. A 611, 133 (2009)]

prospect 2 [Lehrach et al. 1201.5773]

Summary

We have derived model independent formula

$$
\begin{aligned}
w = & -\frac{g_s^3}{(4\pi)^4} 6\mathrm{Im}(sa^*) m_A m_B \\
& \times \Big\{ \left(X T_A T_A X^\dagger\right) f_1(m_A^2, m_B^2, m_S^2) + \left(X X^\dagger T_B T_B\right) f_1(m_B^2, m_A^2, m_S^2) \\
& \quad + \left(X T_A X^\dagger T_B\right) \left[f_2(m_A^2, m_B^2, m_S^2) + f_2(m_B^2, m_A^2, m_S^2) \right] \Big\}
\end{aligned}
$$

- ⋆ we do not specify models
- ⋆ we specify Feynman rules with QCD gauge invariance
- ⋆ you can apply our result to your models immediately

Backup slides

group theory factor : T_S

diagrams with gluons from the scalar field

- (ex.) diagrams with one gluon from the scalar field

$$\propto \left(X^\dagger_{\bar{A}BS}(T^a)_{S'S}X_{\bar{B}A'S'}\right) \equiv \left(X^\dagger T_S X\right)(T^a)_{AA'}$$

We do not need to use factors with *S*

- X and X† contains the information of the representation of *S*
- (ex.)

$$X^\dagger T_S X = X^\dagger T_B X - \frac{1}{N(r_A)}(X T_A T_A X^\dagger) \qquad \mathrm{tr}(T^a T^b) = N(r)\delta^{ab}$$

(the proof is given in our paper)

technique to simplify calculations

Calculation is complicated

- GGG~ contains
  - 3 gluons + 3 external momenta
  - 4 gluons + 2 external momenta
  - 5 gluons + 1 external momenta
  - 6 gluons

technique to simplify calculations

Calculation is complicated

- ⋆ GGG~ contains
  - ✳ 3 gluons + 3 external momenta
  - ✳ 4 gluons + 2 external momenta
  - ✳ 5 gluons + 1 external momenta
  - ✳ 6 gluons

Fock-Schwinger gauge

Refs:
Hisano-Ishiwata-Nagata [1007.2601]
Natsumi Nagata [Master thesis]
Novikov-Shifman-Vainshtein-Zakharov [ITEP-140-1983]
Shifman [Textbook “Vacuum Structure and QCD Sum Rules”]
…

- ⋆ gauge fixing condition

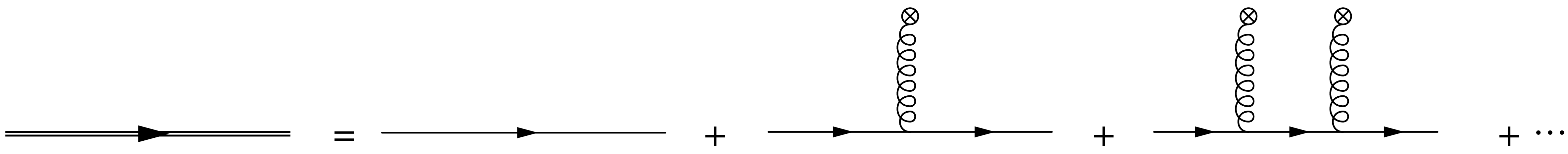
$$x^\mu A_\mu(x) = 0$$

- ⋆ gauge field is expressed by its field strength

$$\begin{aligned} A_\mu(x) =& \frac{1}{2} x^{\nu_1} G_{\nu_1 \mu}(0) \\ &+ \frac{1}{3} x^{\nu_1} x^{\nu_2} D_{\nu_2} G_{\nu_1 \mu}(0) \\ &+ \frac{1}{4}\frac{1}{2!} x^{\nu_1} x^{\nu_2} x^{\nu_3} D_{\nu_3} D_{\nu_2} G_{\nu_1 \mu}(0) \\ &+ \cdots \end{aligned}$$

fermion propagator in FS gauge

gluon is treated as back-ground fields



fermion propagator

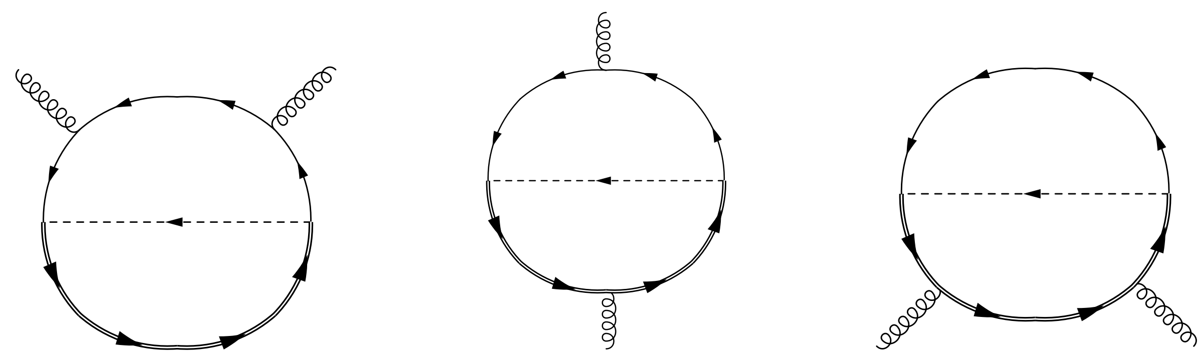
$$
\begin{aligned}
&\int_p \frac{i}{\not{p} - m} e^{-ip(x-y)} \\
&+ \int_{p,k_1} e^{-i(p+k_1)x} \frac{i}{\not{p} + \not{k}_1 - m} (-igT^a\gamma^\mu)\tilde{A}_\mu(k_1) \frac{i}{\not{p} - m} e^{-ipy} \\
&+ \int_{p,k_1,k_2} e^{-i(p+k_1+k_2)x} \frac{i}{\not{p} + \not{k}_1 + \not{k}_2 - m} (-igT^a\gamma^\mu)\tilde{A}_\mu(k_1) \frac{i}{\not{p} + \not{k}_2 - m} (-igT^a\gamma^\mu)\tilde{A}_\mu(k_2) \frac{i}{\not{p} - m} e^{-ipy} \\
&+ \cdots
\end{aligned}
$$

gauge field

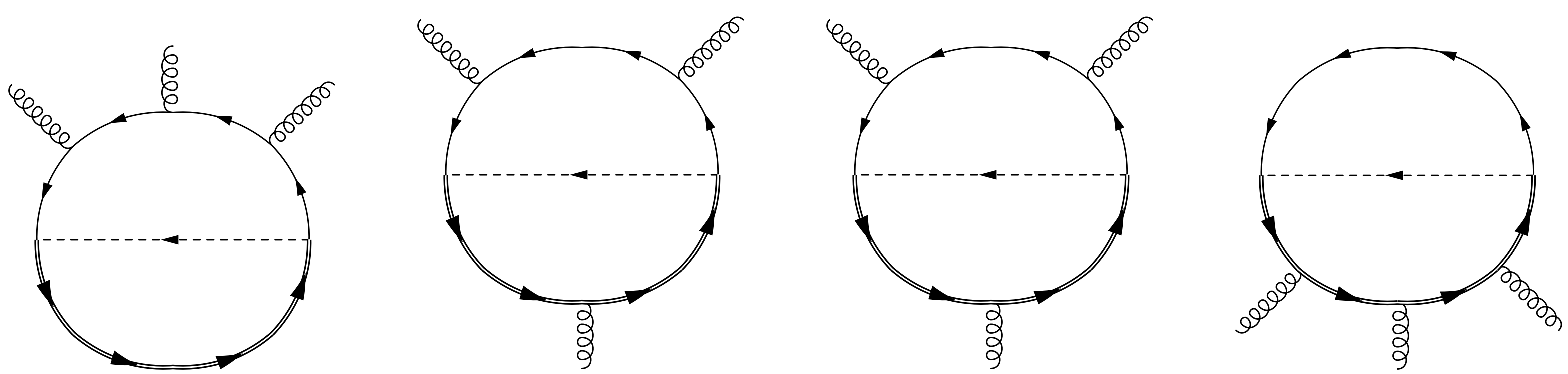
$$
\tilde{A}^a_\mu(k) = \frac{(2\pi)^4}{2} \frac{\partial \delta^4(k)}{i\partial k_{\nu_1}} G^a_{\nu_1 \mu} + \frac{(2\pi)^4}{3} \frac{\partial^2 \delta^4(k)}{i\partial k_{\nu_1} i\partial k_{\nu_2}} D_{\nu_2} G^a_{\nu_1 \mu} + \frac{(2\pi)^4}{4 \cdot 2!} \frac{\partial^3 \delta^4(k)}{i\partial k_{\nu_1} i\partial k_{\nu_2} i\partial k_{\nu_3}} D_{\nu_3} D_{\nu_2} G^a_{\nu_1 \mu} + \cdots
$$

Diagrams

diagrams for DDGG~, DGDG~, GDDG~ (note [D,D] ~ G)



diagrams for GGG~



All the other diagrams do not contribute to GGG~

- at least two gluons from the fermions are needed to obtain $\varepsilon_{\mu\nu\rho\sigma}$
- diagrams with a gluon from the scalar vanish in the FS gauge

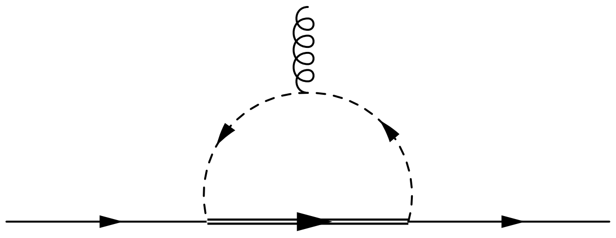
Comparison with other works

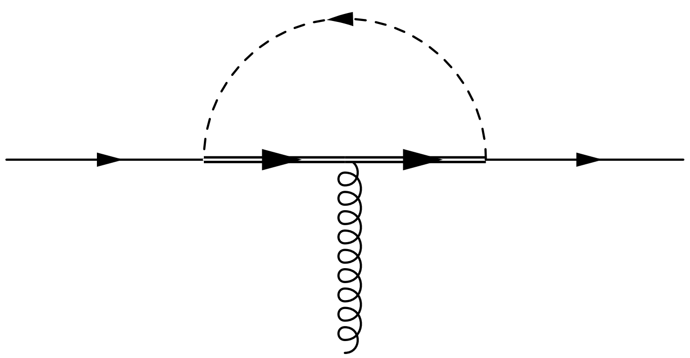
Effective field theory approach [Chang-Kephart-Keung-Yuan '92]

- *A* is in the color fundamental representation
- $m_A << m_B, m_S$
- cEDM of *A* is generated by *B* and *S*

$$\mathcal{L} \supset \bar{Q}\left(-\frac{i}{2}g_s d^c \gamma_5 \sigma^{\mu\nu} G_{\mu\nu}\right) Q.$$

$$C_g^{eff.} = \frac{d^c}{32\pi^2 m_A}$$





Compare with our result

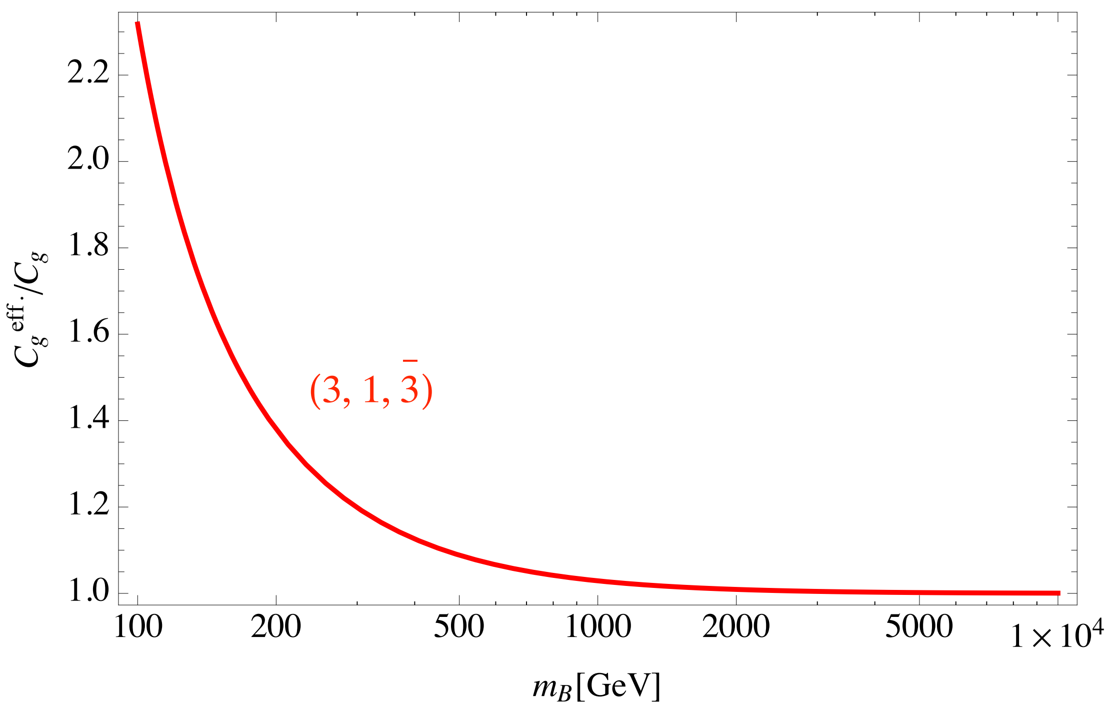
$$\begin{aligned} C_G = &-\frac{1}{(4\pi)^4} 6\mathrm{Im}(sa^*) m_A m_B \\ &\times \left\{ \left(X T_A T_A X^\dagger\right) f_1(m_A^2, m_B^2, m_S^2) + \left(X X^\dagger T_B T_B\right) f_1(m_B^2, m_A^2, m_S^2) \right. \\ &\left. + \left(X T_A X^\dagger T_B\right) \left[f_2(m_A^2, m_B^2, m_S^2) + f_2(m_B^2, m_A^2, m_S^2) \right] \right\}, \end{aligned}$$

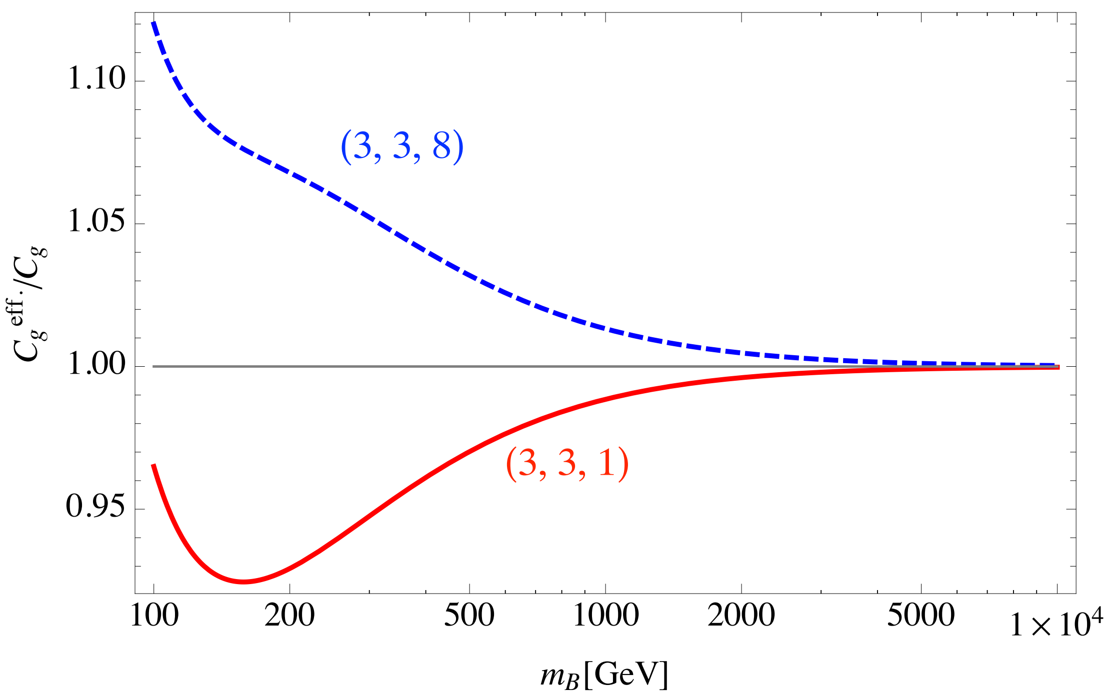
- two results should agree with each other in $m_A << m_B, m_S$ limit

Comparison with other works

Comparison in three difference representations

(m_A = 100 GeV, m_S = m_B)





Agreement in m_A << m_B, m_S limit

- non-trivial check of our calculation