Radiative kaon decays:
where do we stand?

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Rare (radiative) Kaon decays

- Rare kaon decays: extremely useful probes of

  (a) new physics (when SM-suppressed and short-distance dominated).

  (b) the SM itself in the nonperturbative regime (when long-distance dominated).

- Focus of this talk: hadronic decay modes with photons or $\ell^+\ell^-$ pairs (long-distance dominated).

$$K \to \pi\gamma^{(*)}, \pi\pi\gamma^{(*)}$$

- Main goal: determination of ChPT couplings at NLO, i.e., precise knowledge of the SM at low energies.

Theoretical description

- Kaon decays can be described with ChPT, the low-energy EFT of the strong interactions. $\Delta S = 1$ sector:

$$\mathcal{L}_{\Delta S=1} = G_8 f_\pi^4 \, \mathrm{tr}\left[\lambda_6 D_\mu U^\dagger D^\mu U\right] + G_8 f_\pi^2 \sum_j N_j \, W_j(U, D_\mu U, \lambda_6) + \mathcal{O}(p^6)$$

  with

$$U = \exp\left[i\frac{\phi^a \tau^a}{f_\pi}\right]; \qquad D_\mu U = \partial_\mu U + ieA_\mu[Q, U]$$

- The LO is universal, NLO order contains nonperturbative (hadronic) information inside N_i.
- Radiative kaon decays: out of the 37 NLO operators, sensitive to combinations of $W_{14}, ..., W_{18}$ (CP-even) and $W_{28}, ..., W_{31}$ (CP-odd).
- General structure of the amplitudes:

$$\mathcal{M}(K \to X\gamma^{(*)}) = \underbrace{\mathcal{M}_B(\mathcal{O}(p^2))}_{\text{Brems.}} + \underbrace{\mathcal{M}_E(\mathcal{O}(p^4))}_{\text{electric, CP-even}} + \underbrace{\mathcal{M}_M(\mathcal{O}(p^4))}_{\text{magnetic, CP-odd}}$$

Radiative kaon decays

Measured modes:

$K^\pm \to \pi^\pm\pi^0\gamma$	$\sim 10^{-6}$
$K^\pm \to \pi^\pm\gamma\gamma$	$\sim 10^{-6}$
$K^\pm \to \pi^\pm\gamma^*$	$\sim 10^{-7}$
$K_S \to \pi^0\gamma^*$	$\sim 10^{-9}$
$K_S \to \pi^0\gamma\gamma$	$\sim 10^{-8}$

Near future:

$K^\pm \to \pi^\pm\pi^0\gamma^*$	$\sim 10^{-6}$	NA48, NA62
$K_S \to \pi^+\pi^-\gamma^*$	$\sim 10^{-5}$	LHCb
$K_S \to \mu^+\mu^-$	$< 10^{-9}$	LHCb
$K_S \to 4\ell$		LHCb

Measured radiative kaon decays

- Weak chiral couplings are related to the slope of the differential decay rate, most easily accessed through the interference term between Bremsstrahlung and electric emission.
- Experimental effort for the last 15 years, and ongoing:

$K^\pm \to \pi^\pm \gamma^* :$	$a_+ = -0.578 \pm 0.016$	[NA48/2, 2009 – 11]
$K_S \to \pi^0 \gamma^* :$	$a_S = (1.06^{+0.26}_{-0.21} \pm 0.07)$	[NA48/1, 2003 – 04]
$K^\pm \to \pi^\pm \pi^0 \gamma :$	$X_E = (-24 \pm 4 \pm 4)\ \mathrm{GeV}^{-4}$	[NA48/2, 2010]
$K^+ \to \pi^+ \gamma\gamma :$	$\hat{c} = 1.56 \pm 0.23 \pm 0.11$	[NA62, 2014]

- The slopes are linked to ChPT through [Ecker et al; D'Ambrosio et al]

$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = \frac{3}{64\pi^2}\left(\frac{1}{3} - \frac{G_F}{G_8}a_+ - \frac{1}{3}\log\frac{\mu^2}{m_K m_\pi}\right) - 3L_9^r$$

$$\mathcal{N}_S \equiv 2N_{14}^r + N_{15}^r = \frac{3}{32\pi^2}\left(\frac{1}{3} + \frac{G_F}{G_8}a_S - \frac{1}{3}\log\frac{\mu^2}{m_K^2}\right)$$

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17} = -\frac{|\mathcal{M}_K| f_\pi}{2G_8} X_E$$

$$\mathcal{N}_0 \equiv N_{14}^r - N_{15}^r - 2N_{18}^r = \frac{3}{128\pi^2}\hat{c} - 3(L_9^r + L_{10}^r)$$

Status of NLO chiral counterterms

Decay mode	counterterm combination	expt. value
$K^{\pm} \to \pi^{\pm}\gamma^*$	$N_{14} - N_{15}$	$-0.0167(13)$
$K_S \to \pi^0\gamma^*$	$2N_{14} + N_{15}$	$+0.016(4)$
$K^{\pm} \to \pi^{\pm}\pi^0\gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	$+0.0022(7)$
$K^{\pm} \to \pi^{\pm}\gamma\gamma$	$N_{14} - N_{15} - 2N_{18}$	$-0.0017(32)$

- From $K \to \pi\gamma^*$ decays,

$$N_{14} = (-2 \pm 18) \times 10^{-4}; \qquad N_{15} = (1.65 \pm 0.22) \times 10^{-2}$$

- Adding $K \to \pi\gamma\gamma$,

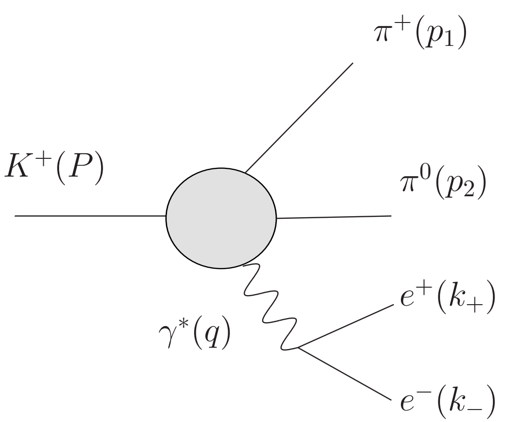
$$N_{18} = (-7.5 \pm 2.3) \times 10^{-3}$$

- So far, only the combination $N_{16} + N_{17}$ constrained. One extra combination needed:

$$K \to \pi\pi\ell^+\ell^-$$

- Most promising channel: $K^{\pm} \to \pi^{\pm}\pi^0 e^+e^-$, under analysis at NA48. Important alternative: $K_L \to \pi^+\pi^- e^+e^-$.

$K^+ \to \pi^+\pi^0 e^+ e^-$



- The hadronic piece contains 3 form factors:

$$H^\mu(p_1,p_2,q) = F_1 p_1^\mu + F_2 p_2^\mu + F_3 \varepsilon^{\mu\nu\alpha\beta} p_{1\nu} p_{2\alpha} q_\beta$$

- Relevant weak coupling combinations:

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17} = +0.0022(7)$$

$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = -0.0167(13)$$

$$\mathcal{N}_E^{(2)} = N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17})$$

e.g.

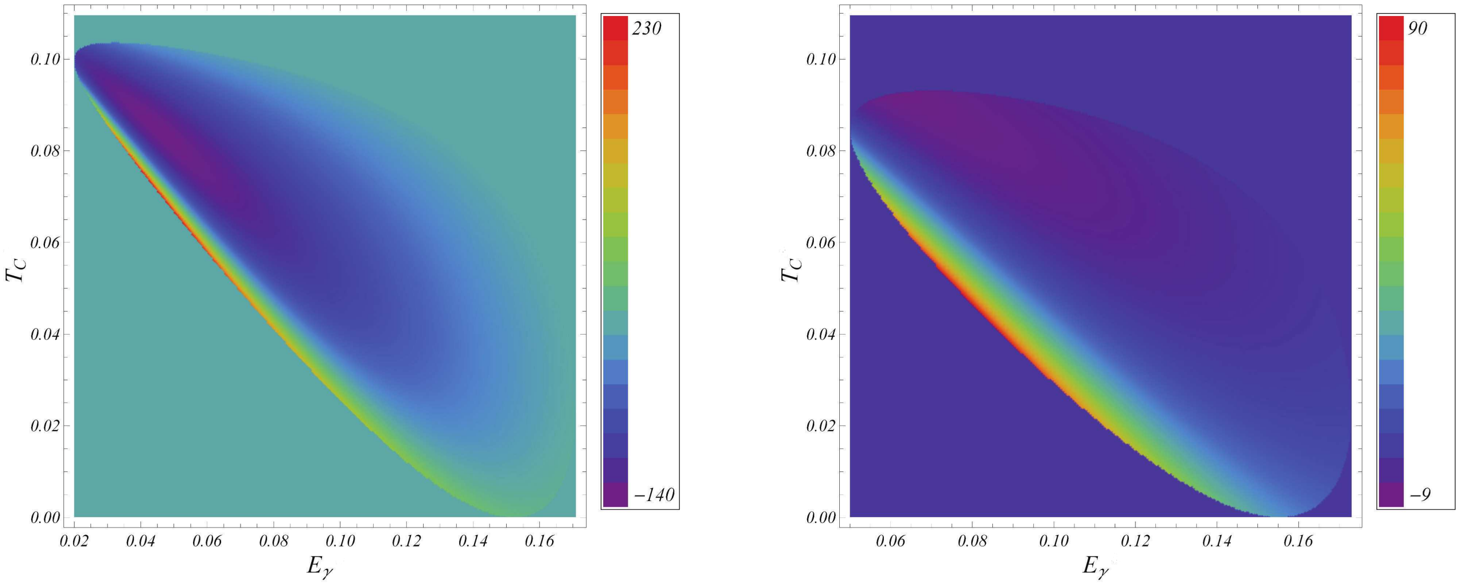
$$F_2 = -\frac{2ie}{2q\cdot p_K - q^2}\mathcal{M}_K e^{i\delta_0^2} + \frac{2ieG_8 e^{i\delta_1^1}}{f_\pi}\left\{ q\cdot p_+ \mathcal{N}_E^{(0)} - \frac{1}{3} q^2 \mathcal{N}_E^{(2)} \right\}$$

$K^+ \to \pi^+\pi^0 e^+ e^-$

- Challenge: how to overcome the $\mathcal{O}(p^2)$ contribution. In terms of integrated branching ratios:

$$\Gamma_M \sim \frac{1}{70}\Gamma_B; \qquad \Gamma_{\rm INT} \sim 10^{-2}\Gamma_B$$

- Best strategy: Bremsstrahlung is peaked at low q^2. Use cuts in the photon energy. [Pichl'01; Cappiello, OC, D'Ambrosio'11,'17]



- The gain depends on the size of $\mathcal{N}_E^{(2)}$, but it can be at least a factor 10.
- Using the already measured radiative decays:

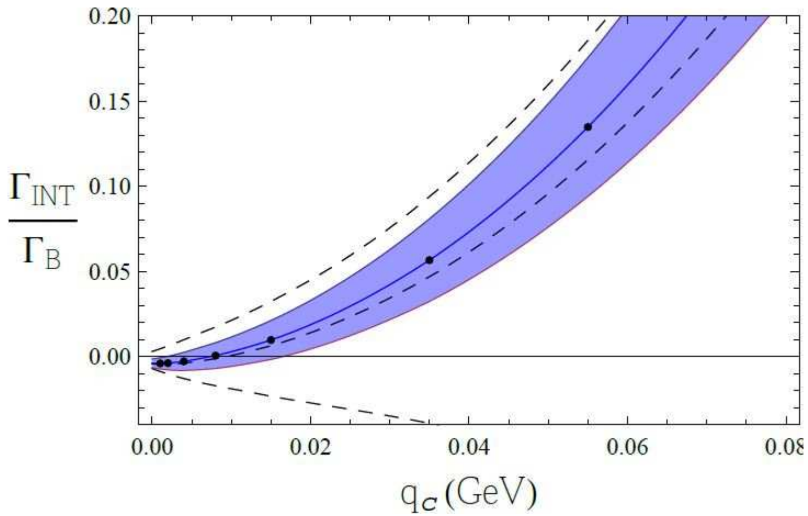
$$\mathcal{N}_E^{(2)} = +0.089(11) + 6N_{17}$$

$K^+ \to \pi^+\pi^0 e^+e^-$

- Prediction: a very large $\mathcal{N}_E^{(2)}$ counterterm.

$$\mathcal{N}_E^{(2)} \sim +\mathcal{O}(10^{-1})$$

- Rather robust: unless N_{17} is large (theoretically disfavored) and negative.
- An analysis shows that the interference has a characteristic pattern depending mostly on $\mathcal{N}_E^{(0)}$ and $\mathcal{N}_E^{(2)}$.



- NA48 has collected 5000 events. Unclear whether this will allow for an informative fit (analysis in progress) or one would need the larger statistics of NA62.

$K_S \to \pi^+\pi^-e^+e^-$

- A similar analysis can be performed for $K_S \to \pi^+\pi^-e^+e^-$. Total branching ratio measured:

$$BR(K_S \to \pi^+\pi^-e^+e^-)_{\mathsf{exp}} = (4.79 \pm 0.15) \times 10^{-5}$$

  Access to interference term requires typically a percent precision. Challenging but in the LHCb agenda.

- On top of $\mathcal{N}_E^{(0)}$ (known), new counterterm, but not independent:

$$\mathcal{N}_E^{(3)} = N_{14} - N_{15} - 3(N_{16} + N_{17}) = -2\mathcal{N}_E^{(1)} + 3\mathcal{N}_E^{(0)} = +0.040(5)$$

- Prediction of the LO and NLO chiral contributions:

$$BR(K_S \to \pi^+\pi^-e^+e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}}$$

- Extraction of N_{17} from neutral kaon decays possible with information on $K_L \to \pi^+\pi^-e^+e^-$:

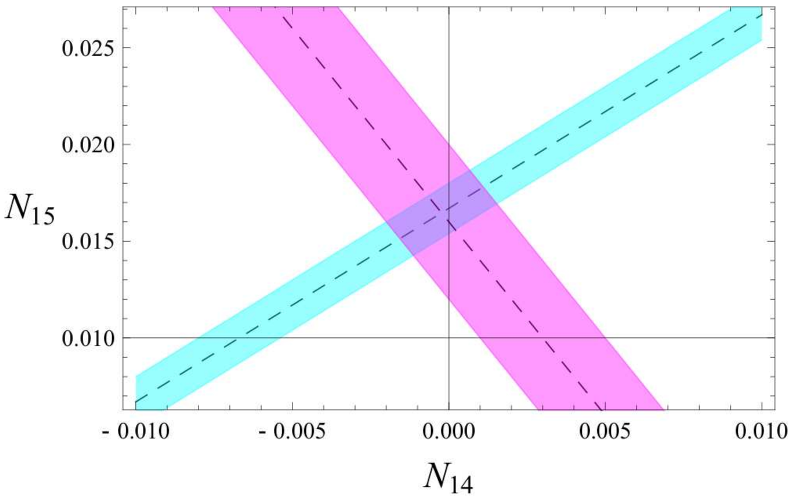
$$\mathcal{N}_E^{(4)} - \mathcal{N}_E^{(3)} = 6N_{17}$$

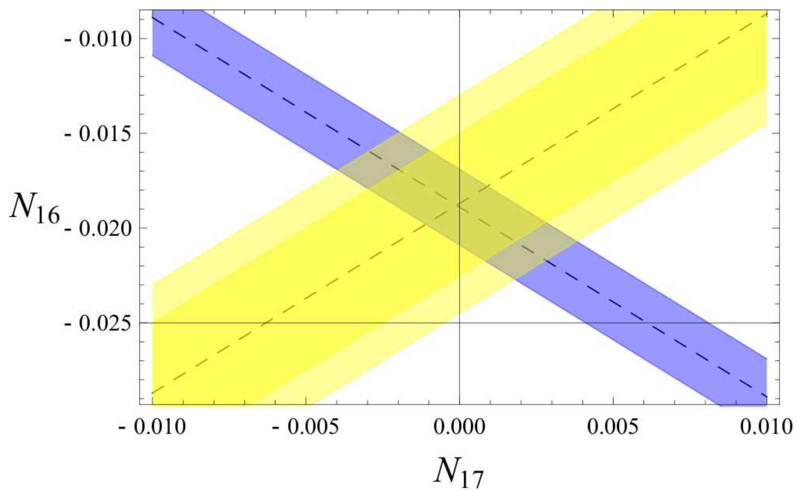
- Muon decay extremely suppressed by phase space:

$$BR(K_S \to \pi^+\pi^-\mu^+\mu^-) = \underbrace{4.17 \cdot 10^{-14}}_{\text{Brems.}} + \underbrace{4.98 \cdot 10^{-15}}_{\text{Int.}} + \underbrace{2.17 \cdot 10^{-16}}_{\text{DE}}$$

Radiative weak couplings

Current situation:





- Strong hierarchies:

$$N_{14} = (-2 \pm 18) \times 10^{-4}; \qquad N_{15} = (1.65 \pm 0.22) \times 10^{-2}$$

- N_{16} vs N_{17} based on an educated guess ($N_{17} \lesssim 10^{-3}$). If experiment contradicts it, rather exceptional failure of all the theoretical frameworks.

Weak counterterms (theory)

- ChPT built on chiral symmetry: operator catalog at each loop order. Coefficients depend on physics above the GeV scale: predictions are model-dependent.
- Main ideas: (explicit) resonance models and weak deformation models.

counterterm combinations	decay mode	WDM/FM/HEW	R^μ
$N_{14} - N_{15}$	$K^\pm \to \pi^\pm \gamma^*$	$-3L_9 - L_{10} - 2H_1$	$-0.02\eta_V$
$2N_{14} + N_{15}$	$K_S \to \pi^0 \gamma^*$	$-2L_{10} - 4H_1$	$0.08\eta_V$
$N_{14} - N_{15} - N_{16} - N_{17}$	$K^\pm \to \pi^\pm \pi^0 \gamma$	$-2(L_9 + L_{10})$	$-0.01\eta_A$
$N_{14} - N_{15} - 2N_{18}$	$K^\pm \to \pi^\pm \gamma\gamma$	$-3(L_9 + L_{10})$	$-0.01\eta_A$
$N_{14} + 2N_{15} - 3(N_{16} - N_{17})$	$K^\pm \to \pi^\pm \pi^0 \gamma^*$	$6L_9 - 4L_{10} + 4H_1$	$0.16\eta_V + 0.01\eta_A$
$N_{14} - N_{15} - 3(N_{16} - N_{17})$	$K_L \to \pi^+ \pi^- \gamma^*$	$-4L_{10} + 4H_1$	$0.04\eta_V + 0.01\eta_A$
$N_{14} - N_{15} - 3(N_{16} + N_{17})$	$K_S \to \pi^+ \pi^- \gamma^*$	$-4L_{10} + 4H_1$	$0.04\eta_V - 0.04\eta_A$
$7(N_{14} - N_{16}) + 5(N_{15} + N_{17})$	$K_S \to \pi^+ \pi^- \pi^0 \gamma$	$10L_9 - 14L_{10}$	$0.48\eta_V + 0.01\eta_A$

$$N_{14} - N_{15} \simeq +\mathcal{O}(10^{-2})\ ; \qquad N_{14} + 2N_{15} - 3(N_{16} - N_{17}) \simeq +\mathcal{O}(10^{-2})\ ;$$

$$2N_{14} + N_{15} \simeq -\mathcal{O}(10^{-2})\ ; \qquad N_{14} - N_{15} - 3(N_{16} - N_{17}) \simeq +\mathcal{O}(10^{-2})\ ;$$

$$N_{14} - N_{15} - N_{16} - N_{17} \simeq \mathcal{O}(10^{-3})\ ; \qquad N_{14} - N_{15} - 3(N_{16} + N_{17}) \simeq +\mathcal{O}(10^{-2})\ ;$$

$$N_{14} - N_{15} - 2N_{18} \simeq \mathcal{O}(10^{-3})\ ; \qquad 7(N_{14} - N_{16}) + 5(N_{15} + N_{17}) \simeq +\mathcal{O}(10^{-1})$$

Conclusions

- First fully experimentally-based determination of all the NLO weak chiral couplings relevant for radiative kaon decays soon to be there.

- An extraction of N_{16} and N_{17} with NA48 statistics requires (a) some expectation for the size of the counterterm combinations to be tested; (b) a strategy to compensate the overwhelming dominance of the Bremsstrahlung contributions.

- So far, results are in qualitative agreement with models (once their limitations are taken into account). Highly nontrivial.

- LHCb and NA62 will continue data taking on radiative kaon decays. More precise knowledge of the SM at low energies achievable through the measurements of $K^{\pm} \to \pi^{\pm}\pi^{0}e^{+}e^{-}$, $K_S \to \pi^{+}\pi^{-}e^{+}e^{-}$, and other modes in the near future.