

# Theory Overview

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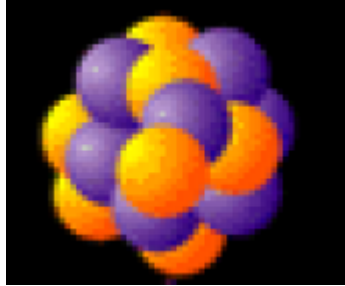
KEK/The Graduate University for Advanced Studies (Sokendai)

The 3<sup>rd</sup> International Conference on Charged Lepton Flavor  
Violation (CLFV2019)

June 17, 2019, Fukuoka, Japan

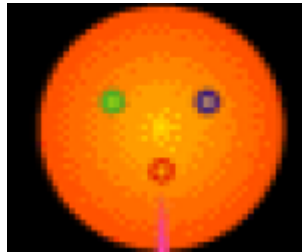
# Entering TeV scale physics

MeV



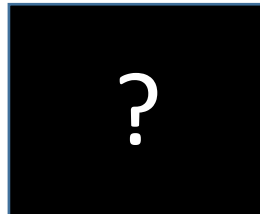
~1930: Discovery of neutrons  
Two New forces (strong, weak) are introduced

GeV



~1970: Theory of three interactions  
based on one additional unknown force  
(electroweak symmetry breaking)

TeV



~2010: Discovery of a Higgs boson  
What is the unknown force?

# The equation of the particle physic in the 20<sup>th</sup> century

The Fermi constant

The Higgs VEV

$$G_F = \frac{1}{\sqrt{2}v^2}$$

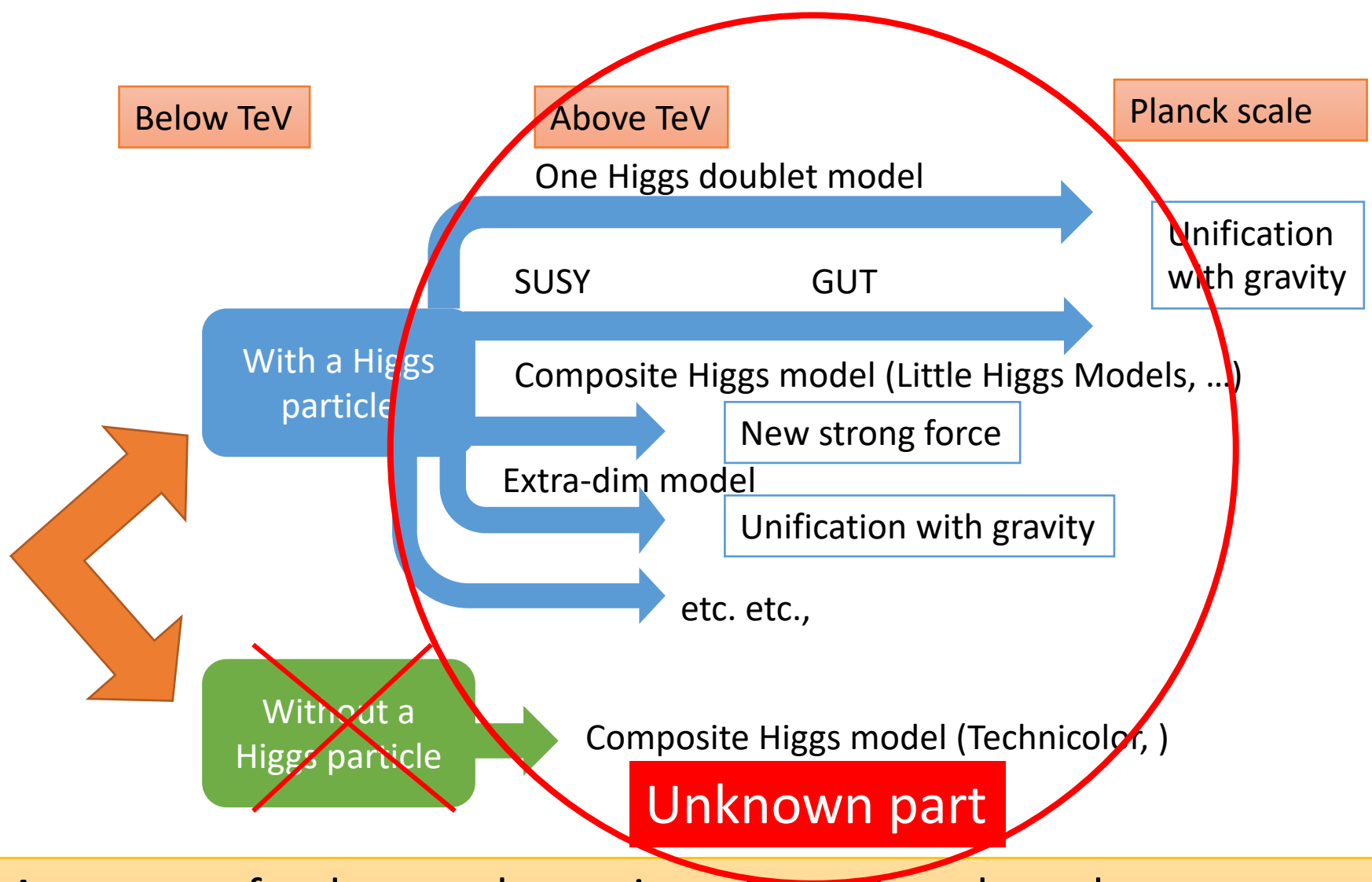
Nambu's Symmetry Breaking

The Higgs vacuum expectation value is determined by the mass and the lifetime of muons

$$\tau_\mu^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

# Particle physics after the Higgs discovery

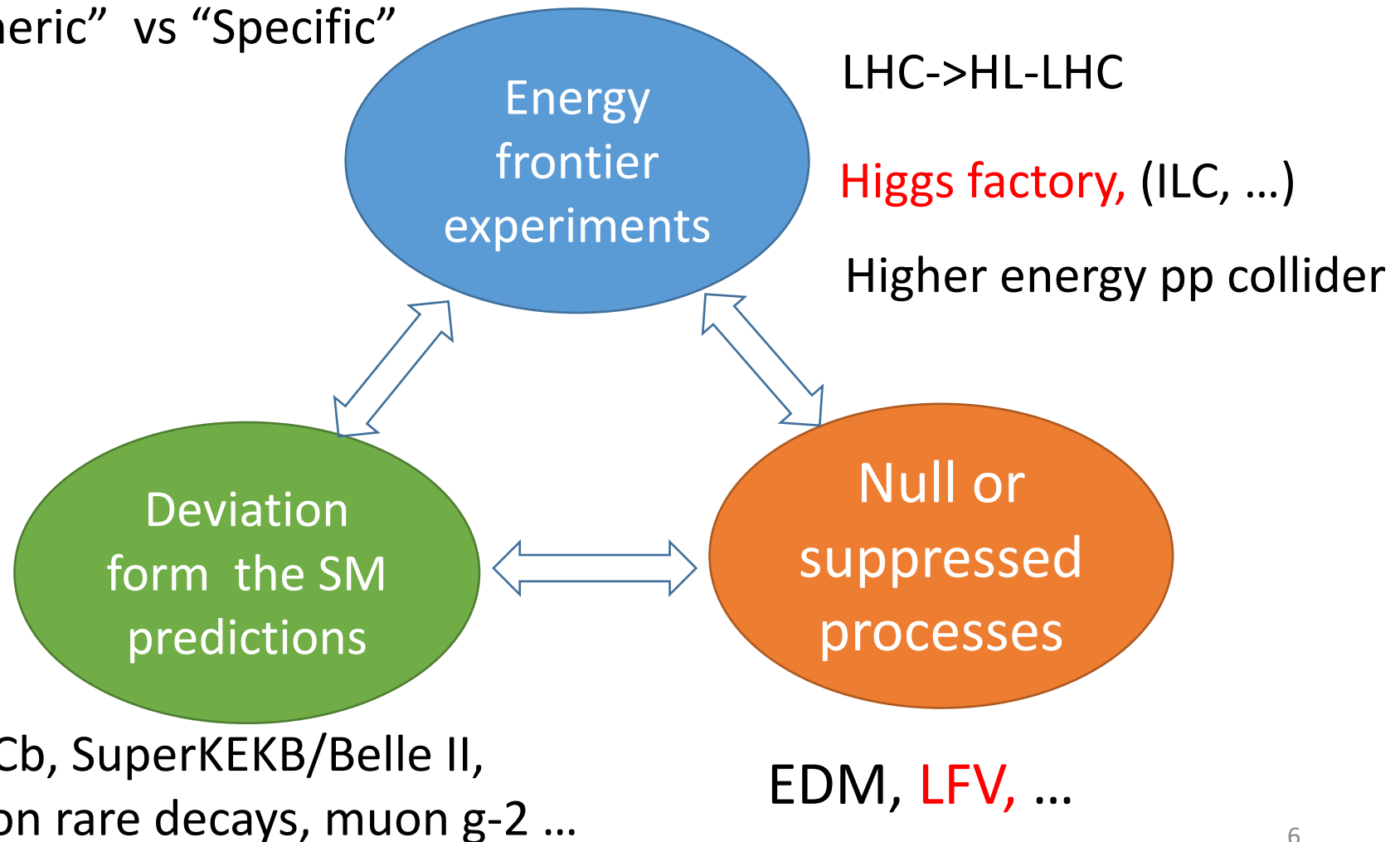
- Enter a new chapter of the particle physics
- The Higgs particle itself is a tool to explore New Physics.
- There are no clear signals beyond the Higgs particle at the LHC experiment.
- Indirect ways to look for new physics becomes more and more important.



Answers to fundamental questions depend on the unknown part.  
What is dark matter?  
How the matter and anti-matter asymmetry was generated?  
Why the neutrino masses are so small? etc.

# Approaches to new physics

“Generic” vs “Specific”



# Why do we believe “new things”?

LHC =TeV physics  
=Electroweak  
symmetry breaking

Something beyond the  
known three gauge  
interaction is  
necessary.

Other puzzles

Origin of the neutrino  
mass

Baryon number of the  
Universe

Dark matter

....

Lepton number  
violation

Lepton flavor  
violation

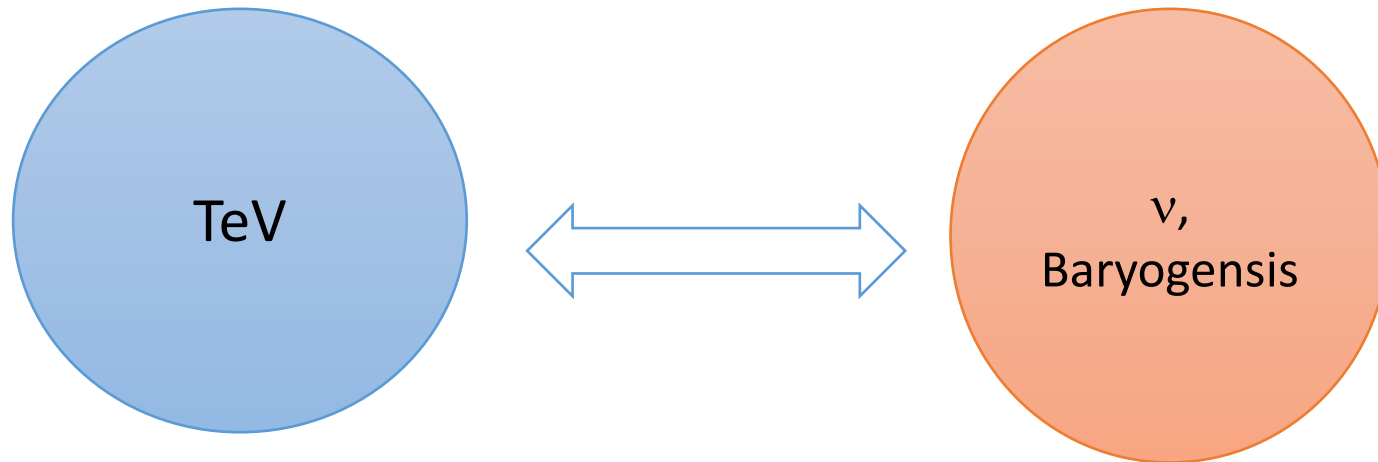
New CP violation

Relevant scale is unknown.

# An important role of Lepton Flavor Violation and EDM searches

- EDM and LFV searches can provide a hint on the relationship between two scales.

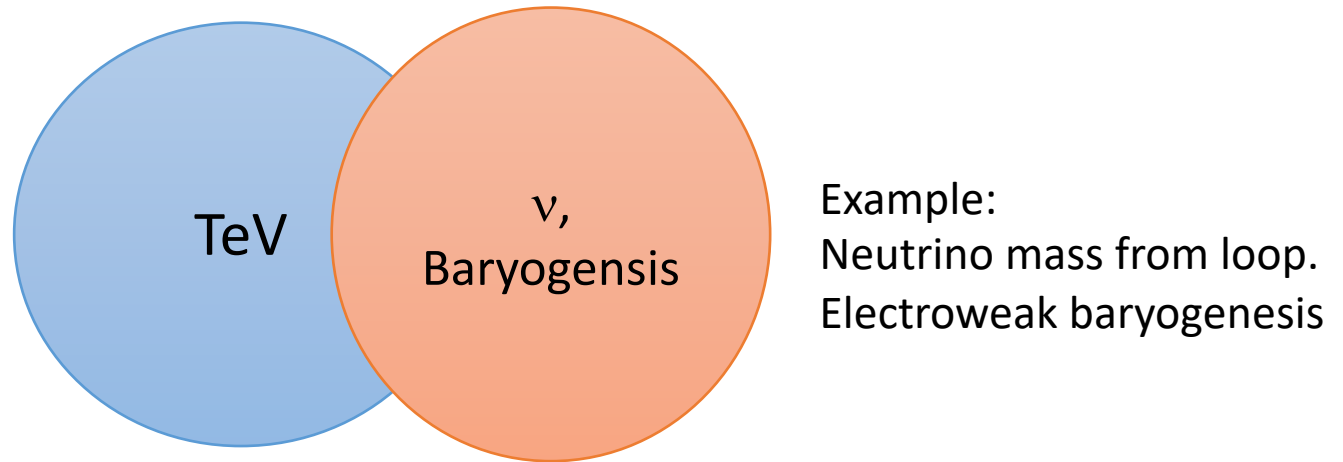
If two scales are well separated,  
EDMs and LFV are suppressed.



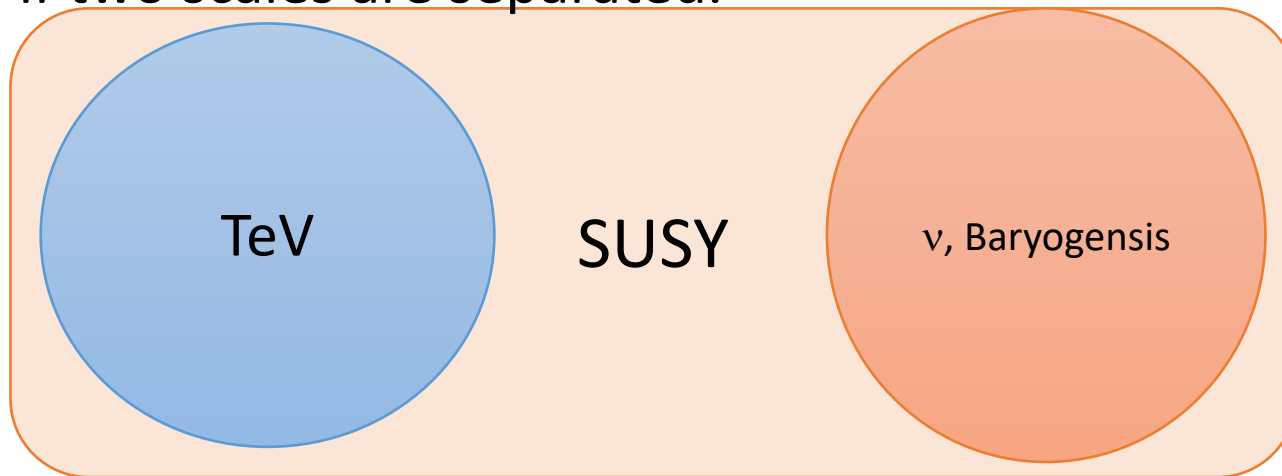
Seesaw neutrino model  
Leptogenesis



If two scales are close, large EDMs and LFV are expected.



In supersymmetric models, large EDMs and LFV are expected even if two scales are separated.



Existence /absence of EDMs and LFV is a clue to fundamental problems such as neutrino mass generation and baryogenesis.

# Three lepton processes: Naïve scaling

g-2

$$\Delta(g - 2)_i \propto m_i^2$$

Current precision  
 $a_e : 0.7 \text{ppb}$ ,  $a_\mu : 0.6 \text{ppm}$   
 $-0.052 < a_\tau < 0.013$

muon g-2 is most sensitive to New Physics.

EDM

$$d_i \propto m_i$$

Current bounds  
eEDM  $O(10^{-27})$   
 $\mu$ EDM  $O(10^{-19})$   
 $\tau$ EDM  $O(10^{-17})$

Electron is most constraining.

LFV

$$B(l_j \rightarrow l_j X) \propto (m_i)^0$$

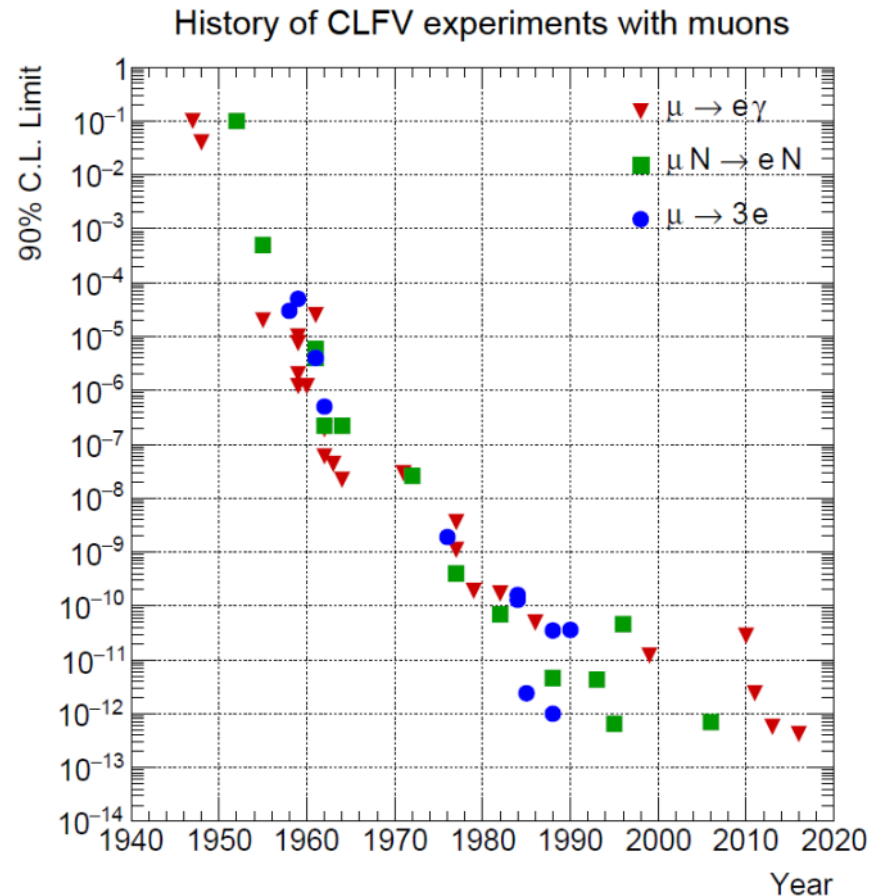
Current bounds  
 $B(\mu \text{LFV}) < O(10^{-13}) - O(10^{-11})$   
 $B(\tau \text{LFV}) < O(10^{-8}) - O(10^{-7})$

Many examples of exceptional cases.

No apparent lepton mass dependence.  
Sensitive to flavor mixing structure.

# Lepton Flavor Violation in Charge Lepton processes

- LFV in charged lepton processes is negligibly small for the Standard Model with simple seesaw neutrinos or Dirac neutrinos.
- Observation of the cLFV is a clear evidence of new physics.



From arXiv:1801.04688

# Three muon LFV processes

$$\mu^+ \rightarrow e^+ \gamma$$

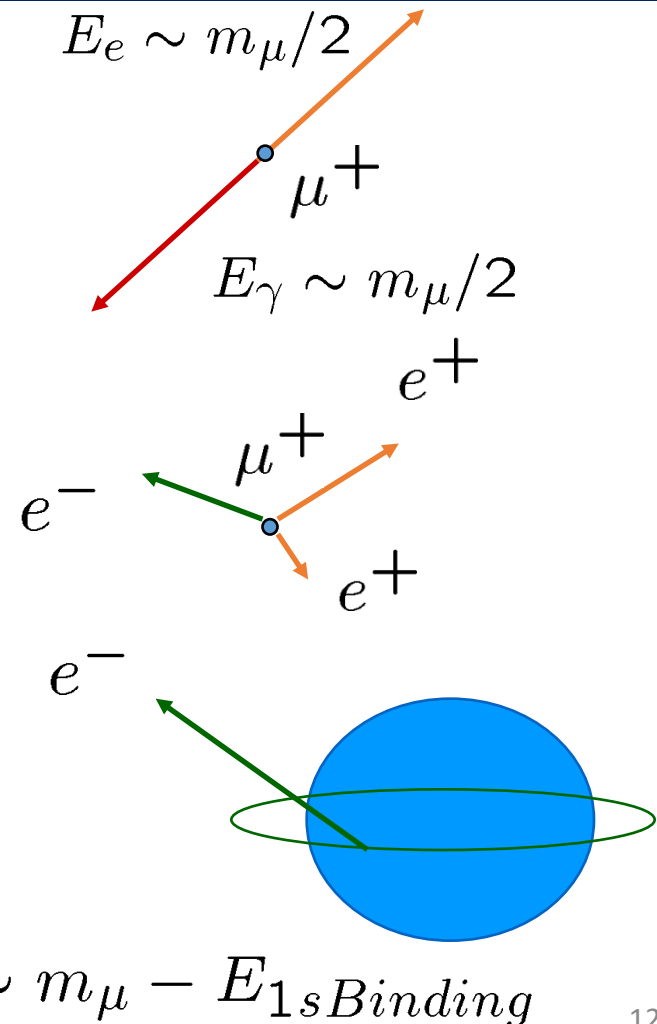
$$B < 4.2 \times 10^{-13} \text{ (MEG)}$$

$$\mu^+ \rightarrow e^+ e^+ e^- \text{ (SINDRUM)}$$

$$B < 1.0 \times 10^{-12}$$

$$\mu^- A \rightarrow e^- A \text{ (SINDRUMII)}$$

$$B < 7 \times 10^{-13} (Au)$$



# Tau LFV processes

Various flavor structures. Many searches can be carried out simultaneously at  $e^+e^-$  colliders.

$$(1) \quad \tau \rightarrow \mu\gamma, \tau \rightarrow e\gamma \quad \longleftrightarrow \quad \mu \rightarrow e\gamma$$

$$(2) \quad \tau \rightarrow lll \quad \longleftrightarrow \quad \mu \rightarrow 3e$$

$$\tau^+ \rightarrow \mu^- \mu^+ \mu^+, \tau^+ \rightarrow \mu^- e^+ e^+,$$

$$\tau^+ \rightarrow \mu^- \mu^+ e^+, \tau^+ \rightarrow e^- \mu^+ \mu^+,$$

$$\tau^+ \rightarrow e^- e^+ e^+, \tau^+ \rightarrow e^- \mu^+ e^+,$$

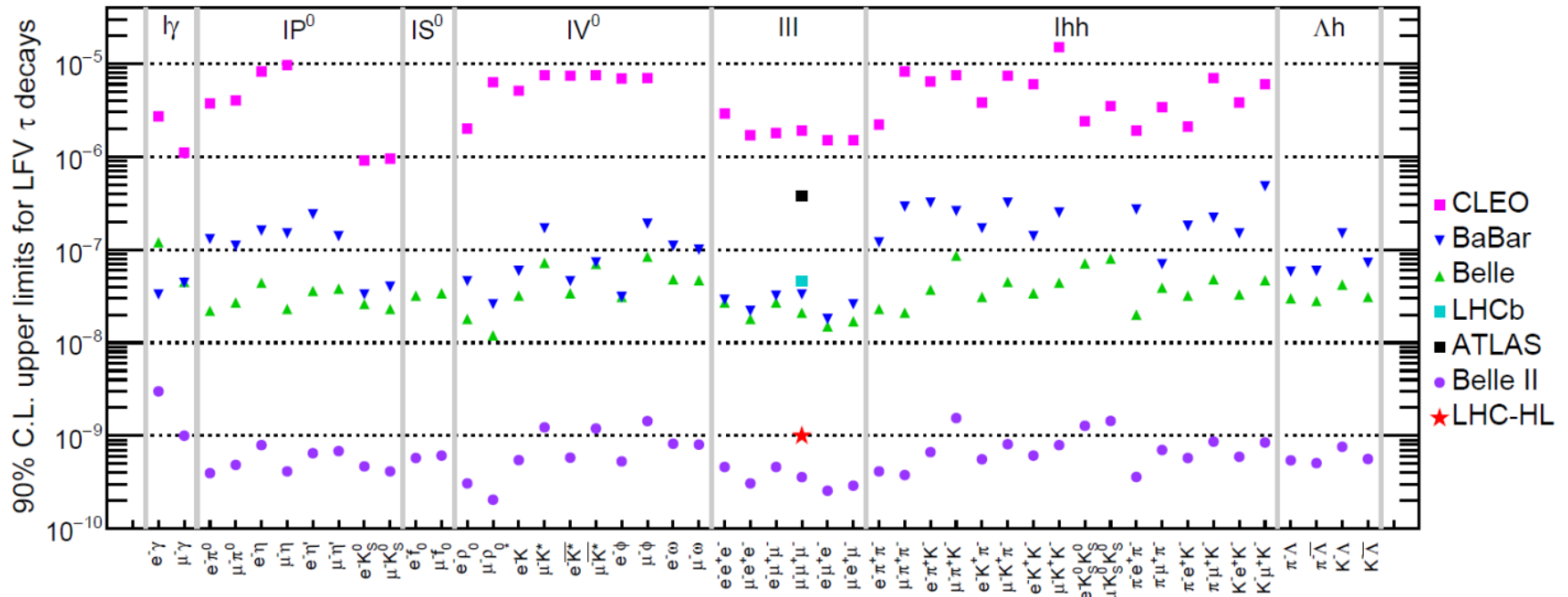
and their CP conjugates

$$(3) \quad \tau \rightarrow \mu\eta, \mu\pi, \mu\rho, \mu\phi, \text{etc.} \quad \longleftrightarrow \quad \mu A \rightarrow eA$$

$$\tau \rightarrow e\eta, e\pi, e\rho, e\phi, \text{etc.}$$

Distinguishing different operators.

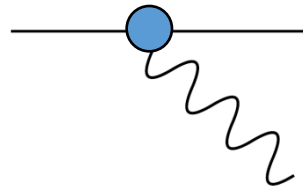
## Tau LFV bounds and prospects



From arXiv:1812.07638

# Effective interactions

$$\mu^+ \rightarrow e^+ \gamma$$

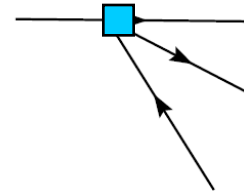
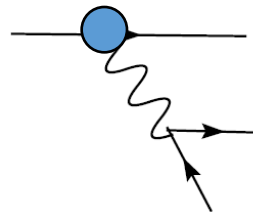


$$-\frac{4G_F}{\sqrt{2}}(m_\mu A_R \bar{\mu} \sigma^{\mu\nu} P_L e F_{\mu\nu} + m_\mu A_L \bar{\mu} \sigma^{\mu\nu} P_R e F_{\mu\nu} + \text{H.c.})$$

$$\mu^+ \rightarrow e^+ \gamma_R$$

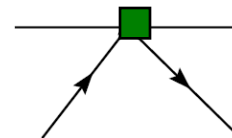
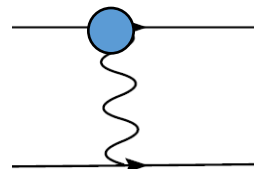
$$\mu^+ \rightarrow e^+ \gamma_L$$

$$\mu^+ \rightarrow e^+ e^+ e^-$$



6 additional operators

$$\mu^- N \rightarrow e^- N$$



Various llqq operators

# Relations between different LFV branching ratios

If the photon penguin process is dominant, there are simple relations among these branching ratios.

$$\begin{aligned} B(\mu \rightarrow 3e) &\sim 6.1 \times 10^{-3} B(\mu \rightarrow e\gamma) \\ B(\mu Ti \rightarrow eTi) &\sim 4.0 \times 10^{-3} B(\mu \rightarrow e\gamma) \\ B(\mu Al \rightarrow eAl) &\sim 2.6 \times 10^{-3} B(\mu \rightarrow e\gamma) \\ B(\tau \rightarrow 3\mu) &\sim 2.2 \times 10^{-3} B(\tau \rightarrow \mu\gamma) \end{aligned}$$

In many case of SUSY modes, this is true.

Other cases:

Additional Higgs exchange diagram (SUSY with large  $\tan \beta$ )

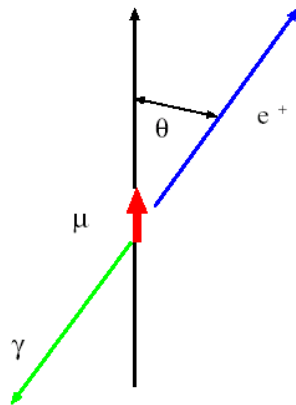
Dominance of tree exchange diagrams (LR symmetric models, etc.)

Loop-induced but Z-penguin dominance (Little Higgs with T-parity)



# Muon polarization and LFV processes

- If the muon is polarized, we can define a P-odd asymmetry for  $\mu \rightarrow e \gamma$  and T-odd and P-odd asymmetries for  $\mu \rightarrow 3e$ . These asymmetries are useful to discriminate different models.



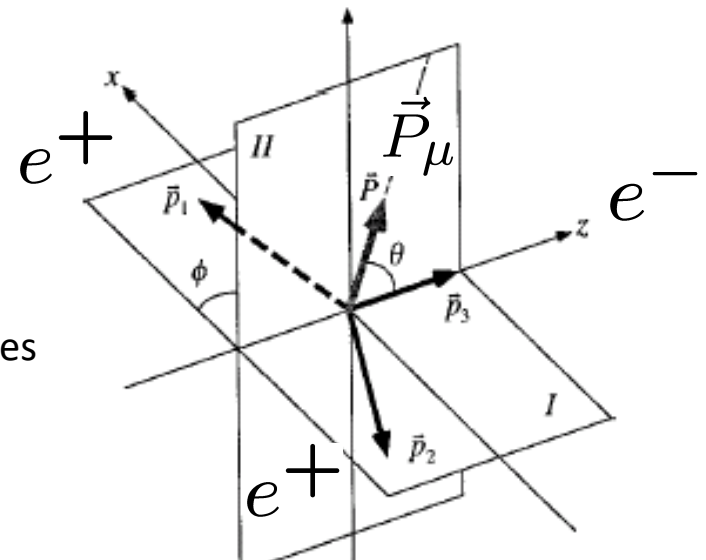
Two P-odd and one T-odd asymmetries

$$\frac{dB(\mu^+ \rightarrow e^+ \gamma)}{d \cos \theta} \propto 1 + A_{\mu \rightarrow e \gamma} P_\mu \cos \theta$$

Example :  $A = -1$  for the SUSY seesaw model

Left-handed slepton mixing  $\Rightarrow$

$$\mu^+ \rightarrow e^+ \gamma_R \Rightarrow (1 - \cos \theta) \text{ distribution}$$



$$A_{P_1} = \frac{N(P_z > 0) - N(P_z < 0)}{N(P_z > 0) + N(P_z < 0)}$$

$$A_{P_2} = \frac{N(P_x > 0) - N(P_x < 0)}{N(P_x > 0) + N(P_x < 0)}$$

$$A_T = \frac{N(P_y > 0) - N(P_y < 0)}{N(P_y > 0) + N(P_y < 0)}$$

# $\mu \rightarrow e \gamma$ and $\mu \rightarrow 3e$ asymmetries in SUSY models

## P and T-odd asymmetries in minimal SUSY GUT models

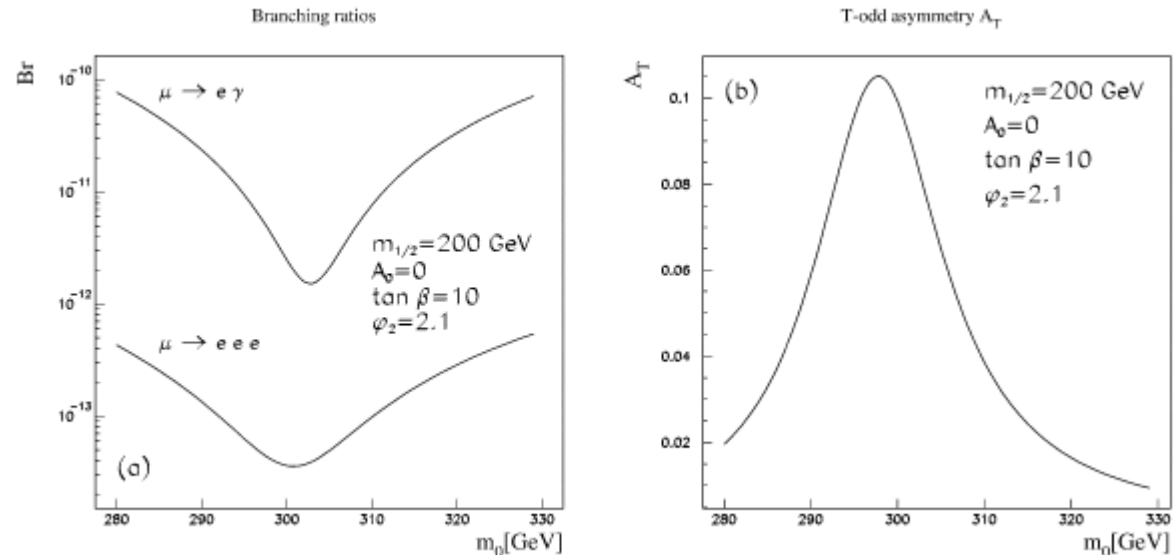
The T-odd asymmetry can be 10 % level for some parameter space of the SU(5) SUSY GUT and the SUSY seesaw model.

	SU (5)	SO (10)
$A_{\mu \rightarrow e \gamma}$	+100%	-100% – +100%
$A_{P_1}$	-30% – +40 %	$\simeq -A_{\mu \rightarrow e \gamma}/10$
$A_{P_2}$	-20% – +20 %	$\simeq -A_{\mu \rightarrow e \gamma}/6$
$ A_T $	$\lesssim 15\%$	$\lesssim 0.01\%$

Y.Okada,K.Okumura,and Y.Shimizu, 2000

## T-odd asymmetry in the SUSY seesaw model

Information on lepton sector CP violation



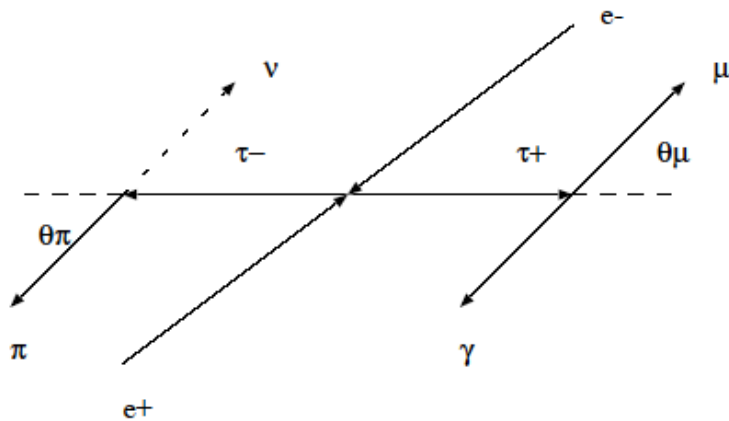
J.Ellis,J.Hisano,S.Lola, and M.Raidal, 2001

# “Polarized” tau decay

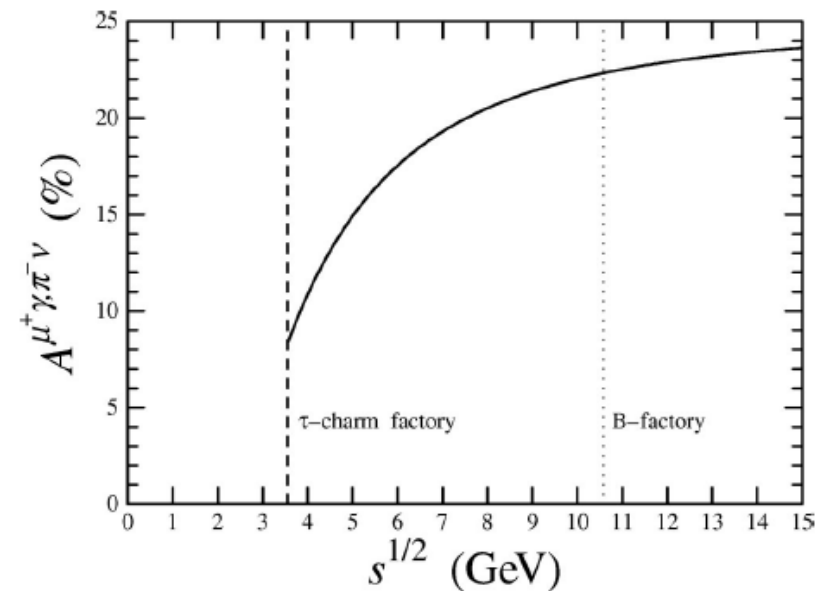
- Angular correlation of tau decay products at  $e^+e^-$  colliders.

Example: 
$$\frac{dB(e^+e^- \rightarrow \tau^+\tau^- \rightarrow \mu^+\gamma + \pi^-\nu)}{d\cos\theta_\mu d\cos\theta_\pi}$$
  

$$\propto 1 - A_{\tau \rightarrow \mu\gamma} \frac{s - 2m_\tau^2}{s + 2m_\tau^2} \cos\theta_\mu \cos\theta_\pi$$

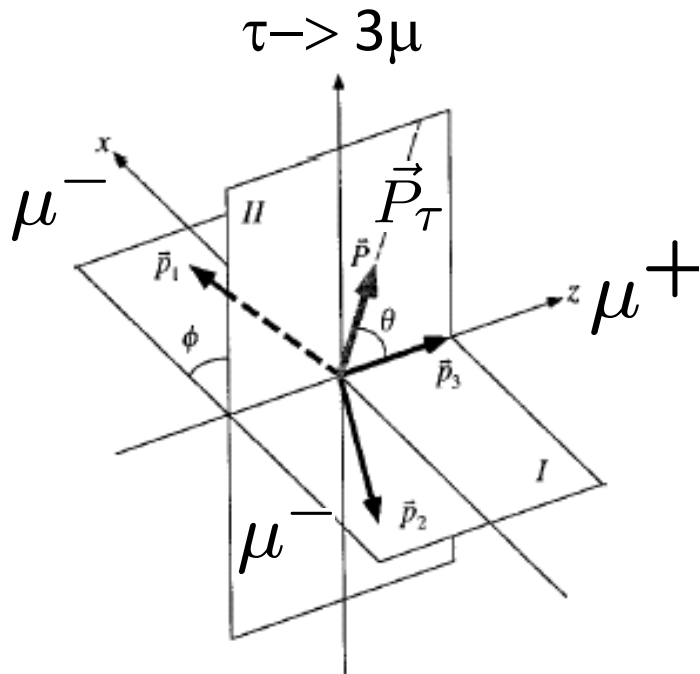


R.Kitano and Y.Okada 2001



Asymmetry for the SUSY seesaw model ( $A=-1$ )

- At LHC, taus from W decays are polarized. We can use asymmetry observables to distinguish different models in  $\tau \rightarrow 3\mu$  decays.



Model	$\frac{1}{\Gamma} \frac{d\Gamma}{d\cos\Theta} \propto \frac{1}{2} + A\cos\Theta$
MSSM with seesaw mechanism	$A = 1/6$
MSSM with $R$ -parity violation:	
“ $R$ ” ( $\lambda_{i22}\lambda_{i23}^* \gg \lambda_{i32}\lambda_{i22}^*$ )	$A = 1/6$
“ $L$ ” ( $\lambda_{i22}\lambda_{i23}^* \ll \lambda_{i32}\lambda_{i22}^*$ )	$A = -1/6$
Littlest Higgs model with $T$ -parity:	
“ $Z$ ” ( $\tilde{Z}_{\text{odd}}^{\tau\mu} \gg \tilde{Y}_{\mu,\text{odd}}^{\tau\mu}$ )	$A = 5/18$
“ $Y$ ” ( $\tilde{Z}_{\text{odd}}^{\tau\mu} \ll \tilde{Y}_{\mu,\text{odd}}^{\tau\mu}$ )	$A = 1/2$
Topcolor-assisted technicolor:	
“ $Z$ ” ( $m_{\pi_t} \gg m_{Z'}$ )	$A = -5/14$
“ $P$ ” ( $m_{\pi_t} \ll m_{Z'}$ )	$A = 0$
Models with doubly charged Higgs bosons:	
Higgs-triplet model	$A = 1/2$
Zee-Babu model	$A = -1/2$

M.Giffels, J.Kallarackal, M.Kramer, B.O'Leary and A.Stahl, 2008

# Calculation of the mu-e conversion rate

- The first calculation of the mu-e conversion rate was done by S. Weinberg and G. Feinberg in 1959.
- O. Shanker made extensive calculations for all interactions based on relativistic wave functions of muon and electrons in 1979.
- A. Czarnecki, W.J. Marciano, and K. Melnikov improved the calculation for selected atoms in 1998.
- Detailed calculations for various nuclei was presented by R. Kitano, M. Koike and Y. Okada in 2002.

# Operators relevant to the coherent mu-e conversion

$$\begin{aligned}
 \mathcal{L}_{\text{eff}}^{(q')} = & -\frac{1}{\Lambda^2} \left[ (C_{DR} m_\mu \bar{e} \sigma^{\rho\nu} P_L \mu + C_{DL} m_\mu \bar{e} \sigma^{\rho\nu} P_R \mu) F_{\rho\nu} \right. && \text{Photonic dipole} \\
 & + \sum_{q=u,d,s} \left( C_{VR}^{(q)} \bar{e} \gamma^\rho P_R \mu + C_{VL}^{(q)} \bar{e} \gamma^\rho P_L \mu \right) \bar{q} \gamma_\rho q && \text{Vector} \\
 & + \sum_{q=u,d,s} \left( C_{SR}^{(q)} m_\mu m_q G_F \bar{e} P_L \mu + C_{SL}^{(q)} m_\mu m_q G_F \bar{e} P_R \mu \right) \bar{q} q && \text{Scalar} \\
 & + \left. (C_{GQR} m_\mu G_F \bar{e} P_L \mu + C_{GQL} m_\mu G_F \bar{e} P_R \mu) \frac{\beta_L}{2g_s^3} G_a^{\rho\nu} G_{\rho\nu}^a + h.c. \right] && \text{gluonic}
 \end{aligned}$$

The gluonic operator can arise by heavy quark loop diagrams.

The gluonic coupling to a nucleon can be expressed by scalar quark densities in a nucleon.

The  $\mu$ -e conversion rate is defined

$$B \equiv \frac{\omega(\mu N(A, Z) \rightarrow e N(A, Z))}{\omega(\mu N(A, Z) \rightarrow \text{capture})}$$

Schematically,

$$\begin{aligned} & \omega(\mu N(A, Z) \rightarrow N(A, Z)) \\ & \sim \left| \sum \int d^3x \bar{\psi}_e \Gamma \psi_\mu^{(1S)} \rho^{(p,n)} \right|^2 \end{aligned}$$

Calculation goes in the following steps:

- (1) Take a matrix element of quark operators in a proton/a neutron state.
- (2) Sum over all the protons and neutrons in a nucleus coherently.
- (3) Evaluate overlap integrals of the above type.

# Theoretical uncertainty depends on a type of operators

## (1) Photonic dipole case: Almost no uncertainty

The calculation only depends on the charge distribution in a nucleus, which is precisely known by electron scattering.

## (2) Vector case:

The main uncertainty comes from the neutron density.

Little uncertainty for light nuclei.

Uncertainty is 5% level for heavy nuclei if the proton scattering data is available (ex. Pb).

## (3) Scalar case:

An addition source of uncertainty is scalar quark densities in a nucleon.

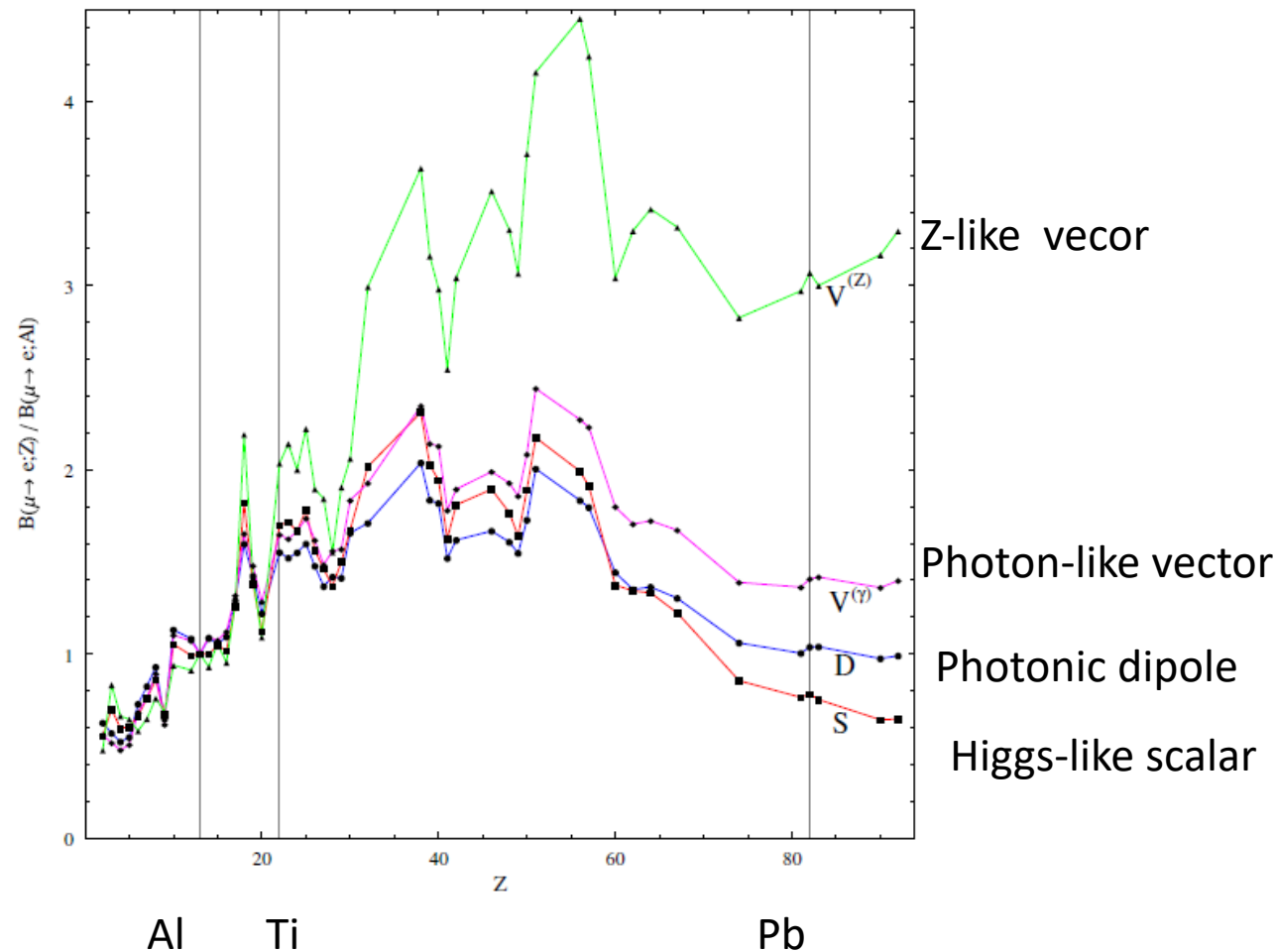
Lattice QCD calculation reduced uncertainty associated with strange quark scalar density.

V. Cirigliano, R. Kitano, Y. Okada, and P. Tuson, 2009

A. Crivellin, M. Hoferichter and M. Procura, 2014



# Atomic number dependence of the mu-e conversion rate for various LFV operators



- Maximal in the intermediate nuclei.
- Different  $Z$  dependence for heavy nuclei.
- Large enhancement in the Z-like vector case (neutron-rich for heavy nuclei).

# Implications to new physics models

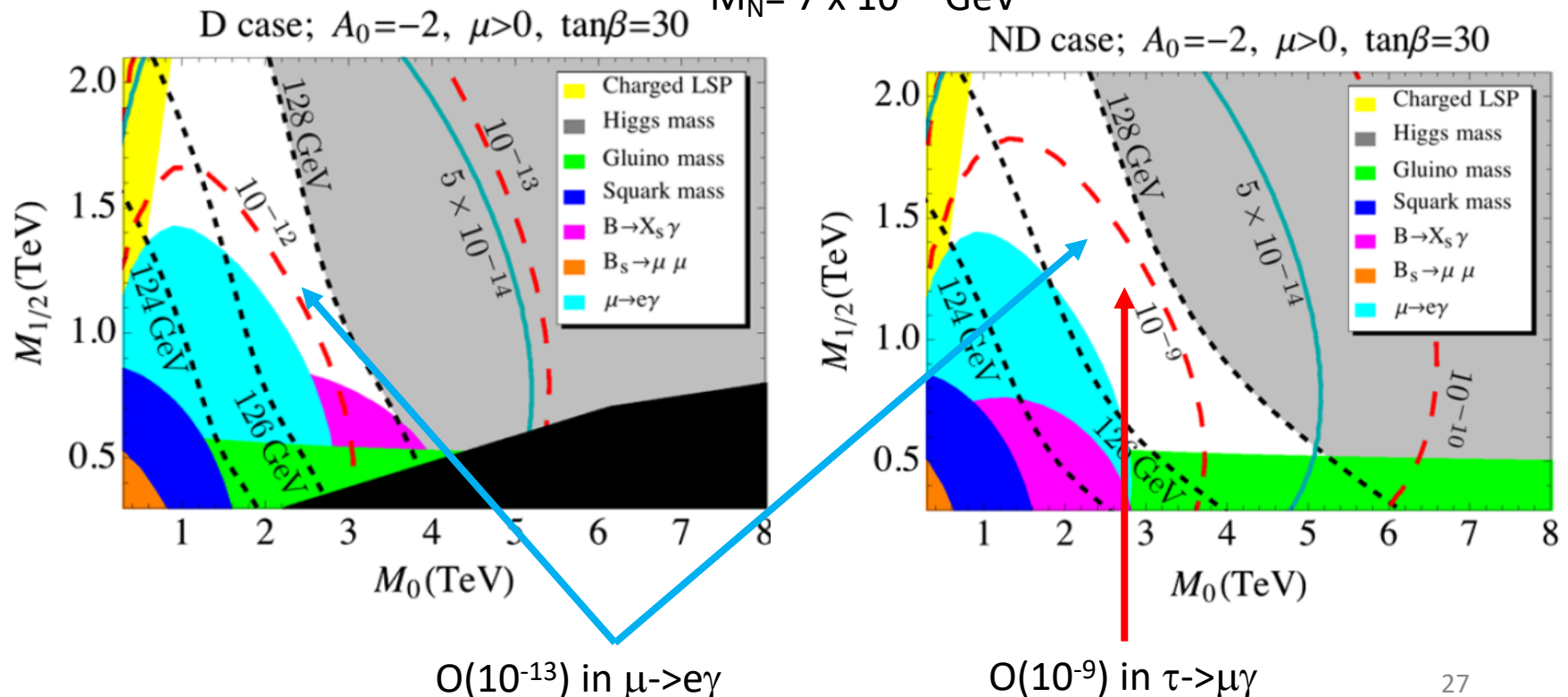
- Observable LFV rates are predicted in many new physics models.
- In particular, the SUSY seesaw model is still a prime candidate producing a large LFV. The predicted rate depends on the scale and the structure of the heavy Majorana neutrino sector.

# LFV in SUSY seesaw model

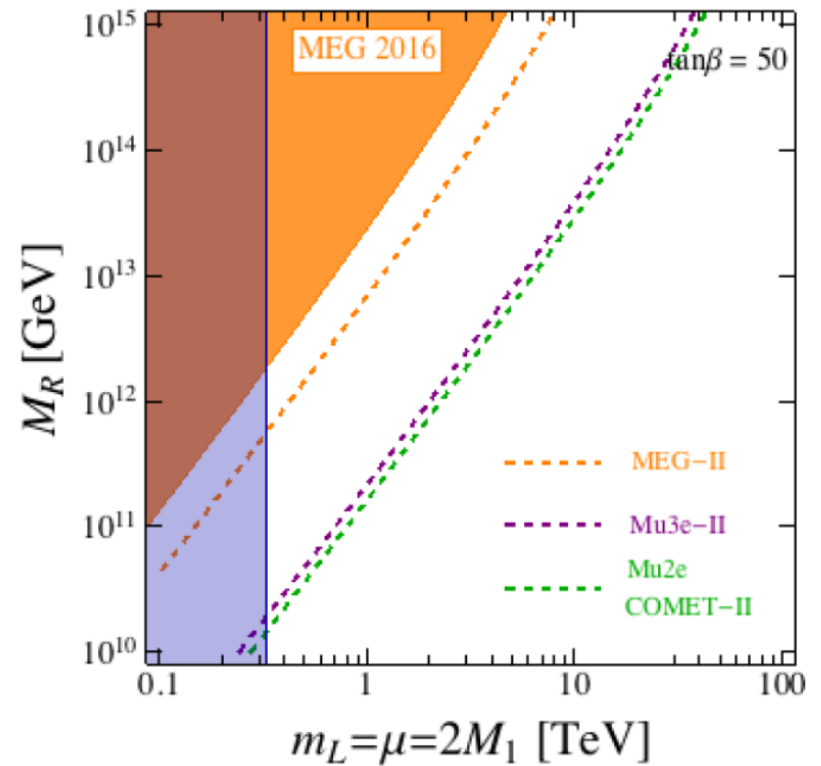
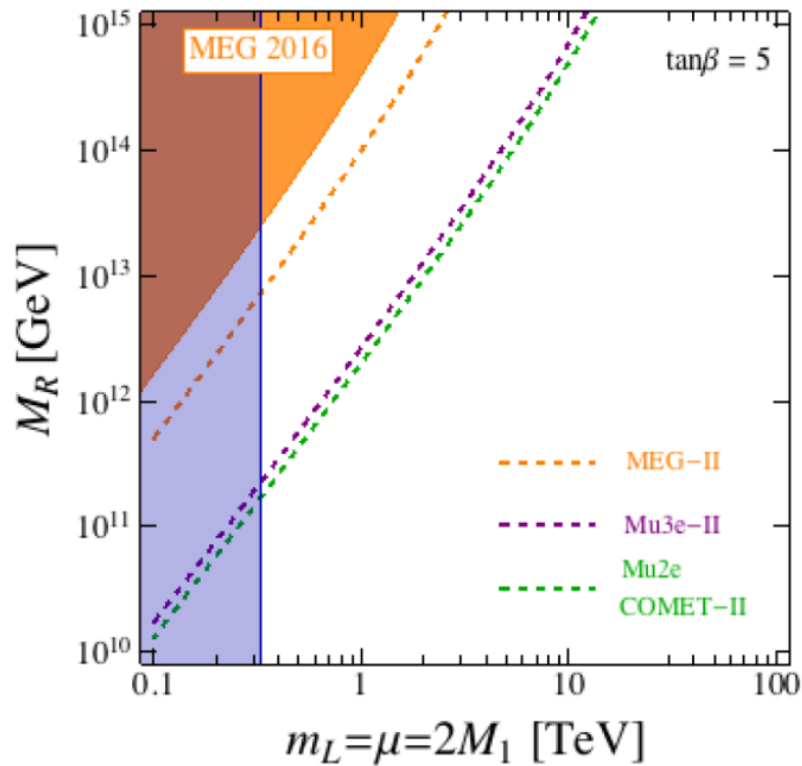
T. Goto, Y. Okada, T. Sindou, M. Tanaka and R. Watanabe, 2015

We have updated the prediction of muon and tau LFV processes in the SUSY seesaw Model, taking into account of various experimental results including the Higgs boson mass, SUSY searches at the 8TeV LHC run and  $\theta_{13}$  in the neutrino experiments.

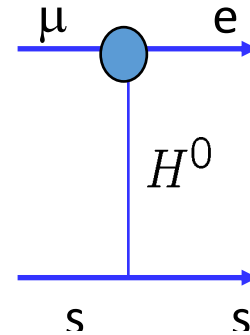
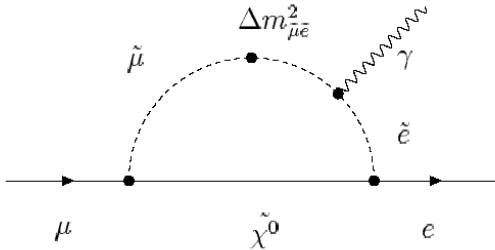
Degenerate case for the Majorana mass matrix  $M_N = 7 \times 10^{12} \text{ GeV}$  A special non-degenerate case



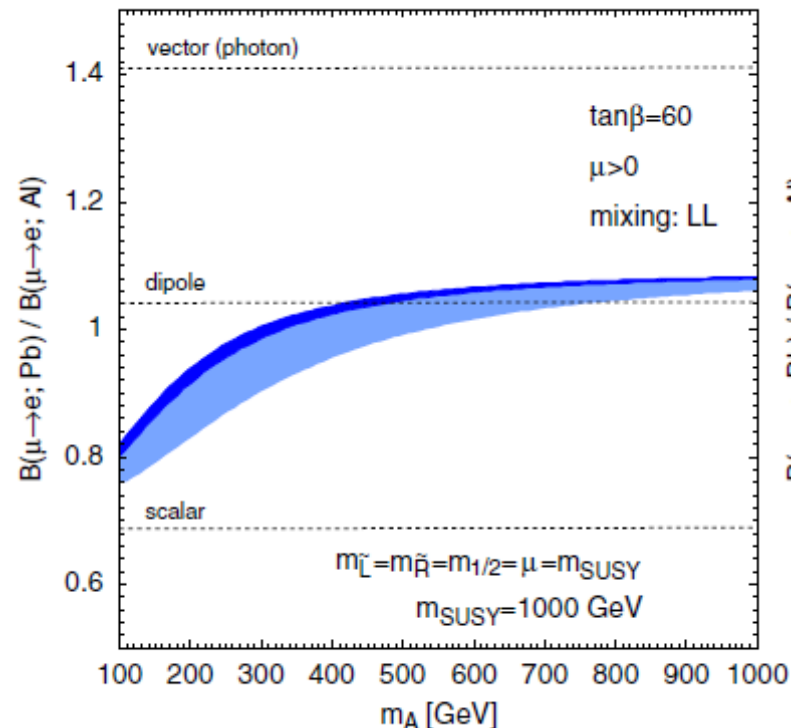
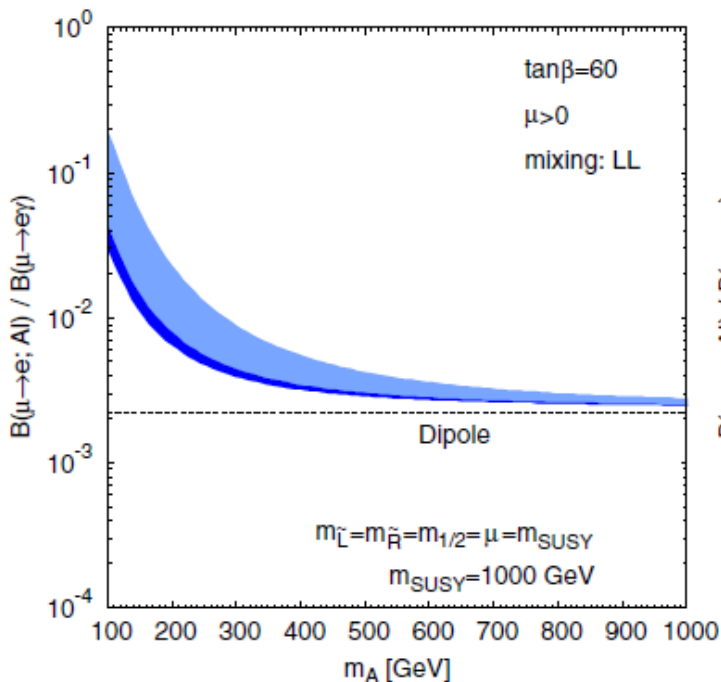
# Parameter space covered by LFV searches (Degenerate Majorana neutrinos)



# Higgs exchange contribution in SUSY seesaw model with a large “ $\tan\beta$ ”



Blue band :  
Uncertainty from “ $y$ ”  
Light:  $0 < y < 0.4$   
Dark:  $0 < y < 0.05$



# Neutrino mass from TeV physics and LFV

- If the origin of neutrino mass comes from TeV physics, a large LFV is expected.
- Each model shows a characteristic feature in branching ratios, angular distributions, etc.

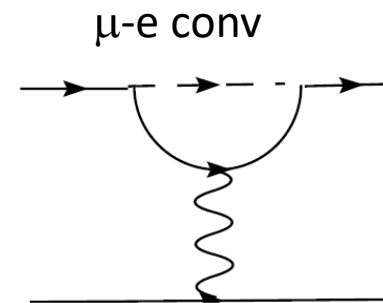
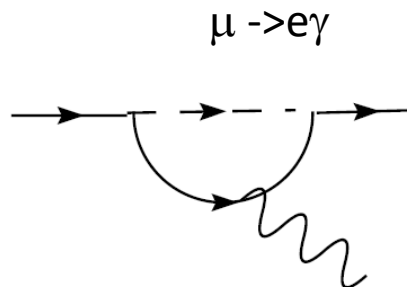
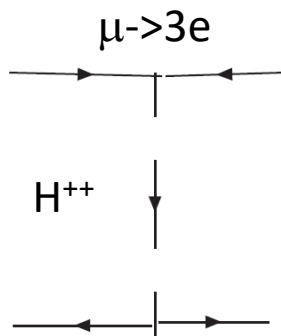
## Examples

Radiative neutrino mass generation (Zee model, etc)

Low energy seesaw model (singlet neutrino, triplet scalar, triplet fermion)

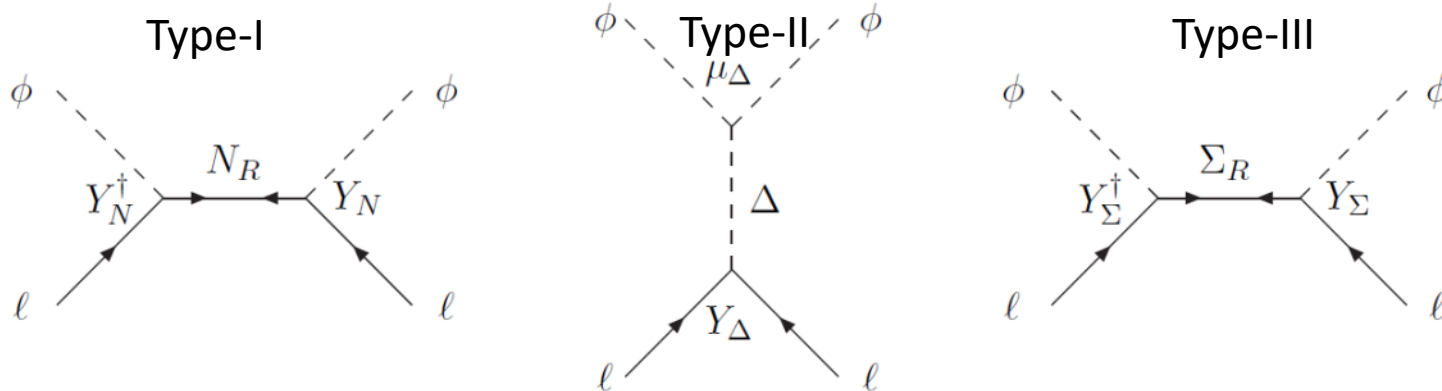
R-parity violating SUSY model

Left-right symmetric model



# TeV scale seesaw model and LFV

A. Abada, C. Biggio, F. Bonnet, M.B.Gavela, and T. Hambye, 2007



Non-unitarity  
in the PMNS matrix

Triplet Higgs  
coupling

Tree-level LFV coupling

Seesaw relation

$$m_\nu \equiv -\frac{v^2}{2} c^{d=5} = -\frac{1}{2} Y_N^T \frac{v^2}{M_N} Y_N$$

$$m_\nu = -2Y_\Delta v^2 \frac{\mu_\Delta}{M_\Delta^2}$$

Inverse seesaw mechanism

$$\begin{pmatrix} 0 & m_{D_1} & 0 \\ m_{D_1} & 0 & M_{N_1} \\ 0 & M_{N_1} & \mu \end{pmatrix} \Rightarrow m_\nu \sim \mu \frac{Y_1^2 v^2}{M_1^2}.$$

M can be close to  
the TeV scale with  
O(1) Yukawa  
coupling constants

Model	$\mu \rightarrow eee$	$\mu N \rightarrow eN$	$\frac{\text{BR}(\mu \rightarrow eee)}{\text{BR}(\mu \rightarrow e\gamma)}$	$\frac{\text{CR}(\mu N \rightarrow eN)}{\text{BR}(\mu \rightarrow e\gamma)}$
MSSM	Loop	Loop	$\approx 6 \times 10^{-3}$	$10^{-3} - 10^{-2}$
Type-I seesaw	Loop*	Loop*	$3 \times 10^{-3} - 0.3$	$0.1 - 10$
Type-II seesaw	Tree	Loop	$(0.1 - 3) \times 10^3$	$\mathcal{O}(10^{-2})$
Type-III seesaw	Tree	Tree	$\approx 10^3$	$\mathcal{O}(10^3)$
LFV Higgs	Loop <sup>†</sup>	Loop* <sup>†</sup>	$\approx 10^{-2}$	$\mathcal{O}(0.1)$
Composite Higgs	Loop*	Loop*	$0.05 - 0.5$	$2 - 20$

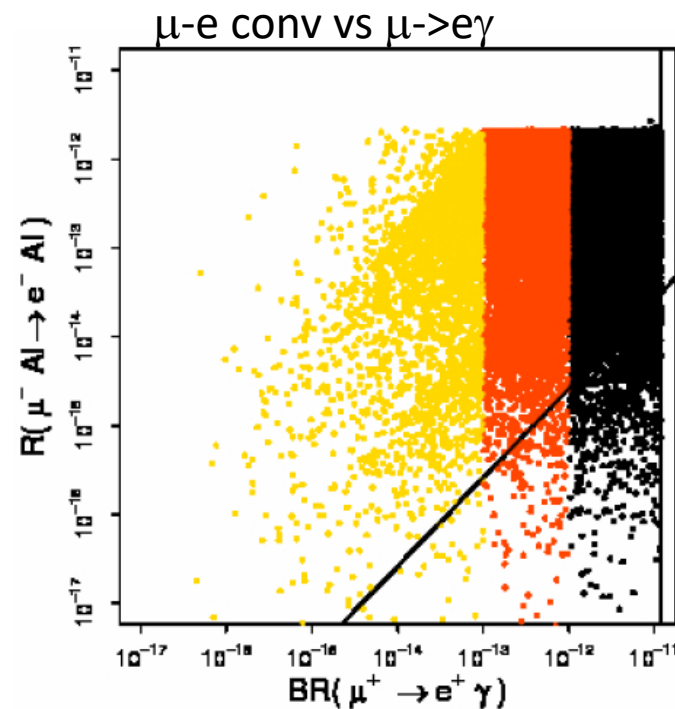
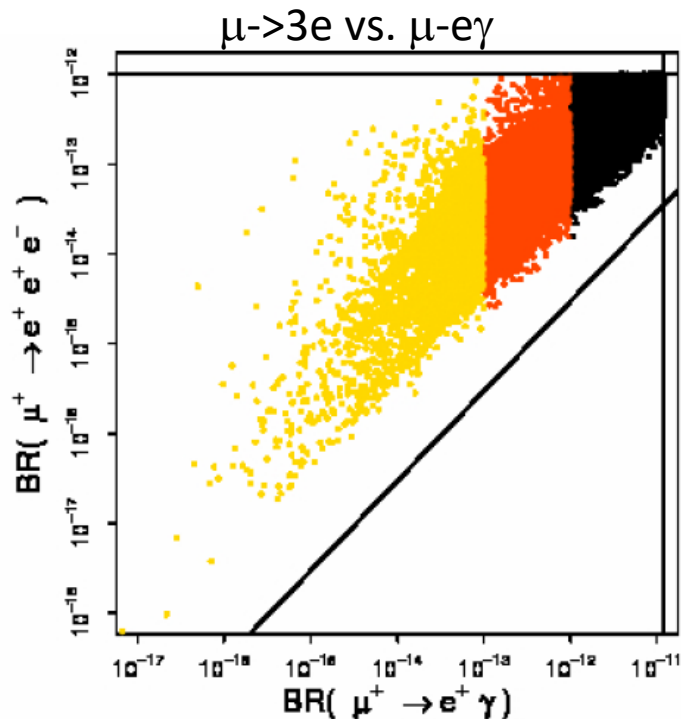
L. Calibbi and G. Signorelli, arXiv:1709.00294



# Little Higgs Model with T parity

- The Higgs boson is a pseudo Nambu-Goldstone boson of some strong dynamics at  $\sim 10$  TeV.
- New gauge bosons and a top partner to stabilize the Higgs potential against large radiative corrections without fine-tuning.
- T-odd heavy quarks and leptons are introduced. New flavor mixing matrixes induce FCNC and LFV.

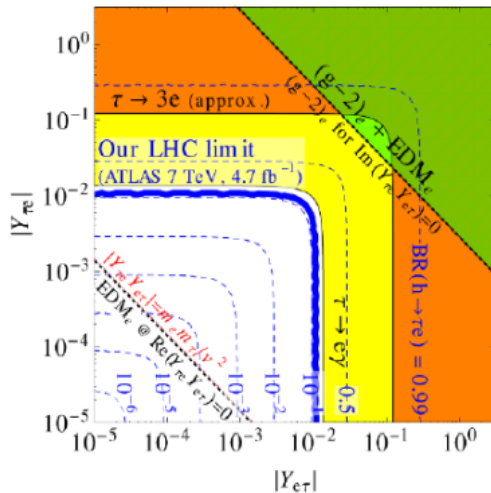
J.Hubisz, S.J.Lee, G.Paz, 2005 ; M.Blanke, et al. 2006-2009;  
S.Rai Choudhury, et al. 2007; T.Goto, Y.Okada, Y.Yamamoto, 2009  
F.del Aguila, J.I.Illana, M.D.Jenkins, 2009, 2010



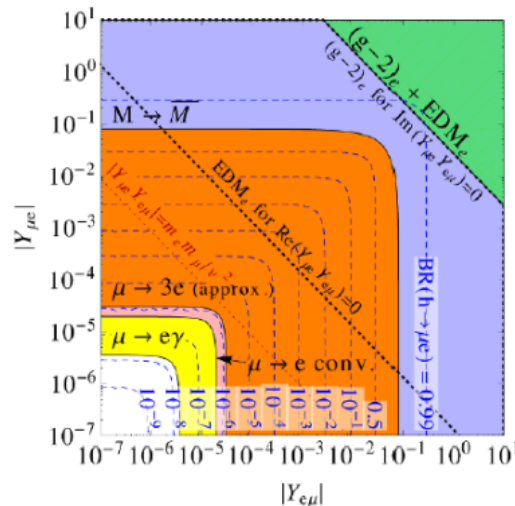
# LFV Higgs decay

- Higgs decays provide opportunity to look for LFV interactions.
- Higgs to tau-mu and tau-e modes are promising taking account of rare decay constraints.

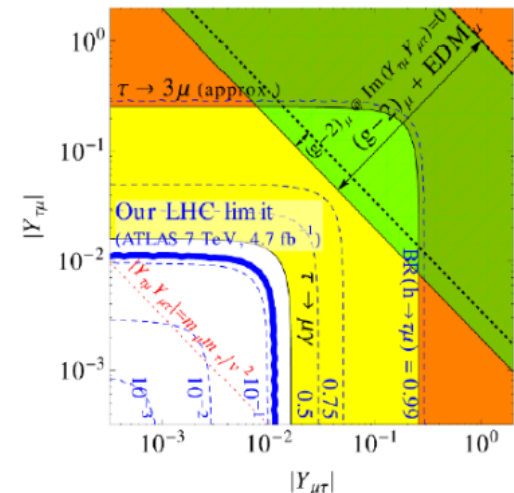
H→τe



H→eμ



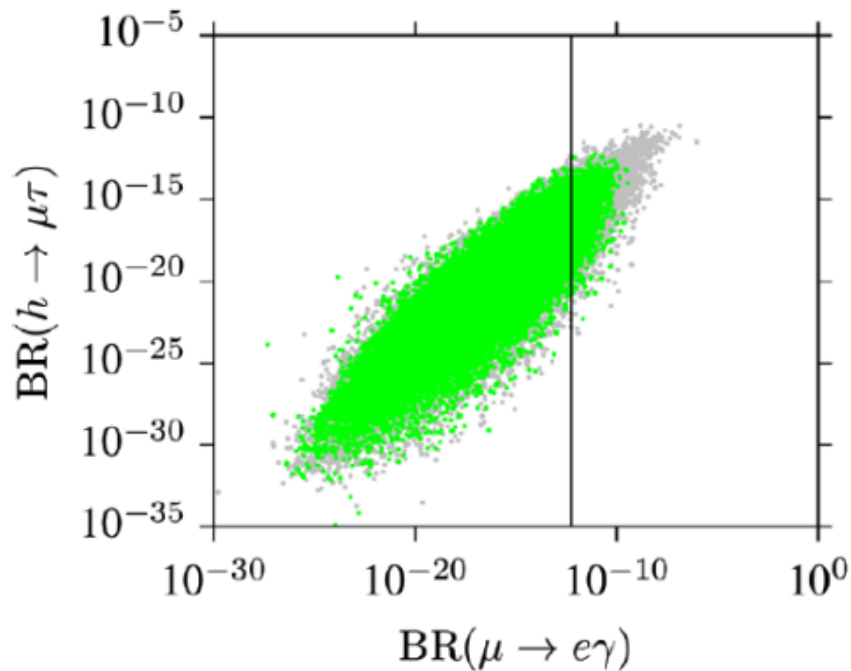
H→τμ



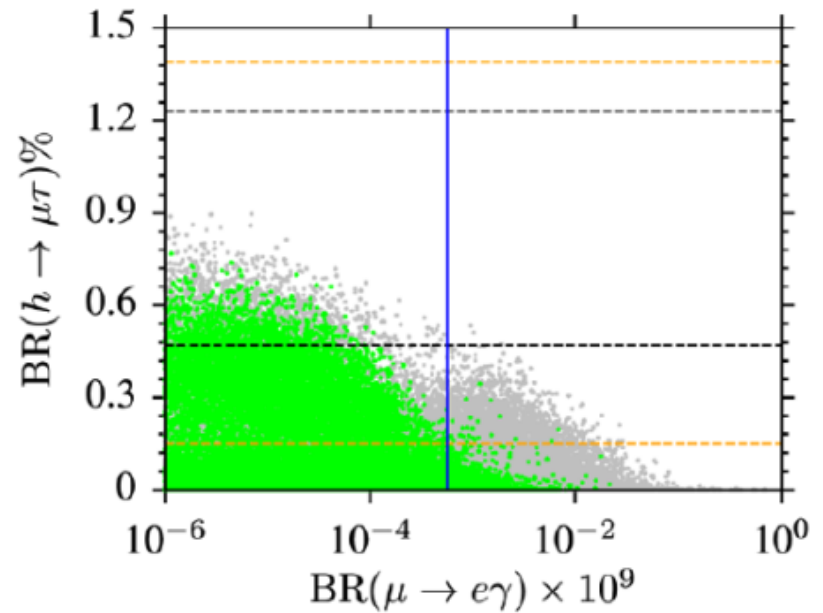
$$\mathcal{L}_Y \supset -Y_{e\mu}\bar{e}_L\mu_R h - Y_{\mu e}\bar{\mu}_L e_R h - Y_{e\tau}\bar{e}_L\tau_R h - Y_{\tau e}\bar{\tau}_L e_R h - Y_{\mu\tau}\bar{\mu}_L\tau_R h - Y_{\tau\mu}\bar{\tau}_L\mu_R h + h.c..$$

# SUSY model example

High scale SUSY seesaw model



SUSY B-L model with inverse seesaw  
(Low scale seesaw model)



A. Hammad S., Khalil, and C.S. Un, 2017

# Summary

- Particle physics has entered a new era.
- Searches for Lepton Flavor Violation offer prime opportunities to explore new physics in generic ways. This is particularly interesting in views of neutrino and/or SUSY.
- Experimental prospects are bright. There are ongoing and planned muon dedicated experiments as well as tau LFV searches at LHC, LHCb and Belle II.