

# $B \rightarrow D^{(*)} \ell \nu$ from lattice QCD with domain-wall quarks

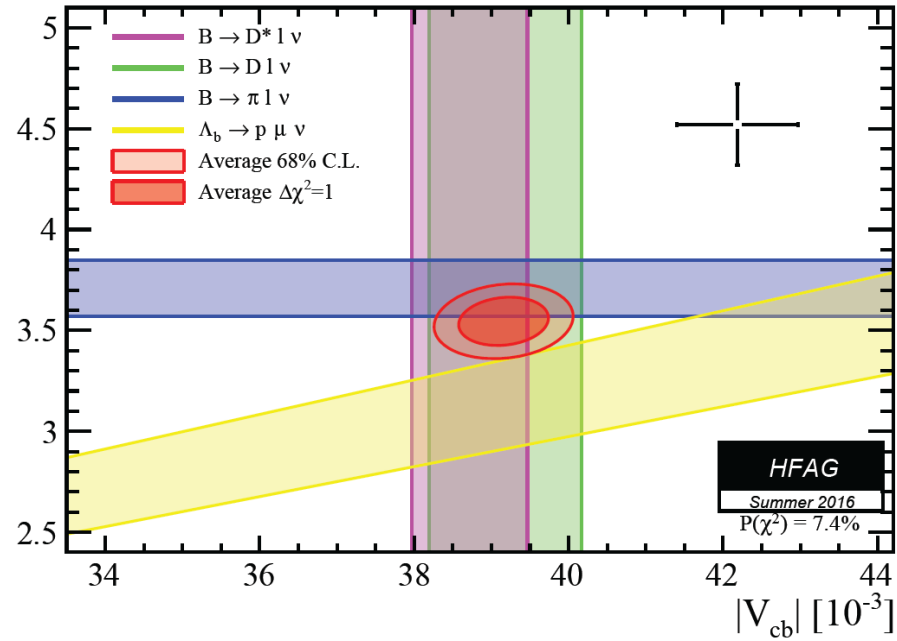
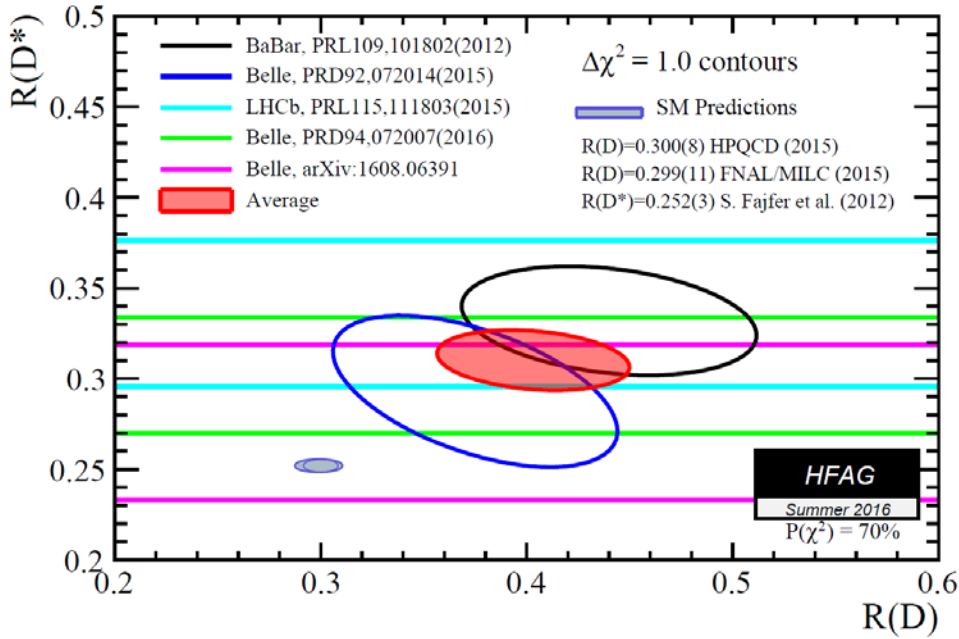
Takashi Kaneko (KEK, SOKENDAI)  
for the JLQCD collaboration

KEK-FF 2019, Feb 14-16, 2019

# introduction

## hint and puzzle

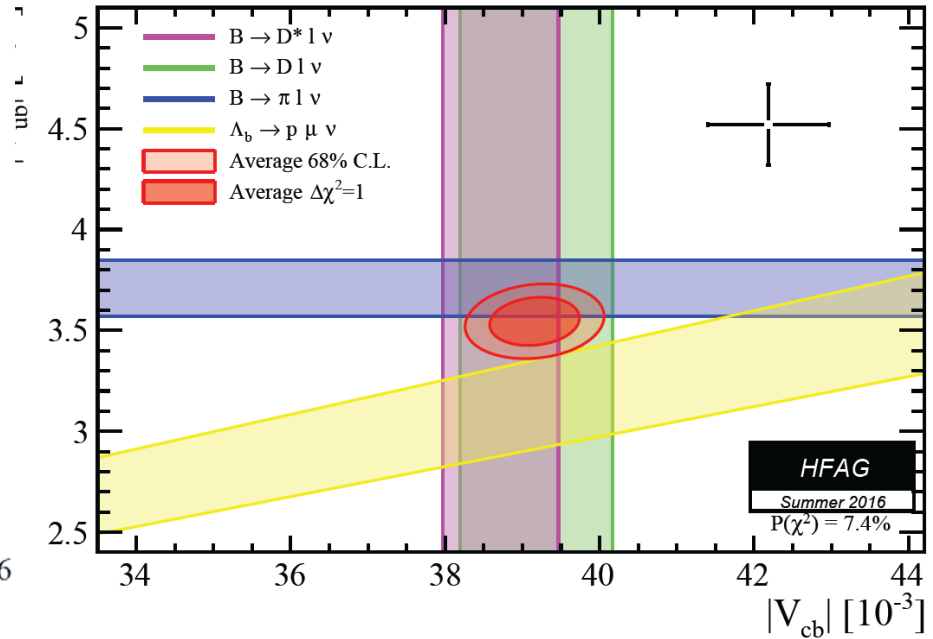
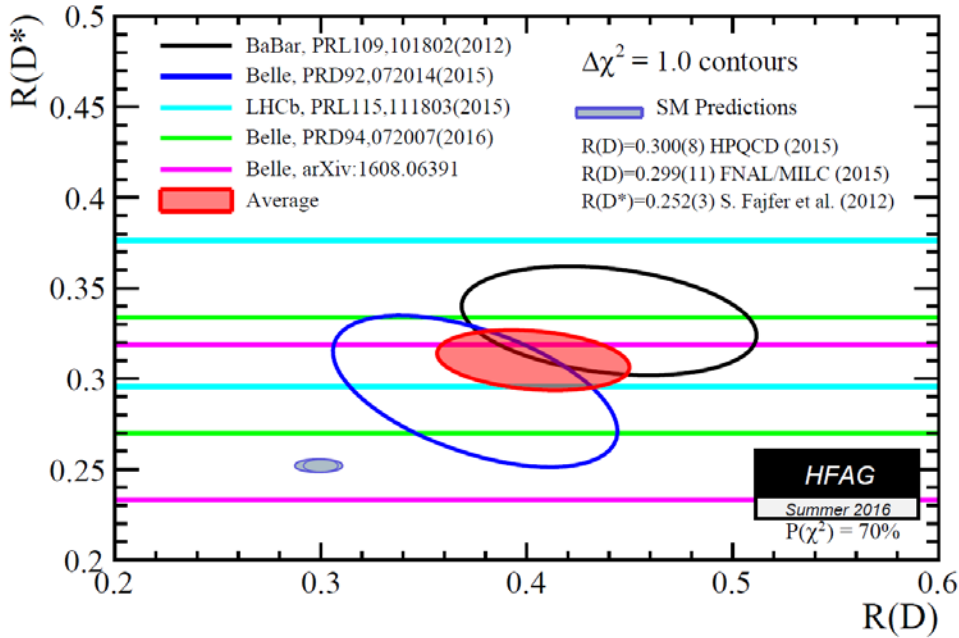
HFLAV'16



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realistic lattice studies only with staggered-type light quarks

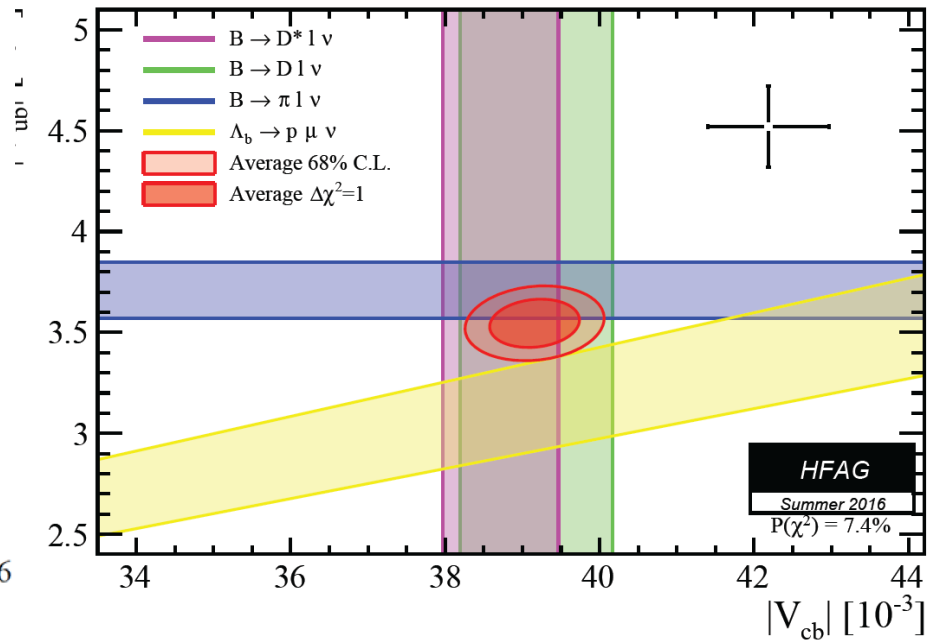
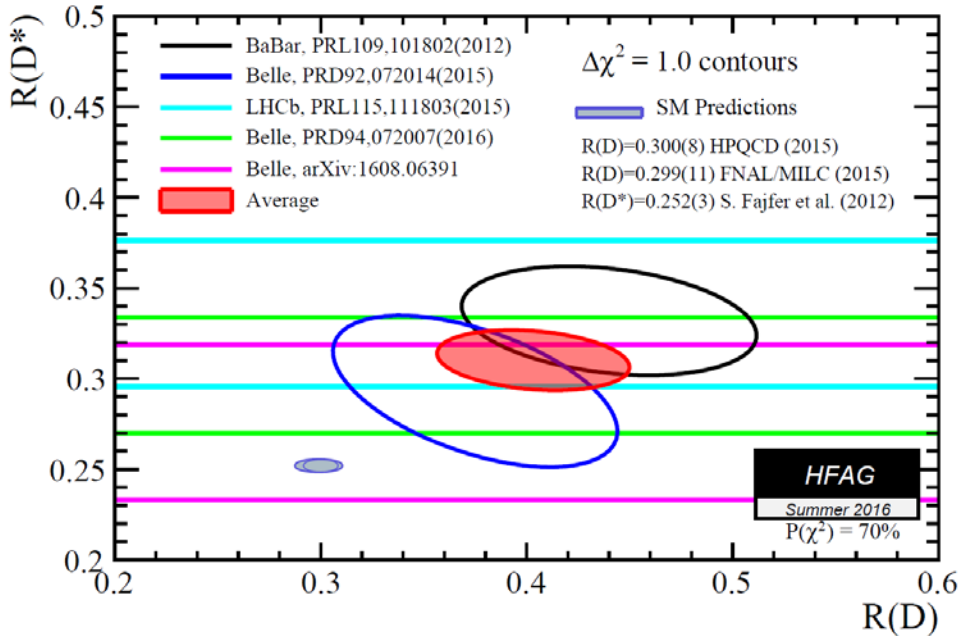
$B \rightarrow D l \nu$ : Fermilab/MILC'15, HPQCD'15, HPQCD'17 ( $w \geq 1$ )

$B \rightarrow D^* l \nu$ : Fermilab/MILC'14, HPQCD'17 ( $w=1$ ) ... and previous talk!

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independent calculations are welcome

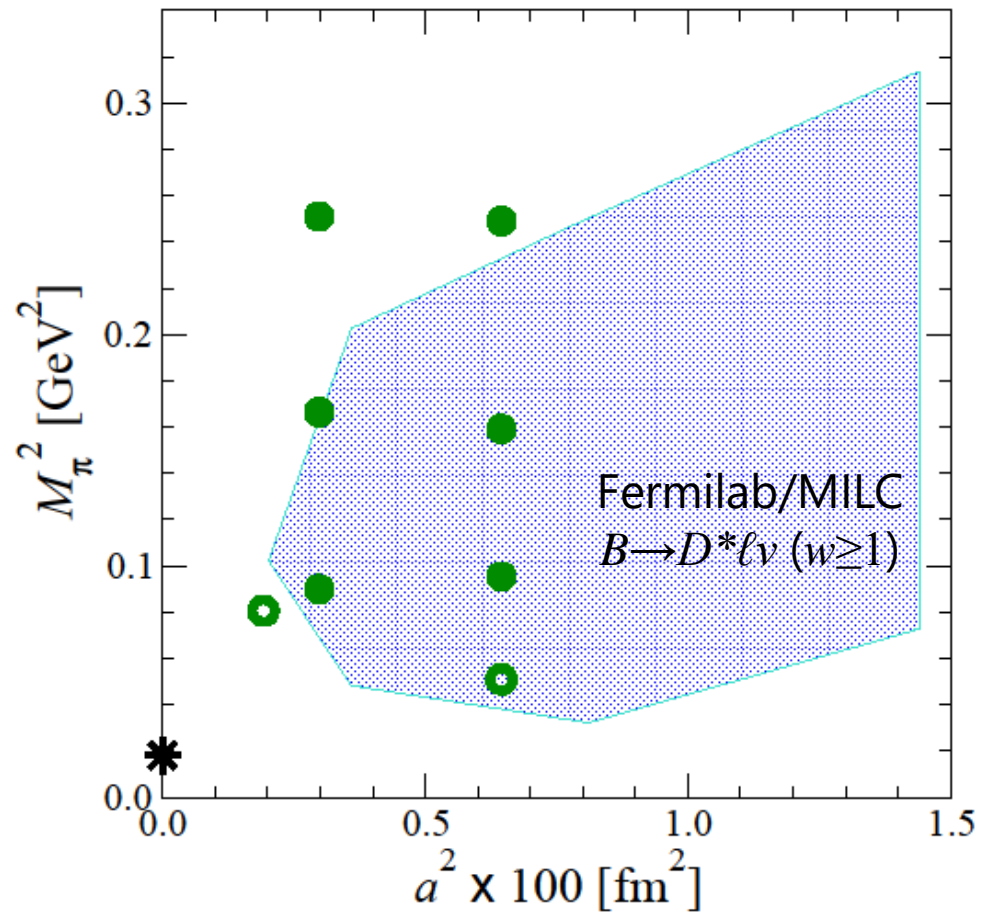
# JLQCD's study

w/ good chiral symmetry

domain-wall quarks

good chiral symmetry

- simple renormalization
- no  $O(a)$  errors



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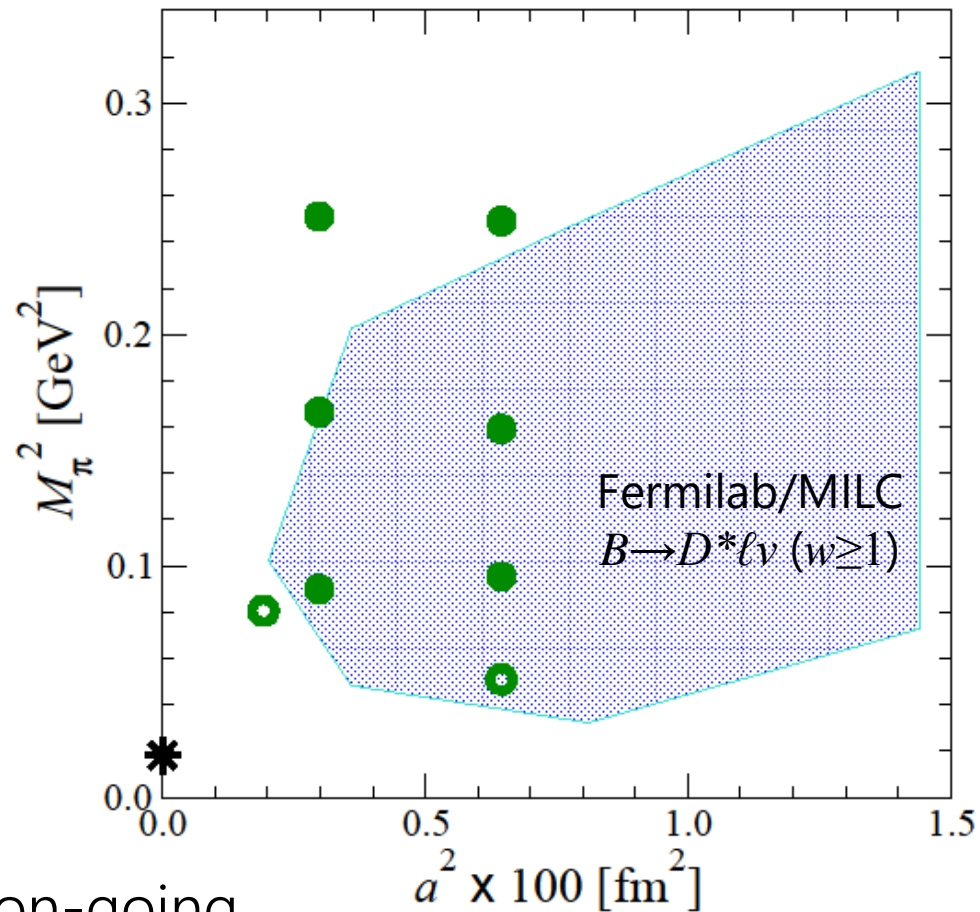
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## simulation parameters

- $a^{-1} \sim 2.5, 3.6, 4.5$  GeV
- $M_\pi \sim 230, 300, 400, 500$  MeV
- $M_\pi L \geq 4$
- $a^{-1} \sim 4.5$  GeV,  $M_\pi \sim 230$  MeV: on-going



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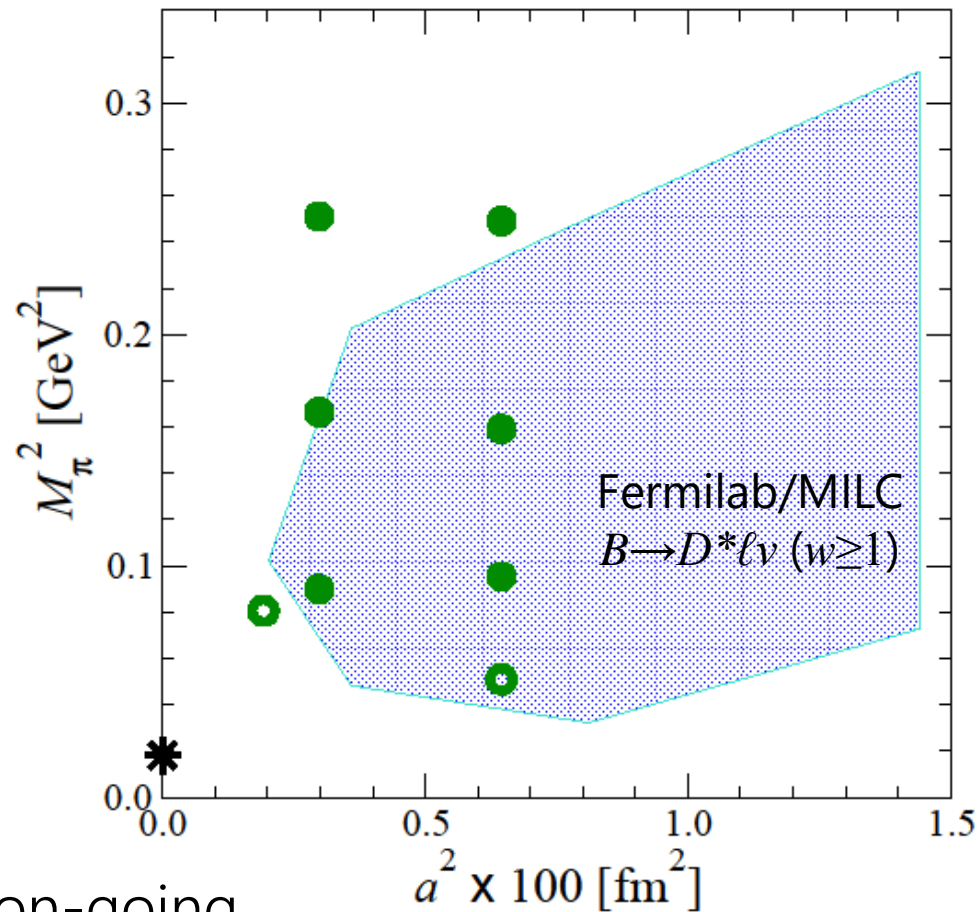
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⇒ preliminary results w/o extrapolations ...

# JLQCD's simulation

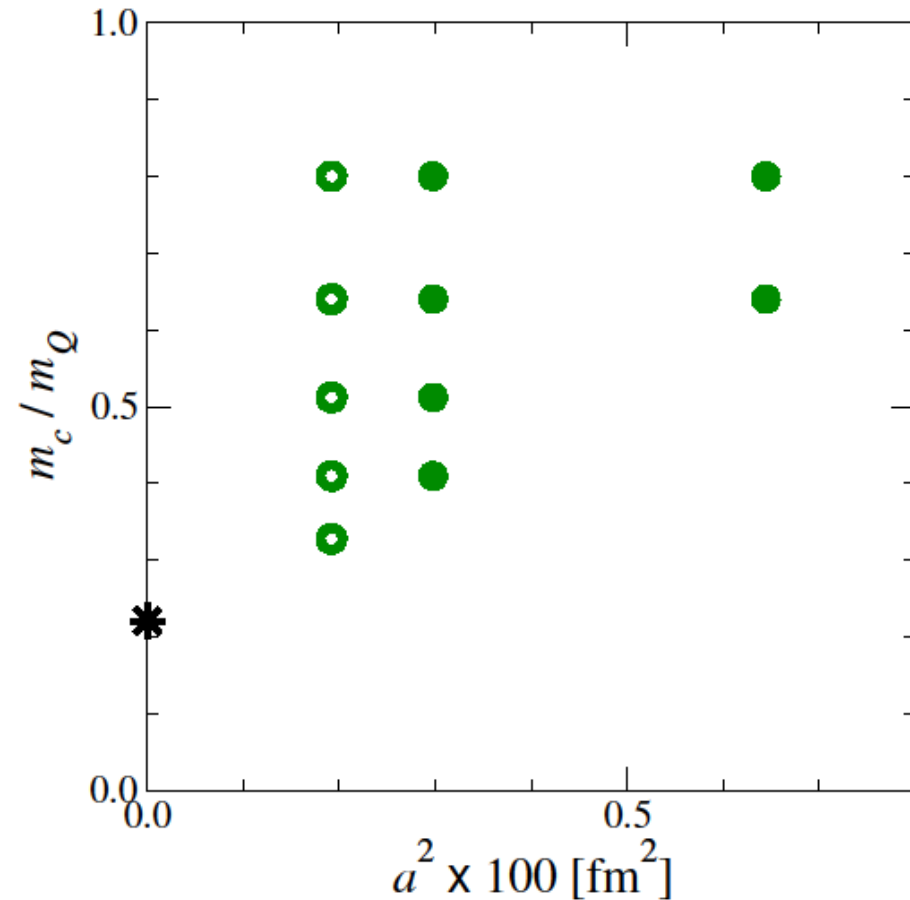
## relativistic lattice QCD

w/ "relativistic" heavy quarks

- simple renormalization
- $m_Q < m_b \Rightarrow$  need extrapolation

$$m_Q / m_c = 1.25, 1.25^2, \dots$$

$$\text{and } m_Q < 0.8 a^{-1}$$





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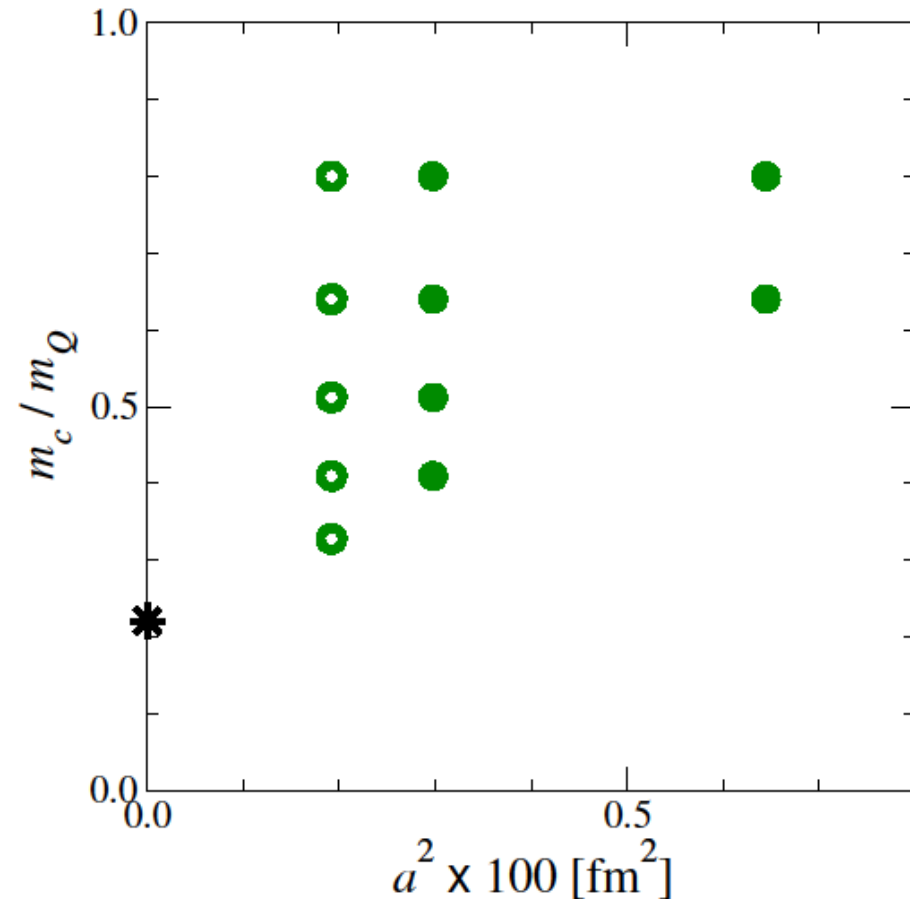
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- NRQCD, Fermilab, RHQ, ...
- need matching to QCD  
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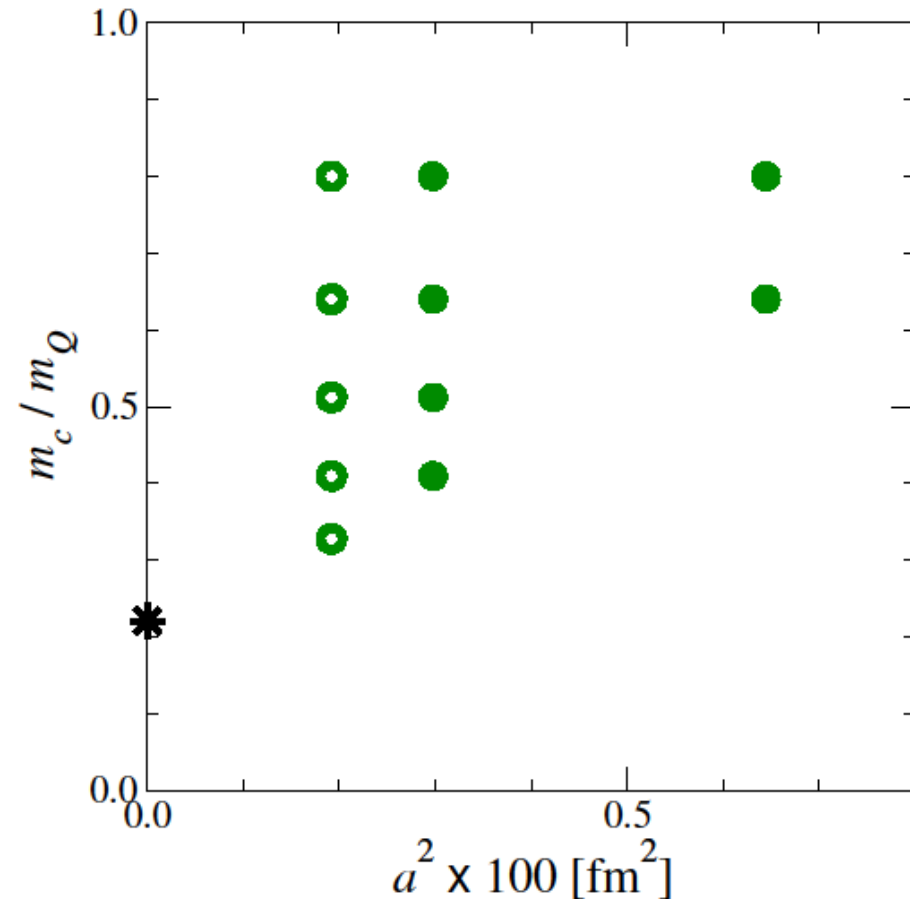
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independent studies w/ (very) different systematics

# $B \rightarrow D^{(*)} \ell \nu$ form factors (FFs)

In the SM

$$\langle D(p') | V_\mu | B(p) \rangle = (v + v')_\mu h_+(w) + (v - v')_\mu h_-(w)$$

$$\langle D^*(p', \varepsilon') | V_\mu | B(p) \rangle = i \varepsilon_{\mu\nu\rho\sigma} \varepsilon'^{* \mu} v'^\rho v^\sigma h_V(w)$$

$$\begin{aligned} \langle D^*(p', \varepsilon') | A_\mu | B(p) \rangle &= \varepsilon'_\mu (1 + w) h_{A_1}(w) \\ &\quad - \varepsilon'^* v \left\{ v_\mu h_{A_2}(w) + v_\mu h_{A_2}(w) \right\} \end{aligned}$$

# ratio method (Hashimoto et al. '99)

a standard way for precision calculation

$$\frac{\begin{array}{c} D^* \qquad B \\ \text{---} V_\mu \text{---} \\ \text{---} A_\mu \text{---} \end{array}}{\begin{array}{c} D^* \qquad B \\ \text{---} A_\mu \text{---} \\ \text{---} V_\mu \text{---} \end{array}} = \frac{\langle D^* | V_\mu^{(\text{lat})} | B \rangle}{\langle D^* | A_\mu^{(\text{lat})} | B \rangle} \rightarrow \frac{h_V(w)}{h_{A_1}(w)}$$

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$\langle B | O_B^\dagger \rangle, \exp[-M_B \Delta t], \dots$  cancel

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$$\frac{\text{Diagram with } V_\mu}{\text{Diagram with } A_\mu} = \frac{\langle D^* | V_\mu^{(\text{lat})} | B \rangle}{\langle D^* | A_\mu^{(\text{lat})} | B \rangle} \rightarrow \frac{h_V(w)}{h_{A_1}(w)}$$

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$Z_A, Z_V$  cancel

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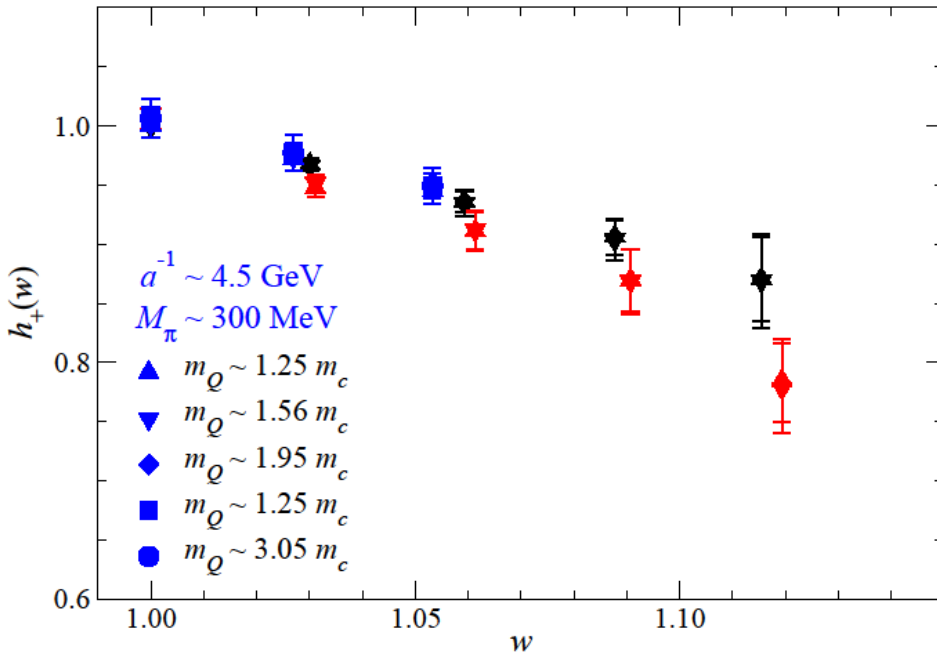
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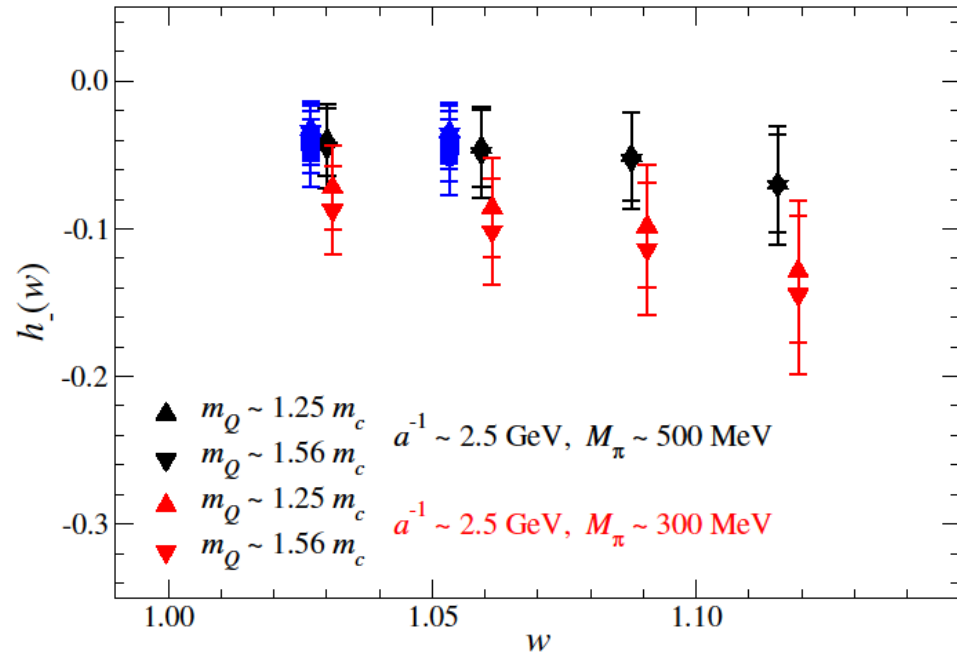
- can calculate SM FFs w/o explicit renormalization
- $\mathbf{p}_B = \mathbf{0}, |\mathbf{p}_{D^{(*)}}|^2 = 0, 1, 2, 3, 4$  in units of  $(2\pi/L)^2$

# B → Dℓν form factors

$h_+$  VS  $w$



$h_-$  VS  $w$

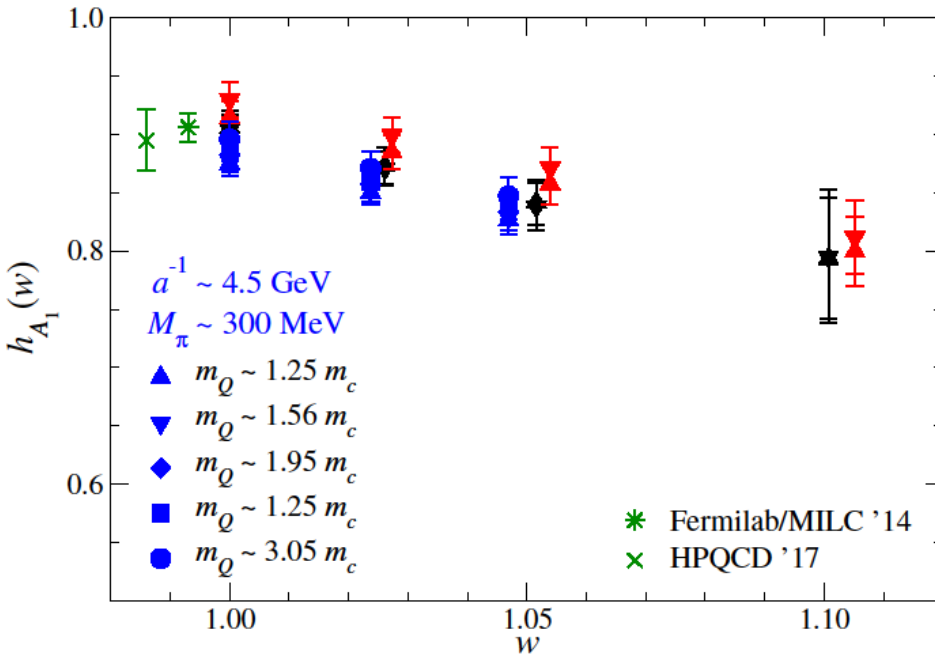


- mild dependence on  $a$ ,  $M_\pi$ ,  $m_Q \Rightarrow$  reasonably close to physical pt.  
larger  $m_Q \Rightarrow$  larger  $h_+$  [smaller  $h_-$ ]  $\Leftrightarrow L_1/2m_Q$  [ $-L_4/2m_Q$ ]  $L_1, L_4 \geq 0$
- typical accuracy:  $\Delta h_+ \leq 1 - 3\%$ ,  $\Delta h_- \sim 40 - 60\%$

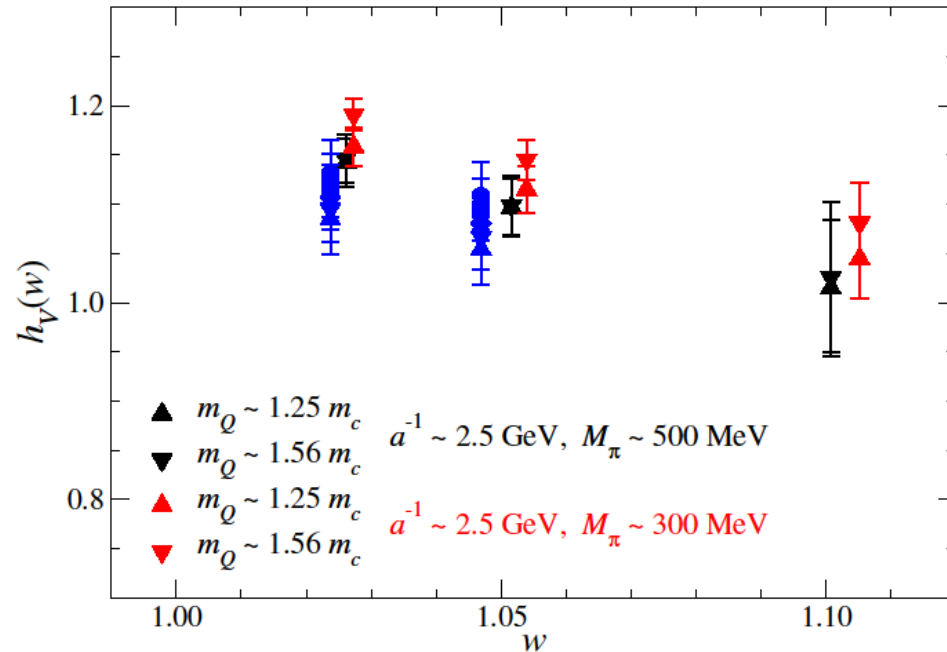


# B → D\*ℓν form factors

$h_{A_1}$  VS  $w$



$h_V$  VS  $w$

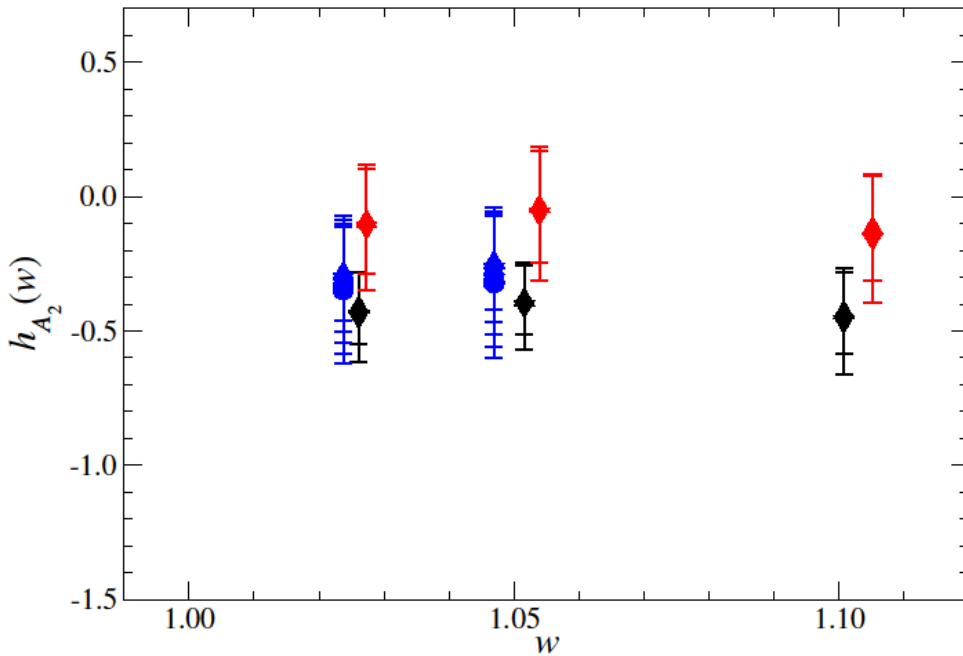


- mild  $a$ ,  $m_Q$ ,  $M_\pi$  dependences / consistent w/ previous studies
- typical accuracy:  $\Delta h_{A_1} \sim 1 - 3\%$ ,  $\Delta h_V \sim 3\%$

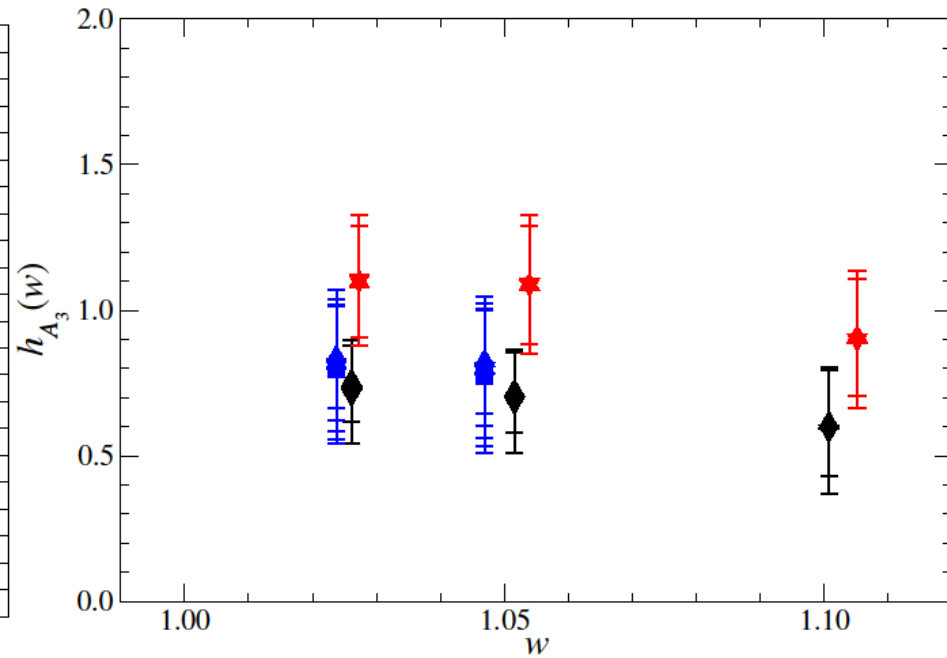
$$\langle D^*(p', \varepsilon') | V_\mu | B(p) \rangle \Rightarrow h_V(w) \quad \langle D^*(p', \varepsilon') | A_\mu | B(p) \rangle \Rightarrow h_{A_1}(w) \quad (\mathbf{p}' \perp \boldsymbol{\varepsilon}')$$

# B → D\*ℓν form factors

$h_{A_2}$  VS  $w$



$h_{A_3}$  VS  $w$



- $h_{+}, h_{A_1}, h_{A_3}, h_V (\rightarrow \xi) \sim O(1)$ ,  $h_{-}, h_{A_2} (\rightarrow 0) \sim 0$
- typical accuracy:  $\Delta h_{A_2} \geq 40\%$ ,  $\Delta h_{A_3} \sim 20 - 30\%$

$$\langle D^*(p', \varepsilon') | A_\mu | B(p) \rangle \Rightarrow \{h_{A_1}(w), h_{A_2}(w), h_{A_3}(w)\}$$

# LQCD vs HQET+QCDSR

## Caprini-Lellouch-Neubert (CLN) parametrization of FFs

- FFs w/ definite spin-parity quantum numbers

$$V_1^{(BD)} = h_+ - \frac{1-r}{1+r} h_- \quad S_1^{(BD)} = h_+ - \frac{1+r}{1-r} \frac{w-1}{w+1} h_-$$

$$A_1^{(BD^*)} = h_{A_1} \quad V_4^{(BD^*)} = h_V$$

$$A_5^{(BD^*)} = \frac{1}{1-r} \left[ (w-r) h_{A_1} - (w-1) (r h_{A_2} + h_{A_3}) \right]$$

- use NLO HQET relations (QCDSR input) ~ small NNLO in ratios

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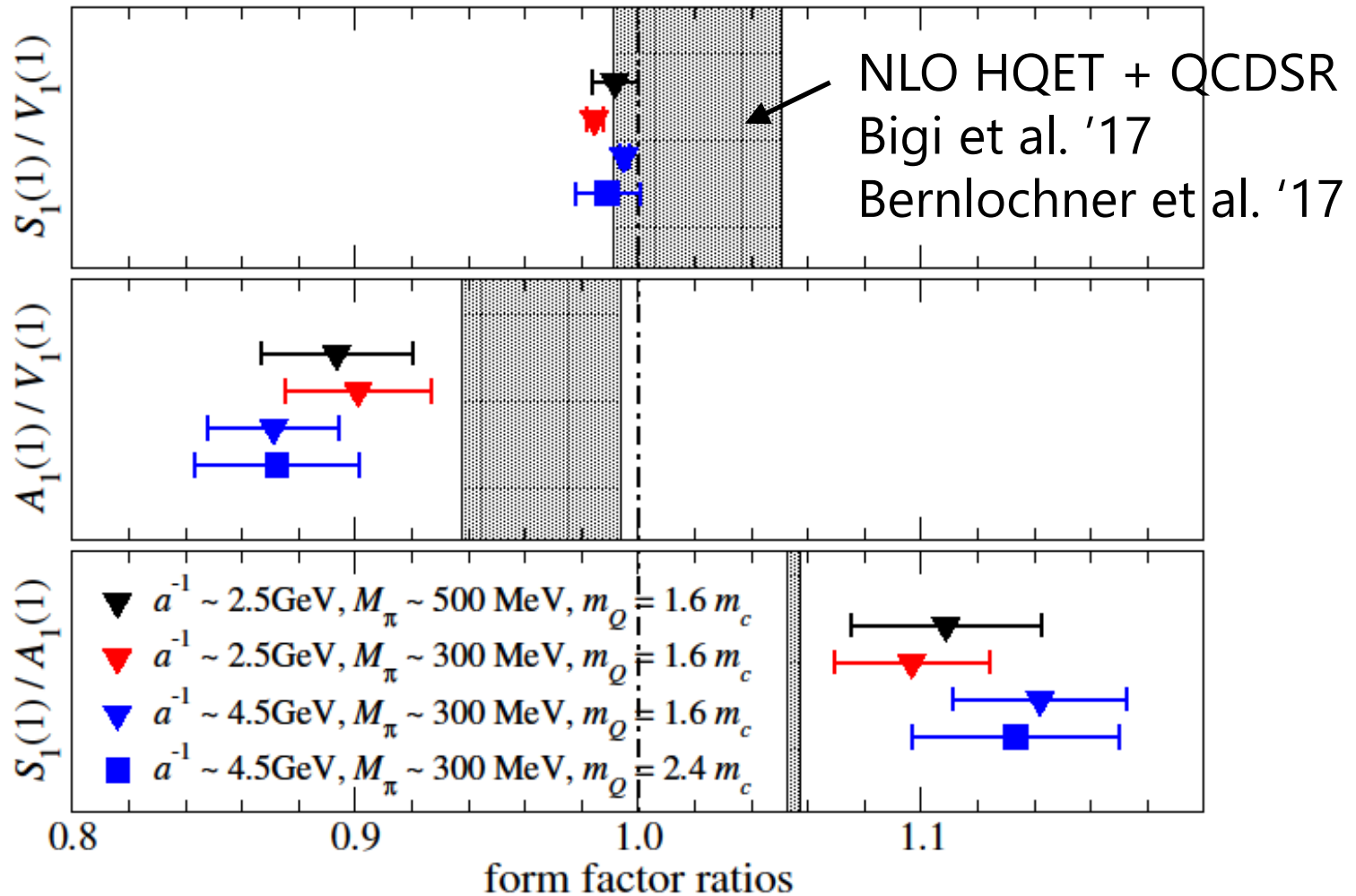
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## Bigi-Gambino-Schacht '17

- comparison b/w HQET+QCDSR and LQCD available at that time

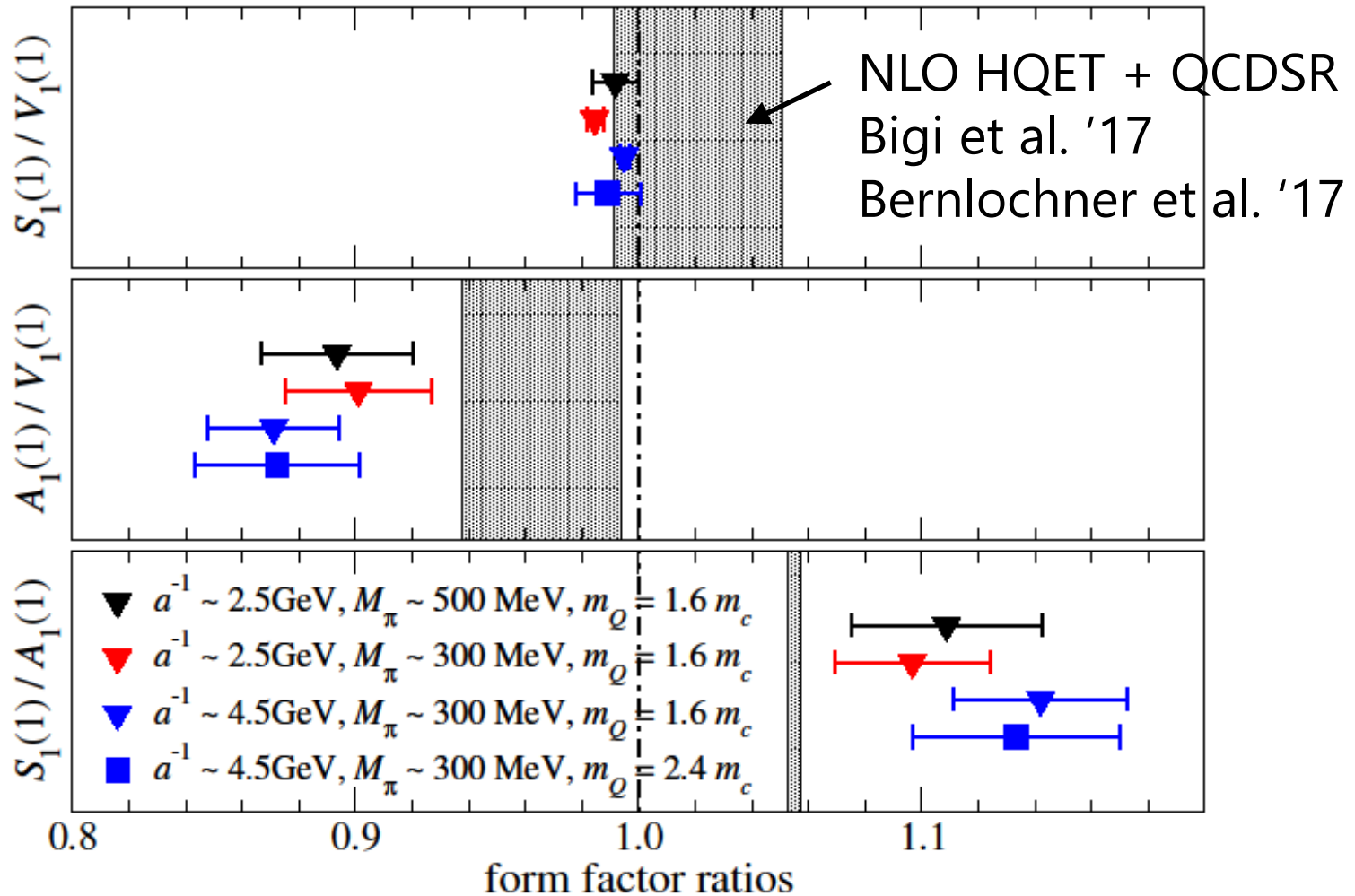
# LQCD vs HQET+QCDSR

at zero recoil



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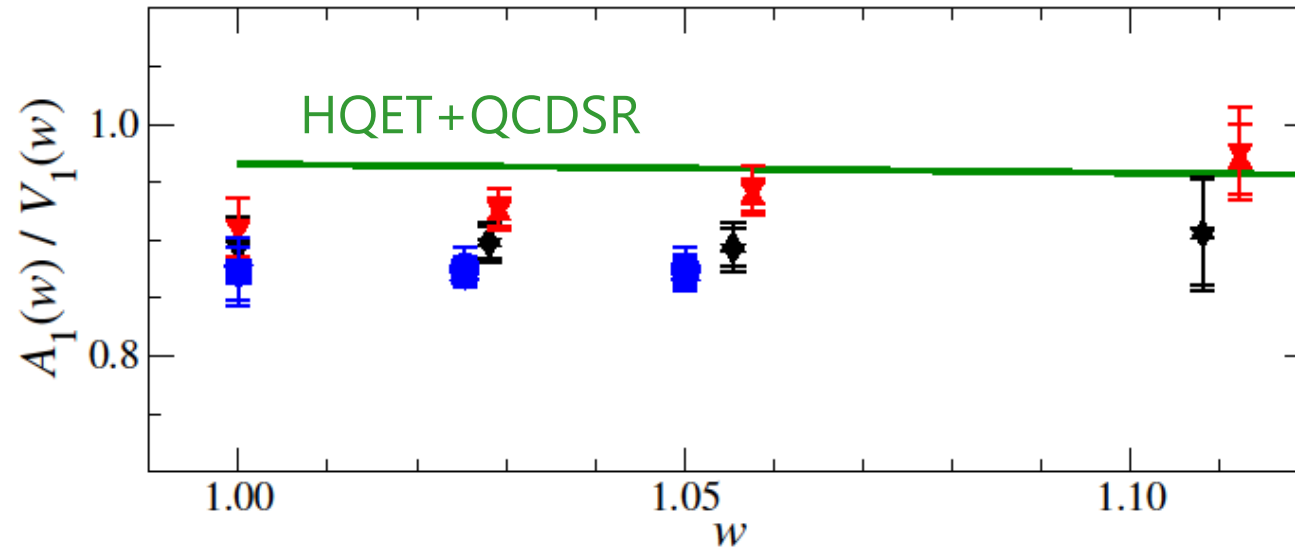
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systematically lower / higher for  $A_1/V_1, S_1/A_1$  ???

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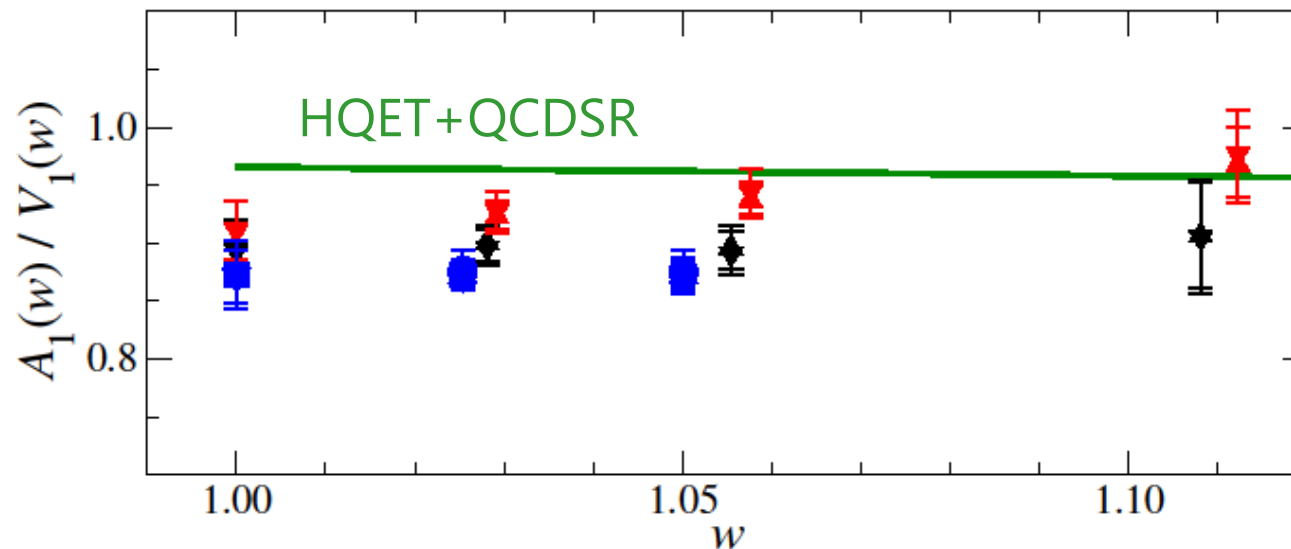
at non-zero recoils



- HQET  $A_1(w)/V_1(w) + V(w)/V(1)$  dispersive bound  $\Rightarrow$  CLN  $A_1(w)$

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- HQET  $A_1(w)/V_1(w) + V(w)/V(1)$  dispersive bound  $\Rightarrow$  CLN  $A_1(w)$
- CLN  $R_2 = (rh_{A_2} + h_{A_3}) / h_{A_1}$  : noisy at the moment
- CLN  $R_1 = h_V / h_{A_1}$   
 $\Rightarrow$  Bernlochner et al. '17: analysis of Belle unfolded / tagged data



# BGL vs CLN w/ Belle data

$$R_1 = h_V / h_{A1}$$

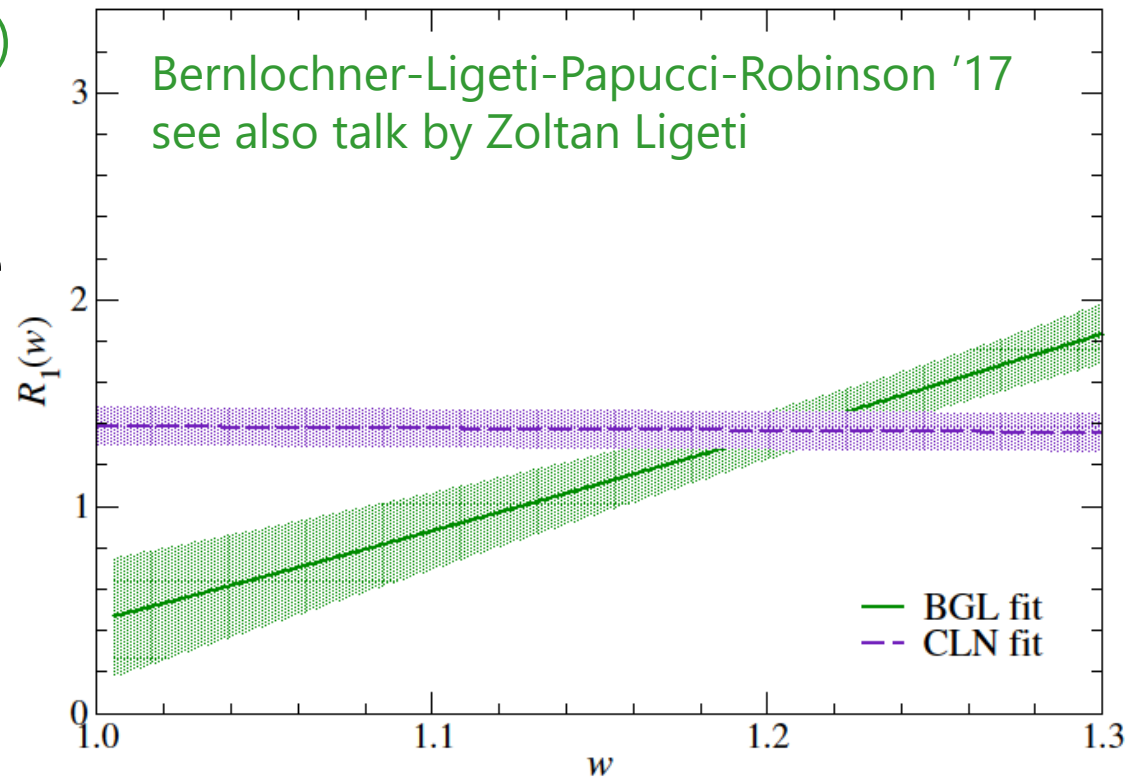
Boyd-Grinstein-Lebed (BGL)

$\Rightarrow |V_{cb}|$  close to inclusive

CLN  $\Rightarrow$  lower than inclusive

Bigi-Gambino-Schacht '17

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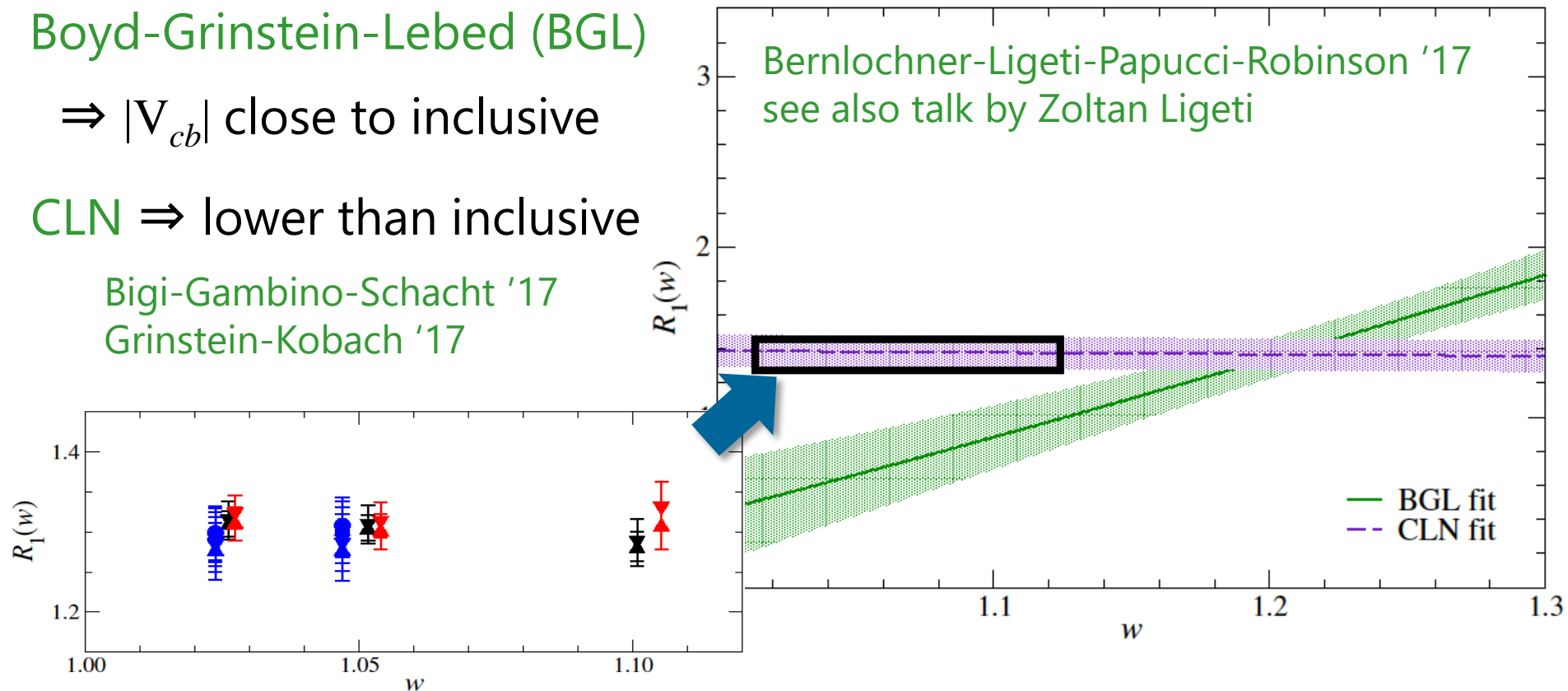
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Bernlochner-Ligeti-Papucci-Robinson '17  
see also talk by Zoltan Ligeti



new data @  $a^{-1} \sim 4.5$  GeV w/ 5  $m_Q$ 's

- $\blacktriangle \blacktriangledown \blacklozenge \blacksquare \bullet$   $a^{-1} \sim 4.5$  GeV,  $M_\pi \sim 300$  MeV,  $m_Q / m_c = 1.25, \dots, 3.05$
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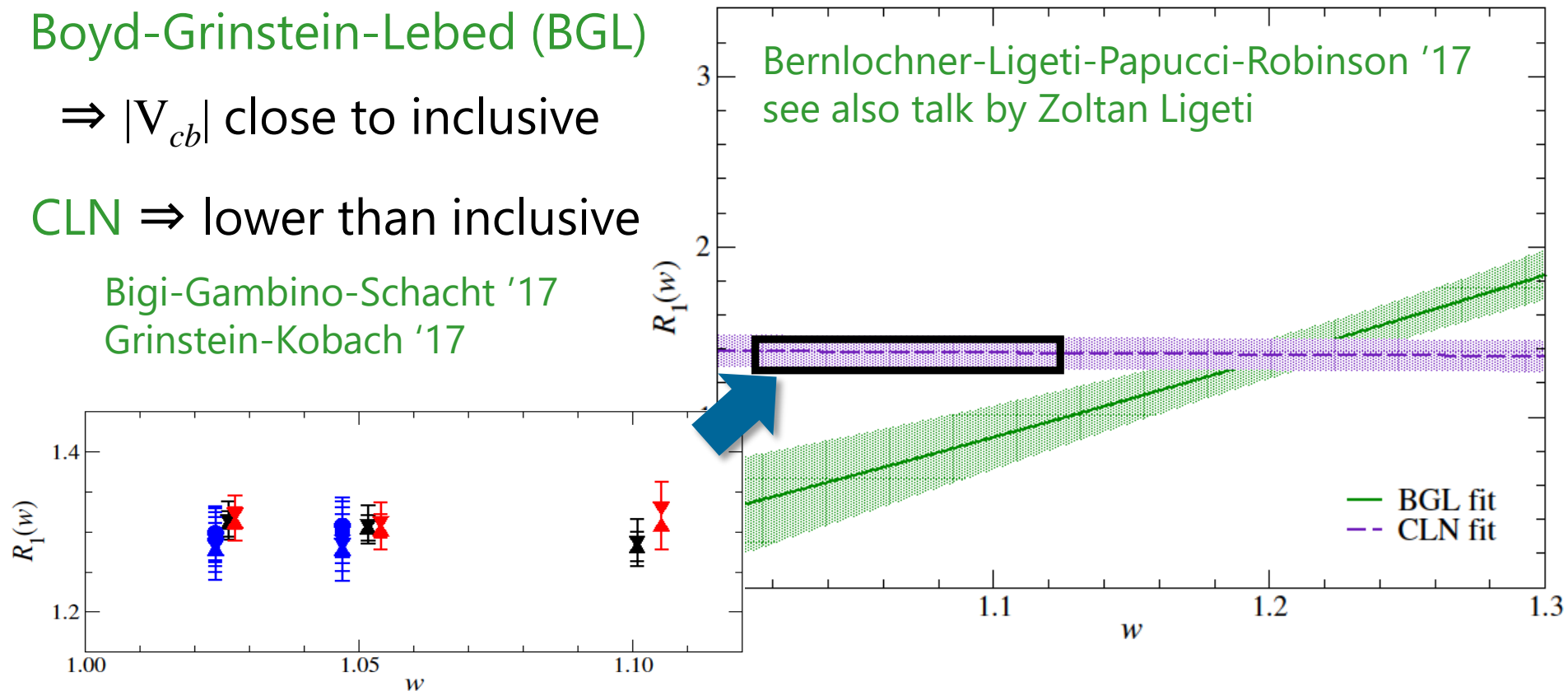
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- consistent w/ CLN and "BGL" fits
- Belle untagged ?

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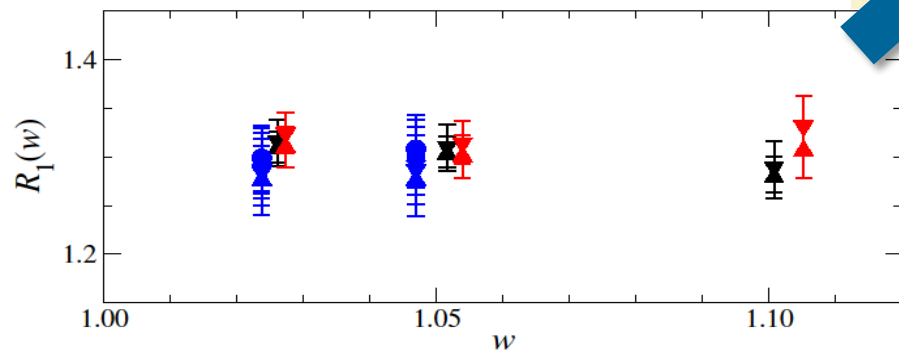
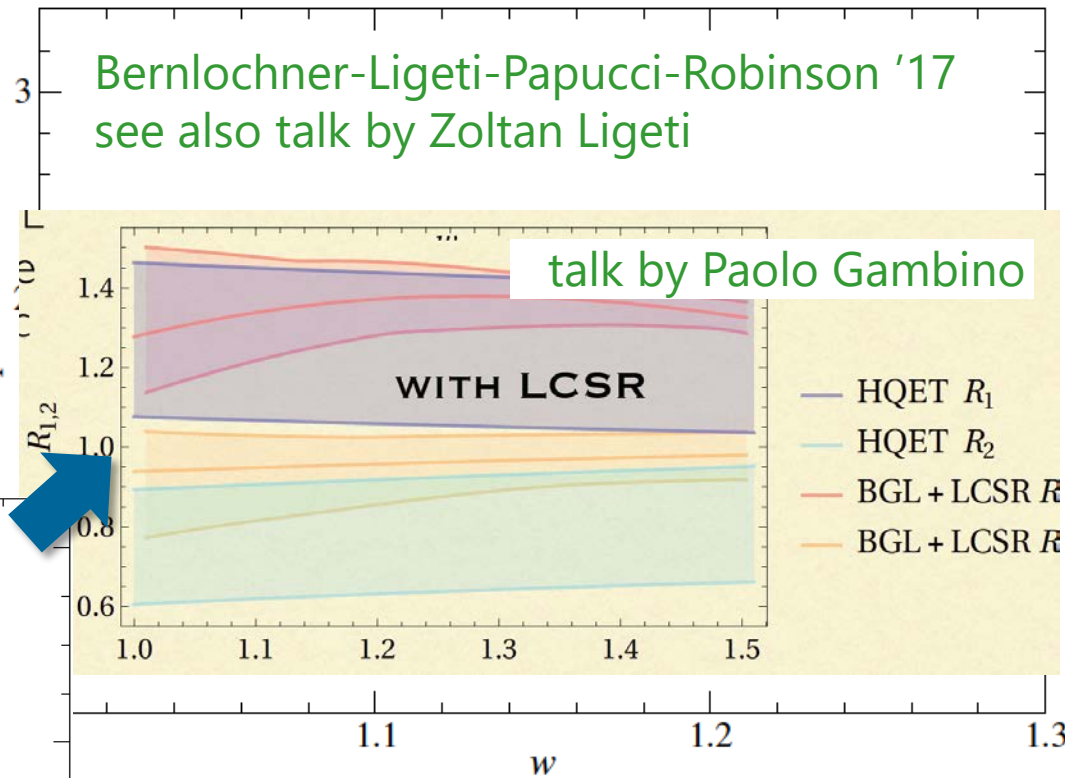
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# Summary

JLQCD's calculation of  $B \rightarrow D^{(*)} \ell \nu$  form factors

- relativistic approach w/ chiral symmetric formulation
  - $\Leftrightarrow$  previous studies: very different systematics
  - $\Rightarrow$  Hashimoto ( $B \rightarrow X_c \ell \nu$ , poster), Colquhoun ( $B \rightarrow \pi \ell \nu$ , Sat)
- extrapolation to the physical point: yet to be done
  - mild  $a$ ,  $M_{\pi'}$ ,  $m_Q$  dependences  $\Leftrightarrow$  reasonably controllable
- interplay w/ phenomenology / experiment
  - LQCD prediction of FFs  $\Rightarrow |V_{cb}|, R(D^{(*)})$
  - heavy quark scaling  $\Leftrightarrow$  data w/ different  $m_Q$ 's
  - FFs beyond the SM  $\Rightarrow$  NP search in the Belle II era