The Reach of Thermal Supersymmetric Dark Matter

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Outline

Motivations

SUSY Dark Matter

Coannihilation
  Gluino Coannihilation
  Stop Coannihilation
Where We Are So Far

- SUSY is most likely somewhat tuned
  - \( \Delta_{BG} \approx \frac{M_{SUSY}^2}{m_Z^2} \)
- Is it time to let that ship sink?
  - We worry because we can detect it
Motivations

Unification and Thresholds

- Gauge couplings unify in SUSY
- $M_{GUT}$ affects on Unification
  - $M_{GUT}$ thresholds $\rightarrow$ unification
- Unification $\rightarrow$ upper limit on $M_{SUSY}$
  - $\beta(\alpha_i)$ change at $M_{SUSY}$
  - $\mu, M_i \gg m_W \rightarrow$ no unification
SUSY Dark Matter

SUSY Well Tempered Neutralinos

- **WIMP miracle**
  - Weak scale masses/interactions give correct density

- **Neutralinos: the perfect WIMP** \( r = \frac{M_1^2}{m_{\tilde{\nu}_R}^2} \rightarrow 0.25 \)
  - density only depends on scattering cross section

\[
\langle \sigma \tilde{B} v \rangle = \frac{3g^3 t_w^3 r (1 + r^2)}{2 \pi m_{\tilde{\nu}_R}^2 x (1 + r)^4}
\]
\[
\langle \sigma_{\text{eff}} H v \rangle \approx \frac{21g^4}{512 \pi \mu^2}
\]
\[
\langle \sigma_{\text{eff}} W v \rangle = \frac{3g^4}{16 \pi M_2^2}
\]

\[
\Omega h^2 \approx 0.12 \left( \frac{m_{\tilde{\nu}_R}}{100 \text{ GeV}} \right)^2
\]
\[
\Omega_H h^2 \approx 0.1 \left( \frac{\mu}{1 \text{ TeV}} \right)^2
\]
\[
\Omega_W h^2 \approx 0.13 \left( \frac{M_2}{2.5 \text{ TeV}} \right)^2
\]
SUSY Dark Matter

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  - Weak scale masses/interactions give correct density
- **Neutralinos: the perfect WIMP** ($r = \frac{M_1^2}{m_{\tilde{e}_R}^2} \rightarrow 0.25$)
  - Density only depends on scattering cross section
  - Thermal Wino ruled out?

\[
\langle \sigma \tilde{N} \bar{v} \rangle = \frac{3g^3 t_w^3 r(1 + r^2)}{2\pi m_{\tilde{e}_R}^2 x(1 + r)^4},
\]

\[
\Omega \tilde{N} h^2 \simeq 0.12 \left( \frac{m_{\tilde{e}_R}}{100 \text{ GeV}} \right)^2
\]

\[
\langle \sigma_{\text{eff}_{\tilde{H}}} \bar{v} \rangle \sim \frac{21g^4}{512\pi \mu^2},
\]

\[
\Omega_{\tilde{H}} h^2 \simeq 0.1 \left( \frac{\mu}{1 \text{ TeV}} \right)^2
\]

\[
\langle \sigma_{\text{eff}_{\tilde{W}}} \bar{v} \rangle = \frac{3g^4}{16\pi M_2^2},
\]

\[
\Omega_{\tilde{W}} h^2 \simeq 0.13 \left( \frac{M_2}{2.5 \text{ TeV}} \right)^2
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SUSY Dark Matter

SUSY Well Tempered Neutralinos

- **WIMP miracle**
  - Weak scale masses/interactions give correct density
- **Neutralinos: the perfect WIMP** \( r = \frac{M_1^2}{m_{\tilde{e}_R}^2} \rightarrow 0.25 \)
  - density only depends on scattering cross section
  - Thermal Wino ruled out? (Cohen, Lisanti, Pierce, Slatyer)
SUSY Dark Matter

SUSY Well Tempered Neutralinos

- **WIMP miracle**
  - Weak scale masses/interactions give correct density
- **Netralinos**: the perfect WIMP \( r = \frac{M_1^2}{m_{\tilde{e}_R}^2} \to 0.25 \)
  - density only depends on scattering cross section
  - Thermal Wino ruled out?
- **Simple thermal relics all but gone** (Badziak, Olechowski, Szczerbiak)

Red: LUX(SI), Green: LUX(SD), Orange: (XENON1T), Yellow: (LZ)
Beyond the Well Tempered Neutralino: Coannihilation

- Reaction rates
  \[ n_\chi n_{\tilde{t}_1} \sigma_{\chi \tilde{t}_1} \sim T^3 m_\chi^{3/2} m_{\tilde{t}_1}^{3/2} \sigma_{\chi \tilde{t}_1} e^{-\frac{m_\chi + m_{\tilde{t}_1}}{T}} \]
  \[ n_\chi n_{SM} \sigma_{\tilde{t}_1 SM} \sim T^{9/2} m_\chi^{3/2} \sigma_{\chi SM} e^{-\frac{m_\chi}{T}} \]

- \( R = \left( \frac{T}{m_{\tilde{t}_1}} \right)^{3/2} \exp \left[ \frac{m_{\tilde{t}_1}}{T} \right] \)

- \((m_{NLSP} - m_{LSP})/m_{LSP} \ll 1 \rightarrow \text{Coannihilation}\)
  - Thermal Fluctuations convert LSP to NSLP

- As \( \tilde{t}_1 \) annihilates, replenished by SM scattering
  - \( n_{\tilde{t}_1 eq} \simeq n_{\chi eq} \rightarrow \text{enhances } \sigma_{\chi \chi \text{eff}} \)

- Scattering of coannihilation partner determines density
Gluino Coannihilation

- Gluino coannihilation largest Sommerfeld enhancement
  - Final states: singlet, octet, and $27_s$ for $C_j = 0, 3, 8$
  - Stronger binding energy more enhancement

$$V = \frac{\alpha_s}{2r} \left[ C_f - C_i - C'_i \right]$$

- Boundstate formation important
  - $R$ hadron production enhances $\tilde{g}\tilde{g}$ annihilation rate
  - $\langle \Gamma \rangle \tilde{R} \gg \langle \Gamma \rangle_{\text{dis}}$ enhanced coannihilation

$$\langle \sigma V \rangle_{\tilde{g}\tilde{g}\rightarrow gg, q\bar{q}} \rightarrow \langle \sigma V \rangle_{\tilde{g}\tilde{g} \text{ incl. } \tilde{R}} \equiv \langle \sigma V \rangle_{\tilde{g}\tilde{g}\rightarrow gg, q\bar{q}} + \langle \sigma V \rangle_{\text{bsf}} \frac{\langle \Gamma \rangle_{\tilde{R}}}{\langle \Gamma \rangle_{\tilde{R}} + \langle \Gamma \rangle_{\text{dis}}},$$
Dependence of Gluino Coannihilation

- Relative importance of Sommerfeld and Bound state
  - No Som/Boun (red)
  - Som only (Orange)
  - All (Black)
  - Boun×2 (Purple)
Dependence of Gluino Coannihilation

- Relative importance of Sommerfeld and Bound state
- Somewhat insensitive to squark mass
  - Squark mass control conversion of $\tilde{\chi} \leftrightarrow \tilde{g}$
Non-Universal CMSSM

- Non-universal input gauginos $\rightarrow$ gluino coannihilation
  - Gluino coannihilation extends to $m_\chi \sim 8.5$ TeV

$$M_1 = M_2, M_3, m_0, \tan \beta, A_0$$
Pure-Gravity Mediation with Vector Multiplets

- Pure-Gravity Mediation
  - GM term $\rightarrow$ linearly independent $B, \mu \rightarrow$ free $\tan \beta$
    $$m_0, \quad \tan \beta$$

- Gauginos mass anomaly mediated
  $$M_i = b_i \frac{g_i^2}{16\pi^2} m_3/2 \quad b_i = \left\{ \frac{33}{5}, 1, -3 \right\}$$

- Additional $10 + \bar{10}$ mass from GM term
  - Additional 10 can couple to $H_u \rightarrow$ larger $\tan \beta, m_h$
  $$K \supset c_H 10\bar{10} + h.c \quad W \supset y'_i H_u Q' U' + ..$$
  - Gaugino mass do not decouple
  - Gluino mass purely from thresholds
  $$M_1 = \frac{48}{5} \frac{g_1^2}{16\pi^2} m_3/2 \quad M_2 = \frac{g_2^2}{4\pi^2} m_3/2 \quad M_3 = 0$$
Gluino Coannihilation in PGM with Vector Multiplets

- Gluino coannihilation extends to \( m_\chi \sim 8.5 \text{ TeV} \)
  - For smaller \( c_H \) gluino thresholds small and \( \tilde{g} \) is LSP
Stop Coannihilation

Stop coannihilation is also Sommerfeld enhanced
- Final states: singlet or octet for $C_f = 0, 3$
- Less enhanced compared to gluino case ($C_3 = \frac{4}{3}$)

$$V = \frac{\alpha_s}{2r} \left[ C_f - C_i - C_i' \right]$$

Boundstate formation important
- Octet $\tilde{t}_R \tilde{t}_R^*$ forms bound state from gluino emission
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\[ V = \frac{\alpha_s}{2r} \left[ C_f - C_i - C'_i \right] \]

- Boundstate formation important
  - Octet $\tilde{t}_R \tilde{t}_R^*$ forms bound state from gluino emission
  - $\langle \Gamma \rangle_{\tilde{R}} \gg \langle \Gamma \rangle_{\text{dis}}$ enhanced coannihilation

\[ \langle \sigma \nu \rangle_{\tilde{t}\tilde{t}^* \rightarrow \text{SM}} \rightarrow \langle \sigma \nu \rangle_{\tilde{t}\tilde{t}^* \text{ incl.} \tilde{R}} \equiv \langle \sigma \nu \rangle_{\tilde{t}\tilde{t}^* \rightarrow \text{SM}} + \langle \sigma \nu \rangle_{\text{bsf}} \frac{\langle \Gamma \rangle_{\tilde{R}}}{\langle \Gamma \rangle_{\tilde{R}} + \langle \Gamma \rangle_{\text{dis}}} \]
Goldstone Boson Enhancements

- Goldstone Boson Equivalence Theorem (GBET)
  - $W_L/Z_L$ remember origins
- GBET leads to enhanced $t \rightarrow Wb$ decay rate
  - Goldstone couples via top Yukawa ($y_t > g_2$)

\[ \Gamma_t \sim \frac{g_2^2}{64\pi} \frac{m_t^3}{m_W^2} = \frac{y_t^2}{32\pi} m_t \]

- In SUSY stops couple to goldstone via $A$-terms
  - $A_t \gg M_{SUSY}$, large enhancement to $W_L/Z_L$ couplings
  - Goldston predominantly in the $H_u$, only $A_t$ matters

\[ \mathcal{L} \supset -y_t (A_t H_u + \mu H_d^\dagger) \bar{Q}_L \tilde{t}_R - |y_t|^2 \left( |\bar{Q}_L|^2 |H_u|^2 + |\tilde{t}_R|^2 |H_u|^2 \right) \]
Stop Coannihilation to Goldstone Boson

In Feynman gauge goldstone boson are manifest

\[ - \tilde{t}_R \tilde{t}_R^* \rightarrow W^+ W^- \ \sim \ \tilde{t}_R \tilde{t}_R^* \rightarrow G^+ G^- \]

\[ \mathcal{L} \supset -y_t X_t \sin \beta G^+ \tilde{b}_L \tilde{t}_R - |y_t|^2 \sin^2 \beta |\tilde{t}_R|^2 |G^+|^2 \]

S-wave annihilation two sources of enhancement

- \( y_t > g_2 \) and \( A_t > \sqrt{m_{t_R}^2 + m_{t_L}^2} \)
- For \( A_t / \sqrt{m_{t_R}^2 + m_{t_L}^2} \geq g_3 / y_t \), most important mode

\[ \langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \rightarrow W^+ W^-} \ \sim \ \frac{g_2^4}{128 \pi m_{t_R}^2} \left( \frac{m_t}{m_W} \right)^4 \left( \left( A_t + \mu \cot \beta \right)^2 - m_{t_R}^2 - m_{t_L}^2 \right)^2 \]
Stop Coannihilation in the CMSSM

- For large $A_t$, $m_\tilde{\chi} \sim 8$ TeV (Similar to gluino case)
  - $m_0$ chosen to give relic density

![Graphs showing the relationship between $m_{1/2}$ and $\delta m$ for $A_0 = 3m_0, \tan \beta = 20$ and $A_0 = 5m_0, \tan \beta = 20$.](image)
Stop Coannihilation in the CMSSM

- For large $A_t$, $m_\chi \sim 8$ TeV (Similar to gluino case)
- $A_t < 0$, $m_\chi \sim 3$ TeV
  - $|A_t(M_{SUSY})| \ll M_{SUSY}$ due to RG running

![Graphs showing the relationship between $m_{1/2}$ and $\delta m$ for different values of $A_0$ and $\tan \beta$.](image)
Stop Coannihilation in the CMSSM

- For large $A_t$, $m_\chi \sim 8 \text{ TeV}$ (Similar to gluino case)
- $A_t < 0$, $m_\chi \sim \text{TeV}$ (Similar to gluino case)
  - $|A_t(M_{\text{SUSY}})| \ll M_{\text{SUSY}}$ due to RG running
- Little dependence on $\tan \beta$
  - Some enhancement from $\mu$ for small $\tan \beta$
The Reach of Thermal Supersymmetric Dark Matter

Coannihillation

Stop Coannihilation

Constraints on the Stop Coannihilation Strip

- Higgs mass constrain coannihilation strip
  - $A_0 < 0$, $|A_t|$ is small and Higgs mass resonable
Constraints on the Stop Coannihilation Strip

- Higgs mass constrain coannihilation strip
  - $A_0 > 0$ and $A_t$ large, Higgs mass calculation unstable

![Graph showing constraints on the Higgs mass and stop coannihilation strip.](image-url)
Constraints on the Stop Coannihilation Strip

- Higgs mass constrain coannihilation strip
  - FeynHiggs 2.14.0 makes things worse

![Graph showing constraints on the Stop Coannihilation Strip with FeynHiggs 2.14.0](image)
Constraints on the Stop Coannihilation Strip

- Higgs mass constraints coannihilation strip
  - FeynHiggs 2.14.0 makes things worse

\[ m_h \text{ along stop coannihilation strip not reliable!!!!} \]
Higgs Mass and Stop Coannihilation Strip

- Corrections to Higgs quartic coupling (Mass)
  - Higgs mass suppressed for very large $A_t$

$$\Delta \lambda \supset \frac{|y_t|^4}{8\pi^2} \left( \tilde{X}_t \tilde{F}_1 \left( \frac{m_{t_L}}{m_{t_R}} \right) - \frac{1}{12} \tilde{X}_t^2 \tilde{F}_2 \left( \frac{m_{t_L}}{m_{t_R}} \right) \right) \quad \tilde{X}_t = \frac{A_t + \mu \cot \beta}{m_{t_R}m_{t_L}}$$

- Coannihilation leading contribution
  - Coannihilation strip extended for large $A_t$

$$\langle \sigma v \rangle_{\tilde{t}\tilde{t}^* \rightarrow W^+W^-} \simeq \frac{g_2^4}{128\pi m^2_{t_R}} \left( \frac{m_t}{m_W} \right)^4 \left( \frac{(A_t + \mu \cot \beta)^2 - m^2_{t_R} - m^2_{t_L}}{m^2_{t_R} + m^2_{t_L}} \right)^2$$

- Length of stop strip maximized for $m_{t_R} = m_{t_L}$
Sub-GUTS and the Stop Coannihilation Strip

- Supersymmetry input scale may be below $M_{GUT}$
  - Mirage mediation $\rightarrow$ apparent sub-GUT spectrum

- Smaller $M_{in}$ leads to less RG running
  - Stop masses less split
  - Higgs mass less suppressed
  - Coannihilation strip extended

- Use FeynHiggs 2.13.0 OS for Higgs mass calculation
  - Most recent available code at the time
  - FeynHiggs 2.14.0 now available but seems worse
Sub-GUT Plane

- Sub-GUT models very different from CMSSM planes
  - Stop masses more split
  - Higgs mass much better

\[ A_0 = 2.75m_0, \tan\beta = 20, \mu > 0 \]
Sub-GUT Plane

- Sub-GUT models very different from CMSSM planes
- Sub-GUT plane
  - Stop LSP region limited
  - Coannihilation region much less tuned

![Graphs showing m_0 vs m_{1/2} with different values of A_0/m_0, tan β, and μ]

A_0/m_0 = 2.75, tan β = 20, μ < 0

M_{in} = 10^9 GeV

A_0/m_0 = 2.75, tan β = 20, μ > 0

M_{in} = 10^9 GeV
Conclusions

- Naturalness somewhat strained
  - But not dead

- Gauge coupling unification still good
  - Upper limit on SUSY breaking scale $\sim 10^6$ GeV

- Thermal dark matter still alive
  - Gluino coannihilation extends to $m_\chi \lesssim 8.5$ TeV
  - Stop coannihilation may extends to $m_\chi \lesssim 8.5$ TeV
  - Sub-GUT models give more natural coannihilation
    - $m_\chi \lesssim 7$ TeV