Cosmological Implications of Quantum Scale Invariance

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EPFL (JSPS Overseas Research Fellow)

arXiv: 1803.***** with M. Shaposhnikov

A link between the EW physics and the Higgs inflation

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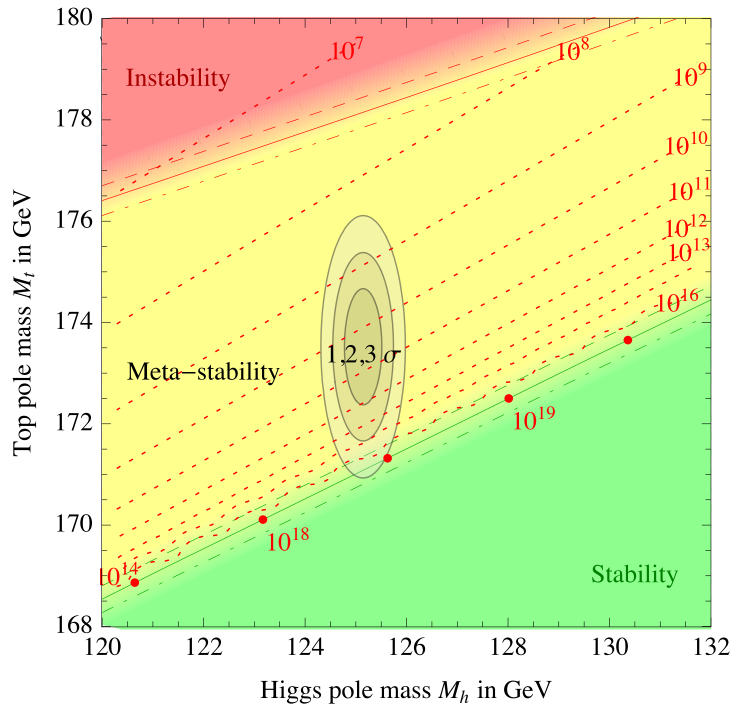
Introduction

$$m_h = 125.09 \pm 0.24\,\mathrm{GeV}$$

The SM can be extended to the Planck scale staying in the perturbative regime.

- The SM cannot account for baryon asymmetry, neutrino oscillation, dark matter etc..

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia (2014)



Introduction

$$m_h = 125.09 \pm 0.24\,\mathrm{GeV}$$

➡ The SM can be extended to the Planck scale staying in the perturbative regime,

・The SM cannot account for baryon asymmetry, neutrino oscillation, dark matter etc..

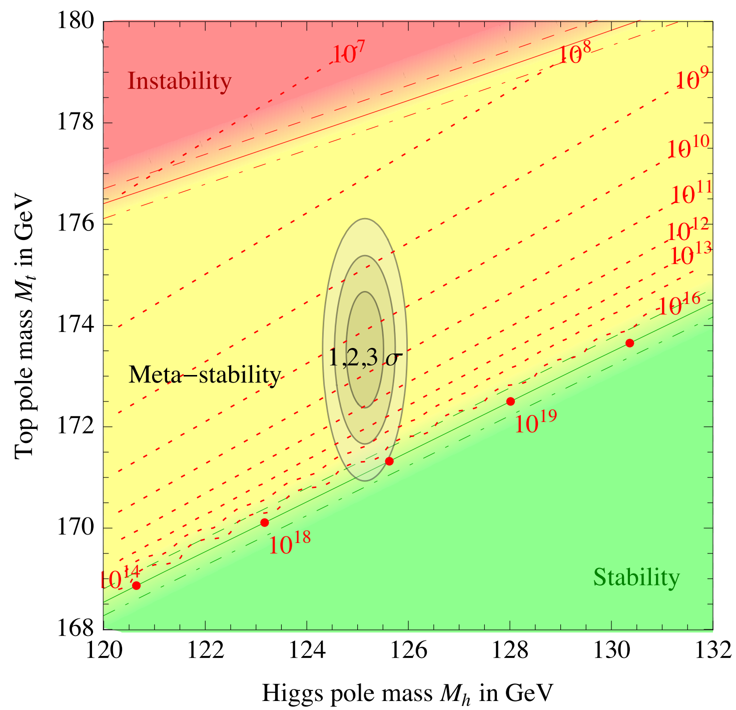
The SM alone ➡ Vacuum **meta-stability**

$$m_t = 173.1 \pm 1.1\,\mathrm{GeV}\ (1\sigma)$$

What does this imply?

A hint of new physics?

D.Buttazzo, G.Degrassi, P.P.Giardino, G.F.Giudice, F.Sala, A.Salvio, A.Strumia (2014)



Introduction

Metastable	Instability during inflation/preheating Inhomogeneity (black hole)

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Metastable	Instability during inflation/preheating Inhomogeneity (black hole)
Degenerate	Prediction of new particles (Hamada's talk) based on UV completion/fundamental principle
Stable *Higgs inflation is possible*	New particles **Renormalization prescription** Link between *EW phys.* and *Inflation*

This talk

Higgs inflation (Tree-level)

F.Bezrukov, M.Shaposhnikov (2007)

Jordan-frame Lagrangian

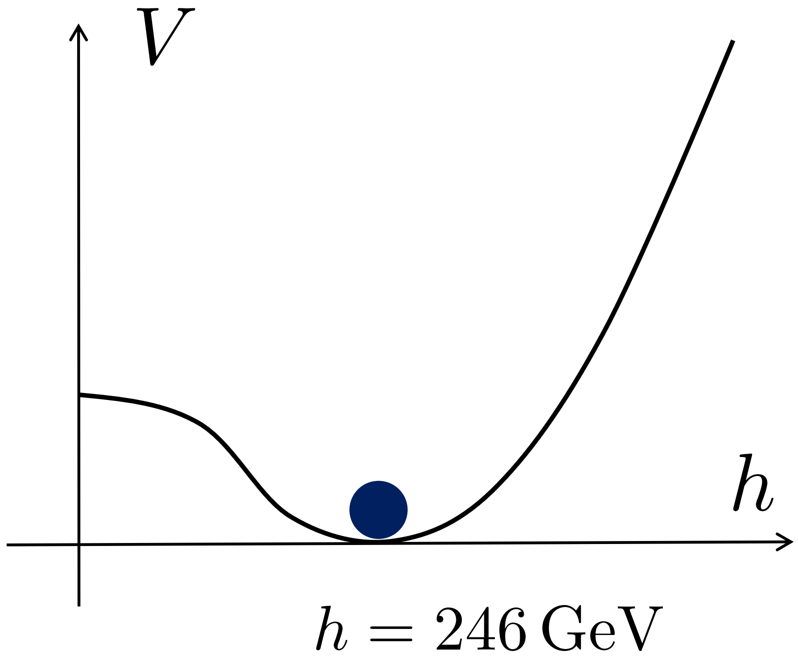
$$\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_{\mathrm{P,eff}}^2}{2} R - \frac{g^{\mu\nu}}{2}\partial_\mu h \partial_\nu h - V(h) + \cdots$$

Einstein + *large non-minimal coupling* + SM

Effective Planck mass

$$M_{\mathrm{P,eff}}^2 = M_{\mathrm{P}}^2 + \xi h^2$$

$$\xi \gg 1$$



$$V = \frac{\lambda}{4} h^4$$ + negative mass term

Higgs inflation

$$\boxed{\widetilde{g}_{\mu\nu} = \Omega^2 \, g_{\mu\nu} \qquad \Omega^2 = 1 + \xi h^2 / M_{\mathrm{P}}^2}$$

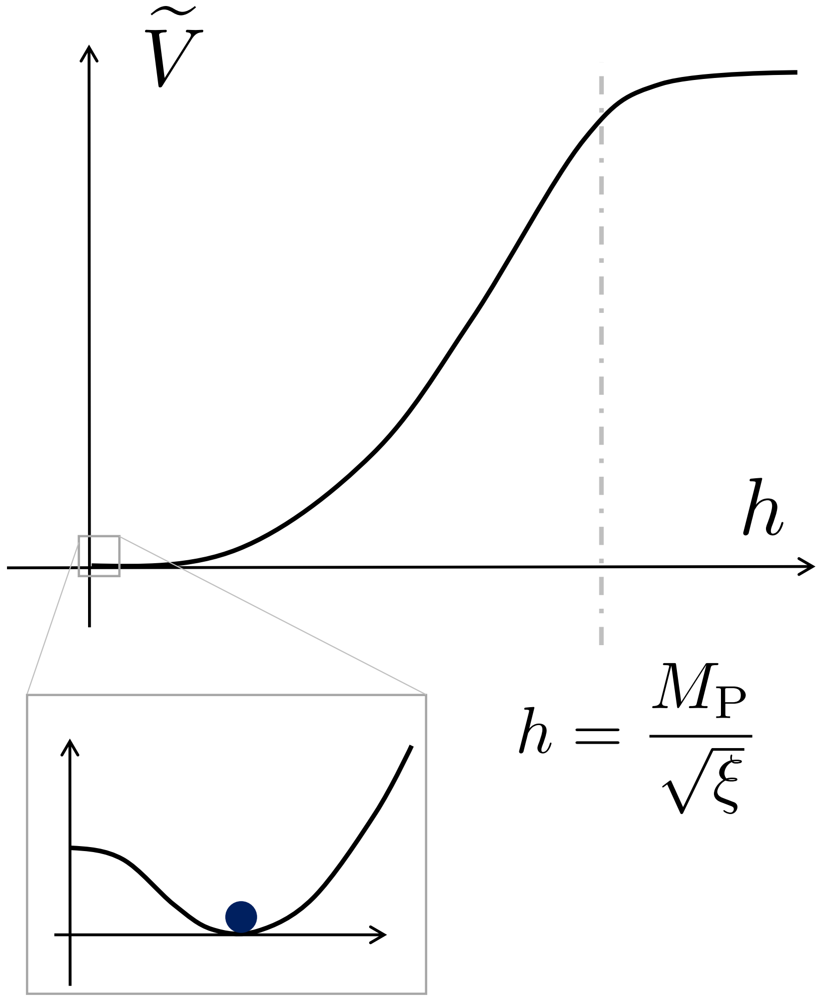
Einstein-frame Lagrangian

$$\frac{\widetilde{\mathcal{L}}}{\sqrt{-\widetilde{g}}} = \frac{M_{\mathrm{P}}^2}{2} \widetilde{R} - \Upsilon \times \frac{\widetilde{g}^{\mu\nu}}{2} \partial_\mu h \partial_\nu h - \widetilde{V}(h) + \cdots$$

Non-canonical kinetic term

Non-polynomial potential

$$\widetilde{V} = \frac{V}{\Omega^4} = \frac{\lambda M_{\mathrm{P}}^4}{4} \frac{h^4}{(M_P^2 + \xi h^2)^2}$$



Higgs inflation

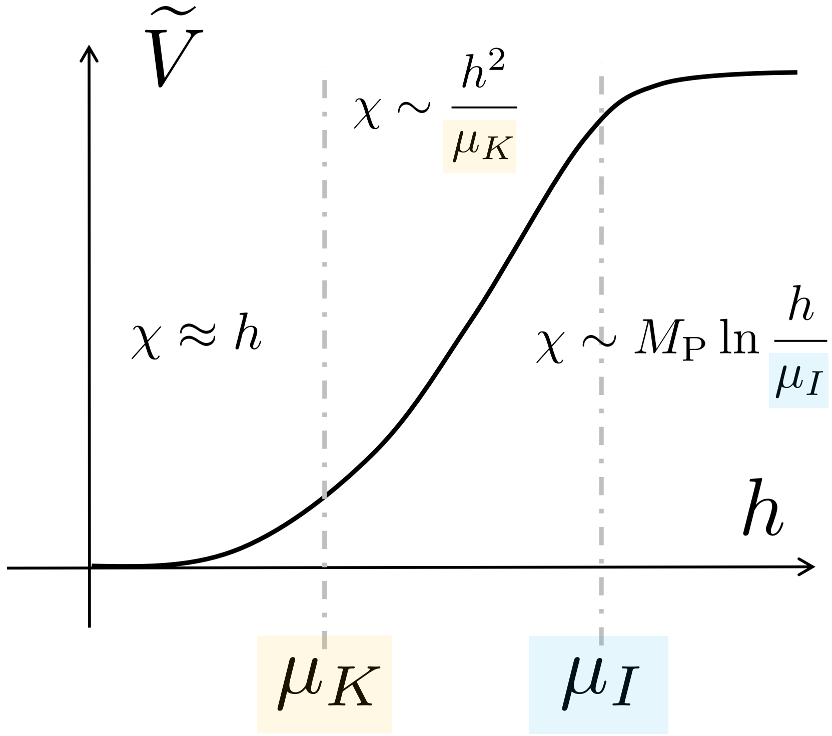
$$\boxed{\widetilde{g}_{\mu\nu} = \Omega^2 g_{\mu\nu} \qquad \Omega^2 = 1 + h^2/\mu_I^2}$$

Einstein-frame Lagrangian

$$\frac{\widetilde{\mathcal{L}}}{\sqrt{-\widetilde{g}}} = \frac{M_{\mathrm{P}}^2}{2}\widetilde{R} - \Upsilon \times \frac{\widetilde{g}^{\mu\nu}}{2}\partial_\mu h \partial_\nu h - \widetilde{V}(h) + \cdots$$

$$\left(\frac{d\chi}{dh}\right)^2 = \Upsilon = \frac{\mu_K^2 + h^2}{\mu_K^2}\,\Omega^{-4}$$

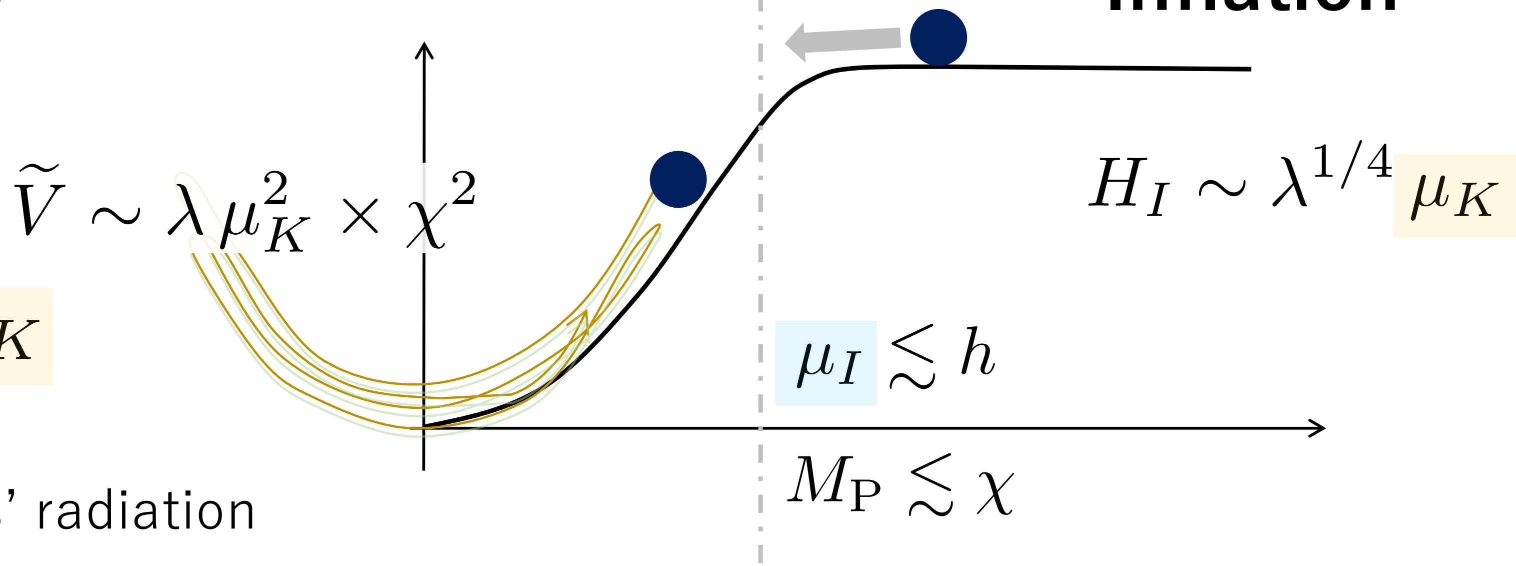
χ : canonical variable



$$\boxed{\mu_K = \frac{M_{\mathrm{P}}}{\xi}\frac{1}{\sqrt{6+1/\xi}}} \ll \boxed{\mu_I = \frac{M_{\mathrm{P}}}{\sqrt{\xi}}} \ll M_{\mathrm{P}}$$

Higgs inflation

Inflation



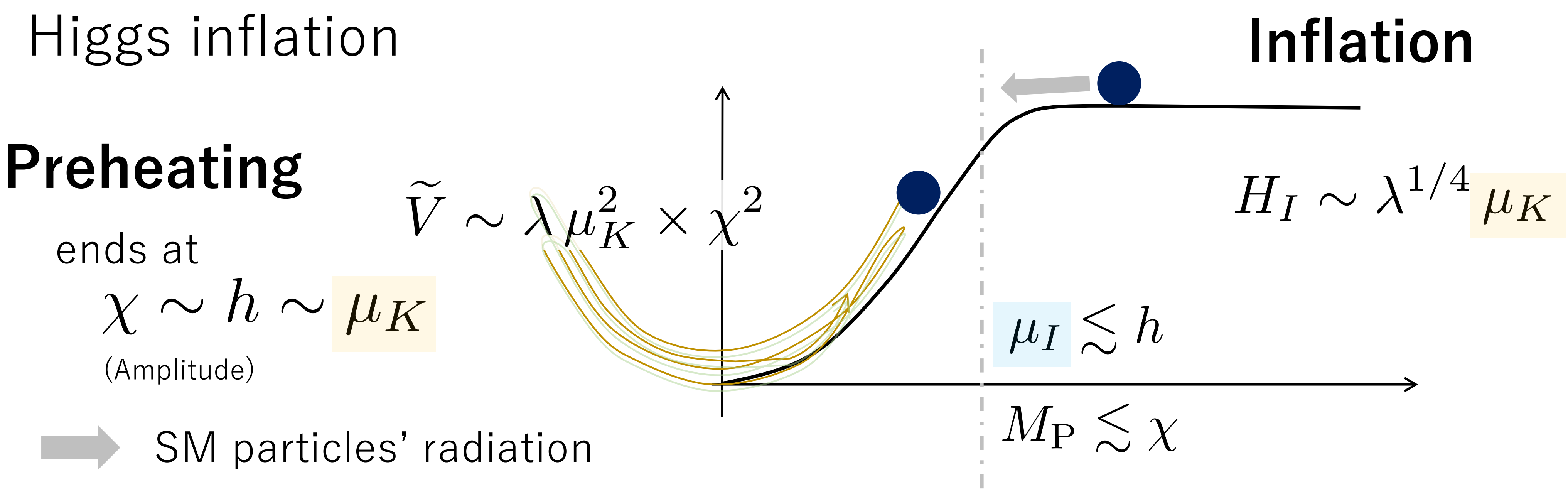
Preheating

ends at

$\chi \sim h \sim \mu_K$

(Amplitude)

SM particles' radiation



$$\xi \simeq 15800 \sqrt{\lambda} \Rightarrow$$

Scalar perturbation amp.

$$A_s \simeq 2.2 \times 10^{-9}$$

Reheating temperature

$$T_{\mathrm{rh}} \sim 10^{14}\,\mathrm{GeV}$$

Spectral tilt

$$n_s \simeq 0.965$$

Tensor to scalar ratio

$$r \simeq 0.003$$

Higgs inflation

$\boxed{\text{Perturbative with such a large } \xi \text{ ??}}$

Tree unitarity

$$\mathcal{M}_N \sim E^{4-N} \quad \textit{at most}$$

N-particle tree amplitude

Higgs inflation

Perturbative with
such a large ξ ??

Tree unitarity

$$\mathcal{M}_N \sim E^{4-N} \quad \textit{at most}$$

N-particle tree amplitude

Tree unitarity violation scale Λ

is **field dependent**

Higgs inflation

Perturbative with
such a large ξ ??

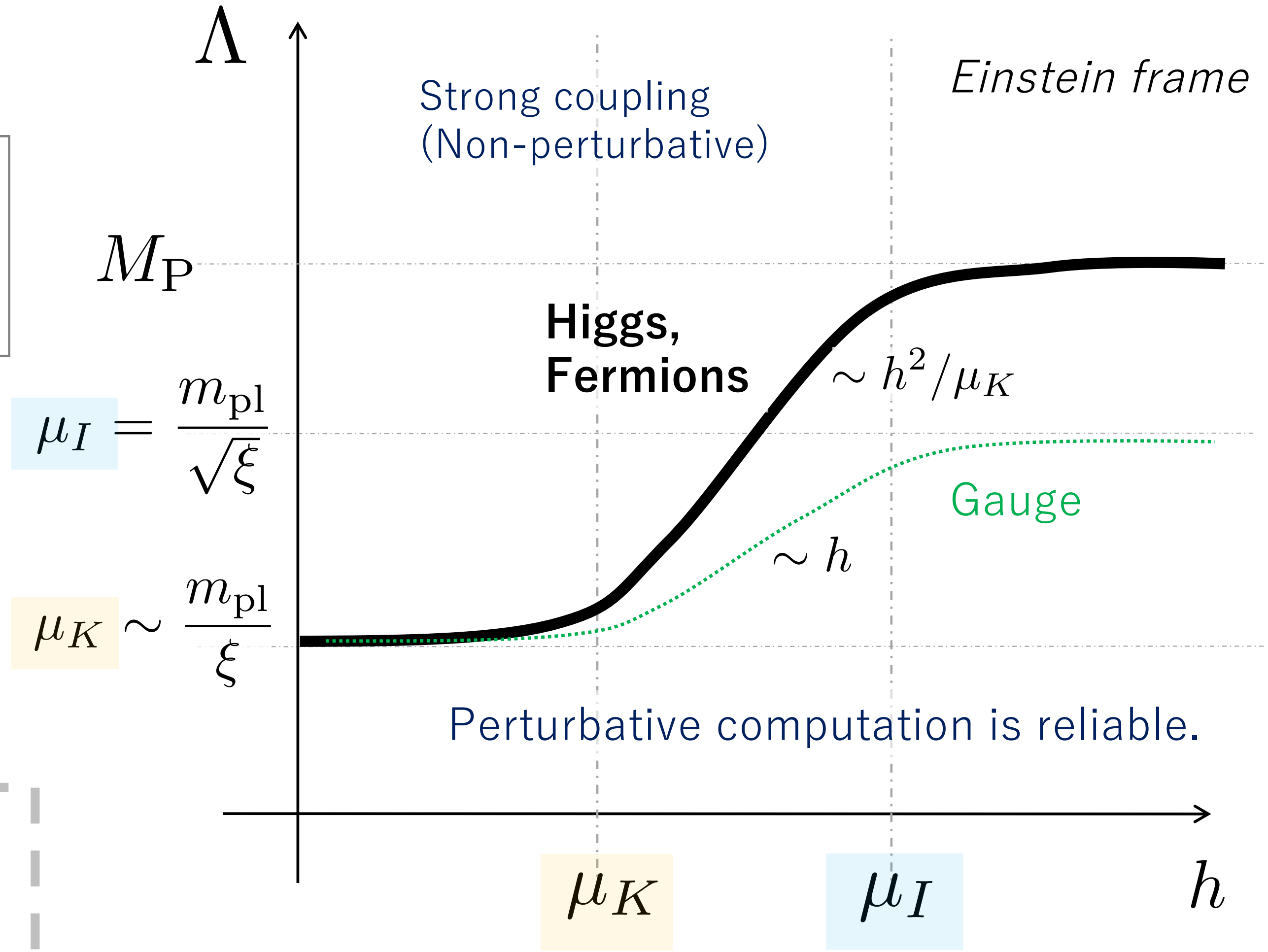
Tree unitarity

$$\mathcal{M}_N \sim E^{4-N} \quad \textit{at most}$$

N-particle tree amplitude

Typical energy scale is
always lower than Λ.

Especially, $H_I \sim \lambda^{1/4} \mu_K \ll \Lambda$ for $\mu_I \lesssim h$.



F.Bezrukov, A.Magnin,
M.Shaposhnikov, S.Sibiryakov (2010)

Higgs inflation

Tree-level ➡ $\lambda \times h^4$

responsible for both { *Low-energy (EW) phys.* / *Inflation* }

Quantum correction ➡

$\lambda_{\rm eff}(h) < 0$ for $h > h_0 \sim 10^{10}\,\mathrm{GeV}$

if $m_t = 173\,\mathrm{GeV}$

Infinite number of non-renormalizable operators

Is the Higgs inflation possible?

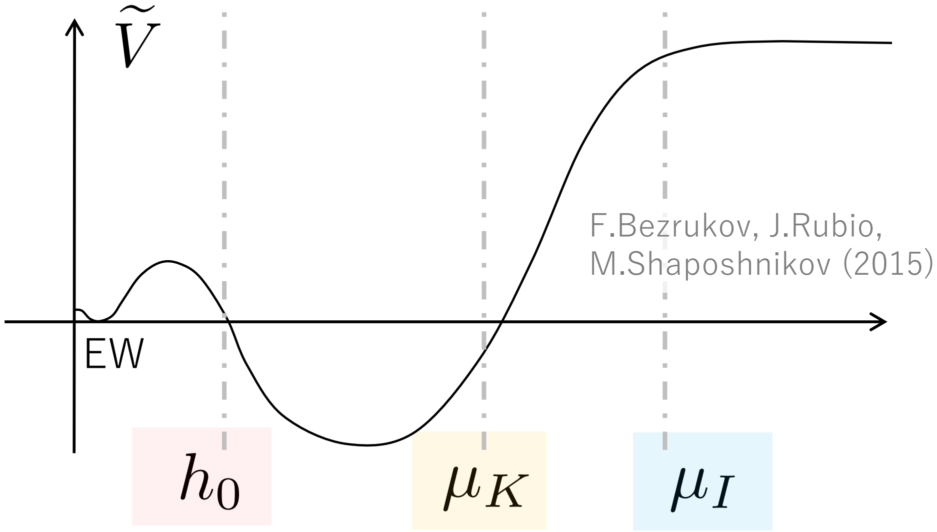
If yes, any link between EW phys. and inflation?

Higgs inflation

Non-renormalizable operators

Higgs inflation
even with **meta-stability**

No clear link between EW and Inflation



In the following, we assume

Non-renormalizable
operators are **negligible**.

$$V = \frac{\lambda_{\rm eff}(h)}{4} h^4$$

must be
responsible for both

- *Low-energy (EW) phys.*
- *Inflation*

From inflation to EW potential

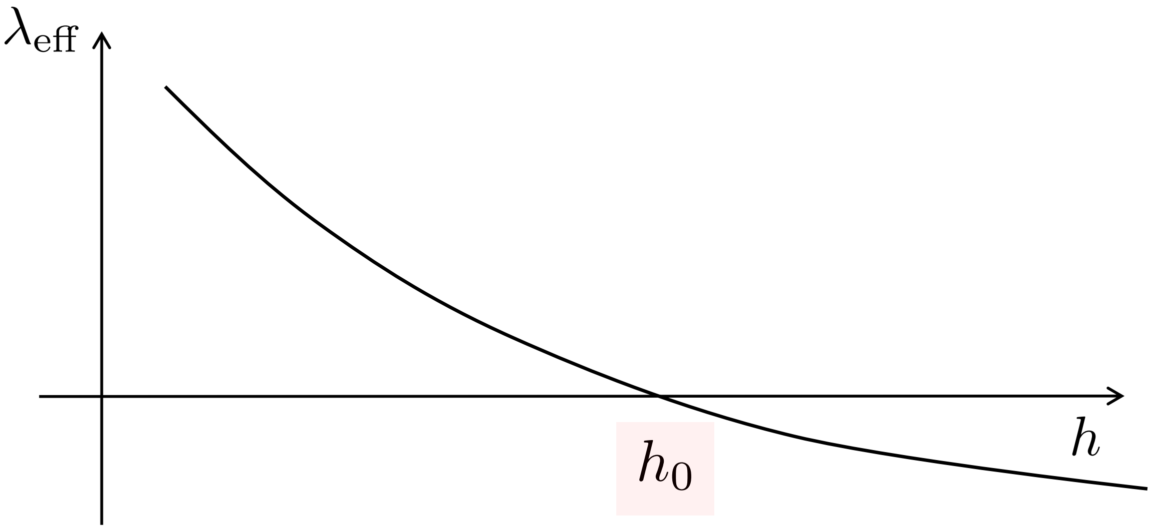
$\lambda_{\rm eff}(h/\boxed{M})$ needs to **stop running** before crossing zero.

proportional to the cutoff scale introduced to define a theory

From inflation to EW potential

$\lambda_{\rm eff}(h/M)$ needs to **stop running** before crossing zero.

$$\lambda_{\rm eff}(h/M) = \overset{0}{\lambda_{\rm eff}(h_0/M)} + \frac{B_0}{2}\ln\left(\frac{h^2}{h_0^2}\right) + \cdots$$



From inflation to EW potential

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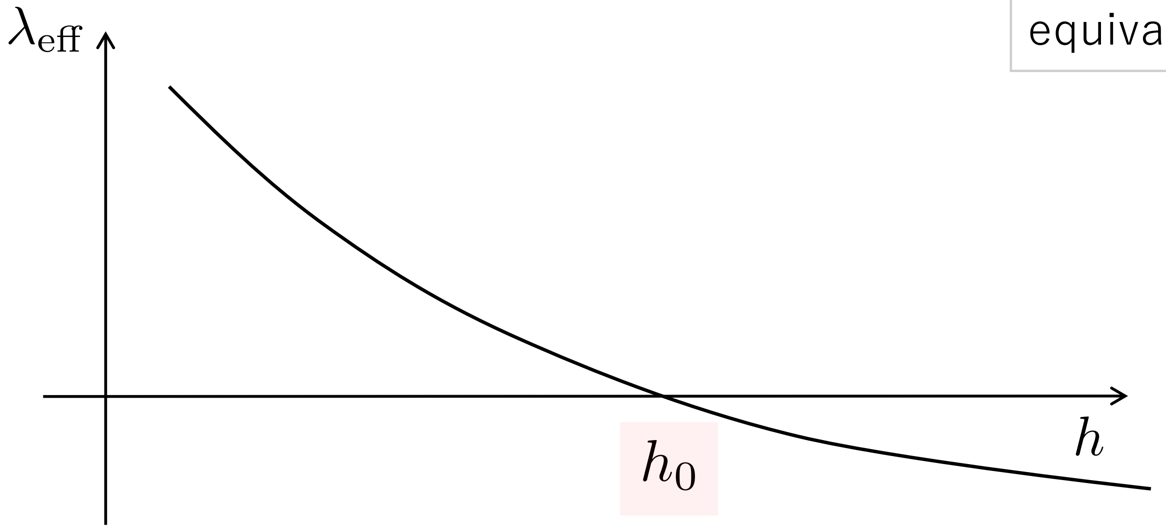
equivalent to introducing a renormalization scale



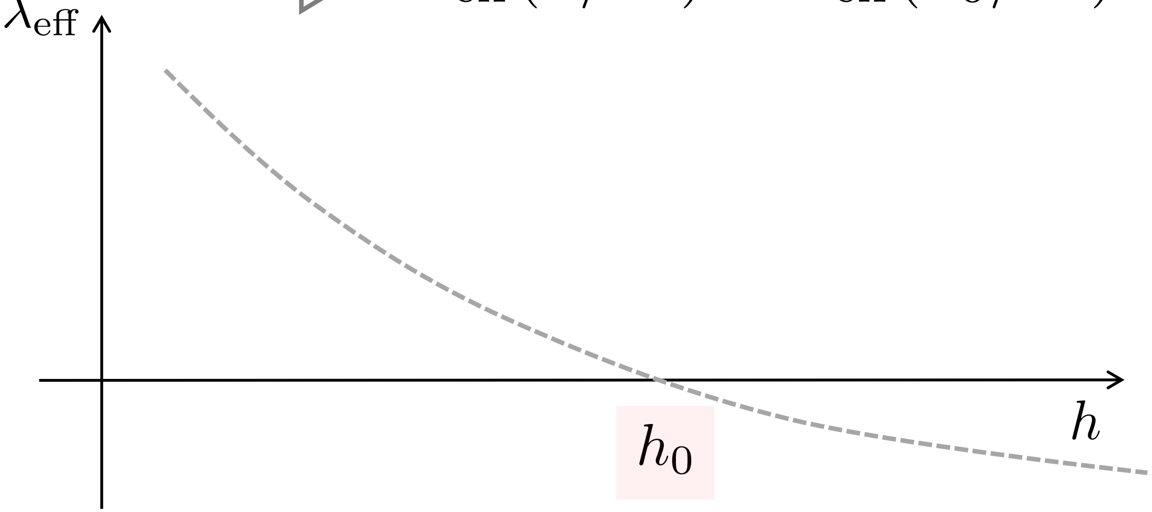
From inflation to EW potential

$\lambda_{\rm eff}(h/M)$ needs to **stop running** before crossing zero.

$$\lambda_{\rm eff}(h/M) = \overbrace{\lambda_{\rm eff}(h_0/M)}^{0} + \frac{B_0}{2}\ln\left(\frac{h^2}{h_0^2}\right) + \cdots$$

Field-dependent

$$\Rightarrow \quad \lambda_{\rm eff}(h/M) = \lambda_{\rm eff}(h_0/M) + \frac{B_0}{2}\ln\left(\frac{h^2}{h_0^2 + c^2 h^2}\right) + \cdots$$



From inflation to EW potential

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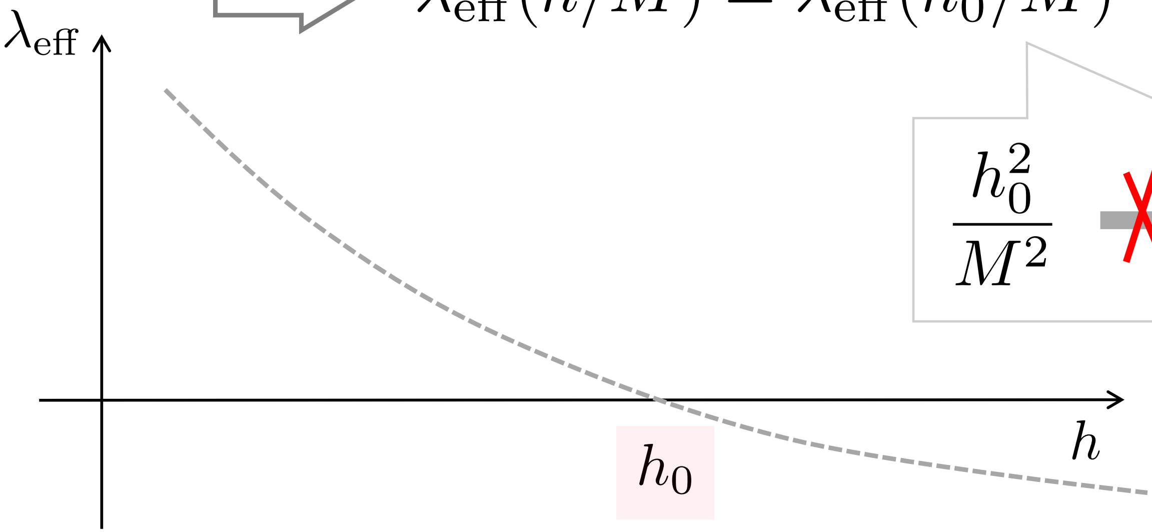
$$\frac{h_0^2}{M^2} \not\longrightarrow \frac{h_0^2 + c^2 h^2}{M^2}$$



From inflation to EW potential

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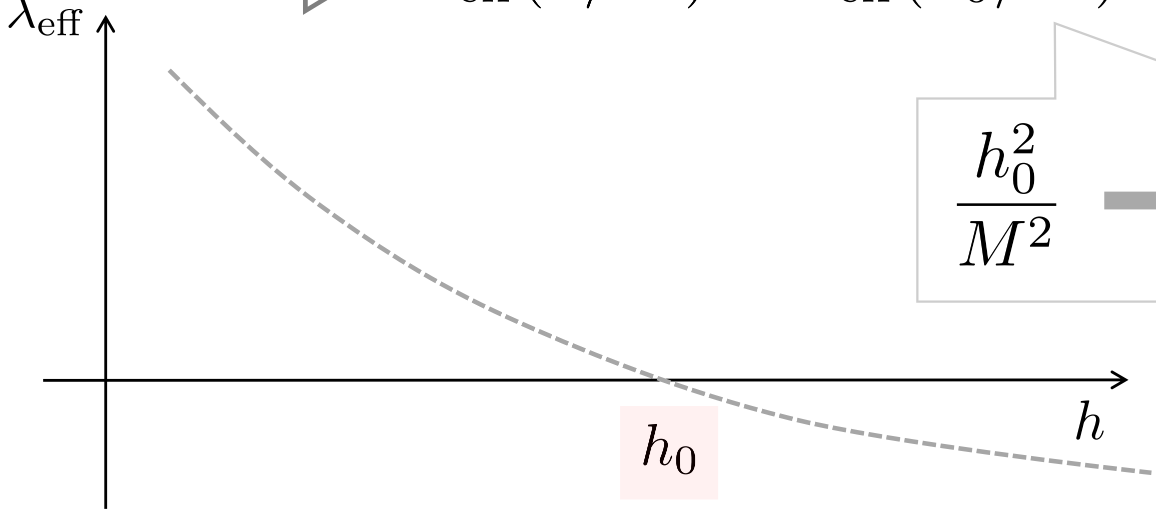
$$\frac{h_0^2}{M^2} \longrightarrow \frac{h_0^2 + c^2 h^2}{M^2(h)} = \frac{h_0^2}{M^2}$$



$$M^2(h) = M^2\,\frac{h_0^2 + c^2 h^2}{h_0^2}$$

From inflation to EW potential

$\lambda_{\text{eff}}(h)$ needs to **stop running** before crossing zero.

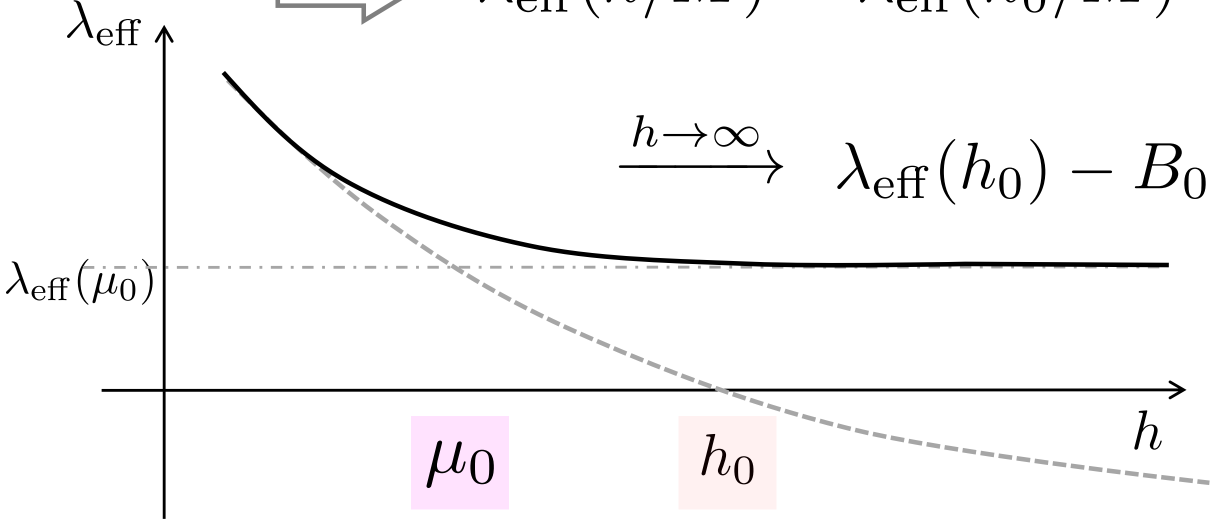
$$\lambda_{\text{eff}}(h/M) = \overset{0}{\lambda_{\text{eff}}(h_0/M)} + \frac{B_0}{2}\ln\left(\frac{h^2}{h_0^2}\right) + \cdots$$

Field-dependent

$$\Rightarrow \quad \lambda_{\text{eff}}(h/M) = \lambda_{\text{eff}}(h_0/M) + \frac{B_0}{2}\ln\left(\frac{h^2}{h_0^2 + c^2 h^2}\right) + \cdots$$

$$\xrightarrow{h\to\infty} \quad \lambda_{\text{eff}}(h_0) - B_0 \ln c + \cdots = \lambda_{\text{eff}}(\underbrace{h_0/c}_{\mu_0})$$

with $c > 1$



From inflation to EW potential

Dimensional regularization $n = 4 - 2\varepsilon$ with

Field-dependent renormalization scale

$$\frac{\lambda h^4}{4} \Rightarrow \mu^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda h^4}{4}$$

$$\mu^2 \propto \mu_0^2 + h^2$$

From inflation to EW potential

Dimensional regularization $n = 4 - 2\varepsilon$ with

Field-dependent renormalization scale

$$\mu^2 \propto \mu_0^2 + h^2$$

$$\frac{\lambda\, h^4}{4} \;\Rightarrow\; \mu^{\frac{2\varepsilon}{1-\varepsilon}} \times \frac{\lambda\, h^4}{4}$$

μ fluctuates to give rise to, for instance,

$$\partial_h \mu^{\frac{2\varepsilon}{1-\varepsilon}} = \frac{2\varepsilon}{1-\varepsilon}\frac{h}{\mu_0^2 + h^2} \times \mu^{\frac{2\varepsilon}{1-\varepsilon}}$$

$\propto \varepsilon$ *at least*

"Abnormal (evanescent)" interaction

⇒ **Non-renormalizable**

M.Shaposhnikov, F.V.Tkachov (2009)

D.M.Ghilencea (2016)

From inflation to EW potential

Jordan frame

Renormalization prescriptions	$\mu^2 \propto$	
I	$M_{\rm P}^2 + \xi h^2 = M_{\rm P,eff}^2$	F.Bezrukov, M.Shaposhnikov (2007)
II	$M_{\rm P}^2$	A.O.Barvinsky, A.Y.Kamenshchik, A.A.Starobinsky (2008)

From inflation to EW potential

Jordan frame

Renormalization prescriptions	$\mu^2 \propto$
I	$M_{\rm P}^2 + \xi h^2 \; = M_{\rm P,eff}^2$
II	$M_{\rm P}^2$

F.Bezrukov, M.Shaposhnikov (2007)

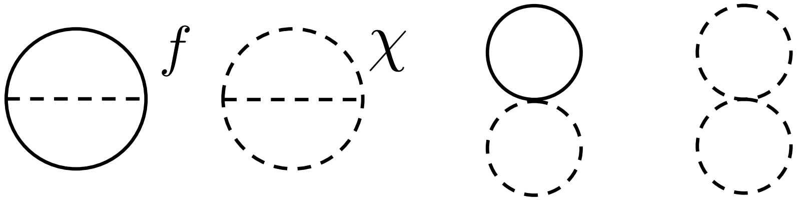
A.O.Barvinsky, A.Y.Kamenshchik, A.A.Starobinsky (2008)

$$\mu_0^2 + h^2 \; \propto M_{\rm P}^2 + \zeta h^2$$

$$\boxed{\zeta = M_{\rm P}^2/\mu_0^2 \gg \xi}$$

From inflation to EW potential

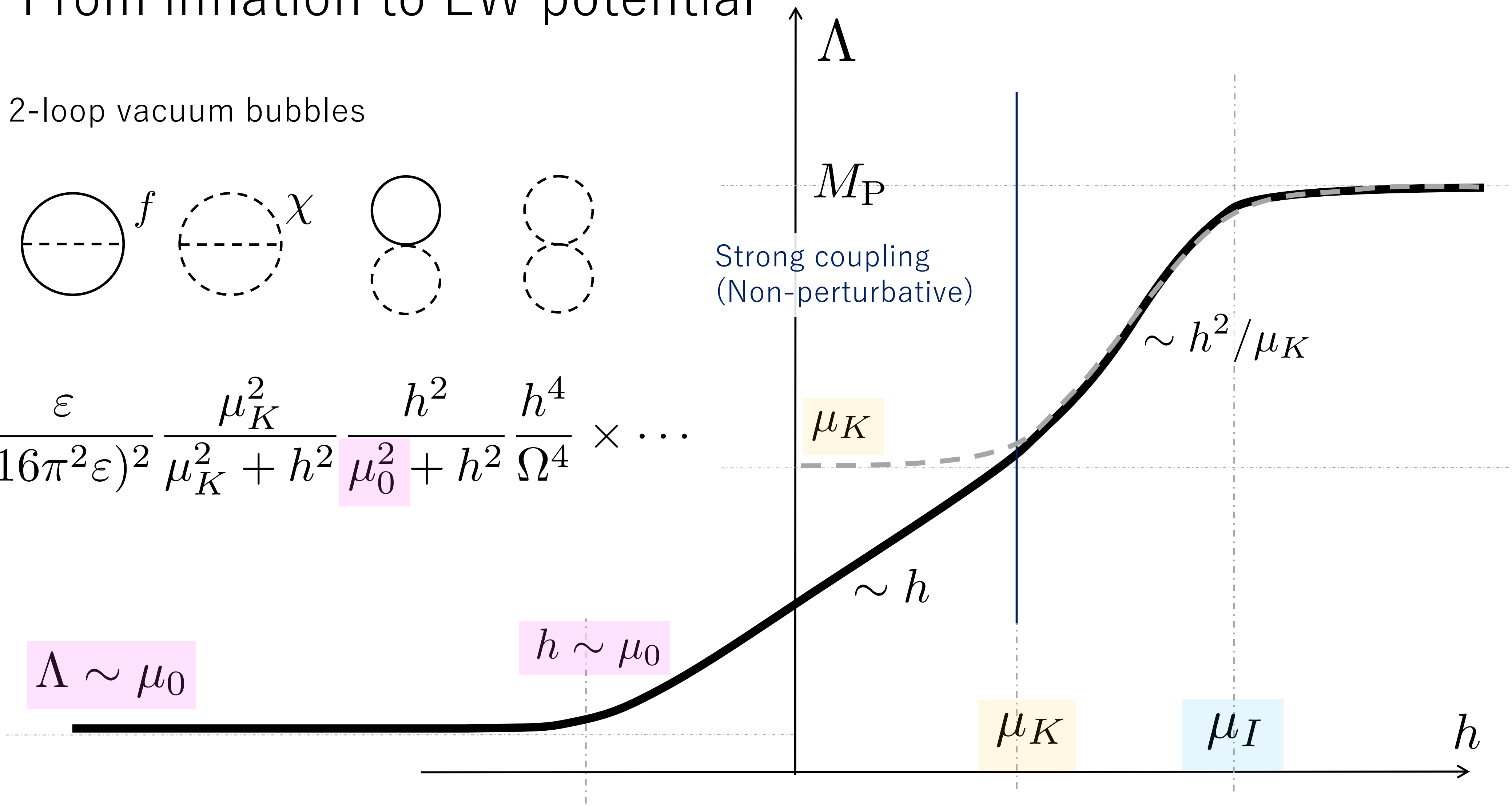
2-loop vacuum bubbles



$$\frac{\varepsilon}{(16\pi^2\varepsilon)^2}\,\frac{\mu_K^2}{\mu_K^2+h^2}\,\frac{h^2}{\mu_0^2+h^2}\,\frac{h^4}{\Omega^4}\times\cdots$$

From inflation to EW potential

2-loop vacuum bubbles



$$\frac{\varepsilon}{(16\pi^2\varepsilon)^2}\,\frac{\mu_K^2}{\mu_K^2+h^2}\,\frac{h^2}{\mu_0^2+h^2}\,\frac{h^4}{\Omega^4}\times\cdots$$

From inflation to EW potential

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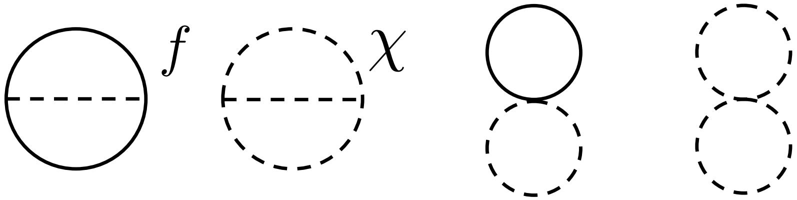
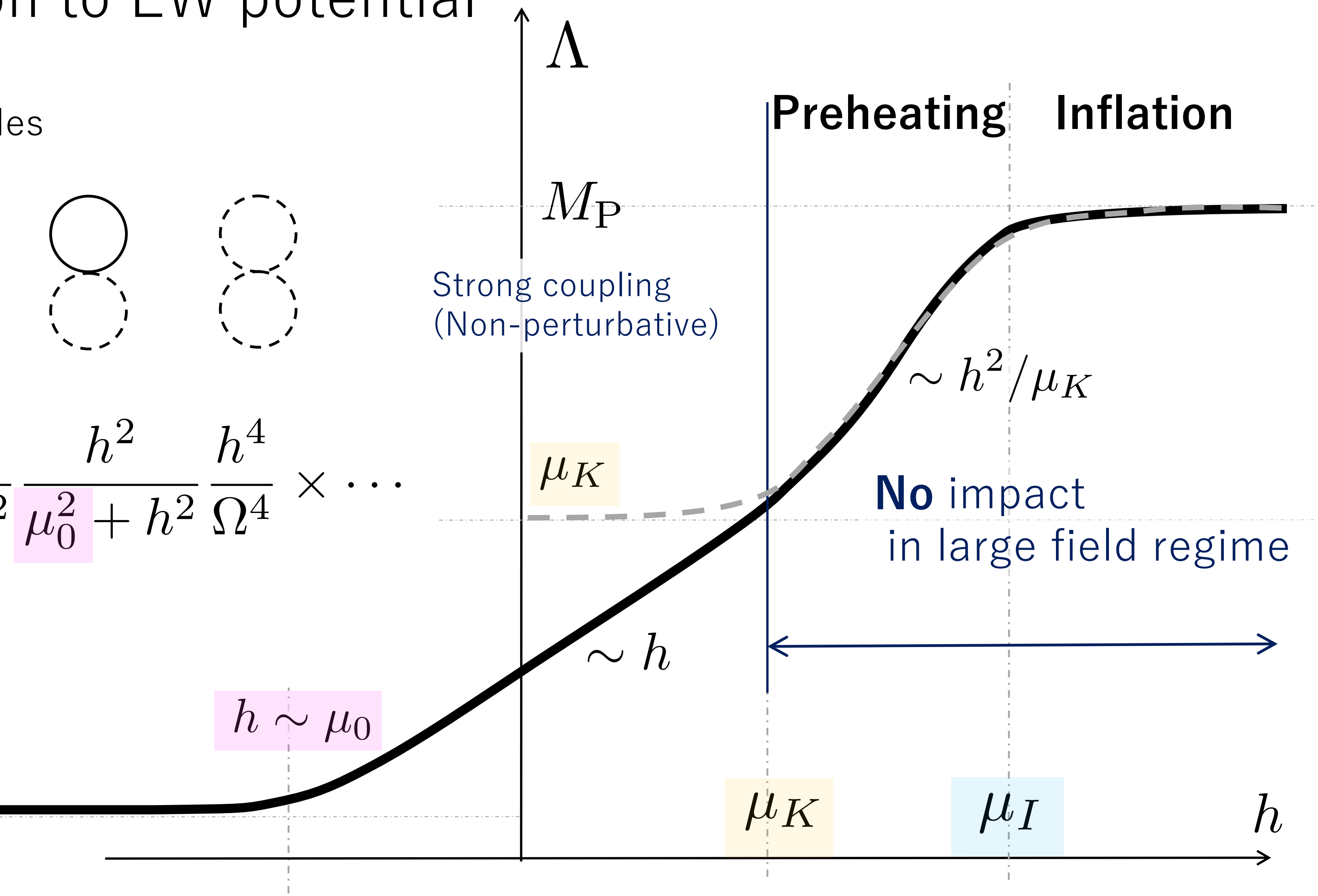


$$\frac{\varepsilon}{(16\pi^2\varepsilon)^2}\,\frac{\mu_K^2}{\mu_K^2+h^2}\,\frac{h^2}{\mu_0^2+h^2}\,\frac{h^4}{\Omega^4}\times\cdots$$



Thermal history

(How many e-folidings from the end of inflation to today?)

Inflation

➡ Preheating ends at $h \sim \mu_K$

➡ Particles ➡ $\langle h \rangle = 0$

Thermal history

Inflation

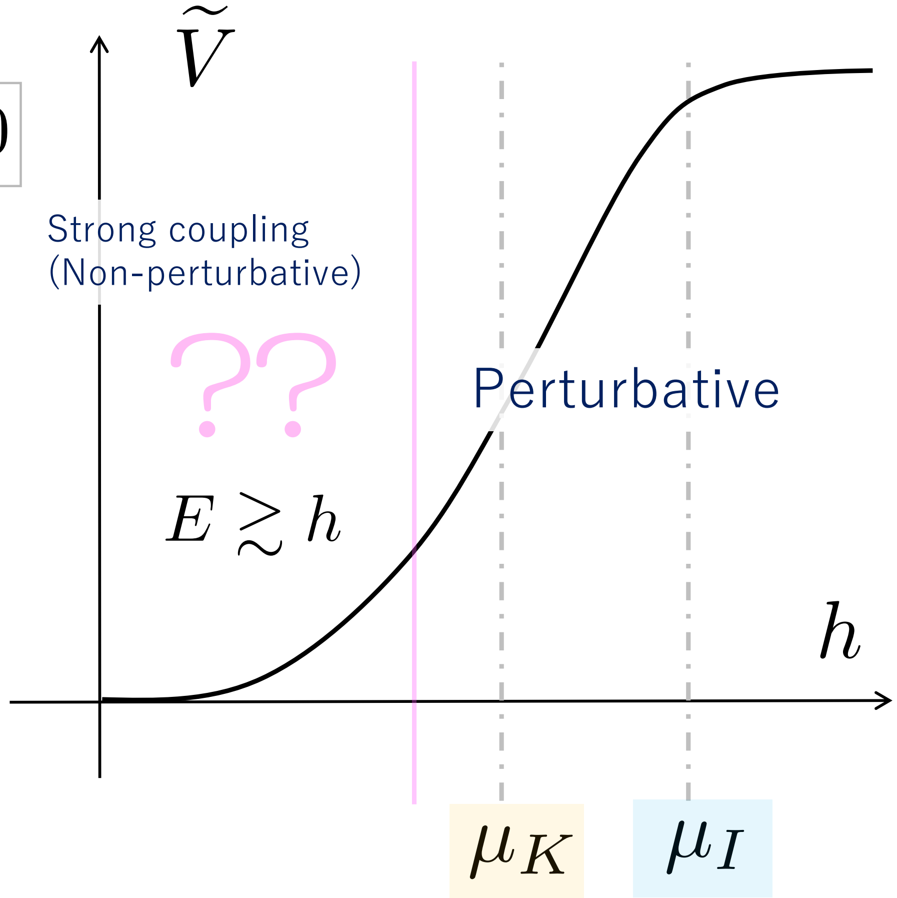
➡ Preheating ends at $h \sim \mu_K$

$\langle h \rangle = 0$

Typical energy scale

$$E \sim \widetilde{V}^{1/4} \sim \lambda^{1/4}\ \mu_K \gg \Lambda \sim \mu_0$$

does NOT necessarily mean instability.



Thermal history

Inflation

→ Preheating ends at $h \sim \mu_K$

Typical energy scale

$$E \sim \widetilde{V}^{1/4} \sim \lambda^{1/4}\ \mu_K \gg \Lambda \sim \mu_0$$

$\langle h \rangle = 0$

$$N_{\rm np} = \ln \frac{a|_{E\sim\mu_0}}{a|_{h\sim\mu_K}}$$

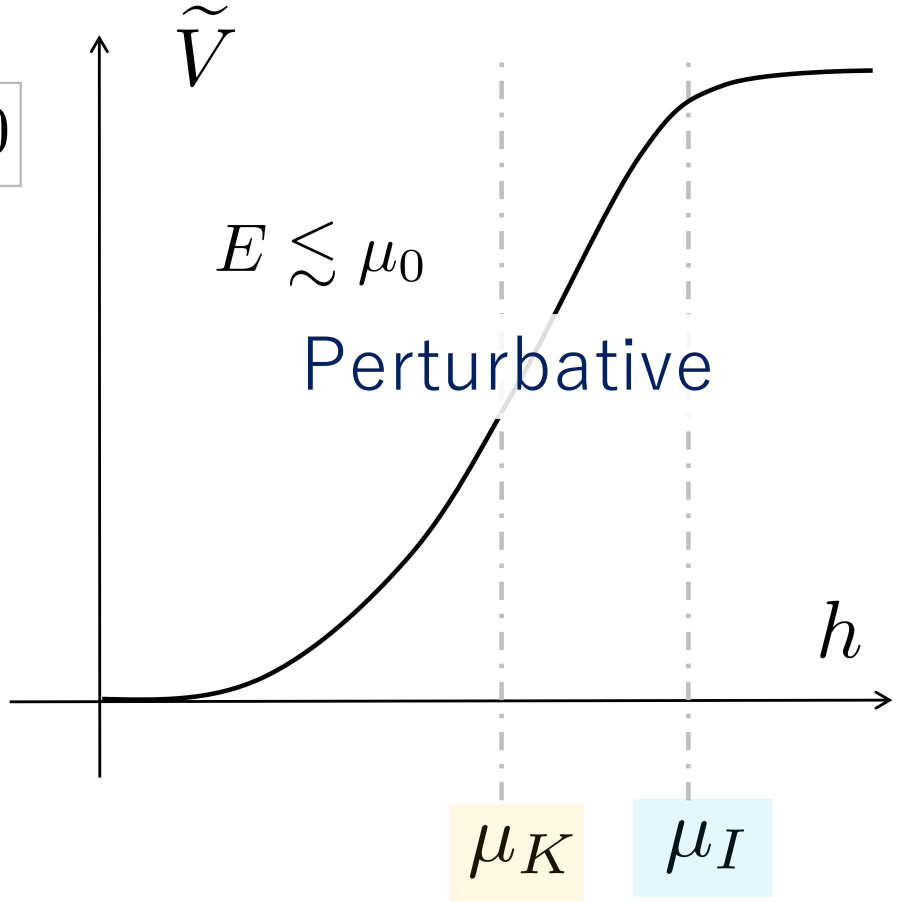
Non-perturbative

$$E \sim \mu_0$$

Dark sector?

Standard model

(quickly thermalized)



Thermal history

- Extra e-foldings gained during the non-perturbative period $\Delta N = N_{\rm np} - N_{\rm th}$

$\Delta N < 5$ ➡ 1 σ region

$\Delta N < 10$ ➡ 2 σ region

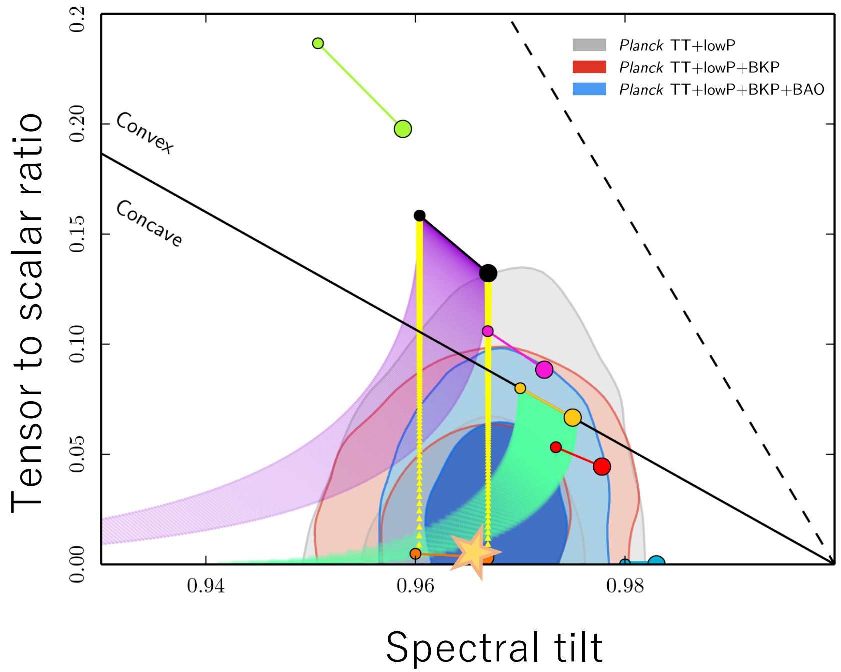
Perturbative, Thermalized

- Baryon asymmetry and Dark matter abundance

If thermalized with the SM below $E \sim \mu_0$,

there is no uncertainty from the non-perturbative period.

- Tree unitarity violation scale should be computed with finite density/temperature effects.



Summary

- We assumed a field dependent renormalization prescription with

$$\mu^2 \propto \mu_0^2 + h^2 \propto M_{\rm P}^2 + \zeta h^2 \qquad \boxed{\zeta = M_{\rm P}^2/\mu_0^2 \gg \xi}$$

- $\lambda \times h^4$ is responsible for both EW phys. and inflation.

  Smallness of the couplings of non-renormalizable terms are stable against quantum correction (technically natural).

- Associated non-perturbative scale Λ is computed.

- Λ is higher than the typical energy scale during the Higgs inflation.

  So the large scale perturbation itself is computable.

- But the universe undergoes the non-perturbative period after the preheating.

  If the extra e-folding is small (< 10), still consistent with the observation.

Thank you