Unification from Scattering Amplitudes

Clifford Cheung

CC, Remmen, Shen, Wen (1709.04932)
CC, Shen, Wen (1705.03025)
CC, Shen (1612.00868)
CC, Kampf, Novotny, Shen, Trnka (1611.03137)

principles	Poincare invariance, spacetime locality, etc.
↓	↓
action	parameters not fixed by symmetry are couplings
↓	↓
observables	calculate via Feynman diagrams

principles

↓

~~action~~

↓

observables

modern scattering amplitudes program

Poincare invariance, spacetime locality, etc.

↓

parameters not fixed by symmetry are couplings

↓

calculate via Feynman diagrams

Some "symmetries" of the action are in truth redundancies of description.

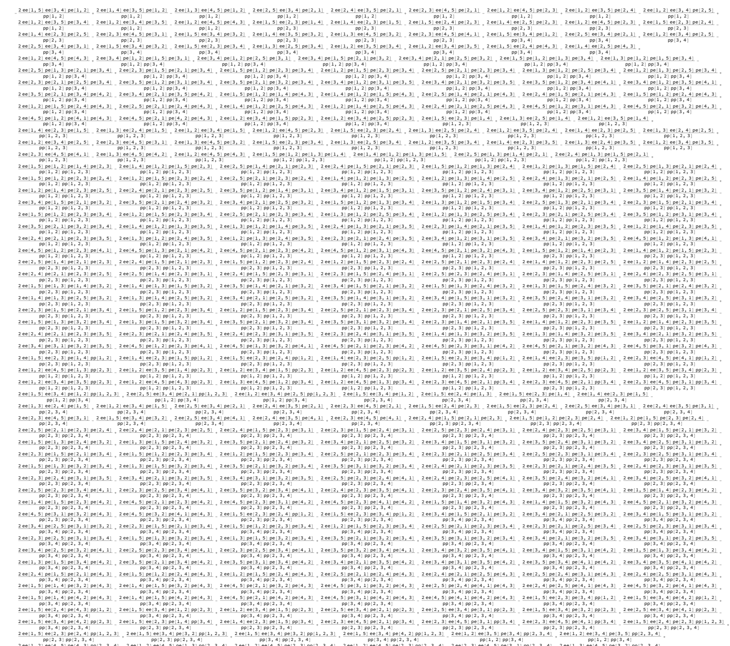
$$A_\mu \to A_\mu + \partial_\mu \theta$$

$$h_{\mu\nu} \to h_{\mu\nu} + \partial_\mu \theta_\nu + \partial_\nu \theta_\mu$$

Gauge invariance is an invention devised to manifest locality and Lorentz invariance for interacting massless particles of spin.

Gauge symmetry yields immense complexity.

$$A(g_1^{h_1} g_2^{h_2} g_3^{h_3} g_4^{h_4} g_5^{h_5}) \quad = $$



Feynman diagrams

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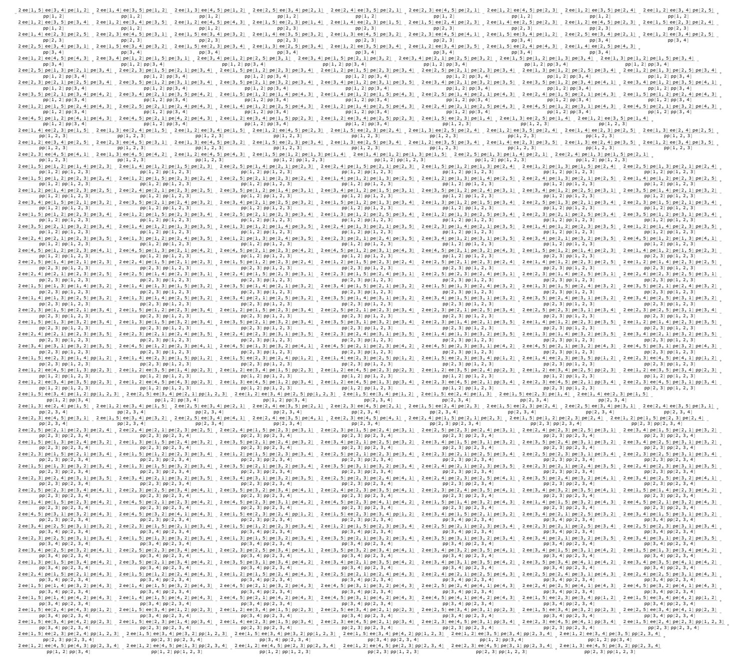

Feynman diagrams

$$A(g_1^+ g_2^+ g_3^+ g_4^+ g_5^+) \quad = \quad A(g_1^- g_2^+ g_3^+ g_4^+ g_5^+) \quad = \quad 0$$

$$A(g_1^- g_2^+ g_3^- g_4^+ g_5^+) \quad = \quad \frac{\langle 13 \rangle^4}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

$$A(g_1^- g_2^- g_3^+ g_4^+ g_5^+) \quad = \quad \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

modern methods

This redundancy becomes ever more taxing at higher orders in perturbation theory.

n-pt	4	5	6	7	8
Feynman diagrams	4	25	220	2485	34300
recursion relations	1	1	3	6	20

The situation is even worse for gravity.

$$
\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow
$$

$$
\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\
&-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})\\
&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad -P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\Big],
\end{aligned} \tag{2.6}
$$

$$
\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow
$$

$$
\begin{aligned}
&\mathrm{Sym}\Big[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})\\
&+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})\\
&+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})\\
&-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})\\
&-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})\\
&+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})\\
&\qquad\qquad -P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\Big].
\end{aligned} \tag{2.7}
$$

3pt graviton
Feynman vertex

4pt graviton
Feynman vertex

DeWitt (1967)

The situation is even worse for gravity.

$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\begin{aligned}
&\mathrm{Sym}\big[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})\\
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&\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})\big], \qquad (2.6)
\end{aligned}$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

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&\qquad\qquad-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})\big]. \qquad (2.7)
\end{aligned}$$

3pt graviton
Feynman vertex

4pt graviton
Feynman vertex

$$A(H_1^- H_2^- H_3^+) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}$$

$$A(H_1^- H_2^- H_3^+ H_4^+) = \frac{\langle 12\rangle^4[34]^4}{stu}$$

modern
methods

Even worse, some “non-symmetries” of the action are also redundancies.

$$\mathcal{L} = \frac{(\partial\phi)^2}{2} \times g(\phi)$$

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$$\mathcal{L} = \frac{(\partial\phi)^2}{2} \times g(\phi) \longleftrightarrow \mathcal{L} = \frac{(\partial\phi)^2}{2}$$

All on-shell scattering amplitudes vanish!

$$f(\phi) \longleftrightarrow \phi \quad \text{where} \quad f'(\phi)^2 = g(\phi)$$

Amplitudes are field redefinition invariant.

“integration variable”

$$Z[J] \sim \int [d\phi]\, e^{iS[\phi] + i\int J\phi}$$

Many Lagrangians yield the same observables.

$$\begin{tikzcd}
\mathcal{L}_1 \arrow[drr] & \mathcal{L}_2 \arrow[dr] & & \cdots \arrow[dl] & \mathcal{L}_\infty \arrow[dll] \\
& & A & &
\end{tikzcd}$$

The action can obscure important physics.

- absolute rigidity of certain QFTs

“only of all possible worlds…”

- unification of these QFTs

“gravity as the mother of all theories…”

rigidity of QFTs

Nature conforms to two physical criteria.

- Poincare Invariance
- Locality

These uniquely define certain theories!

massless spin one = gluon

massless spin two = graviton

Poincare Invariance fixes the 3pt amplitudes.

gluon

$$A(1^-2^-3^+) = \frac{\langle 12\rangle^3}{\langle 13\rangle\langle 32\rangle}$$

$$A(1^+2^+3^-) = \frac{[12]^3}{[13][32]}$$

graviton

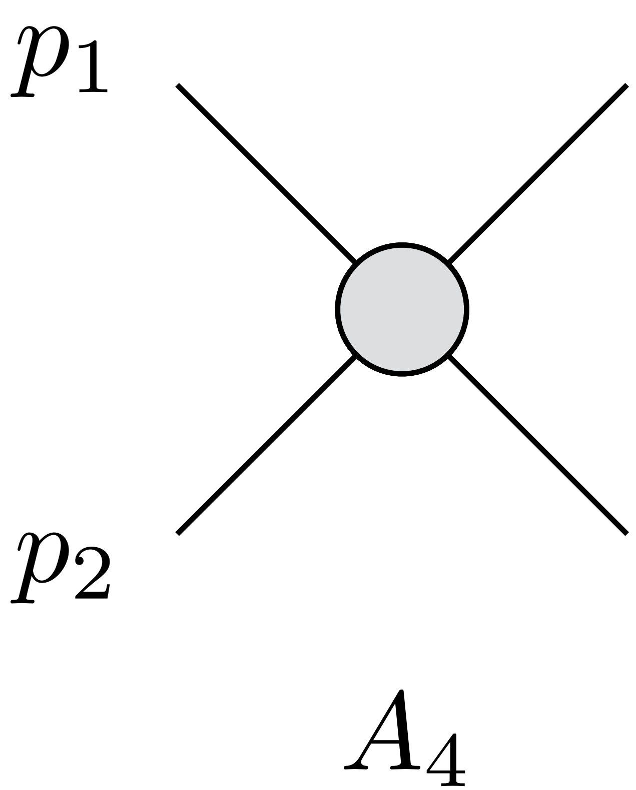
$$A(1^-2^-3^+) = \frac{\langle 12\rangle^6}{\langle 13\rangle^2\langle 32\rangle^2}$$

$$A(1^+2^+3^-) = \frac{[12]^6}{[13]^2[32]^2}$$

simplest example of "gluon^2 = graviton"

Benincasa, Cachazo (0705.4305)

Locality fixes the 4pt amplitudes and higher.



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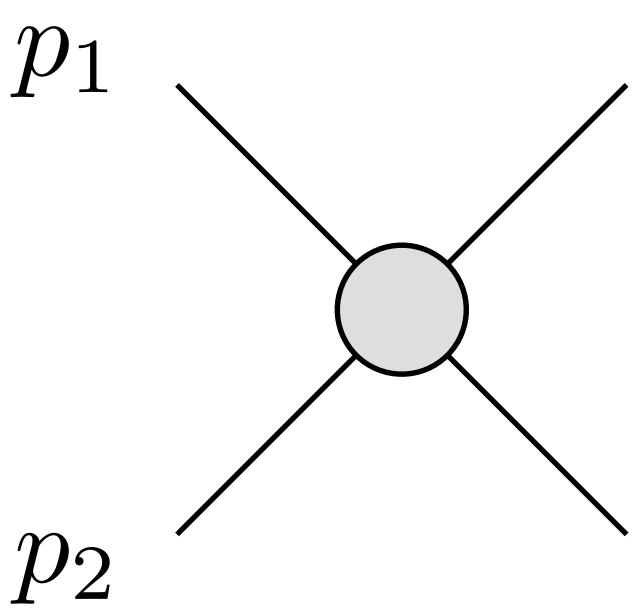
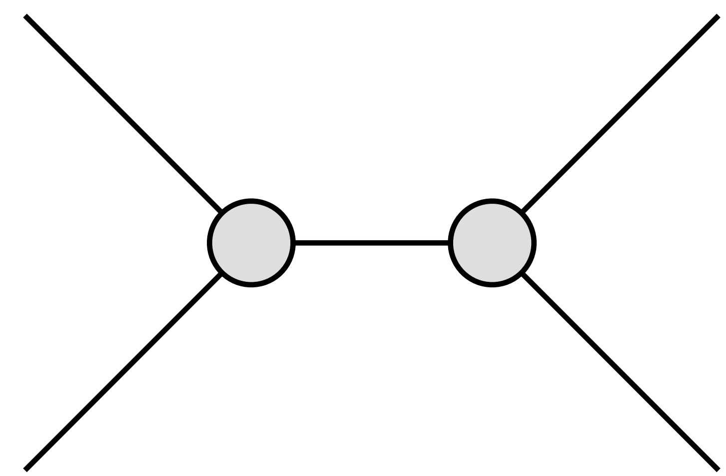


A_4

$$(p_1 + p_2)^2 \to 0$$

(factorization)



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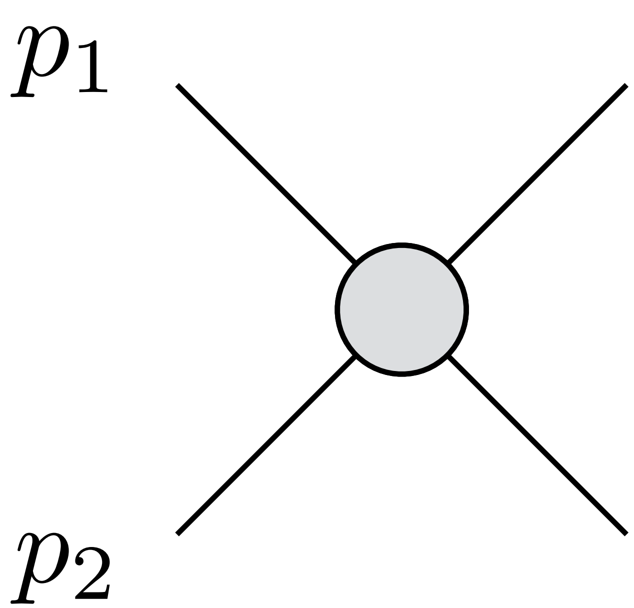

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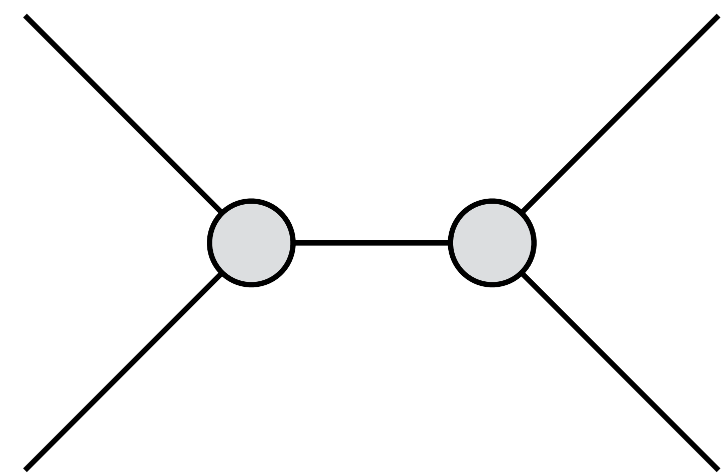
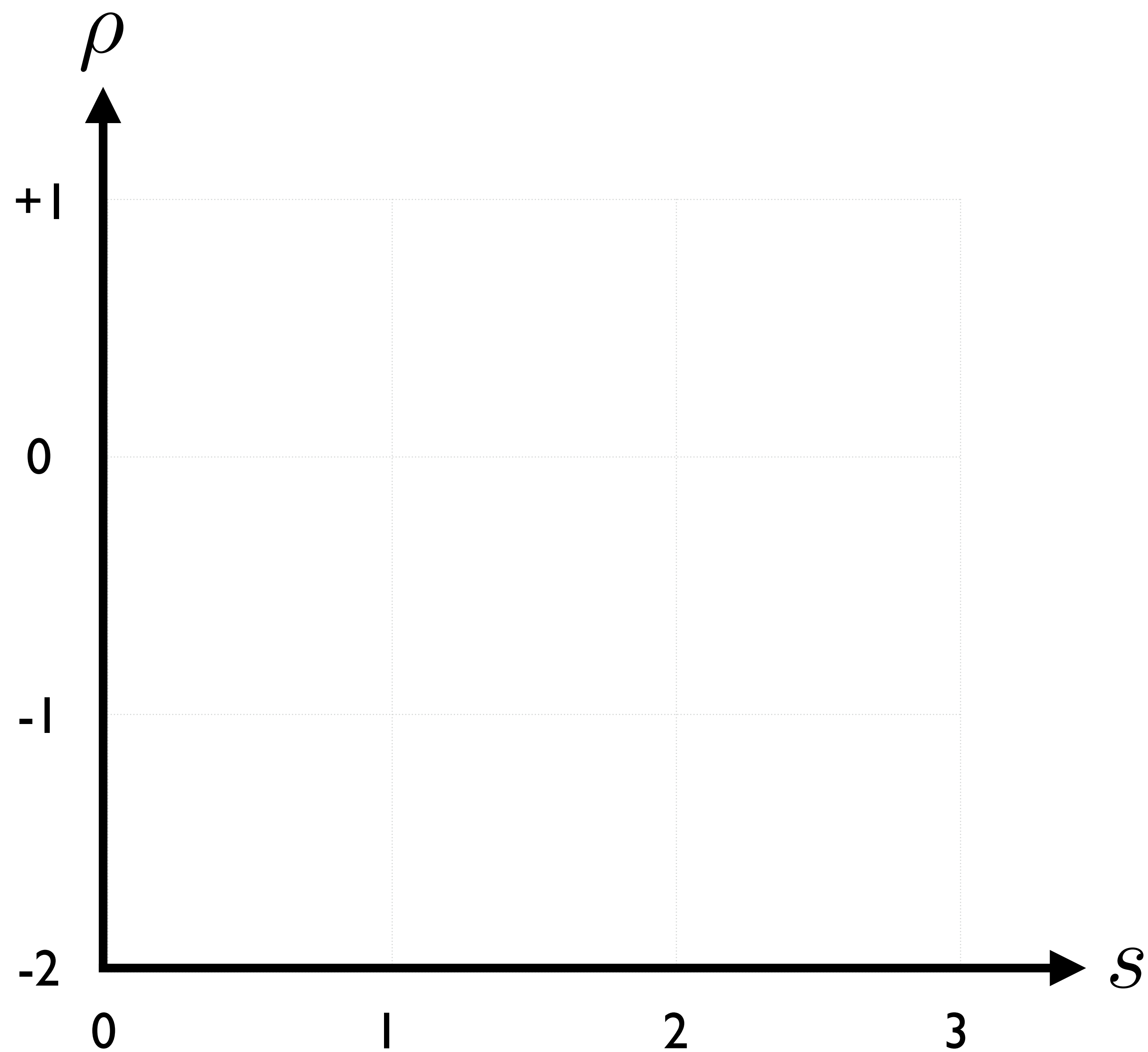


$$A_3 \frac{1}{(p_1 + p_2)^2} A_3$$

On-shell recursion relations automate this.
Optional: matter fields, masses, SUSY.

Britto, Cachazo, Feng, Witten (hep-th/0412308, hep-th/0501052)

Map the “theory space” of massless particles.



Characterize by power counting and spin.

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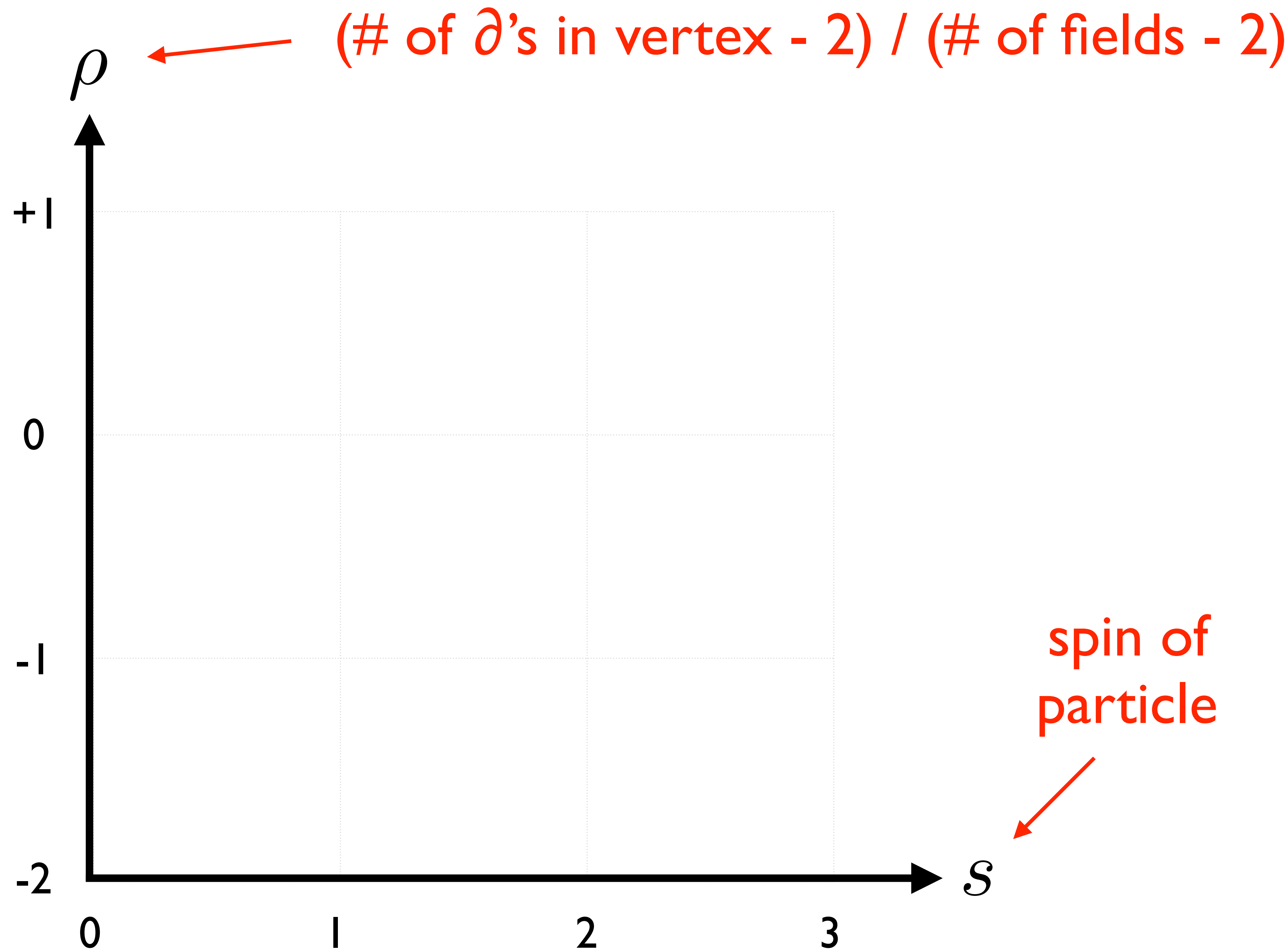

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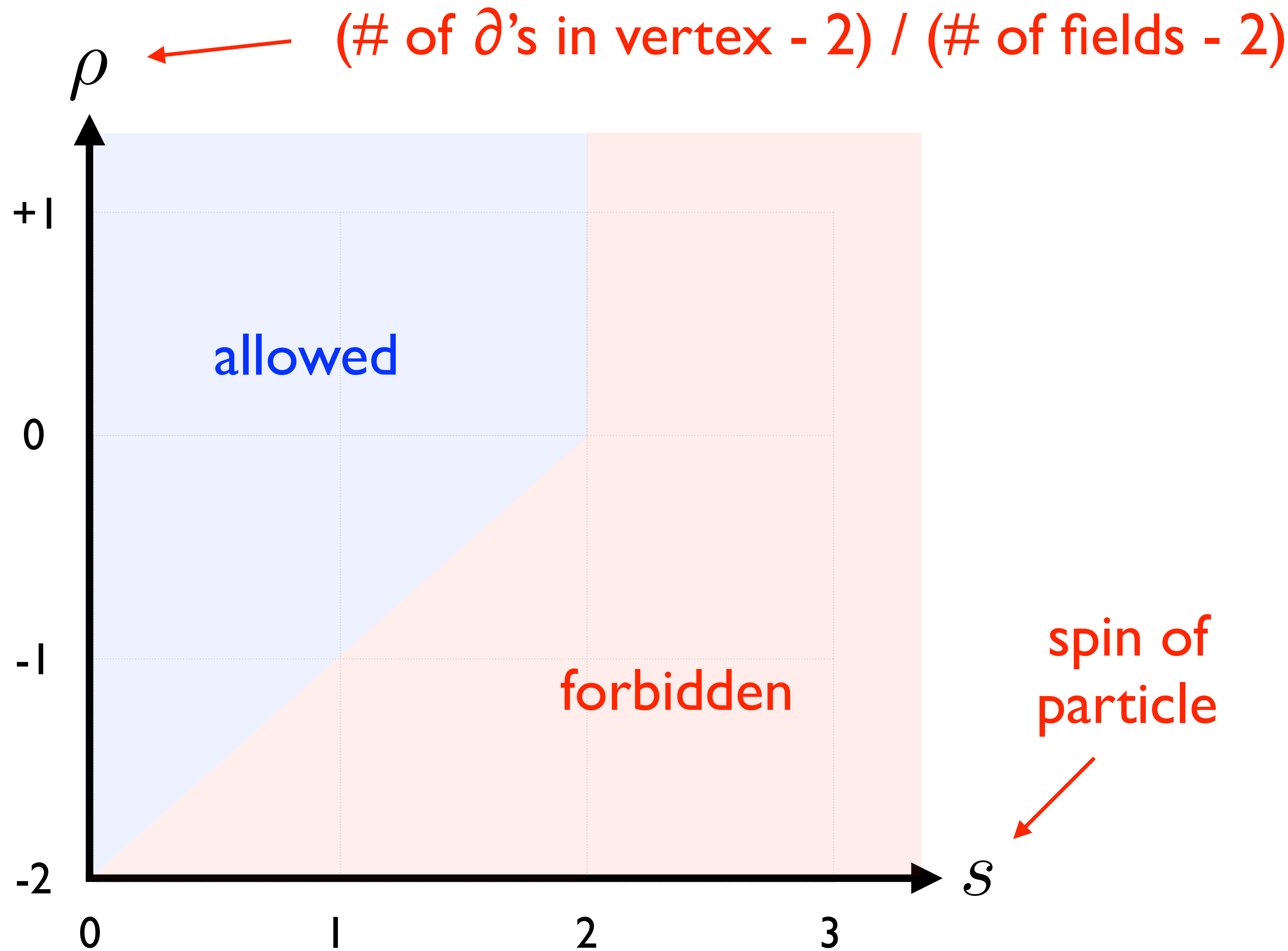

Factorization excludes massless spin > 2.

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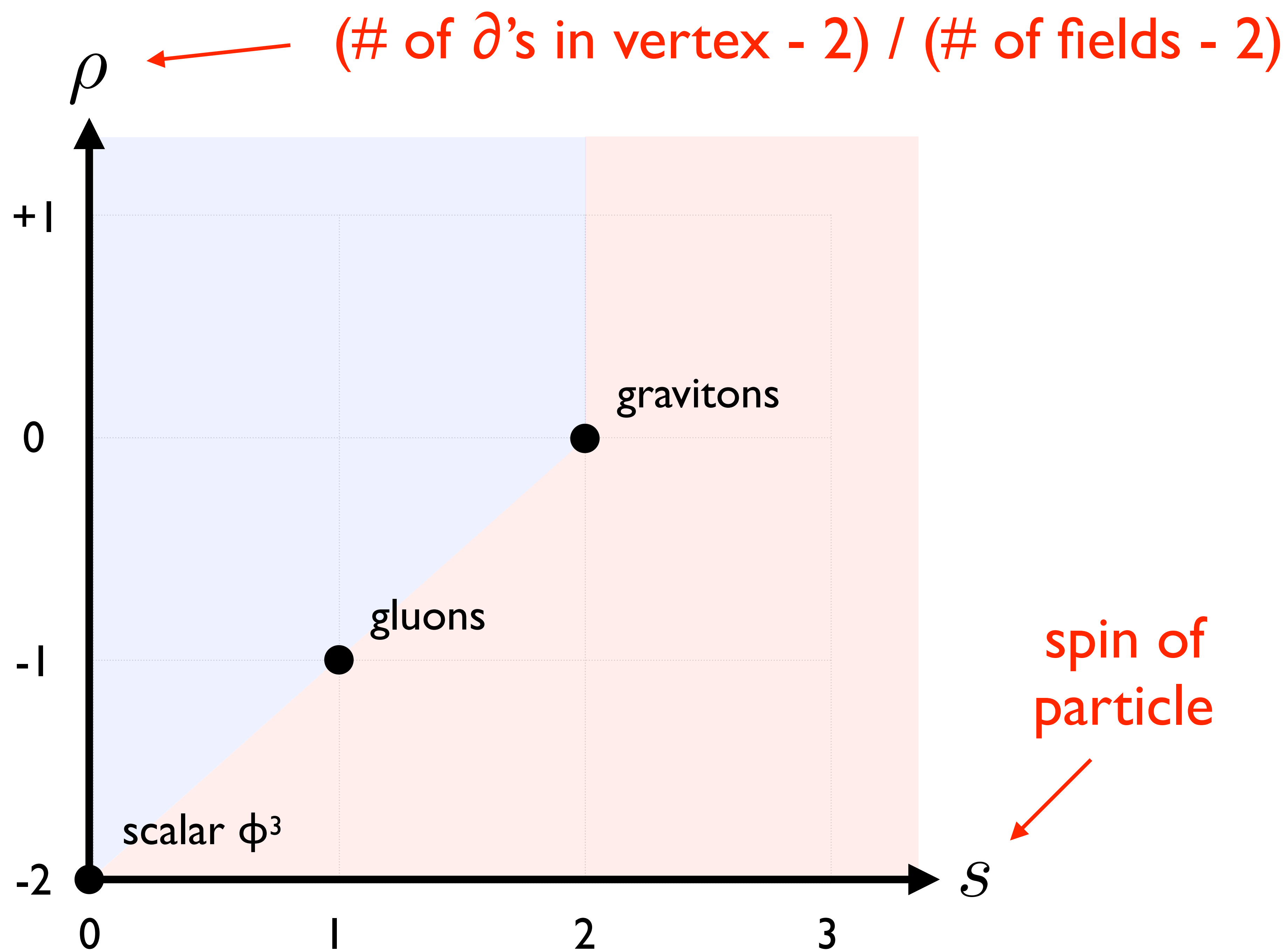

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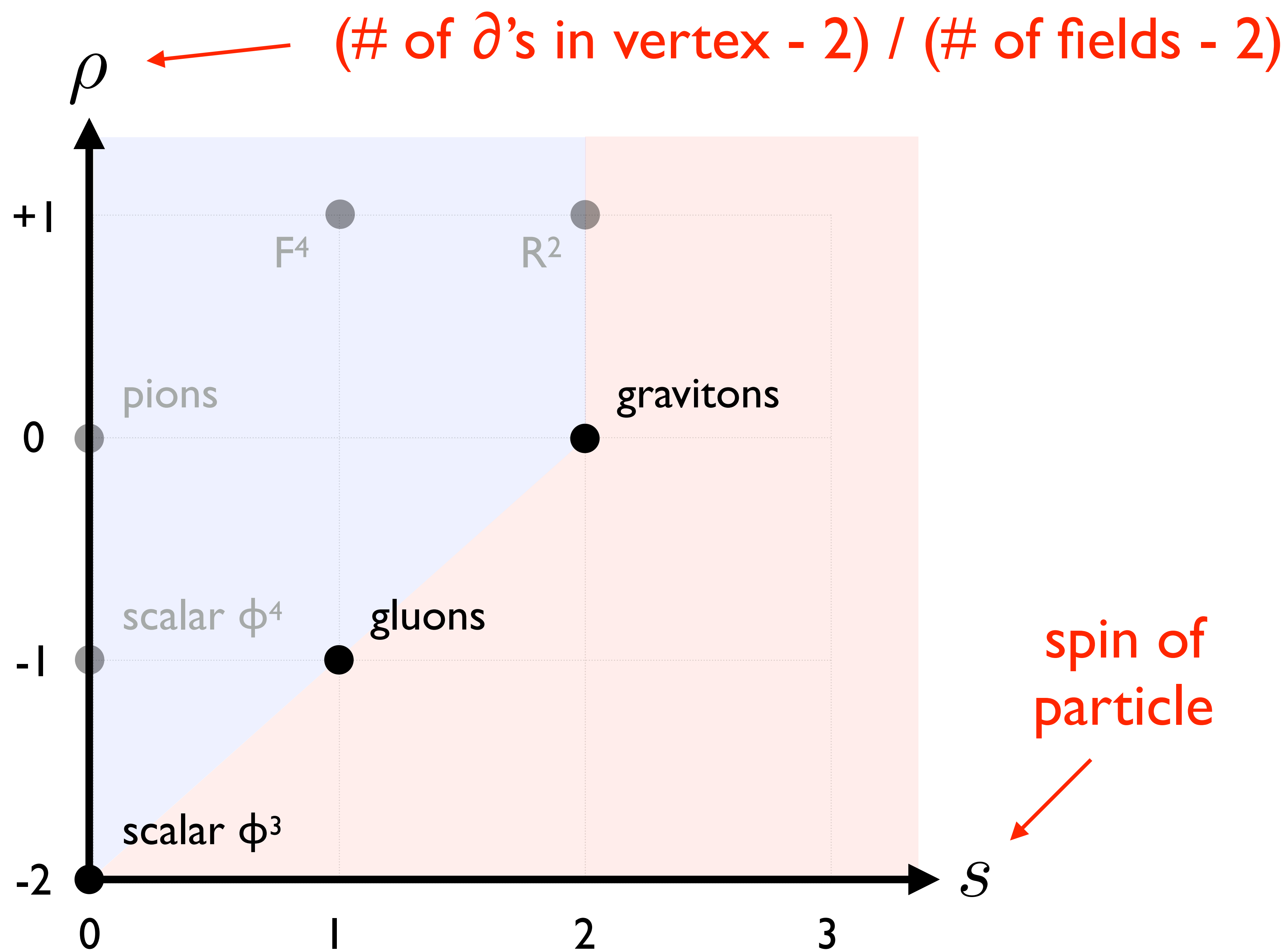

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The tree-level S-matrices of many theories can be constructed from on-shell recursion.

- Yang-Mills theory and gravity
- QCD, SM, SUSY theories
- all renormalizable 4D QFTs

Non-renormalizable operators seem difficult.

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very much
non-renormalizable!

Non-renormalizable operators seem difficult.

What about effective field theories (EFTs)?

- Poincare Invariance
- Locality
- ? ? ?

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What about effective field theories (EFTs)?

- Poincare Invariance
- Locality
- Infrared Structure

EFTs like pion theory hinge upon symmetry breaking. What we can do without an action?

In soft limit, the momentum of an external particle is taken to zero.

generalized "Adler zero"

$$A(p, \cdots) \overset{p \to 0}{\sim} p^{\sigma} \quad \text{where} \quad \sigma \in \mathbb{N}$$

Enhanced infrared behavior corresponds a soft degree greater than expected from number of derivatives per field.

$$\sigma > \rho$$

CC, Kampf, Novotny, Shen, Trnka (1412.4095, 1611.03137)

Consider a single Nambu-Goldstone boson.

$$\mathcal{L} = \sum_n c_n (\partial\phi)^{2n}$$

Amplitudes vanish in the soft limit due to a shift symmetry. There are an infinite number of free coupling constants.

$$A(p, \cdots) \overset{p \to 0}{\sim} p$$

Now demand a faster soft limit.

$$A(p, \cdots) \overset{p \to 0}{\sim} p^2 \quad \longrightarrow \quad \begin{aligned} c_1 &= 1/2 \\ c_2 &= \lambda/8 \\ c_3 &= \lambda^2/16 \\ c_4 &= 5\lambda^3/128 \\ &\vdots \end{aligned}$$

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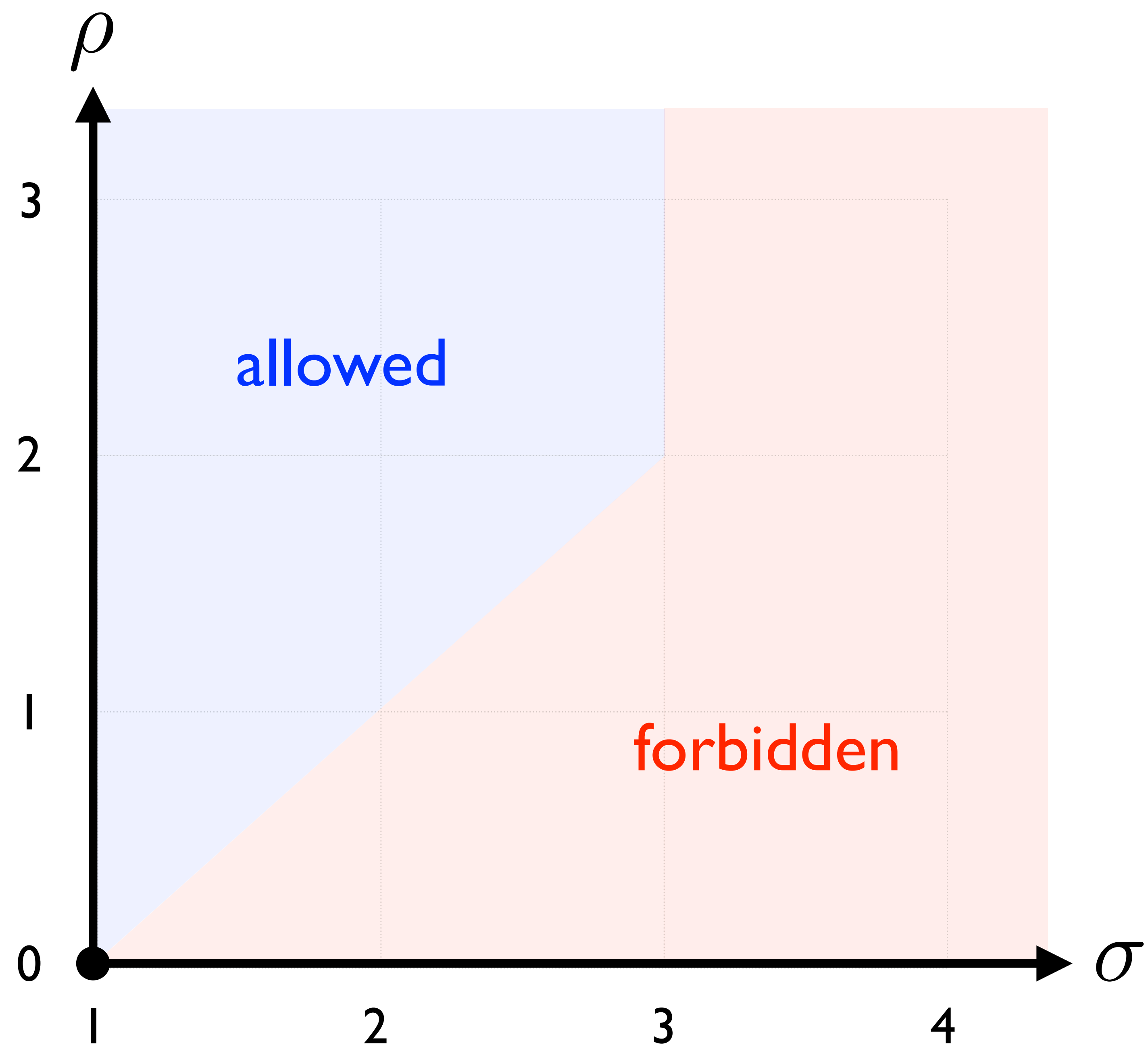
This sums to Dirac-Born-Infeld theory (DBI).

$$\mathcal{L} = -\frac{1}{\lambda}\sqrt{1 - \lambda(\partial\phi)^2} + \text{const}$$

There is an organizing structure of EFTs.



Their amplitudes obey recursion relations.

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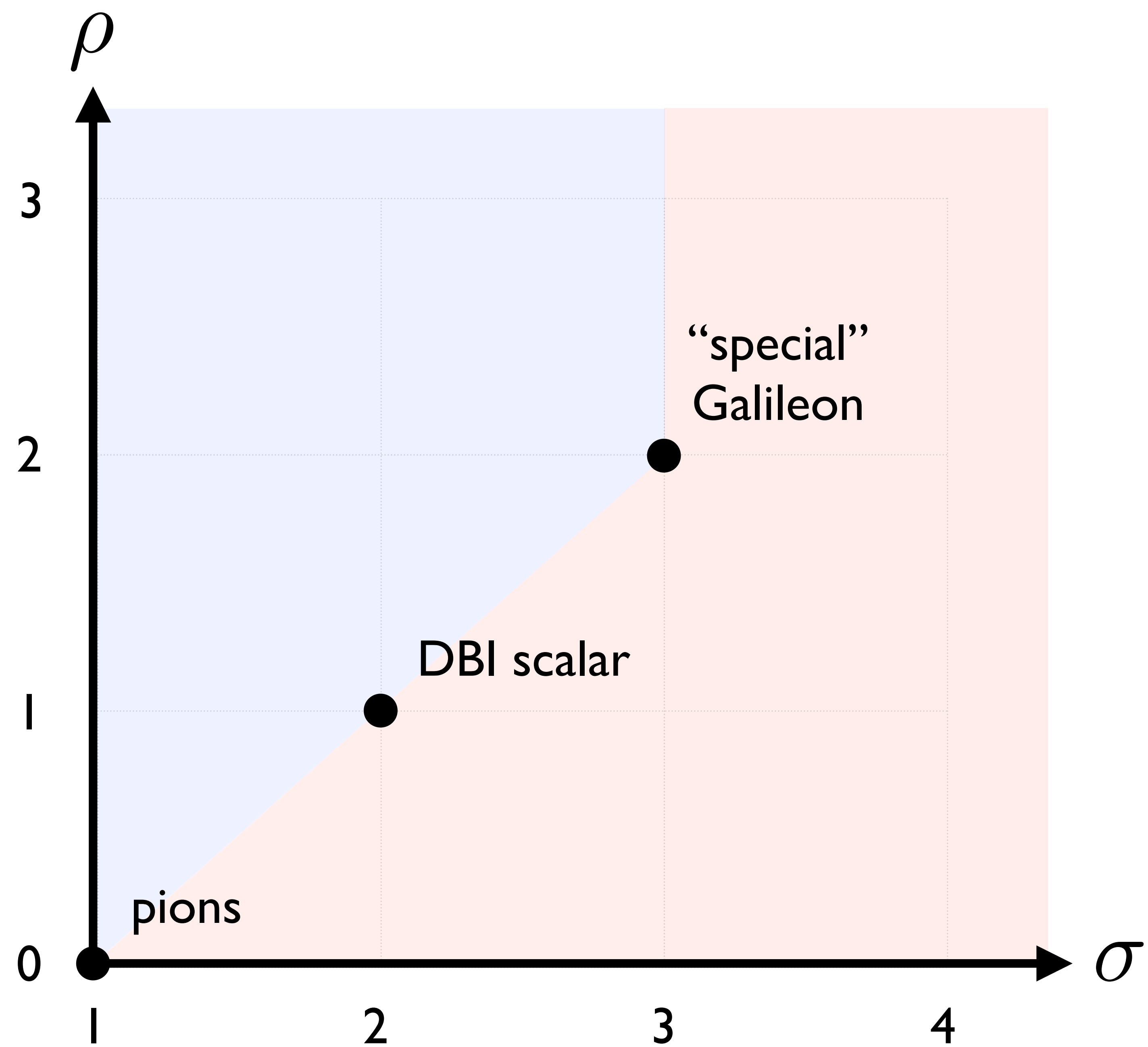

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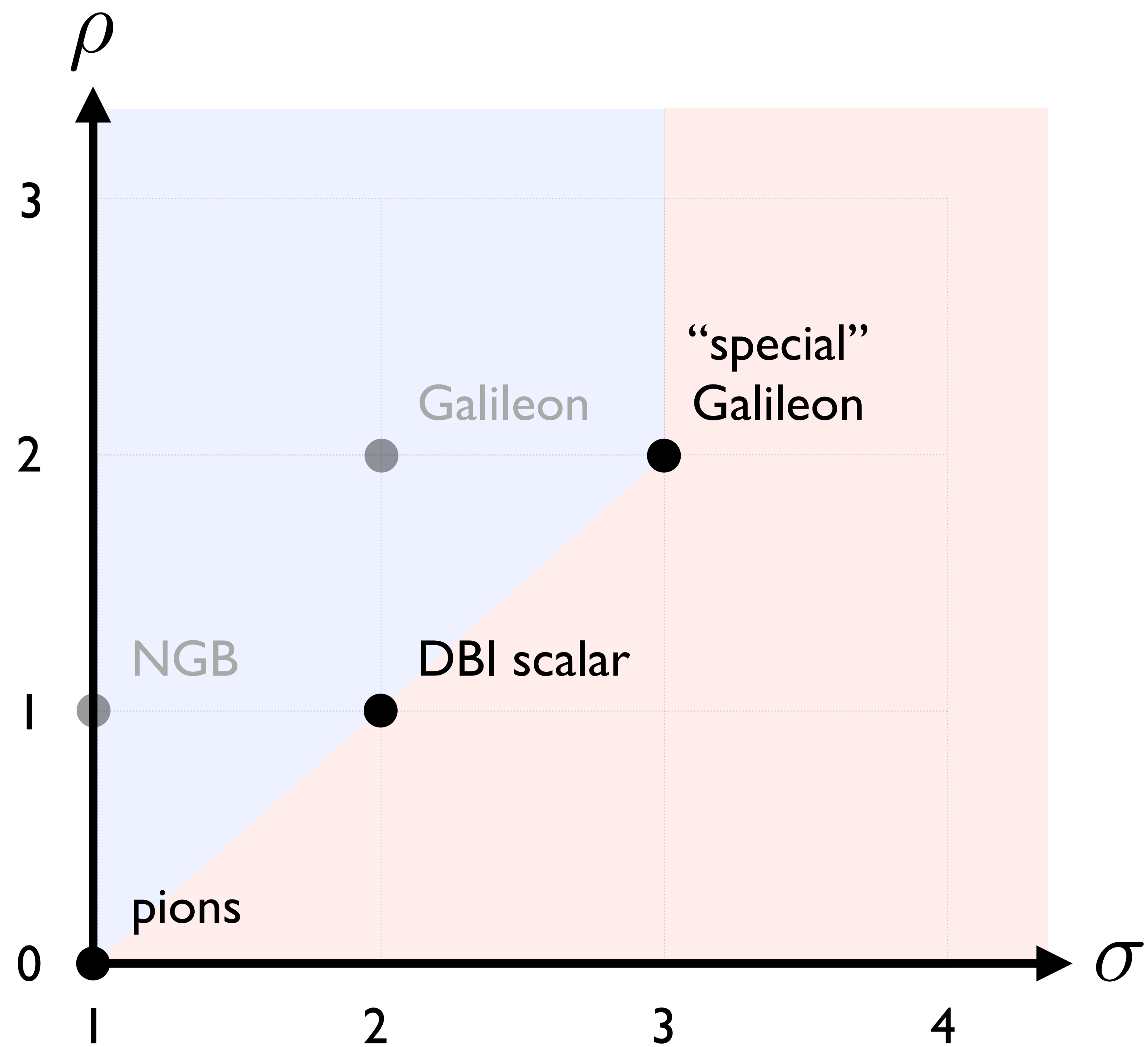

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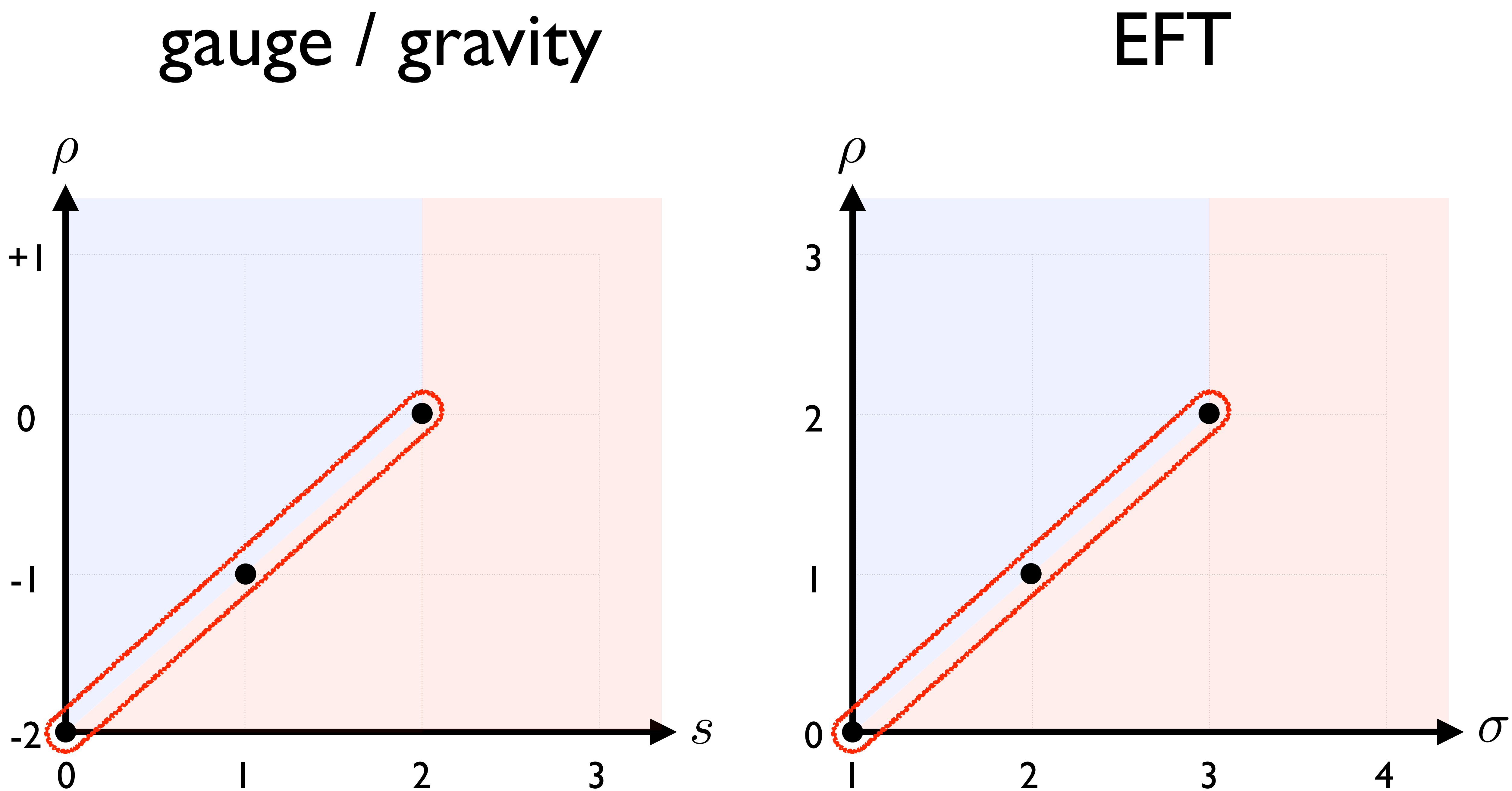
- Yang-Mills theory and gravity
- QCD, SM, SUSY theories
- all 4D renormalizable QFTs
- EFTs like NLSM, DBI, Galileon

CC, Kampf, Novotny, Shen, Trnka (1509.03309)



"Edge" theories are maximally constrained and moreover they are secretly connected!

unification of QFTs

Textbook unification runs from UV to IR.

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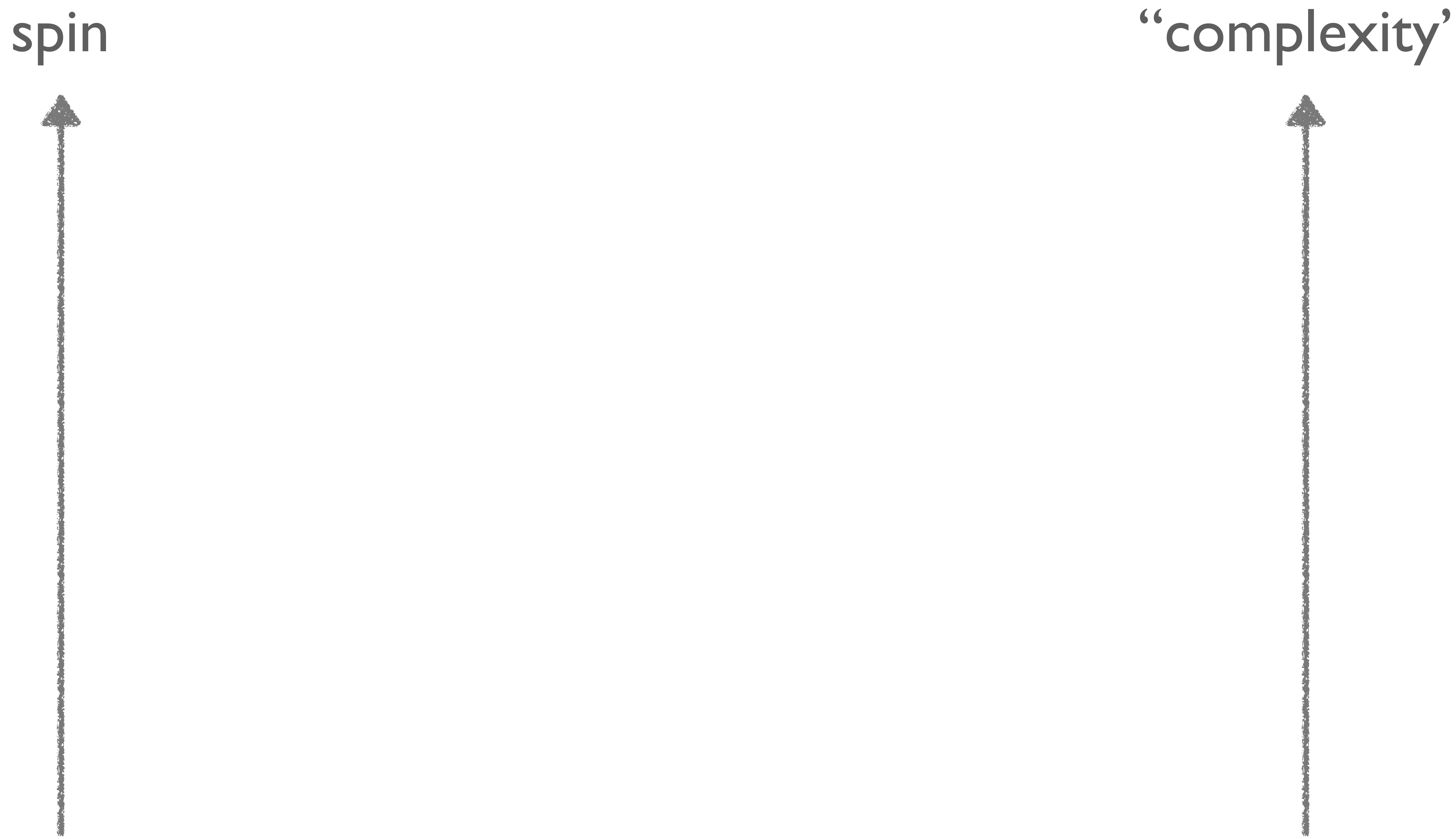
energy

symmetry

$$
\begin{tikzcd}
 & SO(10) \arrow[dl] \arrow[dr] & \\
SU(5) \arrow[dr] & & SU(4) \otimes SU(2)^2 \arrow[dl] \\
 & SU(3) \otimes SU(2) \otimes U(1) &
\end{tikzcd}
$$

Scattering amplitudes reveal a hidden unity!



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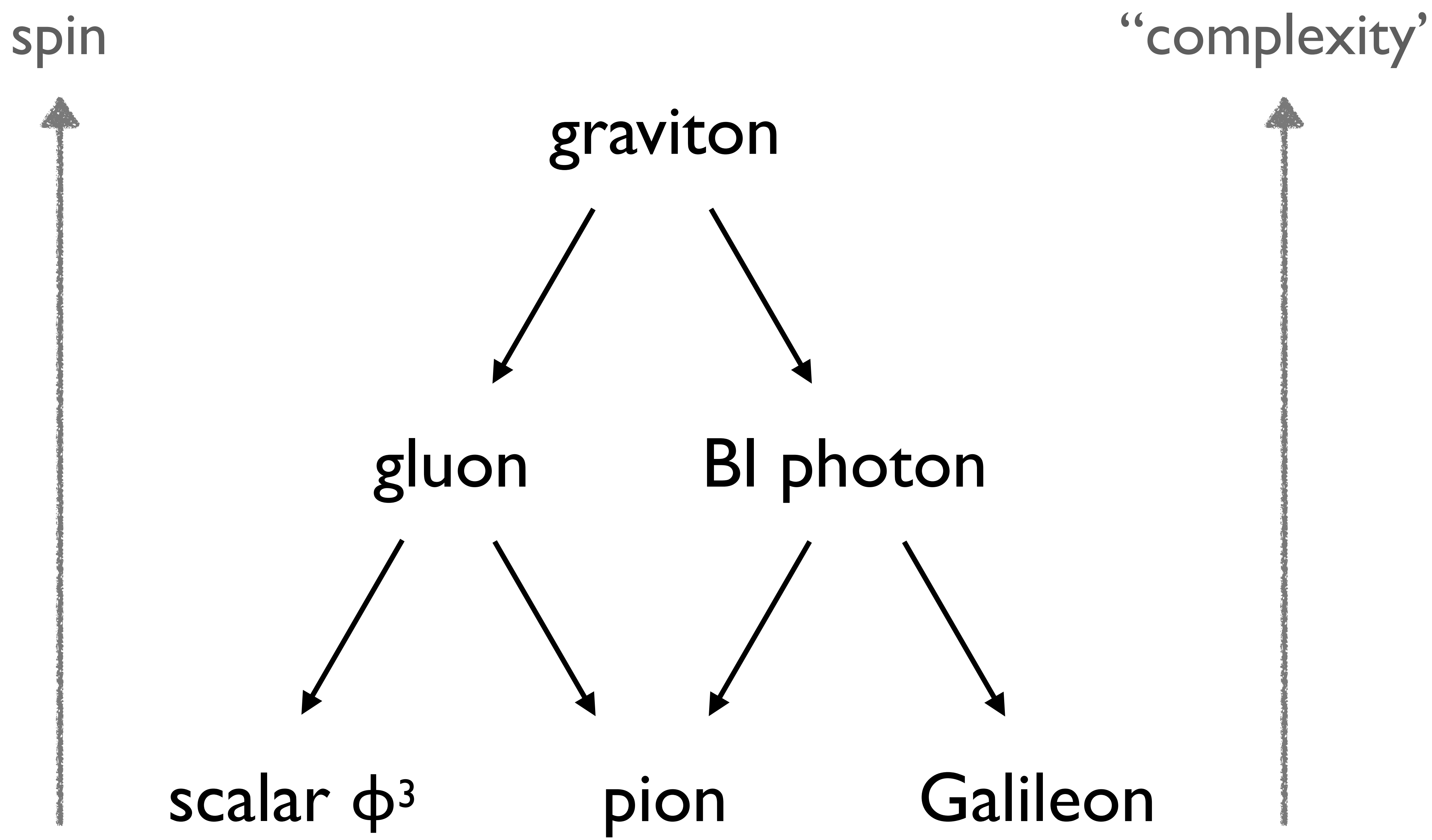

CC, Shen, Wen (1705.03025)

Recast an amplitude as an abstract function.

$$A = e_1^{\mu_1} e_2^{\mu_2} \cdots e_n^{\mu_n} A_{\mu_1 \mu_2 \cdots \mu_n}$$

$$= \text{scalar function of } p_i p_j, p_i e_j, e_i e_j$$

Crucially, we maintain on-shell conditions.

massless helicity basis

$$p_i p_i = p_i e_i = e_i e_i = 0$$

transverse

Physical amplitudes satisfy the Ward identity.

$$A\Big|_{e_i=p_i} = \mathcal{W}_i \cdot A = 0$$

We define an operator that implements the Ward identity and annihilates the amplitude.

$$\mathcal{W}_i = \sum_v p_i v \, \partial_{v e_i}$$

runs over all p_i and e_i

Physical amplitudes conserve momentum.

$$\mathcal{P}_v \cdot A = 0$$

We define an operator that implements momentum conservation.

any p_i and e_i

$$\mathcal{P}_v = \sum_i p_i v$$

Define an operator $\mathcal{T}$ which acts on an amplitude A and builds a new one $\mathcal{T} \cdot A$.

If the operator satisfies the conditions,

$$[\,\mathcal{W}_i\,,\,\mathcal{T}\,] \sim 0 \qquad\qquad [\,\mathcal{P}_v\,,\,\mathcal{T}\,] \sim 0$$

then the resulting object can be physical!

$$\mathcal{W}_i \cdot (\mathcal{T} \cdot A) = 0 \qquad\qquad \mathcal{P}_v \cdot (\mathcal{T} \cdot A) = 0$$

We thus obtain "transmutation" operators.

$$\mathcal{T}_{ij} = \partial_{e_i e_j}$$

$$\mathcal{T}_{ijk} = \partial_{p_i e_j} - \partial_{p_k e_j}$$

$$\mathcal{L}_i = \sum_j p_i p_j \partial_{p_j e_i}$$

CC, Shen, Wen (1705.03025)

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2 gluon → 2 scalar

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We have proven this to be true with on-shell recursion, with explicit checks up to 8pt.

CC, Shen, Wen (1705.03025)

example 1) gluon to charged scalar

$$\mathcal{T}_{12} \cdot \mathcal{T}_{34} \cdot A(g_1 g_2 g_3 g_4) = \frac{p_1 p_3}{p_1 p_2}$$

color-ordered
gluon amplitude

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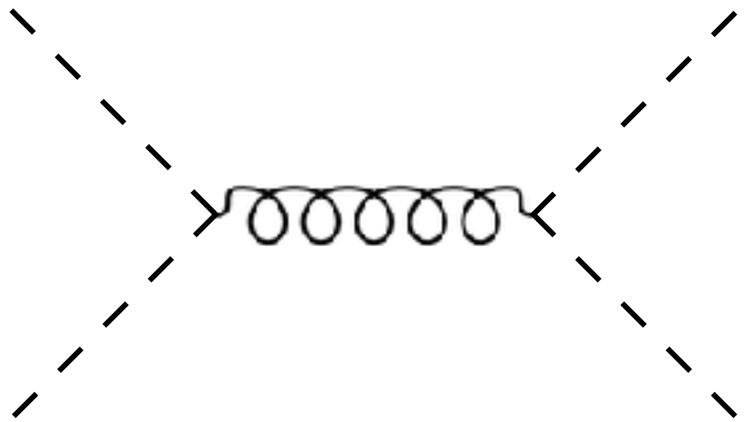
$$= A(\phi_1 \phi_2, \phi_3 \phi_4) =$$ 

scalar matter
scattering via gluons

example 2) gluon to scalar ϕ³

$$\mathcal{T}_{14} \cdot \mathcal{T}_{124} \cdot \mathcal{T}_{234} \cdot A(g_1 g_2 g_3 g_4) = \frac{1}{p_1 p_2} + \frac{1}{p_2 p_3}$$

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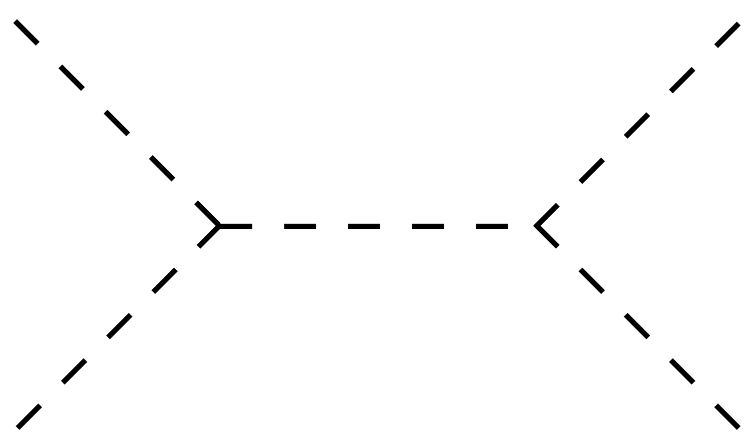
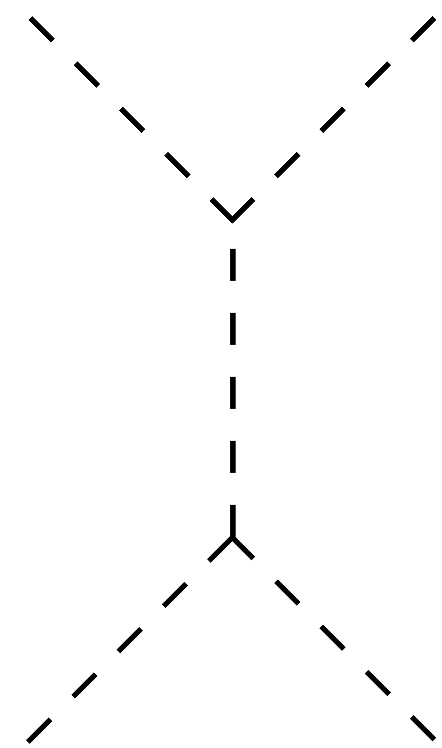
$=$  $+$ 

$$= A(\phi_1 \phi_2 \phi_3 \phi_4)$$

example 3) gluon to pion

$$\mathcal{T}_{14} \cdot \mathcal{L}_2 \cdot \mathcal{L}_3 \cdot A(g_1 g_2 g_3 g_4) = p_1 p_3$$

cyclic invariance
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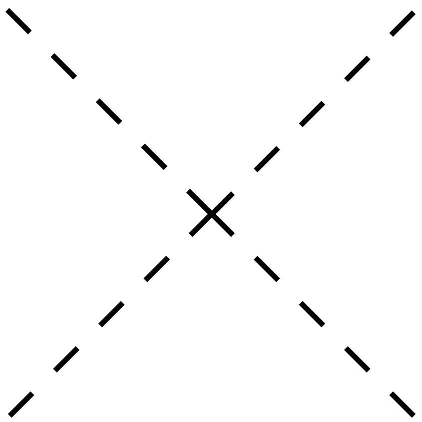
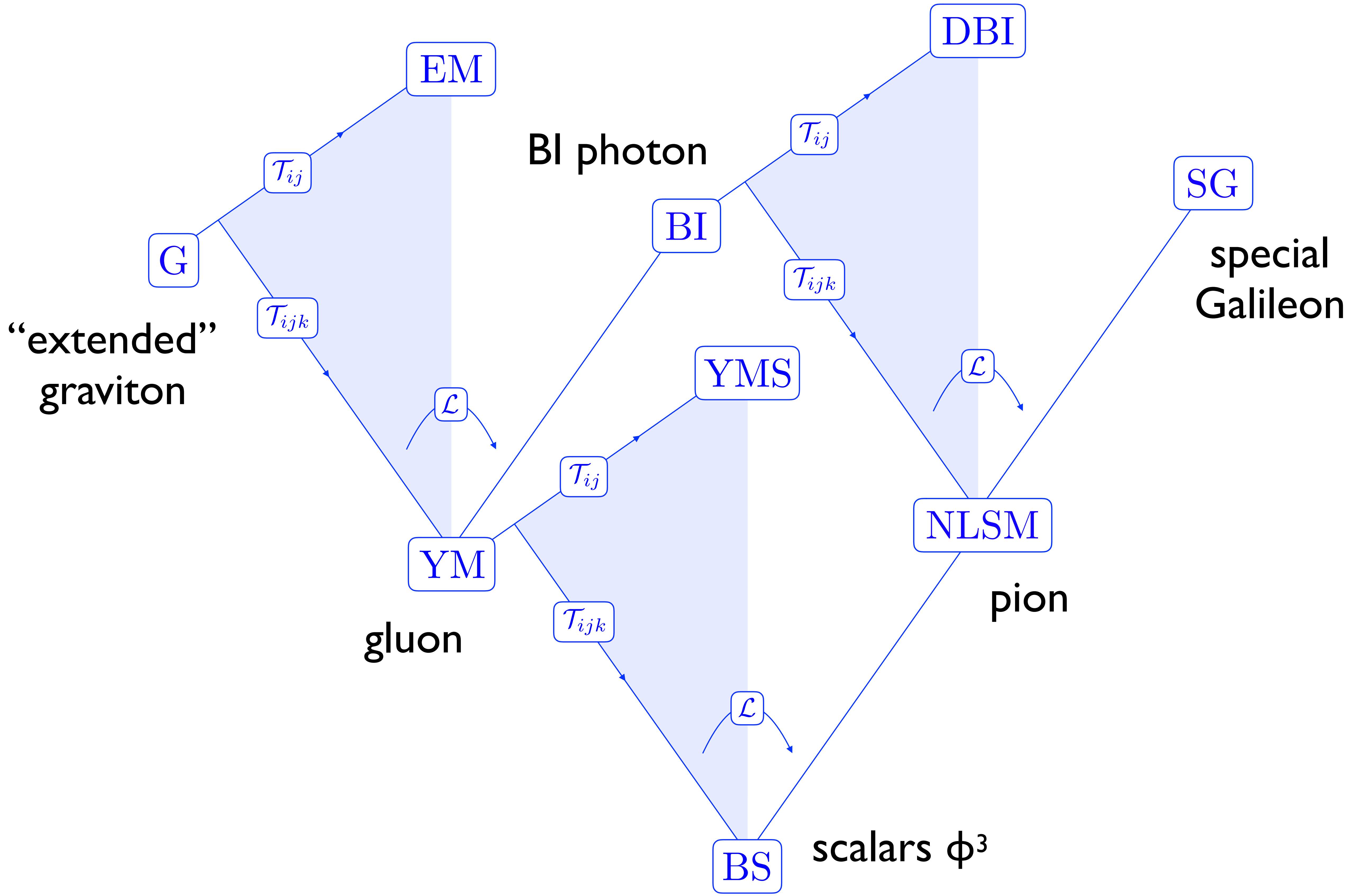


flavor-ordered
pion amplitude



CC, Shen, Wen (1705.03025)

Now translate this procedure to the action.

$$\mathcal{T}_{12} \cdot \mathcal{T}_{34} \cdot A(g_1 g_2 g_3 g_4) = A(\phi_1 \phi_2, \phi_3 \phi_4)$$

This extracts the $(e_1 e_2)(e_3 e_4)$ component and is equivalent to dimensional reduction.

$$e_1^\mu = e_2^\mu = (0, 1, 0)$$

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momenta have support on
d-dimensional subspace

For pions, the reduction is peculiar.

$$\mathcal{T}_{14} \cdot \mathcal{L}_2 \cdot \mathcal{L}_3 \cdot A(g_1 g_2 g_3 g_4) = A(\pi_1 \pi_2 \pi_3 \pi_4)$$

Some states are longitudinally polarized.

$$e_2^\mu = (p_2^\alpha, 0, ip_2^\beta) \qquad\qquad e_3^\mu = (p_3^\alpha, 0, ip_3^\beta)$$

$$\longrightarrow \quad A_\mu = \left(\frac{X_\alpha + Z_\alpha}{\sqrt{2}}, Y, \frac{X_\beta - Z_\beta}{i\sqrt{2}} \right)$$

CC, Remmen, Shen, Wen (1709.04932)

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CC, Remmen, Shen, Wen (1709.04932)

Now apply this reduction to the YM action.

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$$\mathcal{L}_{\mathrm{NLSM}} = \mathrm{Tr}\Bigg(X_\alpha \Box Z^\alpha + \frac{1}{2} Y \Box Y + \\ + i X_{\alpha\beta}[Z^\alpha, Z^\beta] + i Z^\alpha [Y, \partial_\alpha Y]\Bigg)$$

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Recast pions as higher-dimensional gluons!

$$A(\pi_1 \cdots \pi_n) = A(Z_1 \cdots Y_i \cdots Y_j \cdots Z_n)$$

cyclic invariance obscured

Now replace flavor with kinematics.

$$\mathcal{L}_{\mathrm{NLSM}} = \mathrm{Tr}\left(X_\alpha \Box Z^\alpha + \frac{1}{2} Y \Box Y + \right.$$
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$$\downarrow \quad \downarrow \quad \downarrow$$

$$\mathcal{L}_{\text{SG}} = X_{\alpha\bar{\alpha}} \Box Z^{\alpha\bar{\alpha}} + \frac{1}{2} Y \Box Y$$
$$+ X_{\alpha\beta\bar{\alpha}\bar{\beta}} Z^{\alpha\bar{\alpha}} Z^{\beta\bar{\beta}} + Z^{\alpha\bar{\alpha}} Y \partial_\alpha \partial_{\bar{\alpha}} Y$$

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So we arrive at "pion^2 = Galileon".

CC, Shen (1612.00868)

This pion action has several notable features.

- Interactions are purely cubic, in sharp contrast with the typical formulation.
- Permutation invariance and pion parity are obscured.
- Kinematic algebra of pions comes from the higher-dimensional Poincare algebra.
- Weinberg gluon soft theorem maps to the Adler zero condition.

conclusions

- Gauge theory, gravity, and effective field theories can be derived from S-matrix properties rather than vice versa.

- The very simplest theories are secretly unified and can be derived directly from the extended graviton S-matrix.

- This construction translates to a version of dimensional reduction that recasts pions as higher-dimensional gluons.

thank you!

basics of on-shell recursion

Shift momenta with complex parameter z:

$$p_i \to p_i + z q_i \qquad\qquad A \to A(z)$$

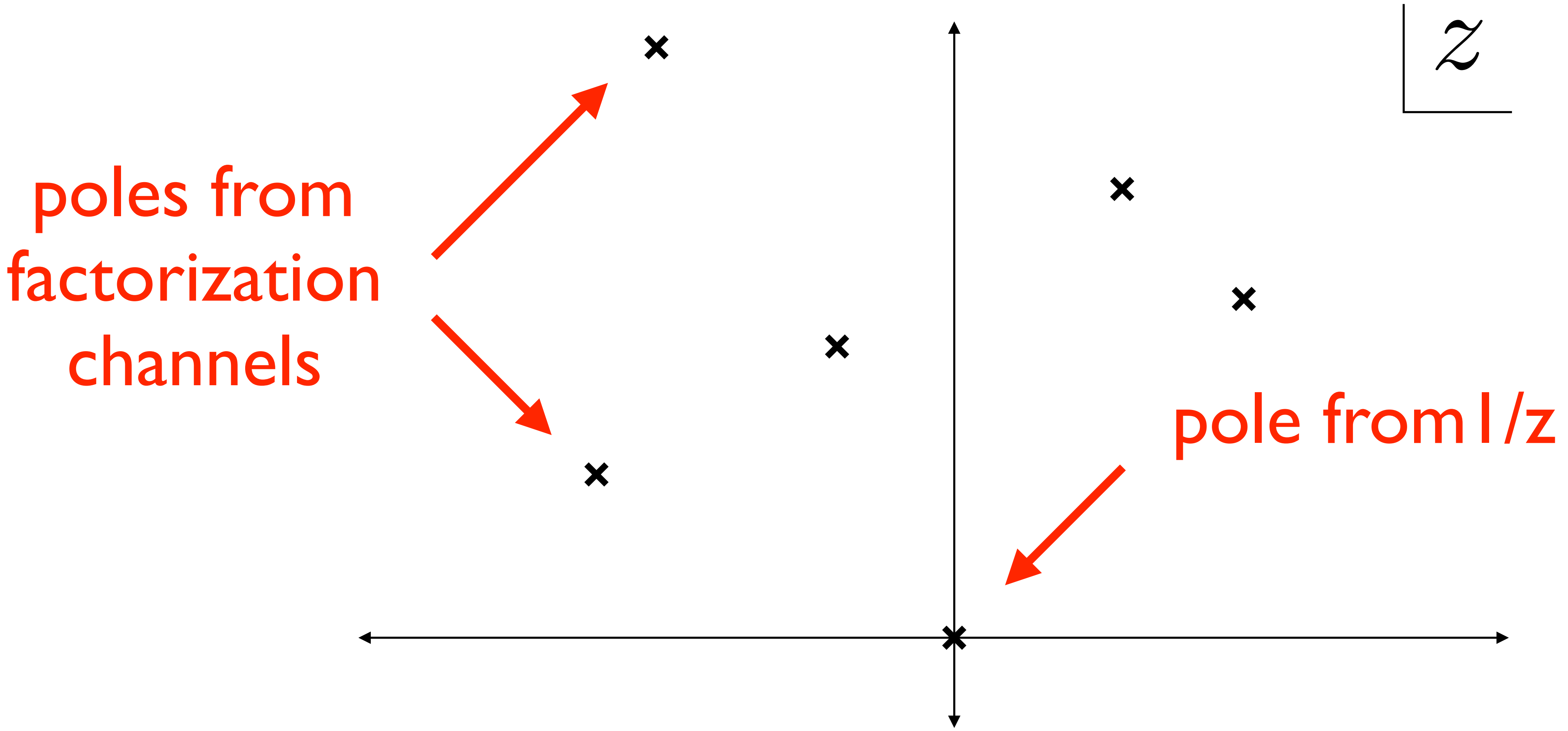
To maintain physical momenta, we impose

momentum conservation: $$\sum_i q_i = 0$$

on-shell: $$q_i \cdot q_i = q_i \cdot p_i = 0$$

Apply Cauchy's them to shifted amplitude.

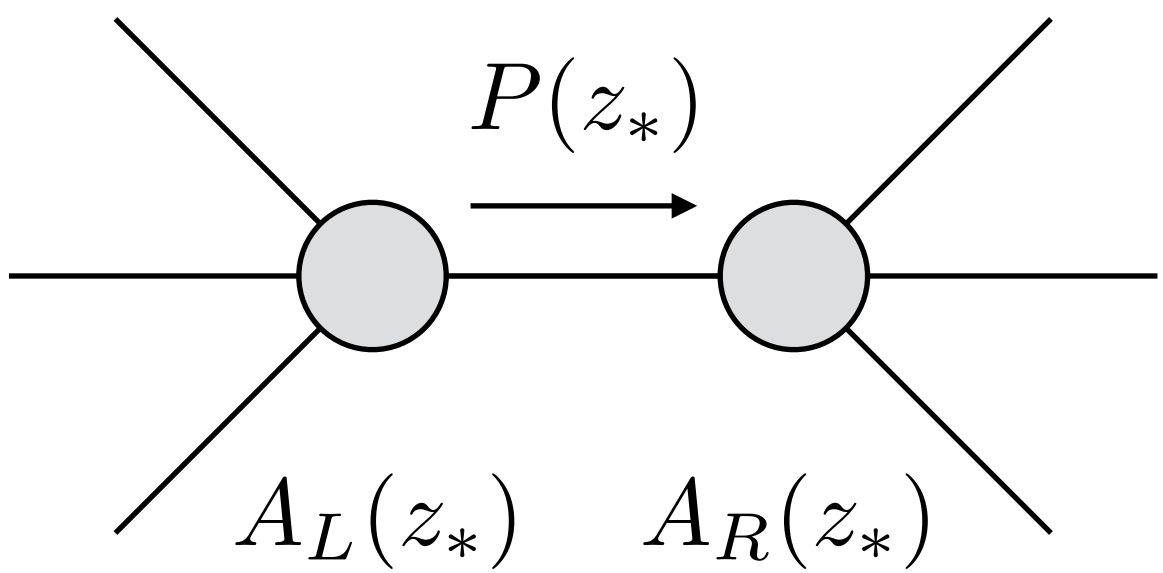
$$A(0) = \frac{1}{2\pi i} \oint_{z=0} \frac{A(z)}{z} = \sum_{z_*} \operatorname*{Res}_{z=z_*} \left(\frac{A(z)}{z} \right)$$



Here z_* is fixed by factorization, $P(z_*)^2 = 0$

$$A(0) = \sum_{z_*} A_L(z_*) \frac{1}{P^2} A_R(z_*) + \operatorname*{Res}_{z=\infty} \left(\frac{A(z)}{z} \right)$$

$$= \sum_{z_*}$$



$+$ ~~boundary term~~

$$A(z) \overset{z \to \infty}{=} 0$$

(required for recursion)