Current Status and Future Prospects for $B \rightarrow X_s l^+ l^-$ at Belle II

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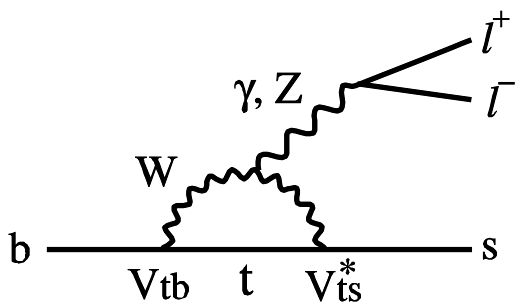
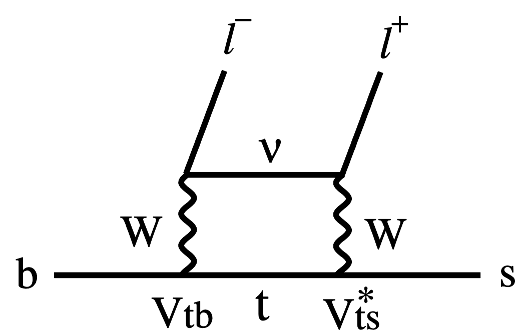
(KEK)

b→sl+l- decays

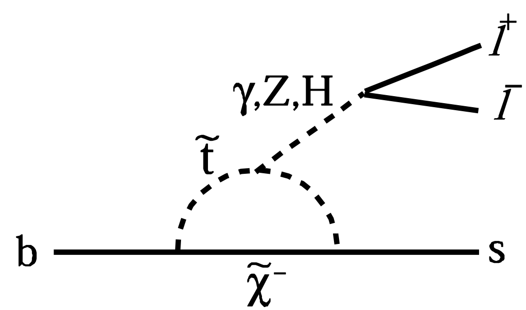
- b→sl+l- decays are FCNC transitions which are sensitive probe for new physics since the amplitudes are loop suppressed.

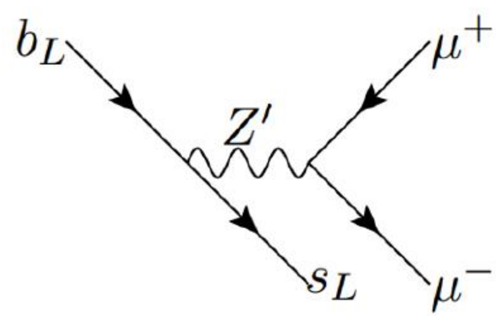
SM

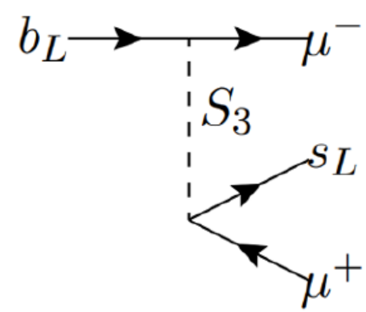




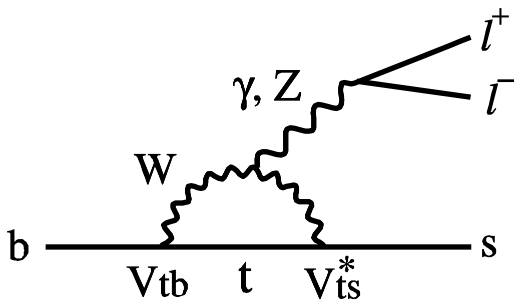
NP

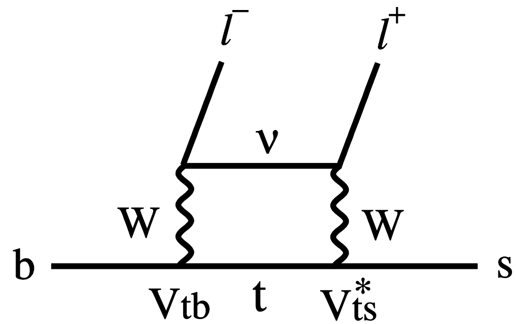


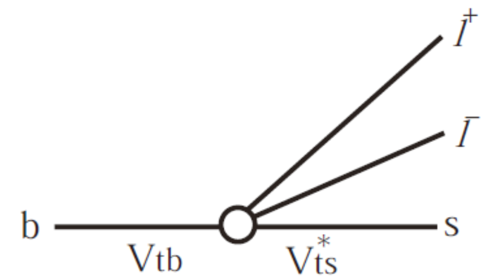




Wilson Coefficients in b→sl+l-







- In the SM, b→s ll can be expressed with real Wilson coefficients in Effective Hamiltoniam Approach
  - b→sll : C_7, C_9 and C_{10}
  - C_7 ~ -0.3, C_9 ~ 4, C_{10} ~ -4
- If NP contributes,
  - Deviation from the SM values
  - Lepton flavor dependent $C_{9e} \neq C_{9\mu}$
  - Scalar and Tensor coefficients, C_S , C_P and C_T
  - Right handed counter coefficients, C_i'
    - $P_{L(R)} \rightarrow P_{R(L)}$
  - Imaginary parts $\mathrm{Im}(C_i)$ and $\mathrm{Im}(C_i')$

$$\mathcal{H}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_{i=1}^{10} C_i(\mu) O_i(\mu)$$

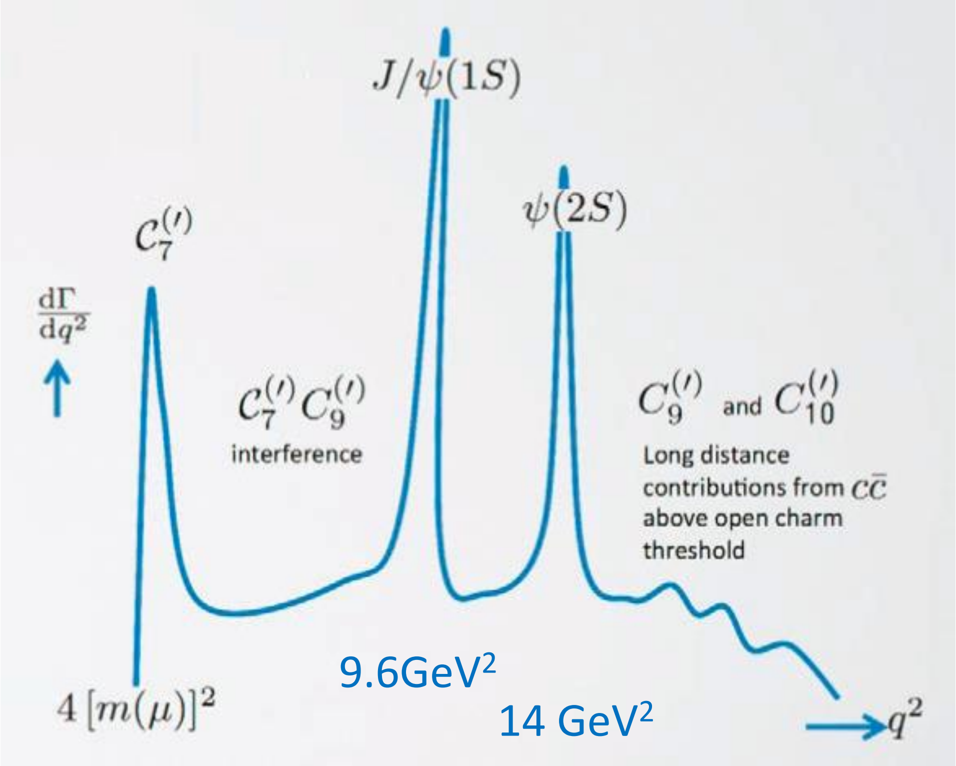
$$\mathcal{O}_7 = \frac{e}{16\pi^2} m_b (\bar{s}\sigma^{\mu\nu} P_R b) F_{\mu\nu},$$

$$\mathcal{O}_9 = \frac{e^2}{16\pi^2} (\bar{s}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \ell),$$

$$\mathcal{O}_{10} = \frac{e^2}{16\pi^2} (\bar{s}\gamma^\mu P_L b)(\bar{\ell}\gamma_\mu \gamma_5 \ell)$$

q^2 distribution in b→sl+l-

- Contribution from C_7 (dipole) is only in low q^2
  - photon pole $1/q^2$
- B→J/psi(l+l-) K* (tree level process) interferes with b→sl+l-
  - Remove this q^2 region
  - For charmonium above psi(2S), leptonic width is small so large interference happens
- Contributions from C_9 (V) and C_{10} (A) are basically for whole q^2
- Interference btw Vector (C_7, C_9) and Axial vector (C_{10}) currents generates A_{FB}.



Current Status of exclusive b→sl+l- decays

Mostly at LHCb

- $R_{K(*)}$
- BF/q^2 distribution
- Angular Analysis

$R_{K(*)}$ gone

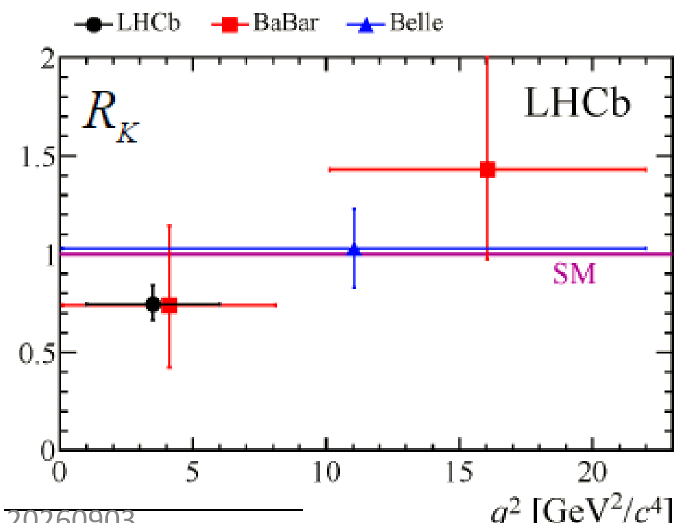
- There was an anomaly with ~3σ significance but gone in 2023.

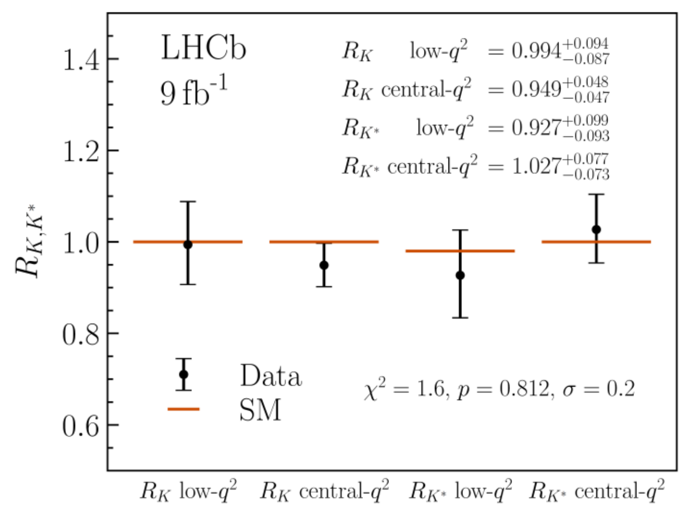
$$R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)}$$

- But observed deficit of the both BF (shown next page)

$\downarrow$ $R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)}{\mathcal{B}(B \to Ke^+e^-)}$ $\downarrow$ SM

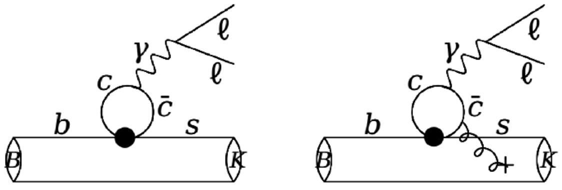
1.0 $R_K = \frac{\mathcal{B}(B \to K\mu^+\mu^-)\downarrow}{\mathcal{B}(B \to Ke^+e^-)\downarrow}$



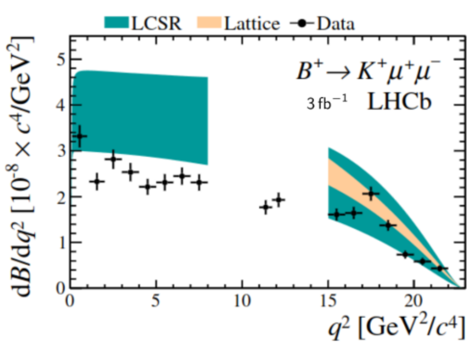


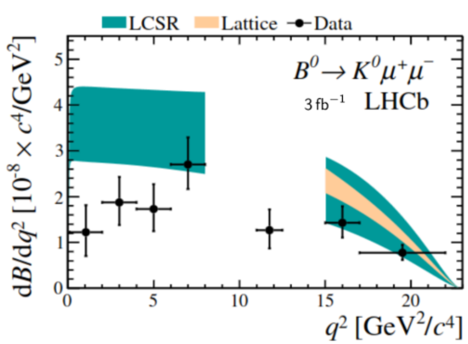
Exclusive BF/q^2 distribution

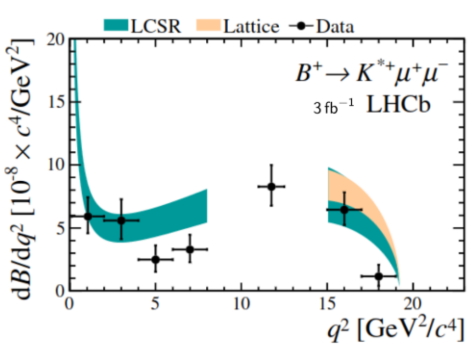
- Deficit observed
  - If simply combined, more than 5σ
- New physics? Form factor (FF) normalization? Charm loop?
  - According to Lattice QCD theorists, FF is solid
  - Charm loop is very hard to calculate in exclusive decays



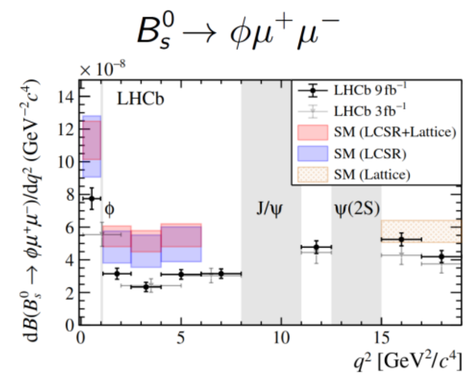
(a) (b)



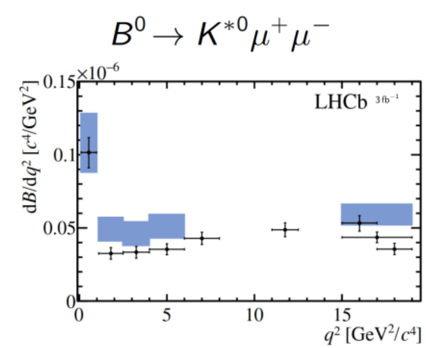




[JHEP 06 (2014) 133]



[PRL 127, 151801 (2021)]

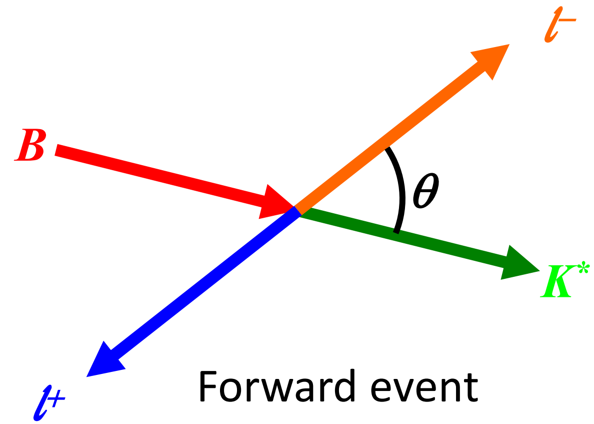


[JHEP 11 (2016) 047][JHEP 04 (2017) 142]

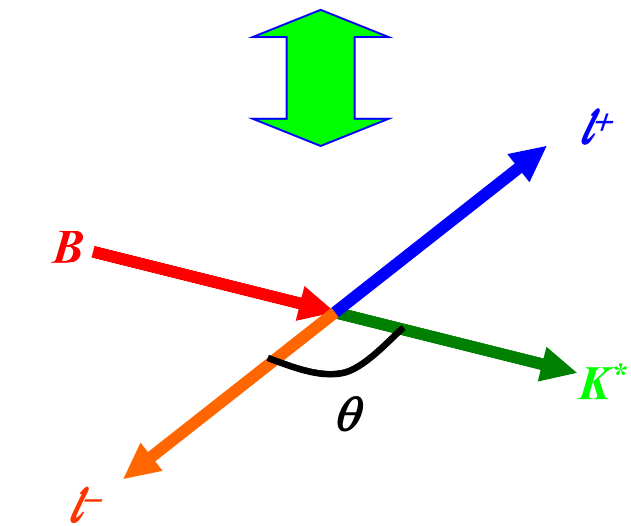
Angular Analysis

- Originally, A_{FB} was considered
  - Interference of vector and axial vector

$$A_{\mathrm{FB}}(q^2) = \frac{\Gamma(q^2, \cos\theta_{B\ell^-} > 0) - \Gamma(q^2, \cos\theta_{B\ell^-} < 0)}{\Gamma(q^2, \cos\theta_{B\ell^-} > 0) + \Gamma(q^2, \cos\theta_{B\ell^-} < 0)}$$



Forward event

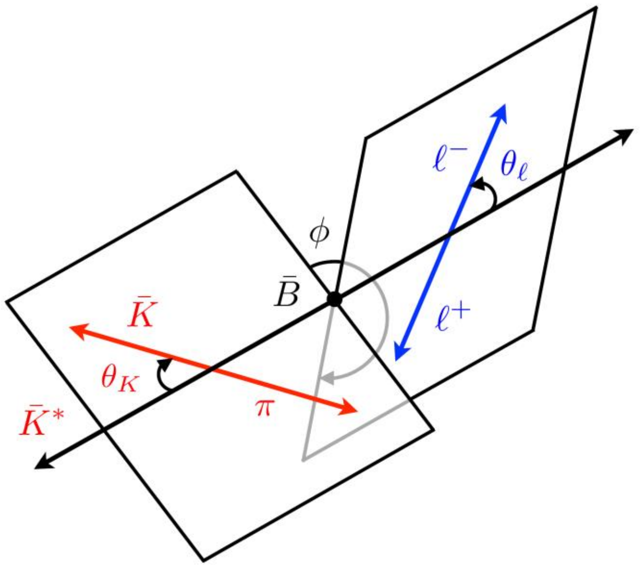


Backward event

- While theorists invented full angular analysis in exclusive B→K*l^+l^- which should be less FF dependent
  - use of interference of three polarization states
  - Next page

Full Angular Analysis

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} I(q^2, \theta_\ell, \theta_K, \phi)$$

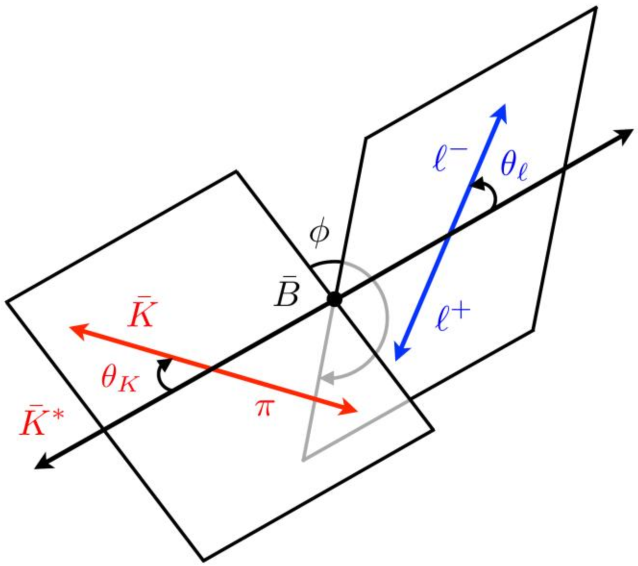


At low q^2

Full Angular Analysis

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} I(q^2, \theta_\ell, \theta_K, \phi)$$

$$\begin{aligned} I(q^2,\theta_\ell,\theta_K,\phi) = {} & I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_\ell \\ & + I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi \\ & + I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi + (I_6^s \sin^2\theta_K + I_6^c \cos^2\theta_K)\cos\theta_\ell \\ & + I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi \\ & + I_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi\,. \end{aligned}$$



$$S_j = (I_j + \bar{I}_j) \Big/ \frac{d\Gamma}{dq^2}$$

$$P_5' = \frac{S_5}{2\sqrt{-S_{2s}S_{2c}}}$$

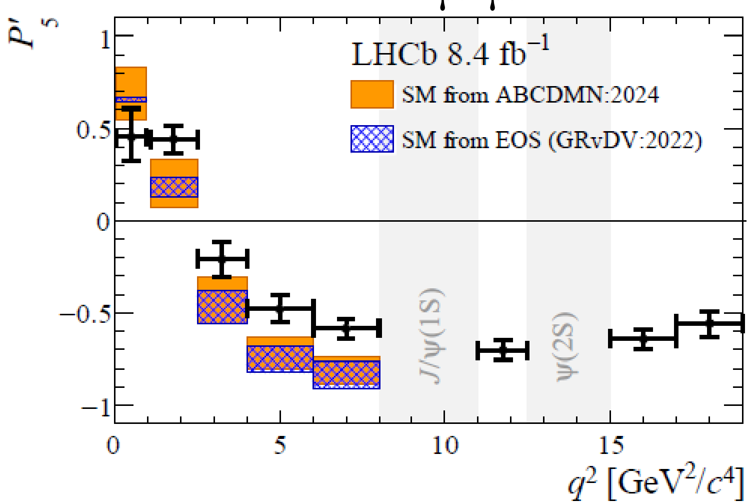
$$P_5' \simeq \frac{\mathrm{Re}\left(C_{10}^* C_{9,\perp} + C_{9,||}^* C_{10}\right)}{\sqrt{\left(|C_{9,\perp}|^2 + |C_{10}|^2\right)\left(|C_{9,||}|^2 + |C_{10}|^2\right)}}$$

At low q^2

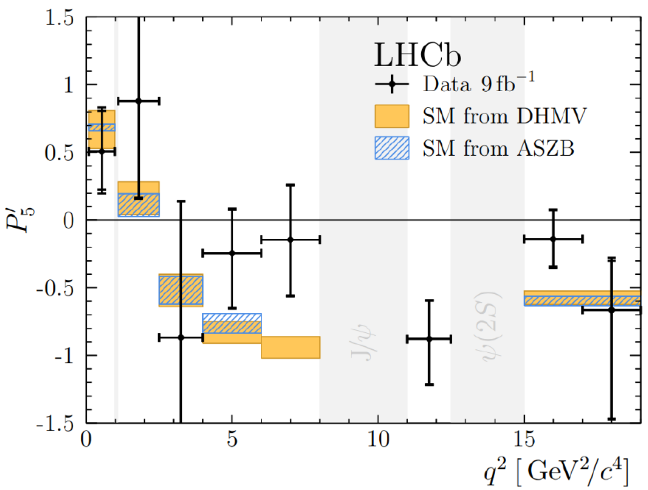
P$_5$'

- There are deviation around q^2=[4,8]GeV^2
  - New physics?
  - Charm loop?

B→K$^{*0}\mu^+\mu^-$



$B^+ \to K^{*+}\mu^+\mu^-$



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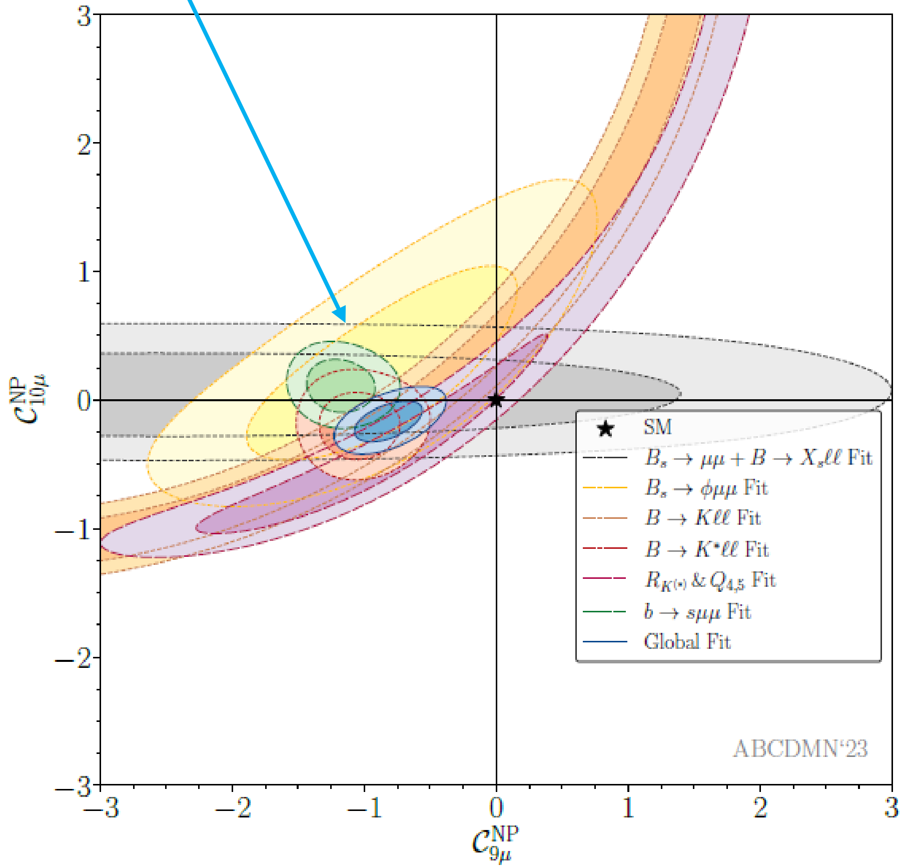
[PRL 126, 161802 (2021)]

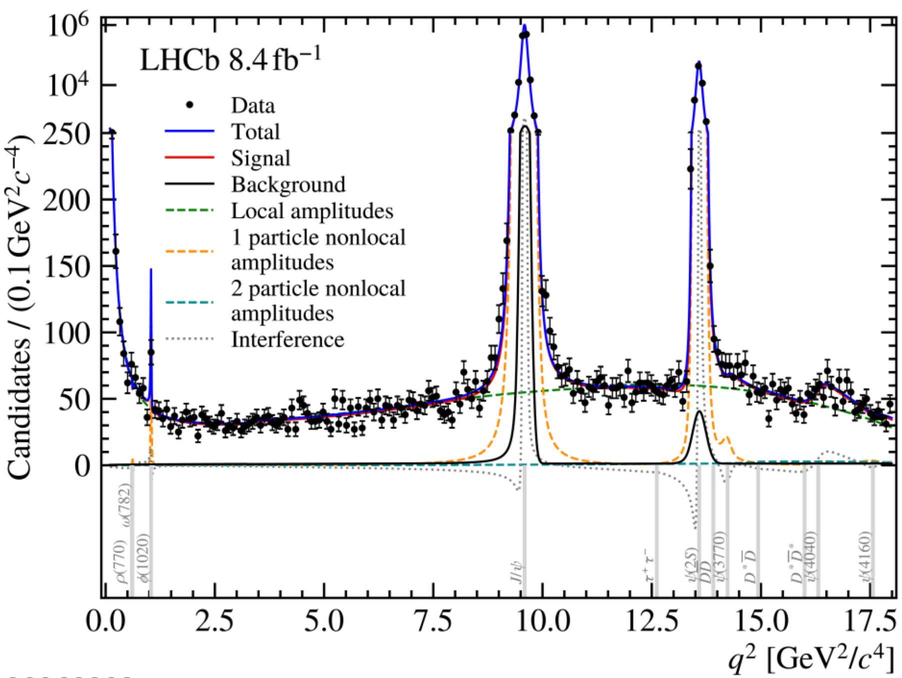
Global Fit to b→s

- Suggest C_9^{NP} = -1 ?
  - C_9^{NP} =0 in SM



Fit with QCD model

- Fit with local QCD contributions
  - Larger uncertainties than previous analysis while consistent with both previous analysis and SM predictions
  - While a lattice QCD theorist said, this kind of QCD effect cannot be formulated



Wilson Coefficient results	
$\mathcal{C}_9$	$3.56 \pm 0.28 \pm 0.18$
$\mathcal{C}_{10}$	$-4.02 \pm 0.18 \pm 0.16$
$\mathcal{C}_9'$	$0.28 \pm 0.41 \pm 0.12$
$\mathcal{C}_{10}'$	$-0.09 \pm 0.21 \pm 0.06$
$\mathcal{C}_{9\tau}$	$(-1.0 \pm 2.6 \pm 1.0) \times 10^2$

Problem

- Do we really understand the charm loop effect in exclusive decays which should modify C_9?
- All exclusive measurements suffer from the charm loop which is hard to calculate (or impossible to calculate according to some theorists).

Problem and Solution

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- All exclusive measurements suffer from the charm loop which is hard to calculate (or impossible to calculate according to some theorists).

- In inclusive decays, theorists can calculate charm loop effect.
- Experimentally, inclusive decays are harder but why not?

Inclusive b→sl⁺l⁻ decays

- Precise theoretical predictions on branching fractions and forward-backward asymmetry in SM are available.
  - Especially for q^2=[1.0,6.0]GeV^2 (and [14.4,max]GeV^2)

q^2 range [GeV^2]	$[1, 6]$
$\mathcal{B}$ $[10^{-7}]$	17.41 ± 1.31

2404.03517

Inclusive $b \to sl^+l^-$ decays

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- While experimentally, it is hard to reconstruct.
  - Fully inclusive method (just two leptons) are currently impossible.
  - Sum-of-exclusive methods are possible while covers limited phase space so extrapolation required.
    - Reconstruction of neutrals (p0 and Ks) are required, it is very hard to

Inclusive $b \to s l^+ l^-$ decays

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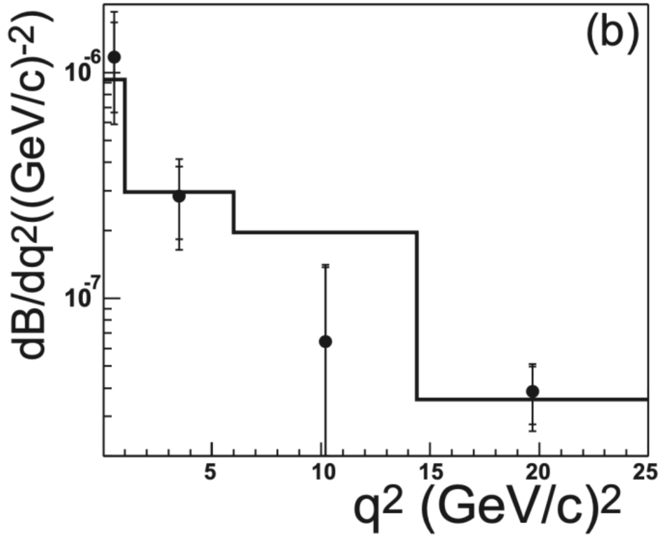
- Firstly, we would measure the inclusive branching fraction with sum-of-exclusive method. And then angular analysis such as forward-backward asymmetry.

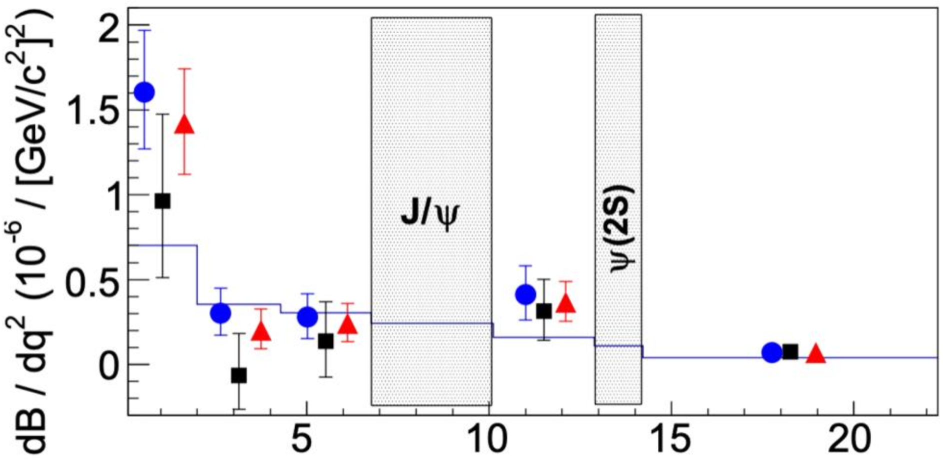
Current Status of the BF

- Belle and BaBar measured it

Belle 140/fb



BaBar 424/fb



$\mathcal{B}(1<q^2<6)\times10^6$	Belle[PRD 72, 092005 (2005)] $N_{B\bar{B}}=152\times10^6$	Babar[PRL 112, 211802 (2014)] $N_{B\bar{B}}=471\times10^6$
Measurements	$1.493\pm0.504^{+0.411}_{-0.321}$	$1.60^{+0.41+0.17}_{-0.39-0.13}\pm0.18$

prediction 1.741±0.131

2404.03517

Measurement at Belle II

- Run1 data: 365/fb data samples
  - Measurement strategy:

$$\mathcal{B}(B \to X_s \ell^+ \ell^-) = \frac{N_{obs}}{\varepsilon \cdot f_{visi} \cdot 2N_{B\bar{B}}}$$

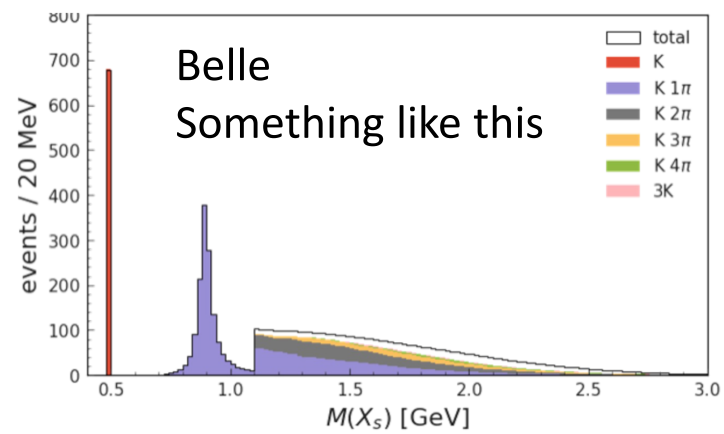
Efficiency: by Monte Carlo simulation

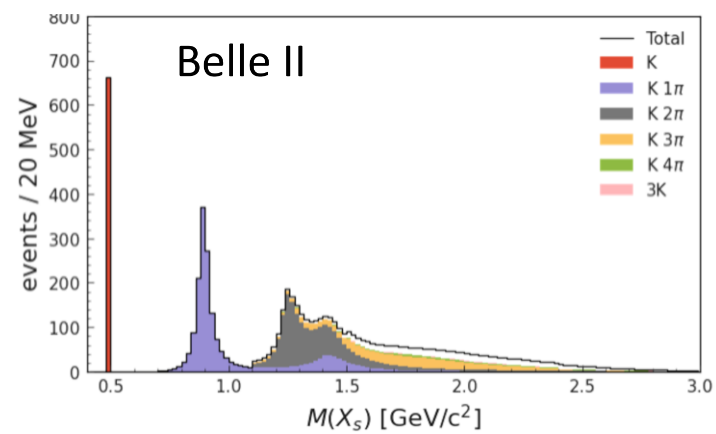
Visible fraction: by Pythia (Lund String model)

- Better modeling of signal reducing the systematic uncertainty
  - Large systematic uncertainty of 24% in Belle measurements

Signal Modelling

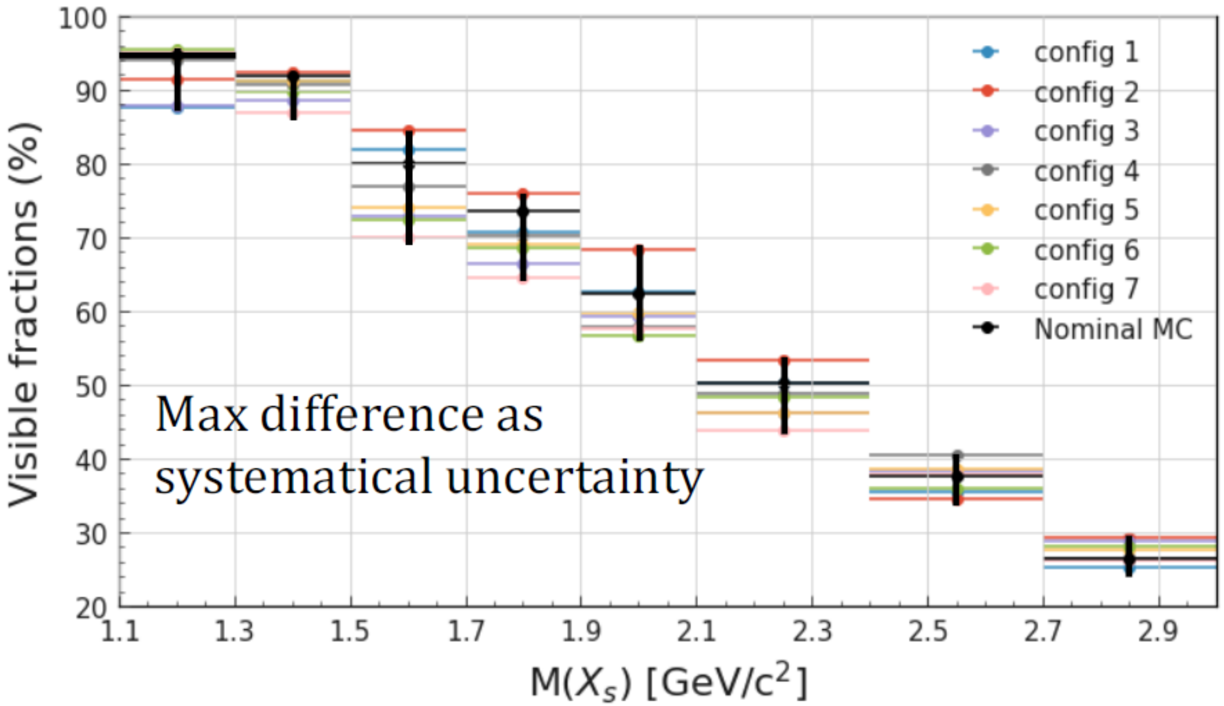
- Belle
  - Exclusive
    - B→Kl+l- and B→K*l+l- with branching fractions from theory.
  - Inclusive
    - B→Xsl+l- from 1.0GeV and Xs decayed by PYTHIA6.
- Belle II
  - Exclusive
    - B→Kl+l- and B→K*l+l- with W.A. branching fractions 17% → 3%
    - Other resonances are inlucded such as K1(1270), K1(1400), K*(1410), K_0*(1430) and K_2*(1430)
  - Inclusive
    - B→Xsl+l- from 1.0GeV and Xs decayed by PYTHIA8.
    - Fragmentation tuning is done with B→Xsγ and B→J/psi Xs 11% → 2%





Visible Fractions

- The PYTHIA parameter sets which reproduce the B→Xsγ fragmentation are estimated.
- Maximum deviations are taken as systematics 7%



Reconstruction

- 20 Xs final states are used (cf. Belle 18, BaBar 10)
- Selections
  - Both electrons and muons
    - FSR inclusion for electron
  - K^+, π^+
  - π^0 from two gamma
  - MVA based $Ks \rightarrow \pi^+\pi^-$ selector
  - B mesons are reconstructed by combining Xs and dilepton
  - M_{Xs} < 1.8GeV to reduce combinatorial backgrounds.

	$B^0, \bar{B}^0$		$\boldsymbol{B^\pm}$	
K	K_S		$K^\pm$	
$K\pi$	$K^\pm\pi^\mp$	$K_S\pi^0$	$K^\pm\pi^0$	$K_S\pi^\pm$
$K2\pi$	$K^\pm\pi^\mp\pi^0$	$K_S\pi^\mp\pi^\pm$	$K^\pm\pi^\mp\pi^\pm$	$K_S\pi^\pm\pi^0$
$K3\pi$	$K^\pm\pi^\mp\pi^\pm\pi^\mp$	$K_S\pi^\mp\pi^\pm\pi^0$	$K^\pm\pi^\mp\pi^\pm\pi^0$	$K_S\pi^\mp\pi^\pm\pi^\pm$
$K4\pi$	$K^\pm\pi^\mp\pi^\pm\pi^\mp\pi^0$	$K_S\pi^\mp\pi^\pm\pi^\mp\pi^\pm$	$K^\pm\pi^\mp\pi^\pm\pi^\mp\pi^\pm$	$K_S\pi^\mp\pi^\pm\pi^\mp\pi^0$
$3K$	$K_SK^\pm K^\mp$		$K^\pm K^\pm K^\mp$	

Dilepton mass veto

- Dalitz decay $\pi^0 \to \gamma e^+ e^-$ and gamma conversion:

$$m(e^+e^-) > 0.2\ \mathrm{GeV/c^2}$$

- Veto Charmonium events $\underline{B \to X_s \psi(\to \ell^+ \ell^-)}$:

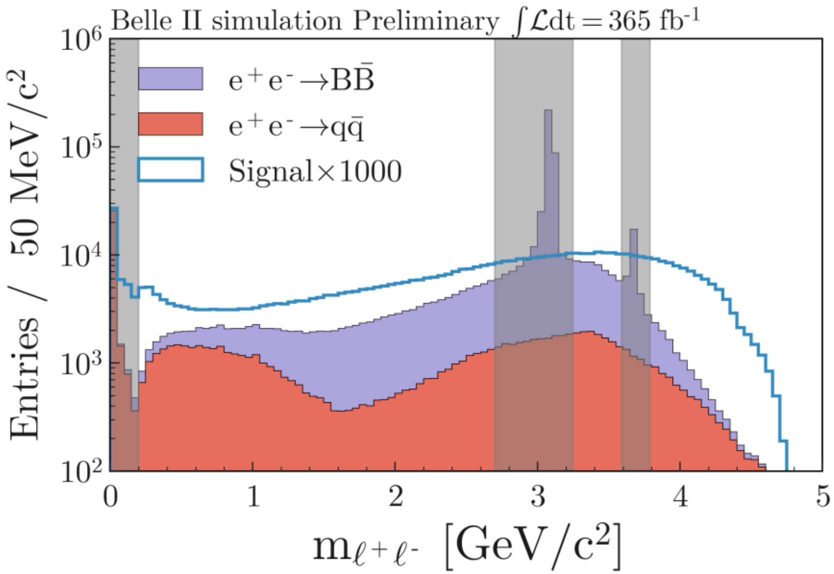
$\mathcal{B} \approx O(10^2)$ of signal process

$$-0.4\ \mathrm{GeV} < m(e^+e^-) - m(J/\psi) < 0.15\ \mathrm{GeV}$$

$$-0.25\ \mathrm{GeV} < m(e^+e^-) - m(\psi(2S)) < 0.10\ \mathrm{GeV}$$

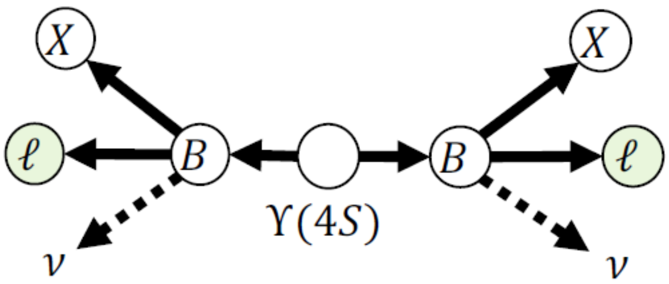
$$-0.25\ \mathrm{GeV} < m(\mu^+\mu^-) - m(J/\psi) < 0.10\mathrm{GeV}$$

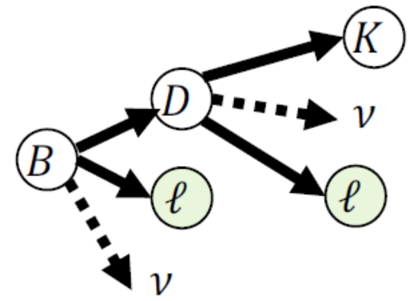
$$-0.15\ \mathrm{GeV} < m(\mu^+\mu^-) - m(\psi(2S)) < 0.10\ \mathrm{GeV}$$



Background Processes

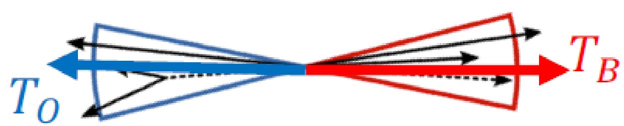
Semi-leptonic events ($B\bar{B}$ events) (event rate ≈ $O(10^3)$ of signal process)



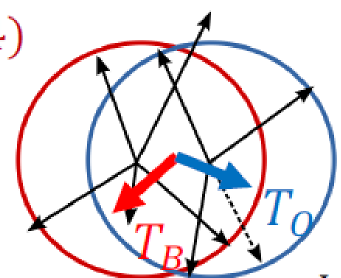


- Accompanied with multi-neutrino, reject with visible energy

Continuum events ($\sigma_{ee\to q\bar{q}}/\sigma_{ee\to B\bar{B}} \approx 3.4$)



$ee \to u\bar{u}, d\bar{d}, c\bar{c}, s\bar{s}$



$ee \to B\bar{B}$

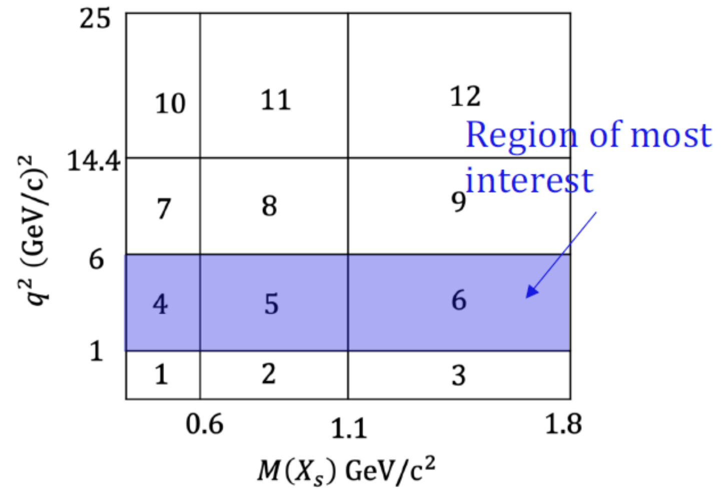
In center-of-mass frame

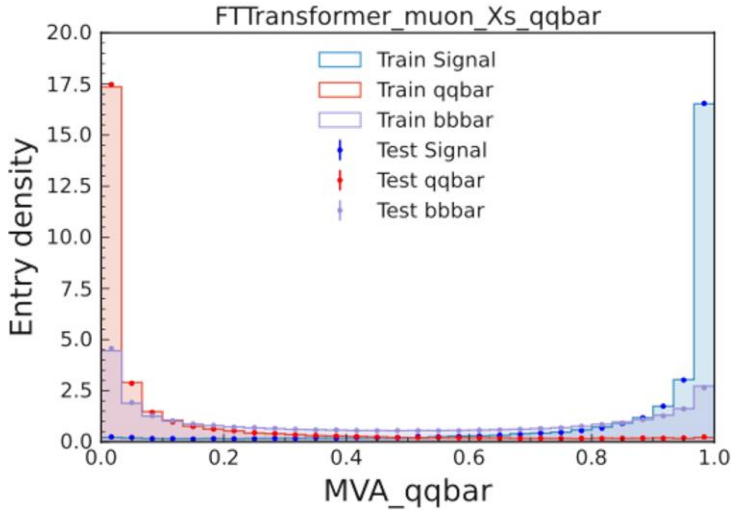
- Event shape is sphere-like for BB, and jet-like for continuum events

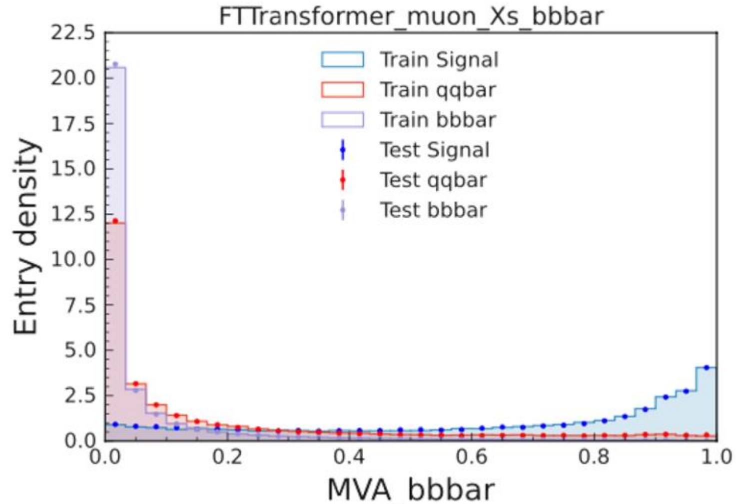
- Other effective variables such as ΔE for rejecting combinatorial BG

Background suppression with MVA

- Use 36variables to train FT-Transformer
- 8 classifiers dependent on lepton flavor (e/μ), Xs mass (>1.1 and <1.1GeV) and BG type (qq/BB)
- 12 regions are separately optimized with Figure of Merit S/√(S+B)
- Reconstruction efficiency is 3.6% which is 20% better than that at Belle.





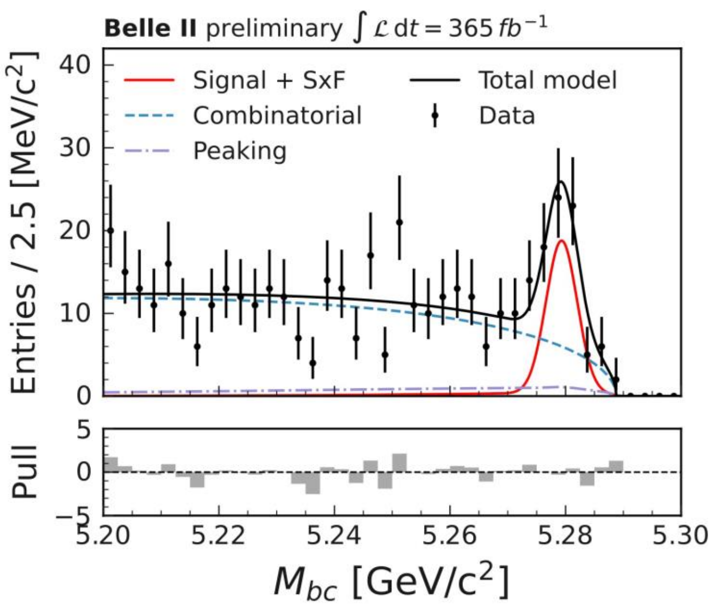


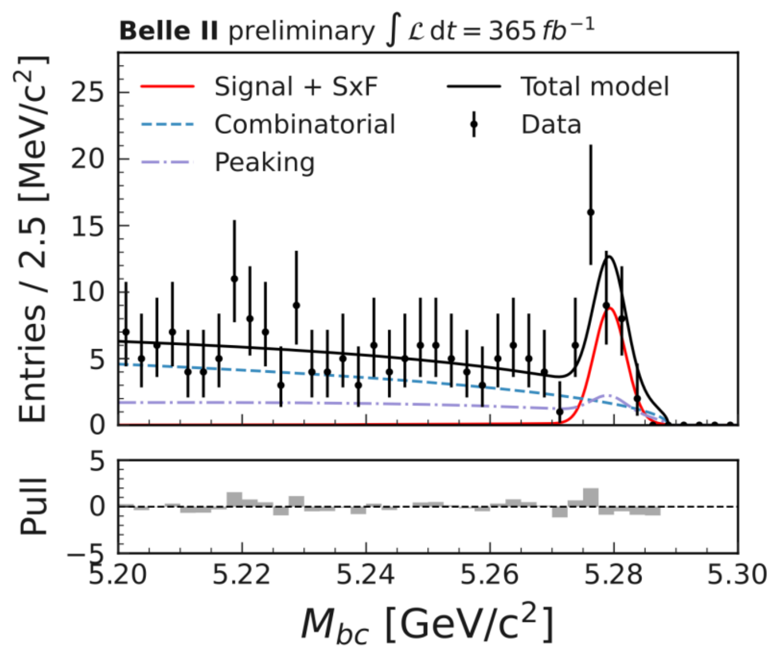
Signal Extraction and Results

- Branching fraction in $1 < q^2 < 6\ \mathrm{GeV}^2/c^4$ region:

$$\mathcal{B}(B \to X_s e^+ e^-) = \left(1.60 \pm 0.33^{+0.15}_{-0.11}\right)\times 10^{-6}$$
$$\mathcal{B}(B \to X_s \mu^+ \mu^-) = \left(1.13 \pm 0.33^{+0.11}_{-0.08}\right)\times 10^{-6}$$
$$\mathcal{B}(B \to X_s \ell^+ \ell^-) = \left(1.36 \pm 0.23^{+0.13}_{-0.10}\right)\times 10^{-6}$$

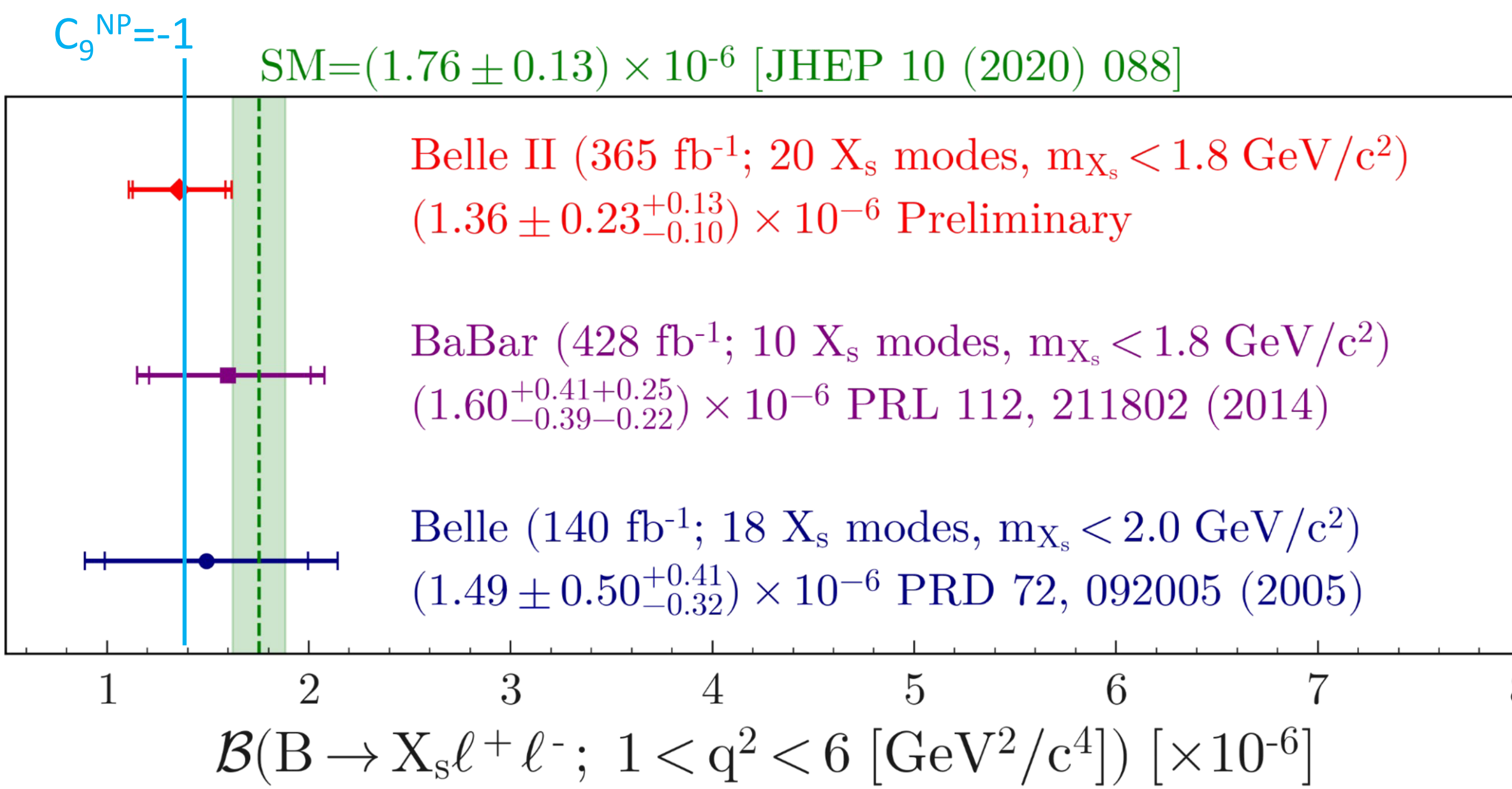
$$R(X_s) = \frac{\mathcal{B}(B \to X_s \mu^+ \mu^-)}{\mathcal{B}(B \to X_s e^+ e^-)} = 0.74 \pm 0.19 \pm 0.04$$





Comparisons

- The result for q^2=[1.0,6.0]GeV^2 is 3(2) times better than Belle(BaBar)
- Consistent with the SM and $C_9^{NP}=-1$ (about 20% deficit)



Comparison of Systematics

- The total systematics uncertainty becomes 1/3 from that at Belle

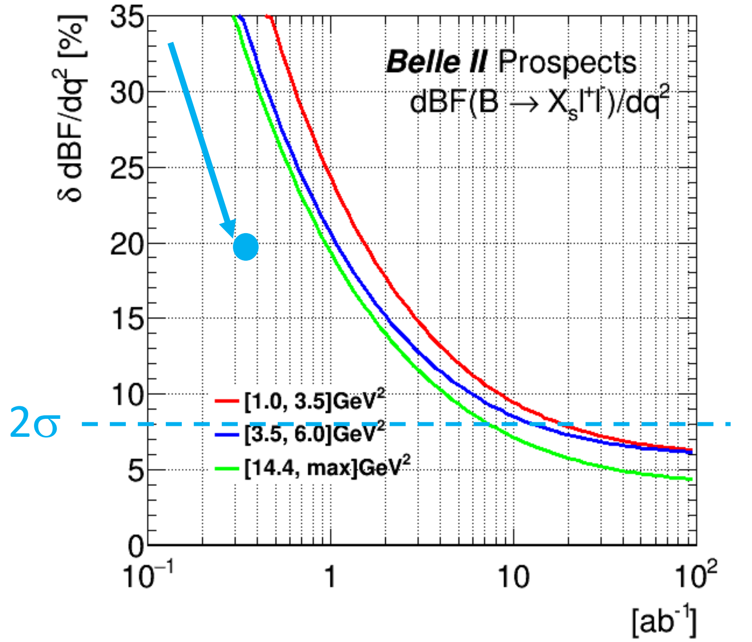
$\mathcal{B}(1 < q^2 < 6\ \mathrm{GeV}^2)$ relative uncertainties	Our study [%] $N_{B\bar{B}} = 387{\times}10^6$	Belle [%] $N_{B\bar{B}} = 152{\times}10^6$	Babar [%] $N_{B\bar{B}} = 471{\times}10^6$
$\mathcal{B}(B \to K\ell\ell)$ ↓	± 2.9	± 14.1	$^{+4.3}_{-4.5}$
$\mathcal{B}(B \to K^*\ell\ell)$ ↓	± 1.4	± 10.5	
Kaon resonances New	± 3.4	--	--
Fragmentation ↓	$^{+1.1}_{-1.7}$	± 11.0	± 11.8
Missing modes More reliable	$^{+6.6}_{-2.5}$	± 5.9	$^{+4.7}_{-4.2}$
$K^* - X_s$ transition ↓	$^{+3.4}_{-2.6}$	± 6.3	--
Fermi motion ↓	$^{+0.3}_{-0.5}$	$^{+8.2}_{-2.3}$	--
Reconstruction ↓	± 2.1	± 11.1	± 1.7
MC statistics ↓	± 0.5	± 2.0	± 0.1
$N_{B\bar{B}}$	± 2.4	± 0.5	± 0.4
Fitting + peaking BG ↓	± 0.9	± 7.6	$^{+7.7}_{-5.1}$
Total $\sigma_{sys.}$	$^{+9.4}_{-7.0}$ ← 1/3!	$^{+27.5}_{-21.5}$	$^{+15.5}_{-13.9}$

Future Prospect on BF

- This measurement 20% better than prospect in physics book
  - Belle II : 20% with 365/fb in q^2=[1,6]GeV^2
    - 28% with 365/fb for q^2=[3.5,6.0]GeV^2

Observables	Belle 0.71 ab^{-1}	Belle II 5 ab^{-1}	Belle II 50 ab^{-1}
$Br(B \to X_s \ell^+ \ell^-)$ ([1.0, 3.5] GeV^2)	29%	13%	6.6%
$Br(B \to X_s \ell^+ \ell^-)$ ([3.5, 6.0] GeV^2)	24%	11%	6.4%
$Br(B \to X_s \ell^+ \ell^-)$ (> 14.4 GeV^2)	23%	10%	4.7%

This measurement [1,6.0]GeV^2



- If we assumeC_9^{NP}=-1, 20% deficit
  - Theoretical uncertainty 7%
  - BF measurement with 7% precision in [1,6]GeV^2 with 4/ab data → 2σ hint while further improvement is not easy.

→ A_{FB} measurement

Future Prospect on A_{FB}

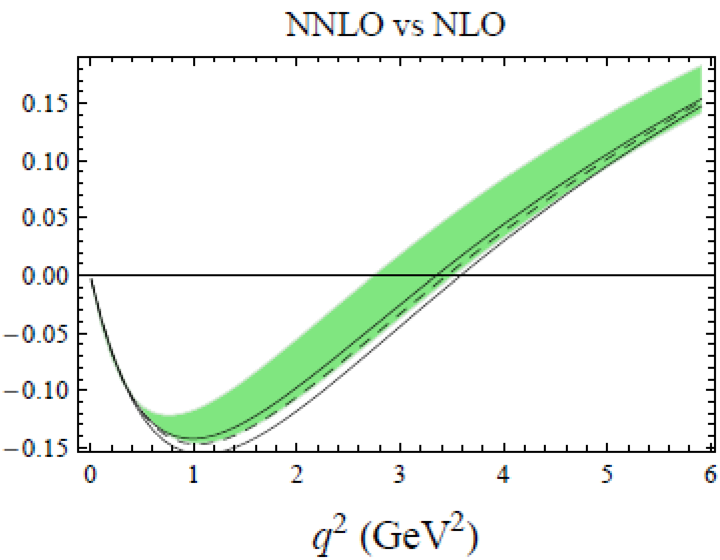
$A_{\mathrm{FB}}(q^2 \in [1,6]\ \mathrm{GeV}^2)$	
SM	$(-0.77 \pm 0.93)\%$
$C_{9\mu}^{\mathrm{NP}} = -1$	$\approx\ -6.5\ \%$

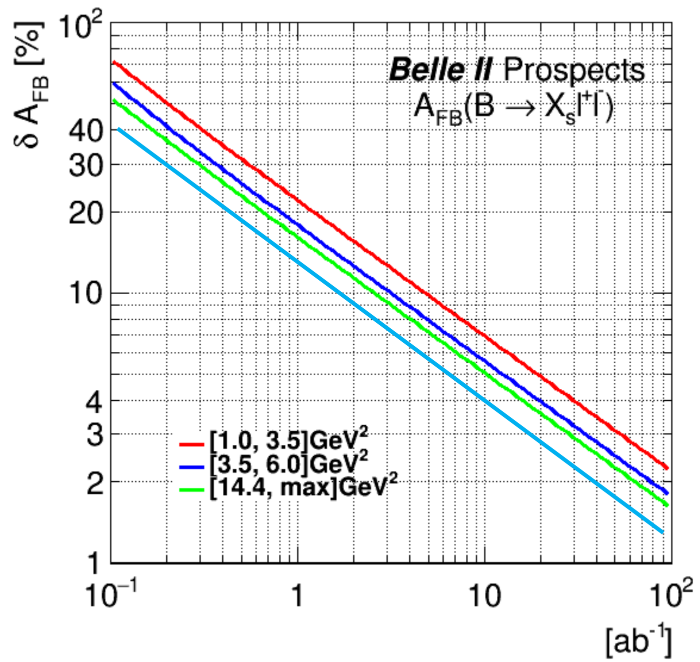
- Both theoretical and experimental uncertainties cancel out
- 6% difference of A_{FB} between SM and C_9^{NP}=-1

Extrapolation assuming this measurement in [1.0,6.0]GeV^2

Observables	Belle 0.71 ab^{-1}	Belle II 5 ab^{-1}	Belle II 50 ab^{-1}
$A_{\mathrm{FB}}(B \to X_s \ell^+\ell^-)$ ([1.0, 3.5] GeV^2)	26%	9.7%	3.1%
$A_{\mathrm{FB}}(B \to X_s \ell^+\ell^-)$ ([3.5, 6.0] GeV^2)	21%	7.9%	2.6%
$A_{\mathrm{FB}}(B \to X_s \ell^+\ell^-)$ (> 14.4 GeV^2)	19%	7.3%	2.4%

$$A_{\mathrm{FB}}(q^2) = \frac{\frac{d\Gamma_F}{dq^2} - \frac{d\Gamma_B}{dq^2}}{\frac{d\Gamma_F}{dq^2} + \frac{d\Gamma_B}{dq^2}}$$

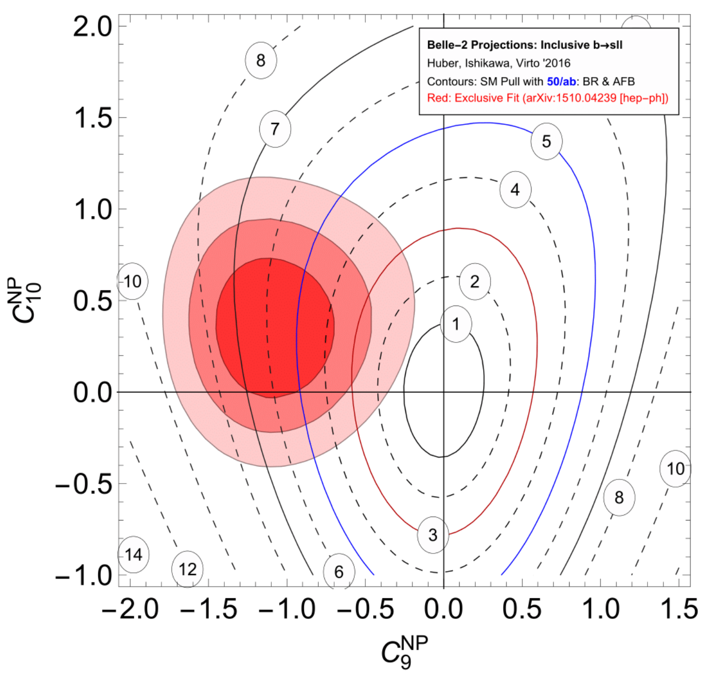




Prospects on Wilson Coefficients

- With 50/ab data, we can conclude that the anomaly is due to NP or charm loop

Physics book
Huber, Ishikawa, Virto



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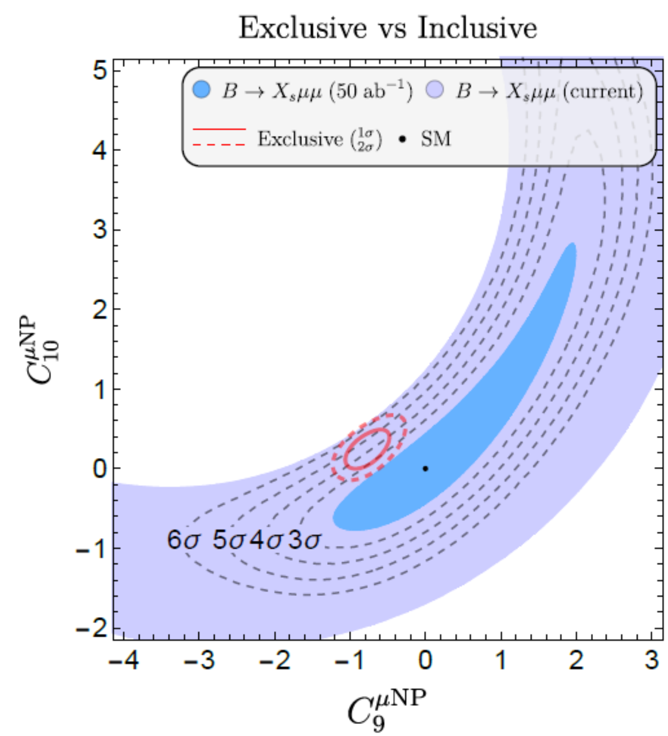
Huber, Hurth, Jenkins, Lunghi, Qin, Vos



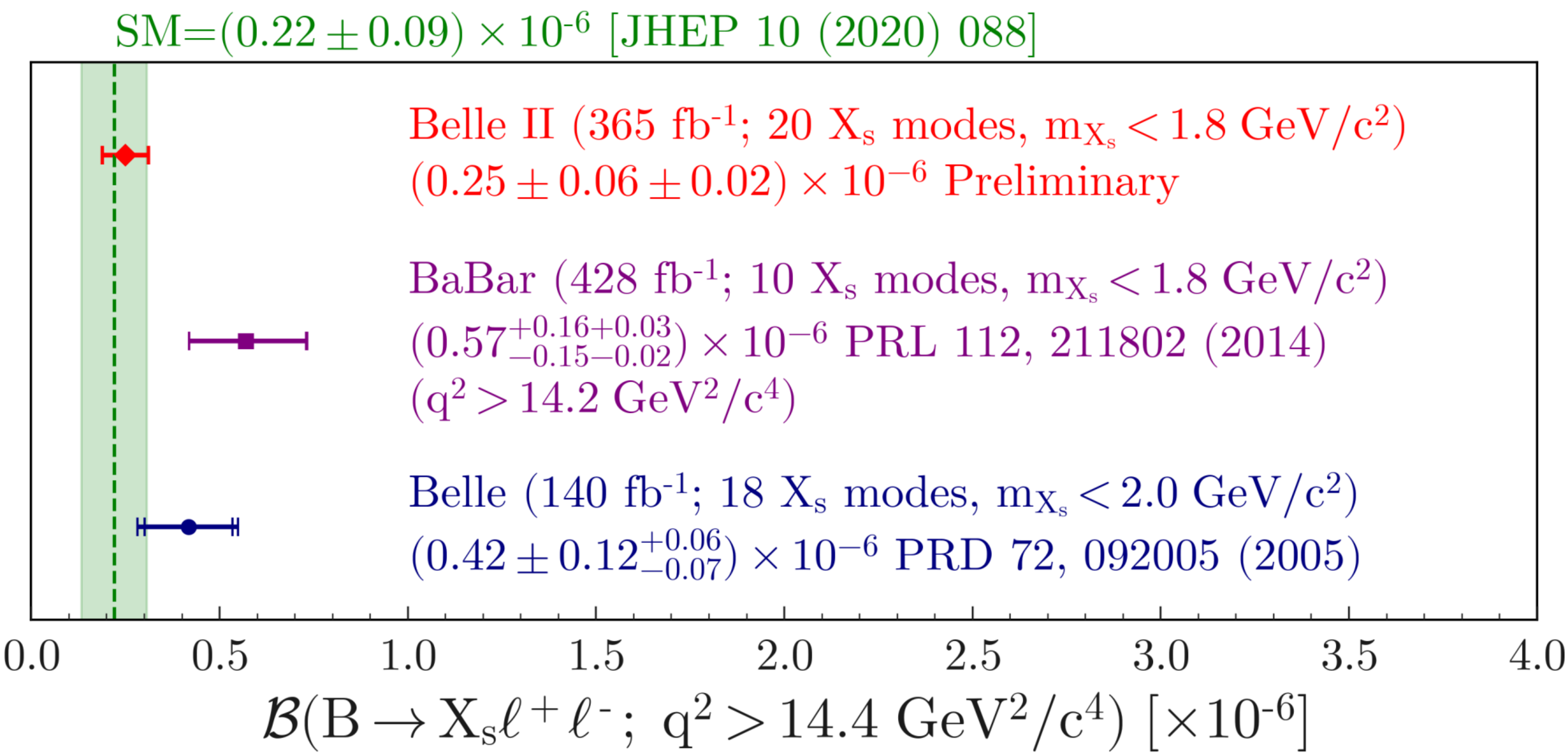
Summary

- Anomaly is seen in exclusive B→K(*)$\mu^+\mu^-$ branching fraction and P_5' which suggest C_9^{NP} ~ -1 while charm loop is the current showstopper.
- Charm loop effect in inclusive decays are theoretically calculable.

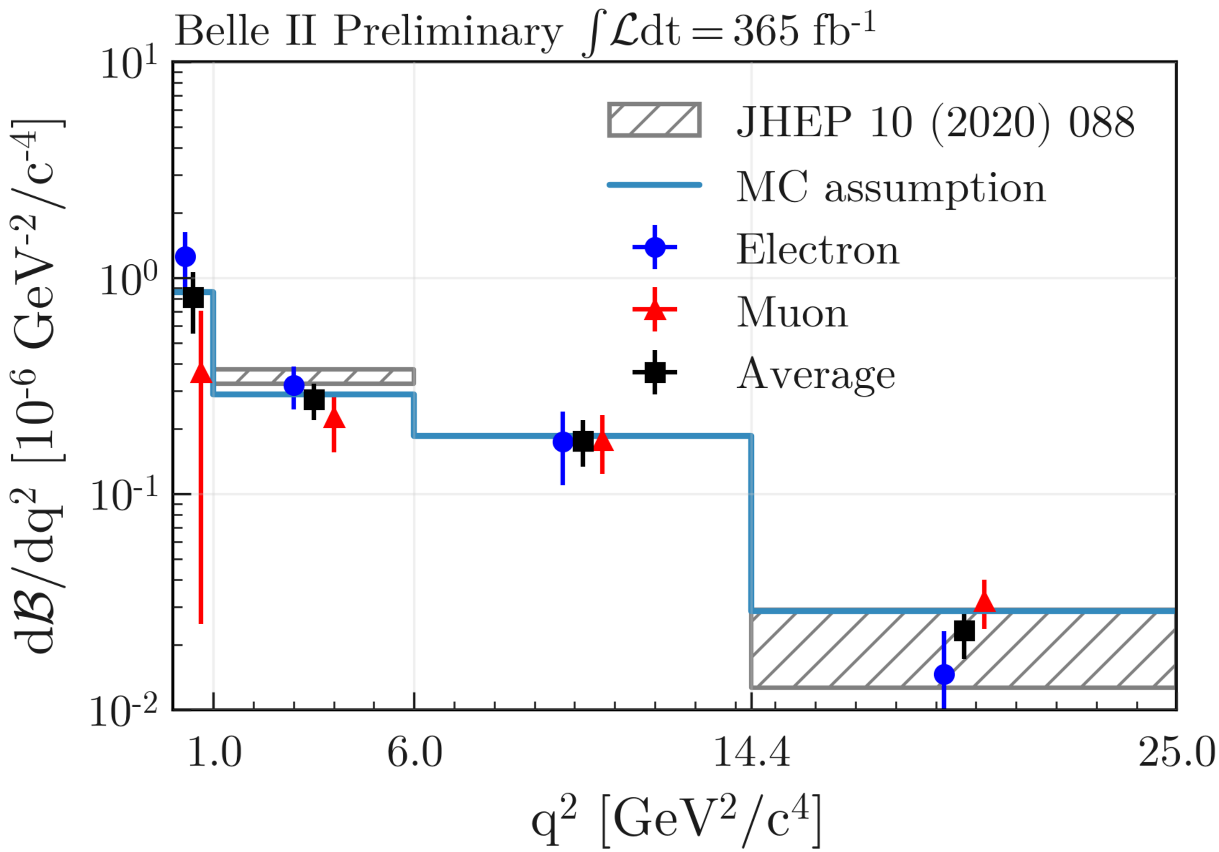
- We have measured the branching fraction of inclusive b→sl+l- with sum-of-exclusive method.
- The BF in q^2=[1,6]GeV^2 is
  - 3(2) times better than Belle (BaBar)
  - consistent with both SM prediction and C_9^{NP} ~ -1 (where 20% deficit expected)

- With 50/ab and combining A_{FB}, we can conclude the anomaly in C_9

Result for q^2>14.4GeV^2



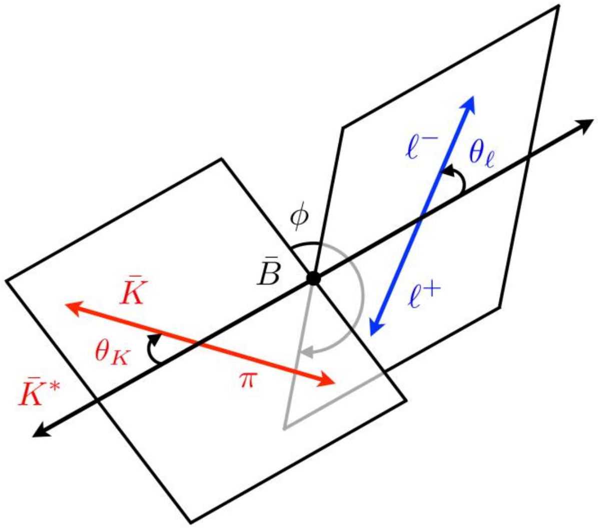
Q2 distribution



Full Angular Analysis

$$\frac{d^4\Gamma}{dq^2\, d\cos\theta_\ell\, d\cos\theta_K\, d\phi} = \frac{9}{32\pi} I(q^2, \theta_\ell, \theta_K, \phi)$$

$$\begin{aligned} I(q^2,\theta_\ell,\theta_K,\phi) = {} & I_1^s \sin^2\theta_K + I_1^c \cos^2\theta_K + (I_2^s \sin^2\theta_K + I_2^c \cos^2\theta_K)\cos 2\theta_\ell \\ & + I_3 \sin^2\theta_K \sin^2\theta_\ell \cos 2\phi + I_4 \sin 2\theta_K \sin 2\theta_\ell \cos\phi \\ & + I_5 \sin 2\theta_K \sin\theta_\ell \cos\phi + (I_6^s \sin^2\theta_K + I_6^c \cos^2\theta_K)\cos\theta_\ell \\ & + I_7 \sin 2\theta_K \sin\theta_\ell \sin\phi + I_8 \sin 2\theta_K \sin 2\theta_\ell \sin\phi \\ & + I_9 \sin^2\theta_K \sin^2\theta_\ell \sin 2\phi\,. \end{aligned}$$



$$S_j = (I_j + \bar{I}_j) \Big/ \frac{d\Gamma}{dq^2}\,, \qquad A_j = (I_j - \bar{I}_j) \Big/ \frac{d\Gamma}{dq^2}$$

$$A_{\rm FB} = \frac{3}{4} S_{6s} + \frac{3}{8} S_{6c}\,, \qquad F_L = -S_{2c} \qquad P_4' = \frac{S_4}{2\sqrt{-S_{2s}S_{2c}}}\,, \qquad P_5' = \frac{S_5}{2\sqrt{-S_{2s}S_{2c}}}\,,$$

$$P_1 = \frac{S_3}{2S_{2s}}\,, \qquad P_2 = \frac{S_{6s}}{8S_{2s}}\,, \qquad P_3 = -\frac{S_9}{4S_{2s}}\,, \qquad P_6' = \frac{S_7}{2\sqrt{-S_{2s}S_{2c}}}\,, \qquad P_8' = \frac{S_8}{2\sqrt{-S_{2s}S_{2c}}}\,.$$

$$P_5' \simeq \frac{\mathrm{Re}\left(C_{10}^* C_{9,\perp} + C_{9,||}^* C_{10}\right)}{\sqrt{\left(|C_{9,\perp}|^2 + |C_{10}|^2\right)\left(|C_{9,||}|^2 + |C_{10}|^2\right)}}$$