Particle Tracking with Space Charge Effect using GPU

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Contents

References

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- Laurent S. Nadolski “Symplectic Integrators applied to Beam Dynamics in Circular Accelerators Basis for Constructing Integration Schemes for Hamiltonian Systems” NPAC 2010–2011 (v1.4)
- *C.K. Birdsall, A.B Langdon, “Plasma Physics via Computer Simulation” DOI: https://doi.org/10.1201/9781315275048*
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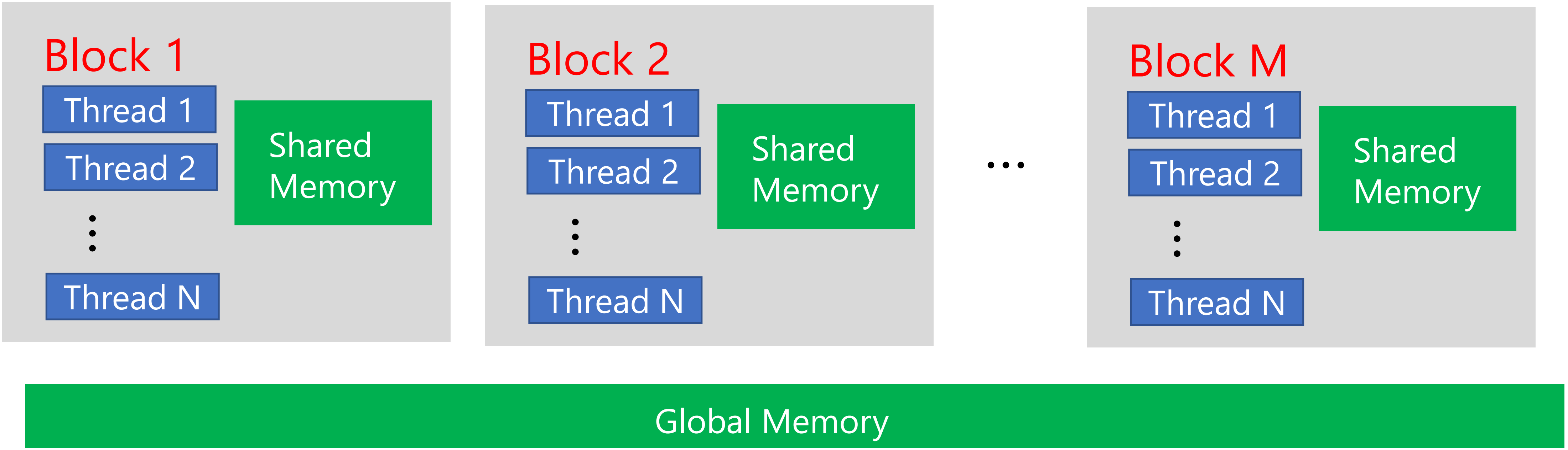
- *F. Christoph Iselin, “The MAD Program Physical Methods Manual “ Version 8.13*

- *S. Machida “空間電荷効果” OHO seminar 2000*
- *K. Oide “ビーム力学入門” 1992*

- *Private Communication to Ohmi-san (KEK) Hochi-san (JAEA)*

This work is also supported by NVIDIA Corporation via NVIDIA GPU Grant Program

GPU (Graphic Processing Unit)



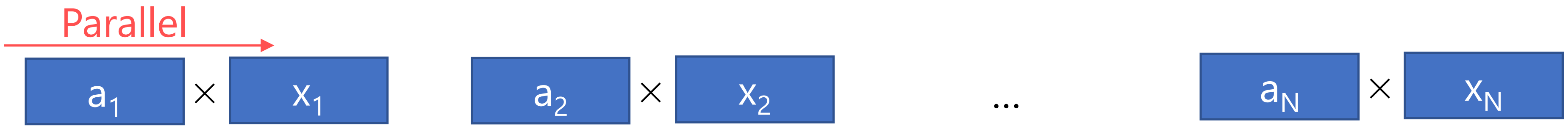
- Each operation can be assigned to each thread.
- **Execution of each thread can be parallelly done**.
- Threads in a common block can access the shared memory.
- Shared memory is limited (12288 double words) but **very fast**

Example

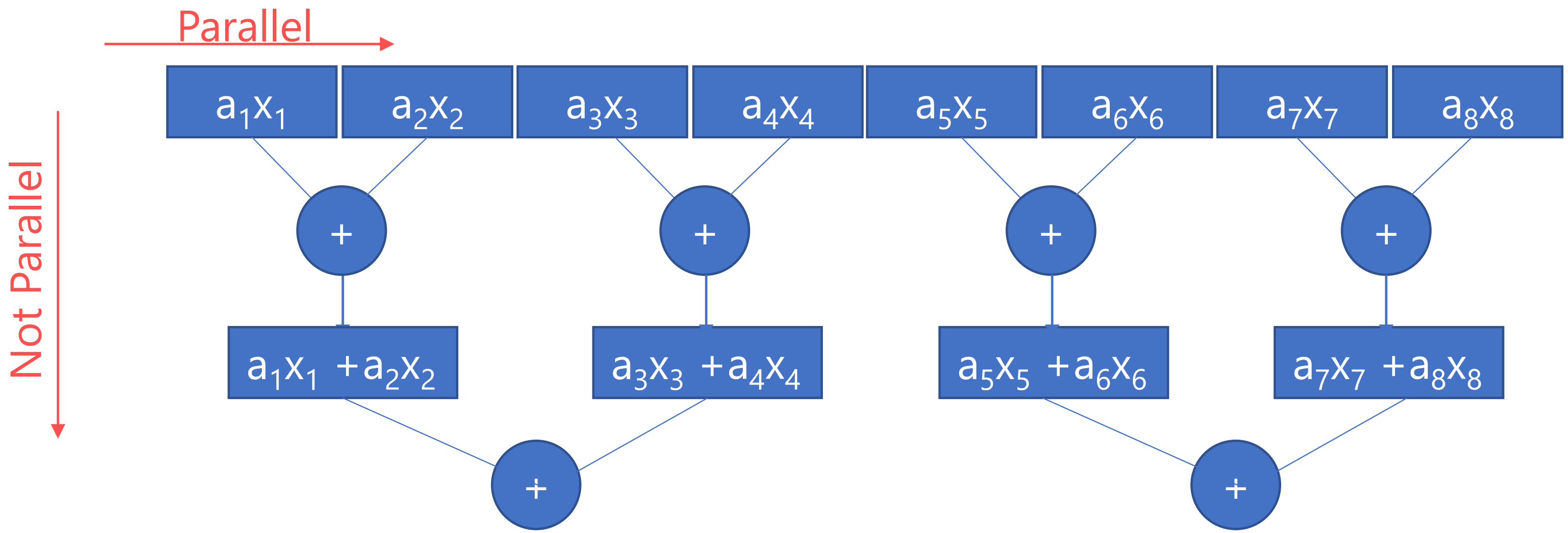
$$\sum_{i=1}^{N} a_i x_i = a_1 x_1 + a_2 x_2 + \cdots + a_N x_N$$

Matrix multiplication, Fourier transformation …

1. Each term can be calculated in parallel



2. Summation also can be partially parallel.



CUDA (Compute Unified Device Architecture)

- C/C++ based Programing Language
- Compiler : “nvcc” (instead of gcc)
- FFT Library : “cuFFT”
- Linear Algebra Library : “cublas”

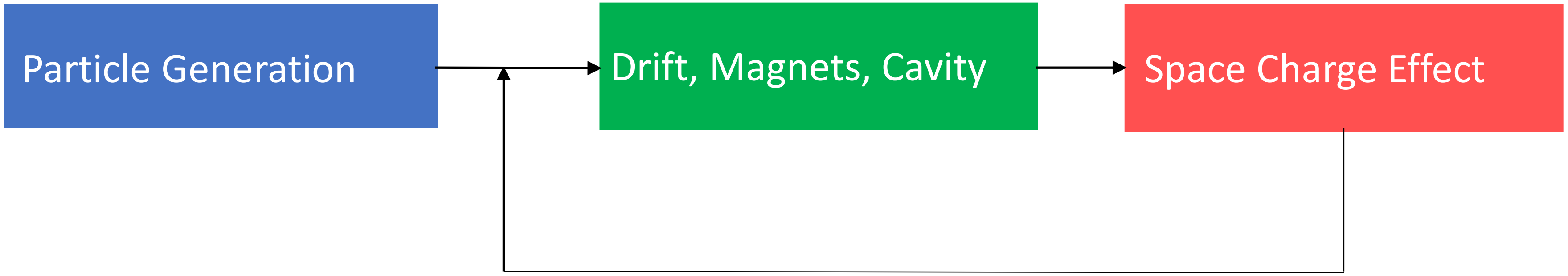
Hardware

NVIDIA Quadro P6000

TESLA V100

	Quadro P6000	TESLA V100
Single Precision (float)	12.63 TFLOP	14 TFLOP
Double Precision (double)	394.8 GFLOP	7 TFLOP
How I Got	NVIDIA gave me	Amazon Web Service 3-4$/hour
How I use	Checking if my code works correctly	Massive calculations

Overview of the Particle Tracking Simulation



Single Particle Mechanics : Maps between two locations can be calculated in parallel

PIC simulation : Maps requires spatial distributions of particles

Single Particle Mechanics

Implemented Components

- QUAD, SEXT, thin multipole, Cavity

  - Using Hamiltonian
  - Approximated by sequential symplectic transformations

- BEND, DRIFT

  - Find the exact solutions of particle motions in uniform magnetic fields

Hamiltonian

$$H(x, p_x, y, p_y, \sigma, p_\sigma; s) = p_\sigma - (1 + hx)\sqrt{(1+\delta)^2 - {p_x}^2 - {p_y}^2} - e\,\frac{A_s(x,y)}{p_0}$$

$$\approx \frac{{p_x}^2 + {p_y}^2}{2} + \frac{{p_\sigma}^2}{2{\gamma_0}^2} - hx - hxp_\sigma - \frac{{p_x}^2 + {p_y}^2}{2} p_\sigma - e\,\frac{A_s(x,y)}{p_0}$$

$$p_\sigma = \frac{E - E_0}{\beta_0 p_0 c}, \quad \sigma = s - \beta_0 t, \qquad \delta = \frac{p - p_0}{p_0} \approx p_\sigma - \frac{1}{2{\gamma_0}^2} {p_\sigma}^2,$$

Then just have to solve the Hamilton equations. But not always solvable

Symplectic Map

In case that a Hamilton equation is difficult to be solved,

$H = H_0 + V$: Both symplectic integrators ($e^{iH_0 s}$ and e^{iVs}) are integrable

The map between the entrance and exit of a components (Length = L) :

$$e^{iH_0 L/2} e^{iVL} e^{iH_0 L/2}$$

or

$$e^{iH_0 aL} e^{-iVbL} e^{-iH_0 cL} e^{iVbL} e^{-iH_0 aL}$$

$$a = \frac{1}{2}\left(1 - \frac{1}{\sqrt{3}}\right), b = \frac{1}{2}, c = \frac{1}{\sqrt{3}}$$

Ex1 Quad Magnets

$$eA_s(x) = -\frac{1}{2}k_1(x^2 - y^2)$$

$$H \approx \underbrace{\frac{{p_x}^2+{p_y}^2}{2} + \frac{1}{2}k_1\left(x^2 - y^2\right)}_{H_0} \underbrace{-\frac{{p_x}^2+{p_y}^2}{2}p_\sigma + \frac{{p_\sigma}^2}{2{\gamma_0}^2}}_{V}$$

- $e^{iH_0 s}$

$$x(s) = x(0)\cos\sqrt{k_1}s + \frac{p_x(0)}{\sqrt{k_1}}\sin\sqrt{k_1}s$$
$$p_x(s) = p_x(0)\cos\sqrt{k_1}s - \sqrt{k_1}x(0)\sin\sqrt{k_1}\,s$$
$$y(s) = y(0)\cosh\sqrt{k_1}s + \frac{p_y(0)}{\sqrt{k_1}}\sinh\sqrt{k_1}s$$
$$p_y(s) = p_y(0)\cosh\sqrt{k_1}s + \sqrt{k_1}y(0)\sinh\sqrt{k_1}\,s$$
$$\sigma(s) = \sigma(0)$$
$$p_\sigma(s) = p_\sigma(0)$$

- e^{iVs}

$$x(s) = -p_x(0)p_\sigma(0)s + x(0)$$
$$y(s) = -p_y(0)p_\sigma(0)s + y(0)$$
$$\sigma(s) = \sigma(0) + \left(\frac{p_\sigma(0)}{{\gamma_0}^2} - \frac{p_x(0)^2 + p_y(0)^2}{2}\right)s$$
$$p_x(s) = p_x(0)$$
$$p_y(s) = p_y(0)$$
$$p_\sigma(s) = p_\sigma(0)$$

Particle Motions of Uniform Magnetic Fields

Equation of Motion

$$\frac{d\vec{P}(t)}{dt} = q\vec{\beta}c \times \vec{B}$$

Solution

$$\tilde{x}(t) = \frac{P_{\tilde{x}}(0)}{qB}\sin\frac{qB}{m\gamma}t + \frac{P_{\tilde{z}}(0)}{qB}\cos\frac{qB}{m\gamma}t + \tilde{x}(0) - \frac{P_{\tilde{z}}(0)}{qB}$$

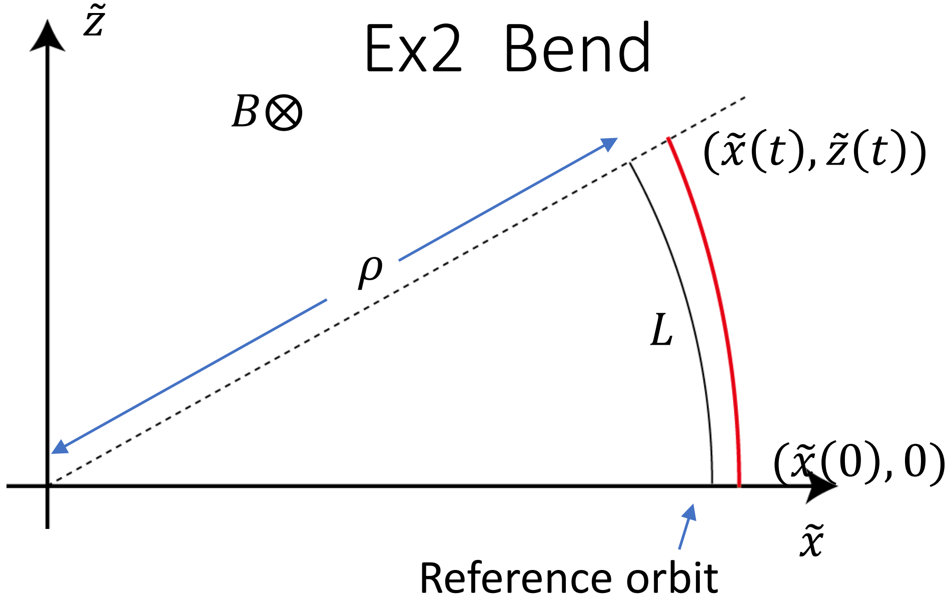
$$P_{\tilde{x}}(t) = P_{\tilde{x}}(0)\cos\frac{qB}{m\gamma}t - P_{z}(0)\sin\frac{qB}{m\gamma}t$$

$$\tilde{z}(t) = \frac{P_{\tilde{z}}(0)}{qB}\sin\frac{qB}{m\gamma}t - \frac{P_{\tilde{x}}(0)}{qB}\cos\frac{qB}{m\gamma}t + \frac{P_{\tilde{x}}(0)}{qB}$$

$$P_{\tilde{z}}(t) = P_{\tilde{z}}(0)\cos\frac{qB}{m\gamma}t + P_{\tilde{x}}(0)\sin\frac{qB}{m\gamma}t$$

, where $P_{\tilde{z}}(0) = p_0\sqrt{1 + 2p_\sigma + \beta^2 {p_\sigma}^2 - p_x(0)^2 - p_y(0)^2}$

$P_{\tilde{x}}(0) = p_0 p_x(0)$



1. Find t when $\tilde{z}(t) = \tan\frac{L}{\rho}\tilde{x}(t)$
2. Convert canonical valuables

$$x(L) = \sqrt{\tilde{x}^2(t) + \tilde{z}^2(t)} - \rho \qquad p_x(L) = \frac{P_{\tilde{x}}(t)\cos\frac{L}{\rho} + P_{\tilde{z}}(t)\sin\frac{L}{\rho}}{p_0}$$

Space Charge Effect

- Overview
- Histogram Making (Charge weighting)
- Poisson Solver
- Electric field

Overview

$$(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2})\varphi(x,y,z) = -\frac{\rho(x,y,z)}{\varepsilon_0}$$

2D approximation

$\sigma_x, \sigma_y \ll \sigma_z$

$$g(z)\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)u(x,y) = -\frac{g(z)f(x,y)}{\varepsilon_0}$$

1. Make histogram $f(x,y)\ and\ g(z)$ ← The most time-consuming part
2. Solve 2D Poisson equation $u(x,y)$ with boundary conditions[a]
3. Calculate gradient (kick) $-\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)Ag(z)u(x,y)\times L$

$A = \frac{e}{m_p{\gamma_p}^3{\beta_p}^2c^2}$[b], $L = Moving\ Distance$

a. Using “polar” or “rectangular” boundary condition depending on the duct shape.
b. This coefficient A is not always correct (come back later)

Space Charge Effect

- Overview
- Histogram Making (Charge weighting)
- Poisson Solver
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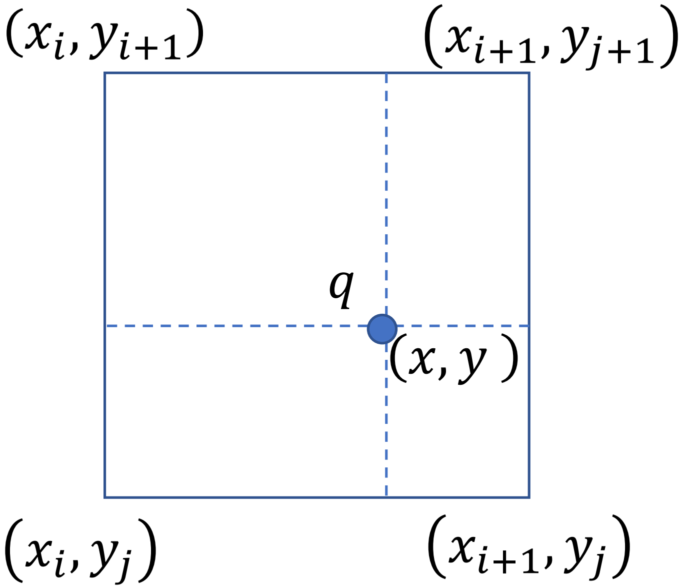
Making Histograms

Cartesian Coordinate



$$Q(x_i, y_j) \mathrel{+}= q\,\frac{(x_{i+1} - x)(y_{j+1} - y)}{\delta x \delta y}$$

$$Q(x_{i+1}, y_j) \mathrel{+}= q\,\frac{(-x_i + x)(y_{j+1} - y)}{\delta x \delta y}$$

$$Q(x_i, y_{j+1}) \mathrel{+}= q\,\frac{(x_{i+1} - x)(-y_j + y)}{\delta x \delta y}$$

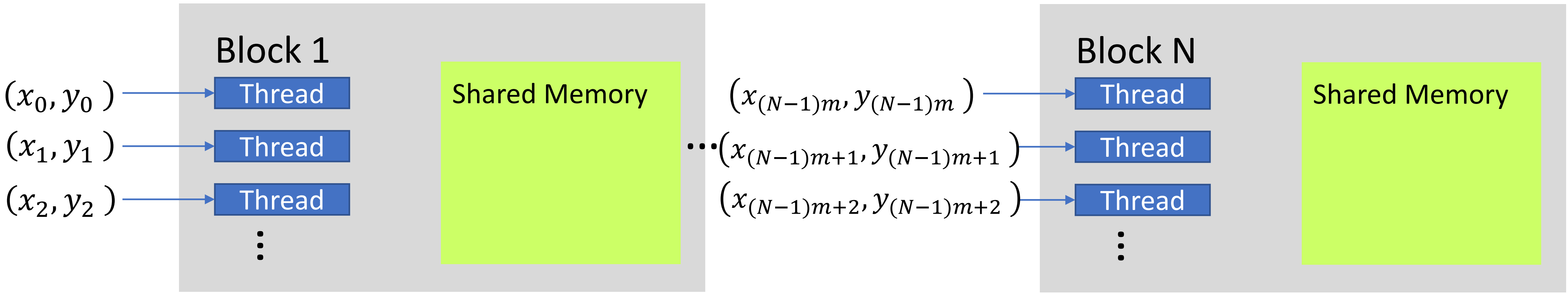
$$Q(x_{i+1}, y_{j+1}) \mathrel{+}= q\,\frac{(-x_i + x)(-y_j + y)}{\delta x \delta y}$$

Sub-histograms using shared memory

GROUP1	GROUP2		GROUPN
(x_0, y_0)	(x_m, y_m)		$(x_{(N-1)m}, y_{(N-1)m})$
(x_1, y_1)	(x_{m+1}, y_{m+1})	…	$(x_{(N-1)m+1}, y_{(N-1)m+1})$
(x_2, y_2)	(x_{m+2}, y_{m+2})		$(x_{(N-1)m+2}, y_{(N-1)m+2})$
$\vdots$	$\vdots$		$\vdots$
(x_{m-1}, y_{m-1})	(x_{2m-1}, y_{2m-1})		(x_{Nm-1}, y_{Nm-1})

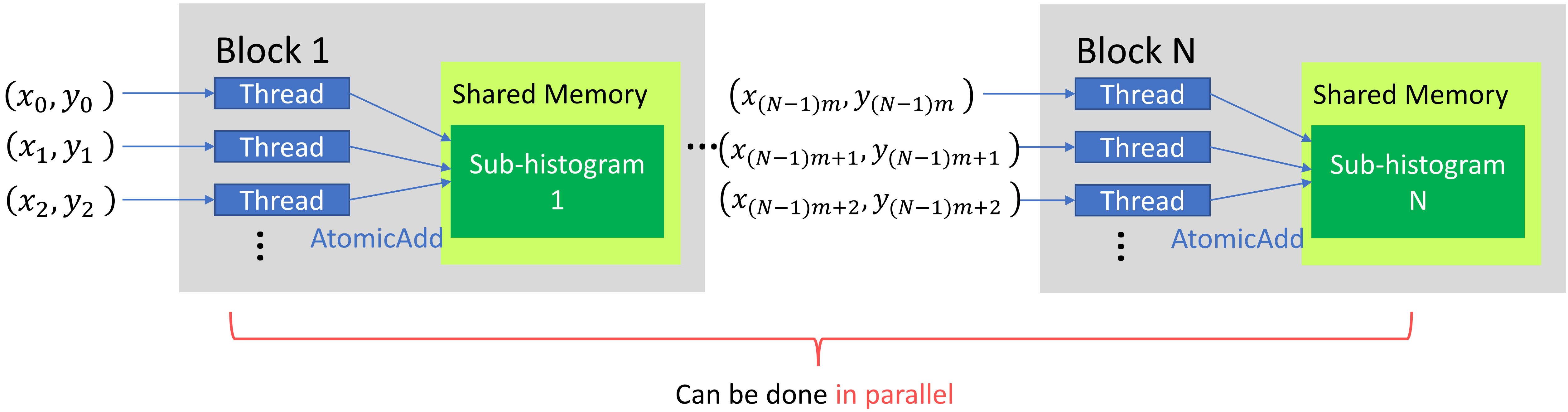
1. Divide (macro) particles into N groups

Sub-histograms using shared memory



1. Divide (macro) particles into N groups
2. Assign each group to each block and each particle to each thread

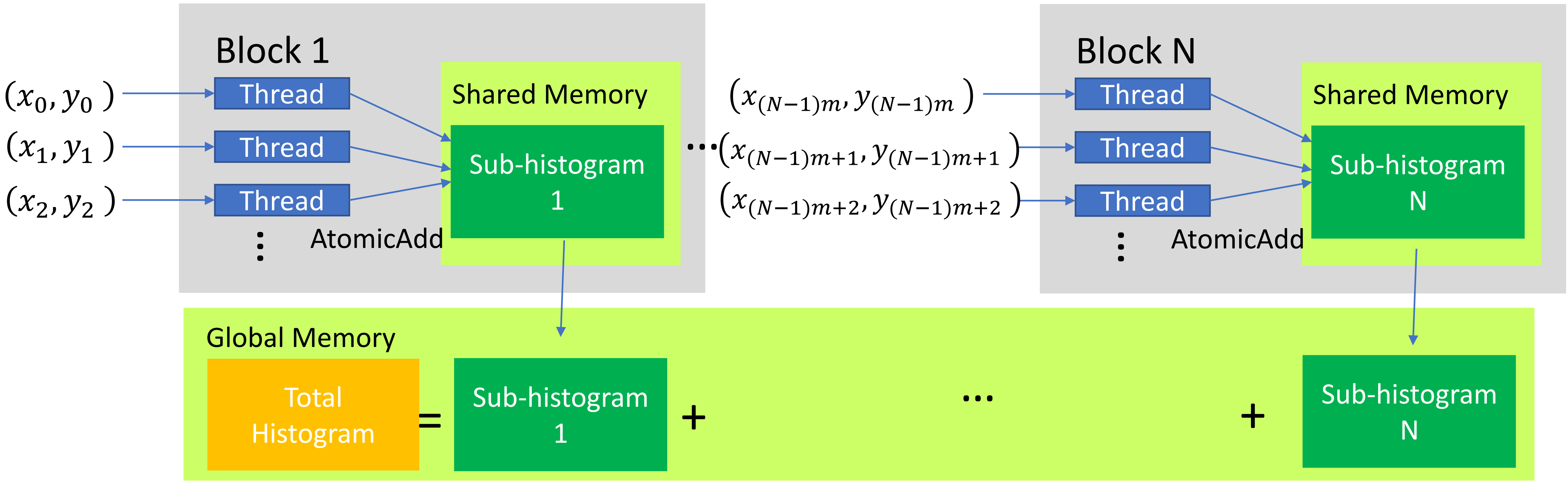
Sub-histograms using shared memory



1. Divide (macro) particles into N groups
2. Assign each group to each block and each particle to each thread
3. Sub-histogram in each shared memory be made in parallel.

Note: Each thread in a common block adds entries in shared sub-histogram. But use "exclusive add" operation (called AtomicAdd function)

Sub-histograms using shared memory



1. Divide (macro) particles into N groups
2. Assign each group to each block and each particle to each thread
3. Sub-histogram in each shared memory be made in parallel.
4. Copy sub-histograms to the global memory and make summation

Space Charge Effect

- Overview
- Histogram Making (Charge weighting)
- Poisson Solver
- Electric field

Poisson Solver (Rectangular Coordinates)

<u>2D Poisson Equation (Rectangular Coordinates)</u>

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f(x, y)$$

Boundary Condition : $u(x, 0) = u(x, L_x) = u(y, 0) = u\left(y, L_y\right) = 0$

Discretization with $\frac{\partial y}{\partial x} = \frac{y_{i+1} - y_i}{\Delta x}$

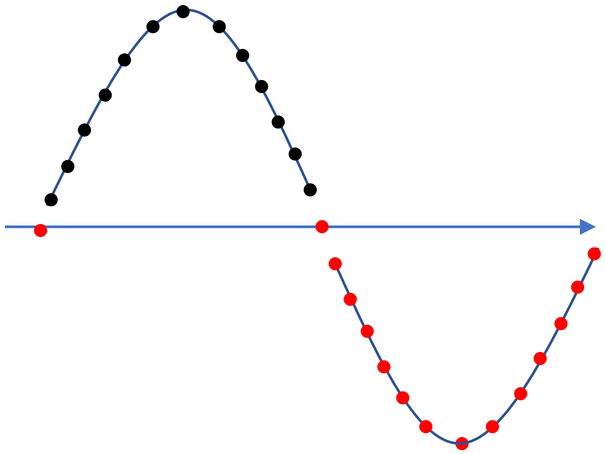
$i, j = 1, \cdots\cdots, m$

$$\frac{u_{i-1,j} - 2u_{i,j} + u_{i+1}}{\Delta x^2} + \frac{u_{i,j-1} - 2u_{i,j} + u_{i,j+1}}{\Delta y^2} = f_{i,j}$$

<u>Odd Extension for x</u>

$$U_{l,j} = (0, u_{1,j}, u_{2,j}, \cdots, u_{m,j}, 0, -u_{m,j}, -u_{m-1,j}, \cdots, -u_{1,j})$$

$$l = 0, \cdots\cdots, 2(m+1) - 1$$



Poisson Solver (Rectangular Coordinates)

$$DFT_x(\frac{U_{l-1,j} - 2U_{l,j} + U_{l+1,j}}{\Delta x^2})_p = u_{1,j} + e^{-i\frac{p}{2(m+1)}2\pi}\left(u_{2,j} - 2u_{1,j}\right) + e^{-i\frac{2p}{2(m+1)}2\pi}\left(u_{3,j} + u_{1,j} - 2u_{2,j}\right)$$

$$= 0 + \left(1 - 2e^{-i\frac{p}{2(m+1)}2\pi} + e^{-i\frac{2p}{2(m+1)}2\pi}\right)u_{1,j} + \left(e^{-i\frac{3p}{2(m+1)}2\pi} - 2e^{-i\frac{2p}{2(m+1)}2\pi} + e^{-i\frac{3p}{2(m+1)}2\pi}\right)u_{2,j} \cdots$$

$$= \left(e^{i\frac{p}{2(m+1)}2\pi} + e^{-i\frac{p}{2(m+1)}2\pi} - 2\right)\left(0 + e^{-i\frac{p}{2(m+1)}2\pi}u_{1,j} + e^{-i\frac{2p}{2(m+1)}2\pi}u_{2,j} + \cdots\right)$$

$$= -4\, sin^2 \frac{p\pi}{2(m+1)} \sum_{l=0}^{2(m+1)-1} e^{-i\frac{pl}{2(m+1)}2\pi} U_{l,j}$$

Odd Extension for y

$$V_{l,l'} = (0, U_{l,1}, U_{l,2}, \cdots, U_{l,m}, 0, -U_{l,m}, -U_{l,m-1}, \cdots, -U_{l,1})$$

$$l' = 0, \cdots\cdots, 2(m+1) - 1$$

$$DFT_y(DFT_x(\frac{V_{l-1,l'} - 2V_{l,l'} + V_{l+1,l'}}{\Delta x^2})_p)_q = -\frac{4}{\Delta x^2} sin^2 \frac{p\pi}{2(m+1)} \sum_{l=0}^{2(m+1)-1} \sum_{l'=0}^{2(m+1)-1} e^{-i\frac{pl}{2(m+1)}2\pi} e^{-i\frac{ql'}{2(m+1)}2\pi} V_{l,l'}$$

Poisson Solver (Rectangular Coordinates)

$$DFT_y(DFT_x(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2})_p)_q \rightarrow -4\left(\frac{1}{\Delta x^2}sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2}sin^2\frac{q\pi}{2(m+1)}\right)\sum_{l=0}^{2(m+1)-1}\sum_{l'=0}^{2(m+1)-1}e^{-i\frac{pl}{2(m+1)}2\pi}e^{-i\frac{ql'}{2(m+1)}2\pi}V_{l,l'}$$

$$= -4\left(\frac{1}{\Delta x^2}sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2}sin^2\frac{q\pi}{2(m+1)}\right)DFT_y(DFT_x(V)_p)_q$$

(FFT result for the left-hand side of the equation)

Odd Extension of $f_{i,j}$

$$F_{l,0} = (0,0,0,\cdots,0,0,0,0,\cdots,0)$$

$$F_{l,1} = (0,f_{1,1},f_{2,1},\cdots,f_{m,1},0,-f_{m,1},-f_{m-1,1},\cdots,-f_{1,1})$$

$$F_{l,2} = (0,f_{1,2},f_{2,2},\cdots,f_{m,2},0,-f_{m,2},-f_{m-1,2},\cdots,-f_{1,2})$$

$$F_{l,m} = (0,f_{1,m},f_{2,m},\cdots,f_{m,m},0,-f_{m,m},-f_{m-1,m},\cdots,-f_{1,m})$$

$$F_{l,m+1} = (0,0,0,\cdots,0,0,0,0,\cdots,0)$$

$$F_{l,m+2} = (0,-f_{1,m},\text{-}f_{2,m},\cdots,-f_{m,m},0,f_{m,m},f_{m-1,m},\cdots,f_{1,m})$$

$$\vdots$$

$$DFT_y(DFT_x(F_{i,j})_p)_q \rightarrow \sum_{l=0}^{2(m+1)-1}\sum_{l'=0}^{2(m+1)-1}e^{-i\frac{pl}{2(m+1)}2\pi}e^{-i\frac{ql'}{2(m+1)}2\pi}F_{l,l'}$$

(FFT result for the right-hand side of the equation)

Poisson Solver (Rectangular Coordinates)

$$DFT_y(DFT_x(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2})_p)_q = DFT_y(DFT_x(F_{i,j})_p)_q$$

$$\longrightarrow \quad -4\left(\frac{1}{\Delta x^2} sin^2 \frac{p\pi}{2(m+1)} + \frac{1}{\Delta y^2} sin^2 \frac{q\pi}{2(m+1)}\right) DFT_y(DFT_x(V)_p)_q = DFT_y(DFT_x(F_{i,j})_p)_q$$

$p \neq 0 \;\; or \;\; q \neq 0$

$$DFT_y(DFT_x(V)_p)_q = -\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2} sin^2 \frac{p\pi}{2(m+1)} + \frac{1}{\Delta y^2} sin^2 \frac{q\pi}{2(m+1)}\right)}$$

$p = 0 \;\; and \;\; q = 0$

$$DFT_y(DFT_x(V)_0)_0 = \sum_{l=0}^{2(m+1)-1} \sum_{l'=0}^{2(m+1)-1} V_{l,l'} = 0$$

($V_{l,l'}$ is odd extension of $u_{i,j}$)

$$V_{l,l'} = -iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2} sin^2 \frac{p\pi}{2(m+1)} + \frac{1}{\Delta y^2} sin^2 \frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

⬅ 4 FFTs using GPU

$$V_{l,l'} \to u_{i,j}$$

Poisson Solver (Rectangular Coordinates)

<u>Summary</u>

$$V_{l,l'} = -iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2} sin^2 \frac{p\pi}{2(m+1)} + \frac{1}{\Delta y^2} sin^2 \frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

$$V_{l,l'} \rightarrow u_{i,j}$$

1. Odd extension of charge distributions

Poisson Solver (Rectangular Coordinates)

<u>Summary</u>

$$V_{l,l'} = -iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2}sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2}sin^2\frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

$$V_{l,l'} \rightarrow u_{i,j}$$

1. Odd extension of charge distributions
2. Apply 2D FFT to the extended distributions

Poisson Solver (Rectangular Coordinates)

<u>Summary</u>

$$V_{l,l'} = -iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2} sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2} sin^2\frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

$$V_{l,l'} \rightarrow u_{i,j}$$

1. Odd extension of charge distributions
2. Apply 2D FFT to the extended distributions
3. Divide the FFT results by $-4\left(\frac{1}{\Delta x^2} sin^2 \frac{p\pi}{2(m+1)} + \frac{1}{\Delta y^2} sin^2 \frac{q\pi}{2(m+1)}\right)$

Poisson Solver (Rectangular Coordinates)

<u>Summary</u>

$$V_{l,l'} = -iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2} sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2} sin^2\frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

$$V_{l,l'} \rightarrow u_{i,j}$$

1. Odd extension of charge distributions
2. Apply 2D FFT to the extended distributions
3. Divide the FFT results by $-4\left(\frac{1}{\Delta x^2} sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2} sin^2\frac{q\pi}{2(m+1)}\right)$
4. Apply 2D iFFT to 3

Poisson Solver (Rectangular Coordinates)

Summary

$$V_{l,l'} = -iDFT_y(iDFT_x(\frac{DFT_y(DFT_x(F_{i,j})_p)_q}{4\left(\frac{1}{\Delta x^2} sin^2\frac{p\pi}{2(m+1)}+\frac{1}{\Delta y^2} sin^2\frac{q\pi}{2(m+1)}\right)})_l)_{l'}$$

$$V_{l,l'} \to u_{i,j}$$

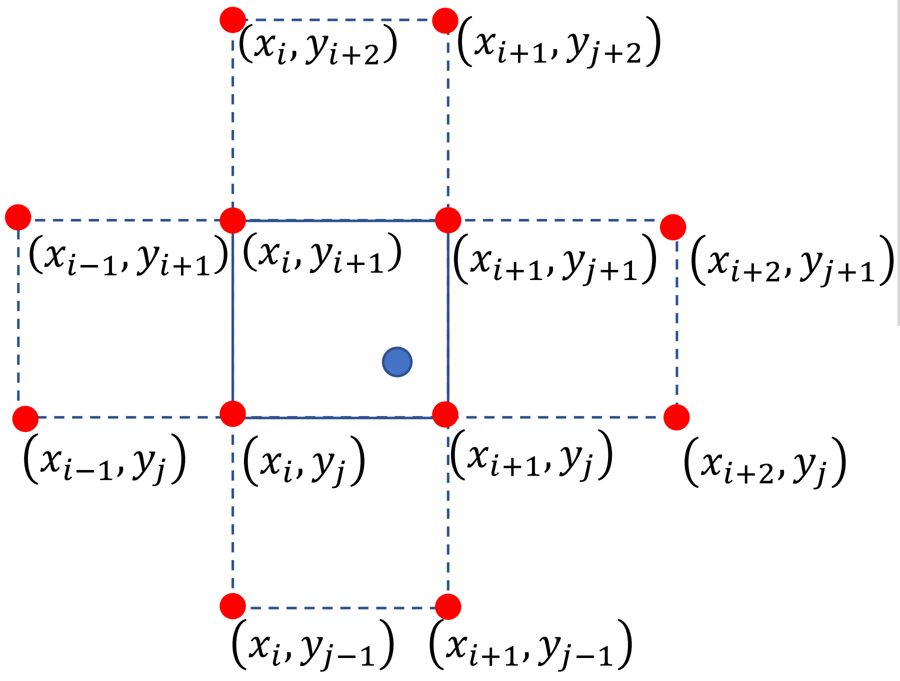
1. Odd extension of charge distributions
2. Apply 2D FFT to the extended distributions
3. Divide the FFT results by $-4\left(\frac{1}{\Delta x^2} sin^2\frac{p\pi}{2(m+1)} + \frac{1}{\Delta y^2} sin^2\frac{q\pi}{2(m+1)}\right)$
4. Apply 2D iFFT to 3
5. Extracted the solutions from the extended solutions.

Space Charge Effect

- Overview
- Histogram Making (Charge weighting)
- Poisson Solver
- Electric field

Electric Field

Rectangular Coordinate



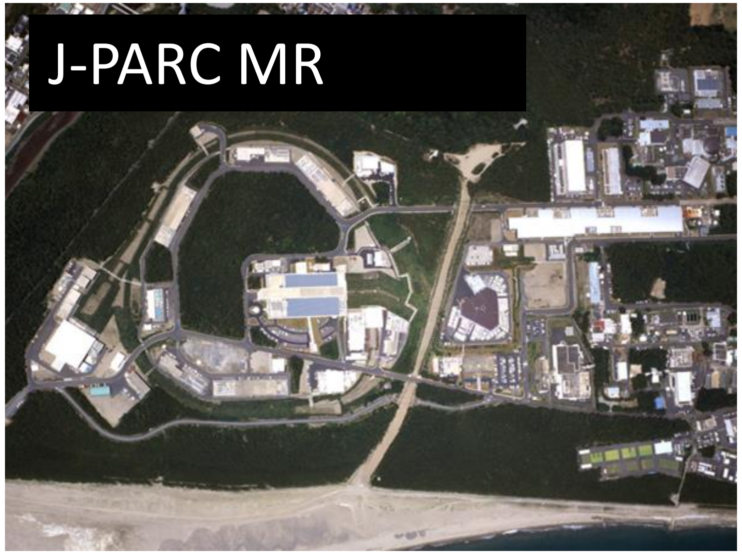
Interpolate potential using Bezier Surface

$$\varphi(x,y) = \sum_{m=0}^{3}\sum_{n=0}^{3} \varphi(x_{i+m-1}, y_{j+n-1}) \frac{3!}{m!\,(3-m)!}\left(\frac{x-x_i}{\delta x}\right)^m \left(\frac{x_{i+1}-x}{\delta x}\right)^{3-m} \times \frac{3!}{n!\,(3-n)!}\left(\frac{y-y_j}{\delta y}\right)^n \left(\frac{y_{j+1}-y}{\delta y}\right)^{3-n}$$

$$\vec{E}(x,y) = -grad\ \varphi(x,y) = -\left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right)\varphi(x,y)$$

Application to J-PARC Main Ring

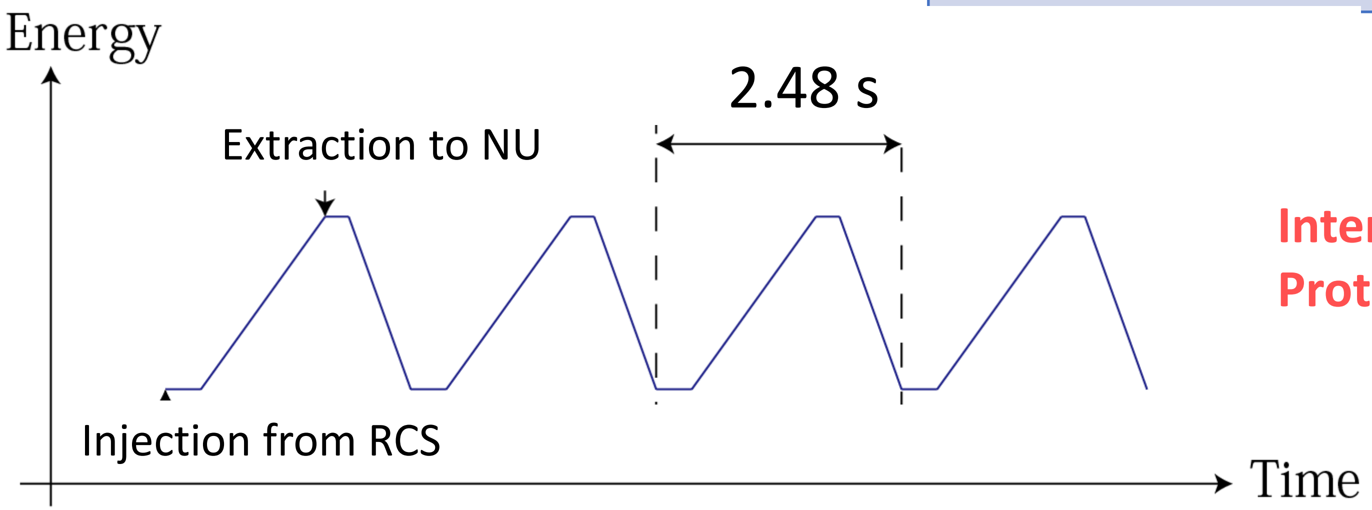
J-PARC Main Ring (MR)



Specifications of J-PARC MR

Circumference	1568 m
Injection Energy	3 GeV
Extraction Energy	30 GeV
Transverse Tune	(21.35, 21.45)
Synchrotron Tune	0.002 → 0.00015
Transition γ_t	j32.5
RF frequency	1.67→1.72 MHz
Harmonics	9

Present Operation Cycle for NU



Intensity ~ 500 kW (8 bunches)
Protons per bunch > 3×10^{13}

Simulation of J-PARC Main Ring

Conditions

- (ν_x, ν_y) = (21.35, 21.45)
- 3×10^{13} ppb
- 200000 macro-particles
- Rf 1st 160 kV 2nd 110 kV
- Bunch factor @ Inj = 0.2
- 2σ emittance @ Inj 16π for horizontal and vertical
- Aperture 65π for horizontal and vertical
- Number of Components：~1600 / turn
- Number of Locations for SC calculations ：~1000 points /turn

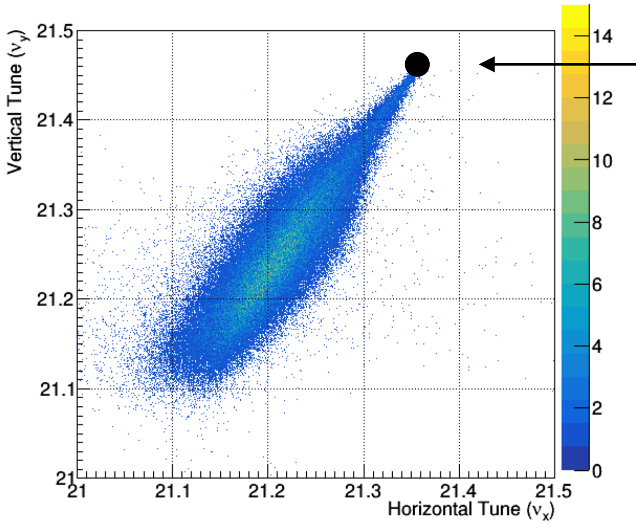
→

2700 sec /10000 turns using NVIDIA TESLA V100

At least 10 times faster than CPU simulation that we usually use for J-PARC MR

Preliminary Result

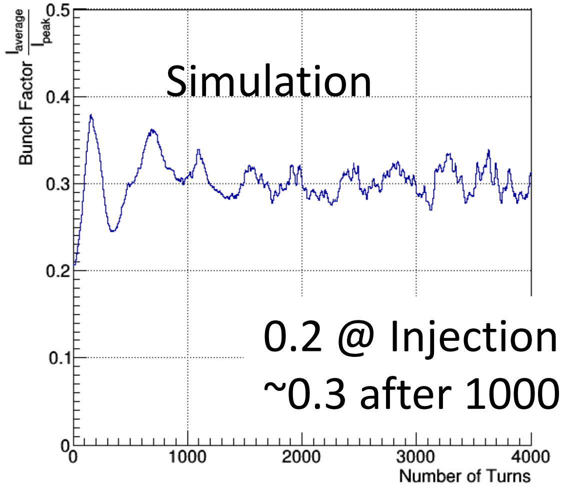
Transverse Tunes

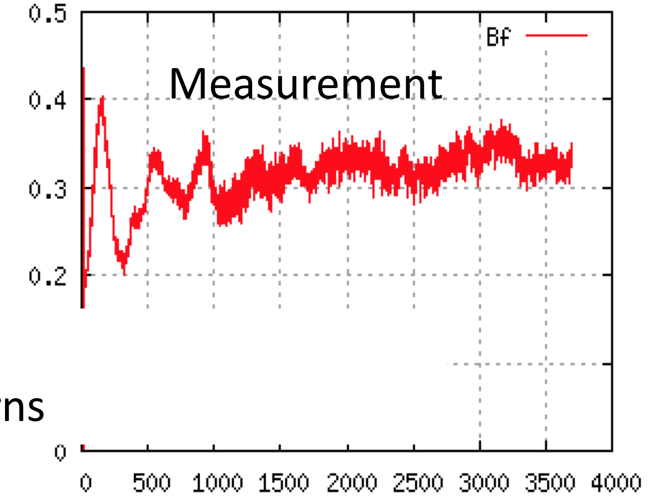


Operating Point
（21.35,21.45）

Tune Spread
Δν = 0.3-0.4

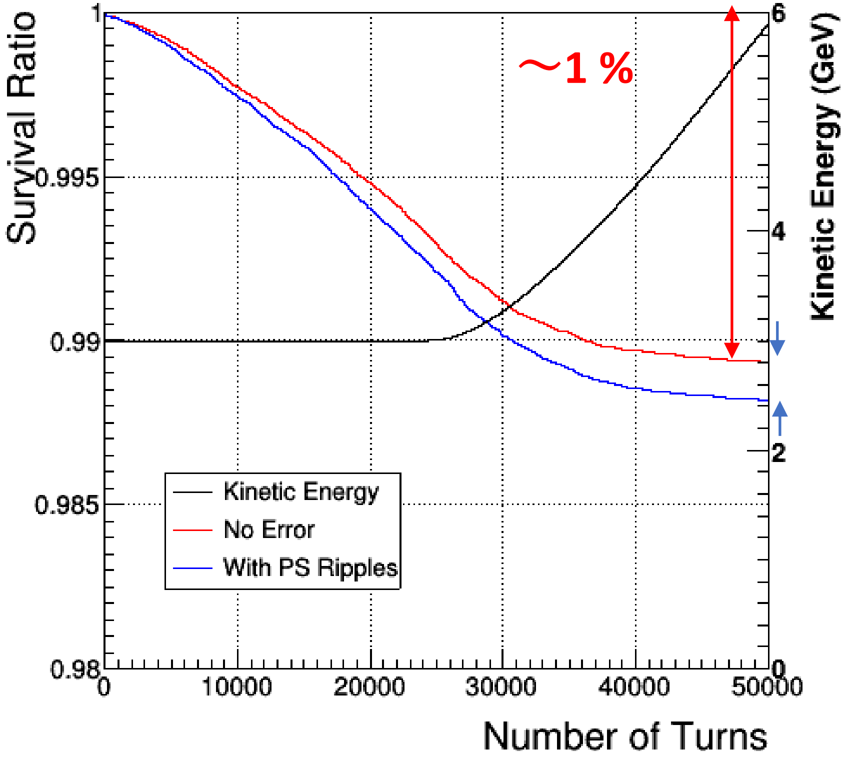
Bunch Factor $I_{average}/I_{peak}$





0.2 @ Injection
~0.3 after 1000 turns

Beam Survival



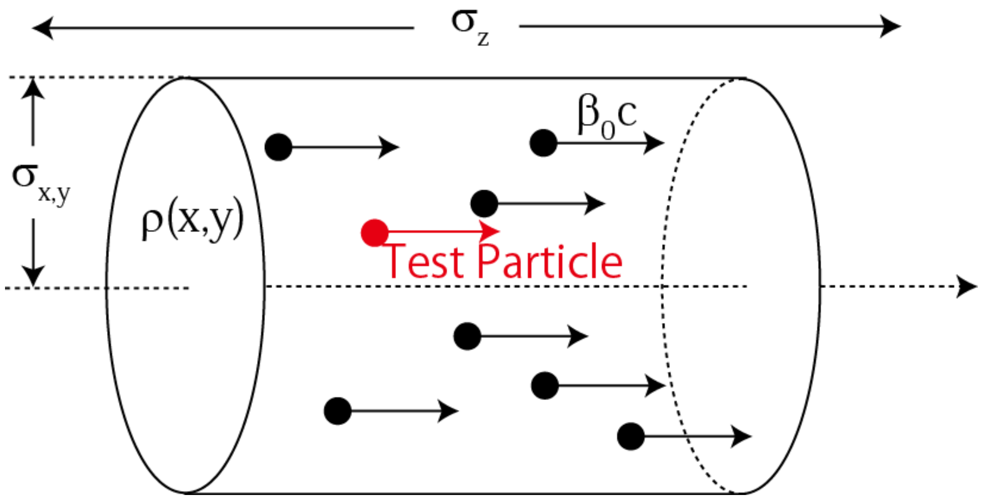
Seems correctly working.

Current Issues

The calculations here were already implemented in my code.
But they are not included in the results shown in the last slide. This is because they does not reproduce our measurements so far.

Origin of the coefficient $\frac{e}{m_p {\gamma_p}^3 {\beta_p}^2 c^2}$ (1)

2D Approximation ($\sigma_z \gg \sigma_{x,y}$)



Electric Field

- $div\vec{E} = \frac{\rho(x,y)}{\varepsilon_0}, \vec{E} = -grad\ \varphi_E(x,y)$

$$\Delta\varphi_E(x,y) = -\frac{\rho(x,y)}{\varepsilon_0}$$

Magnetic Field

- $rot\vec{B} = \frac{i(x,y)}{\varepsilon_0 c^2}, \vec{B} = rot\ \vec{A}(x,y),\ i(x,y) = \beta_0 c\rho(x,y)$

$$\Delta A_z(x,y) = -\frac{\rho(x,y)\beta_0}{\varepsilon_0 c} \rightarrow \Delta\varphi_B(x,y) = -\frac{\rho(x,y)}{\varepsilon_0} \qquad \left(\varphi_B(x,y) = \frac{c}{\beta_0} A_z(x,y)\right)$$

(Using Coulomb Gauge)

In free space, $\varphi_E(x,y)$ and $\varphi_B(x,y)$ are same. (satisfy same equation)

Origin of the coefficient $\dfrac{e}{m_p {\gamma_p}^3 {\beta_p}^2 c^2}$ (2)

The forces on a test particle by electric and magnetic fields are written as

$$\overrightarrow{F_E}(x,y) = -q\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right)\varphi_E(x,y)$$

$$\overrightarrow{F_B}(x,y) = q\,(0,0,\beta_0 c)\times\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y},\frac{\partial}{\partial z}\right)\times\big(0,0,A_z(x,y)\big) = -q\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right)\left(-{\beta_0}^2\varphi_B(x,y)\right)$$

Using $\vec{F}(x,y) = \overrightarrow{F_E}(x,y) + \overrightarrow{F_B}(x,y) = \frac{d}{dt}\vec{P} = P_0\beta_0 c\frac{d}{ds}\big(x\dot{(s)},\ y\dot{(s)}\big)$ (Equation of Motion),

$$\frac{d}{ds}\big(x\dot{(s)},\ y\dot{(s)}\big) = -\frac{q}{m{\beta_0}^2{\gamma_0}^3c^2}\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right)\varphi(x,y)$$

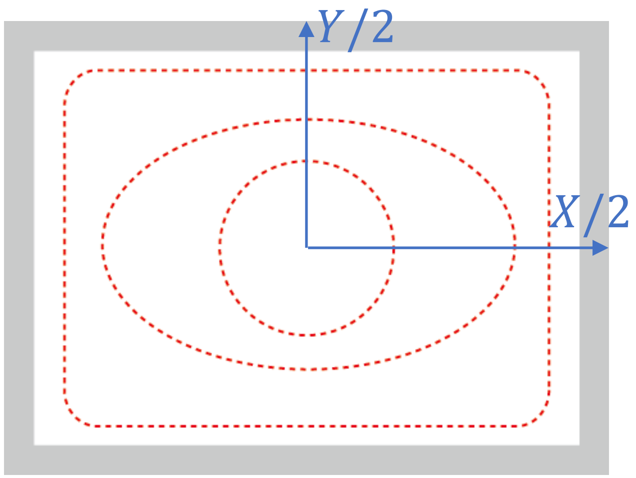
This is correct only when the beam is in infinite free space. $\varphi_E(x,y) = \varphi_B(x,y)$

In general, however, electric and magnetic fields satisfy different boundary conditions

$$\boldsymbol{\varphi_E(x,y) \neq \varphi_B(x,y)}$$

Boundary Conditions with Beam Ducts

Electric Field



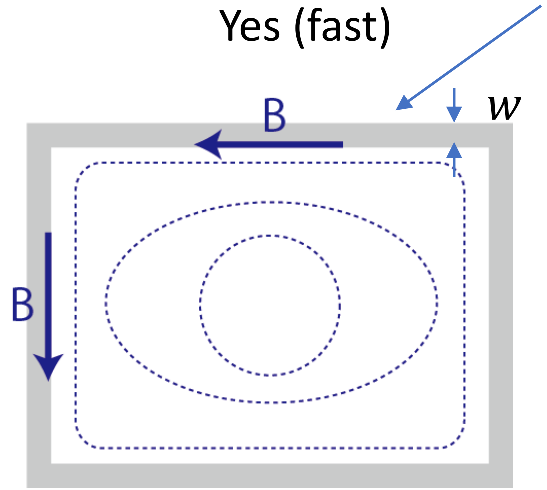
Same potential at duct surfaces

$$\varphi_E(\pm X/2, y) = \varphi_E(x, \pm Y/2) = \text{Const.}$$

Magnetic Field

Skin depth δ(frequency of the beam) << Duct depth w

Yes (fast)



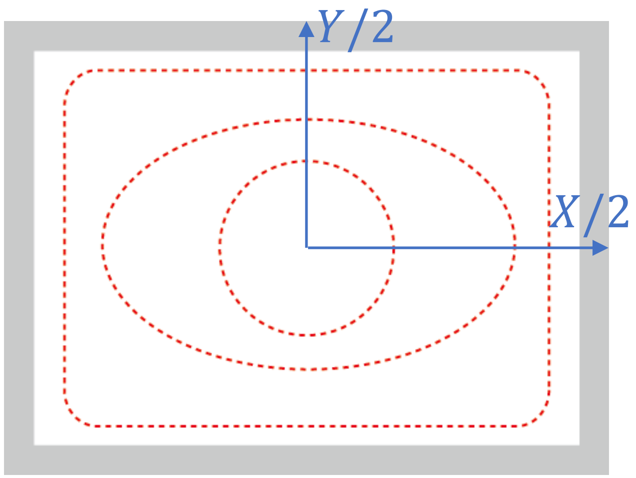
B is tangential to duct surfaces

$$B_y\left(x, \pm\frac{Y}{2}\right) = -\frac{\partial}{\partial x}\varphi_B(x, \pm\frac{Y}{2}) = 0$$

$$B_x\left(\pm\frac{X}{2}, y\right) = \frac{\partial}{\partial y}\varphi_B(\pm\frac{X}{2}, y) = 0$$

Boundary Conditions with Beam Ducts

Electric Field



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φ_B and φ_E can be commonly calculated

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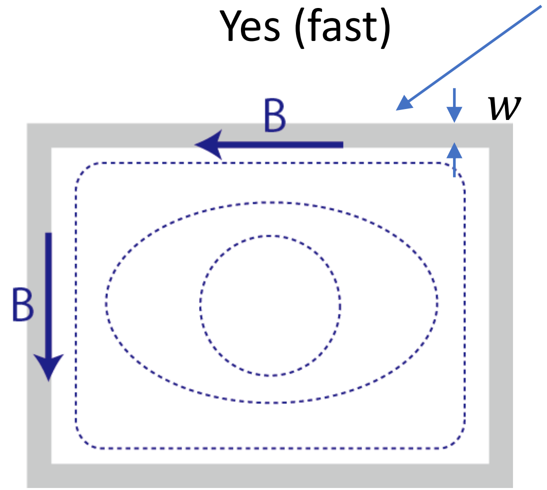

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Boundary Conditions with Beam Ducts

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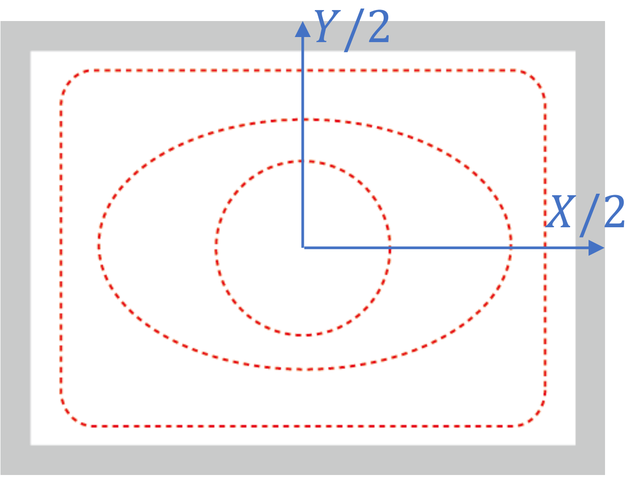

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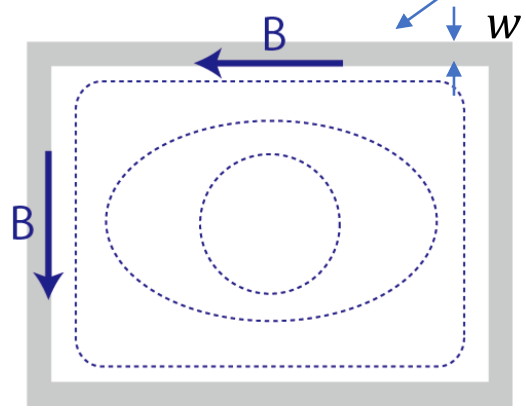

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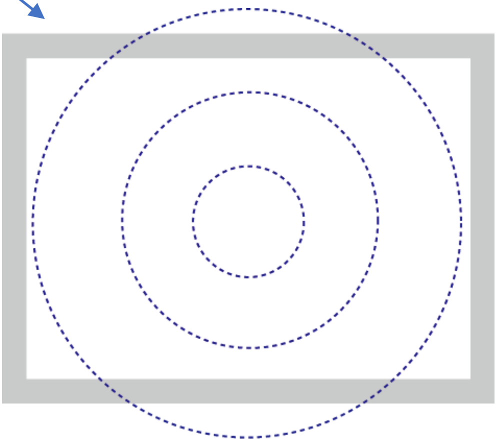
No (slow)

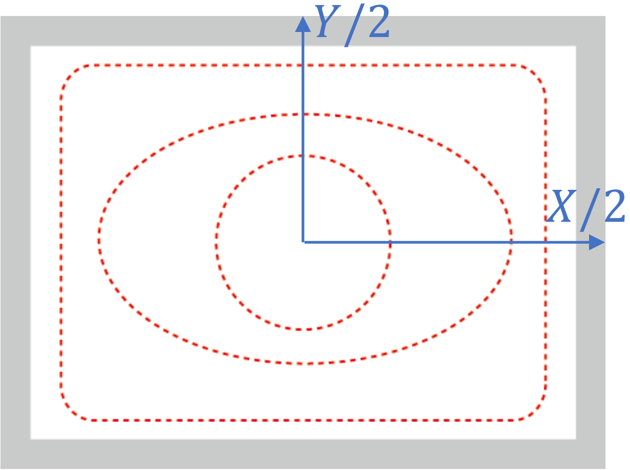


$$\varphi_B(\pm\infty) = 0$$

Same as free space

Boundary Conditions with Beam Ducts

Electric Field



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Magnetic Field

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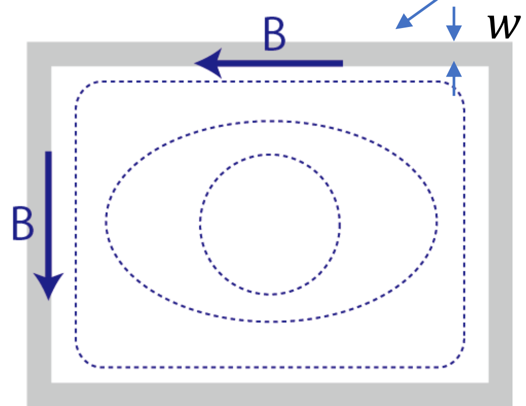

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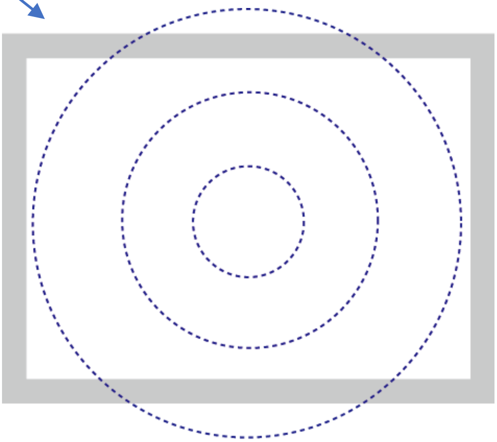


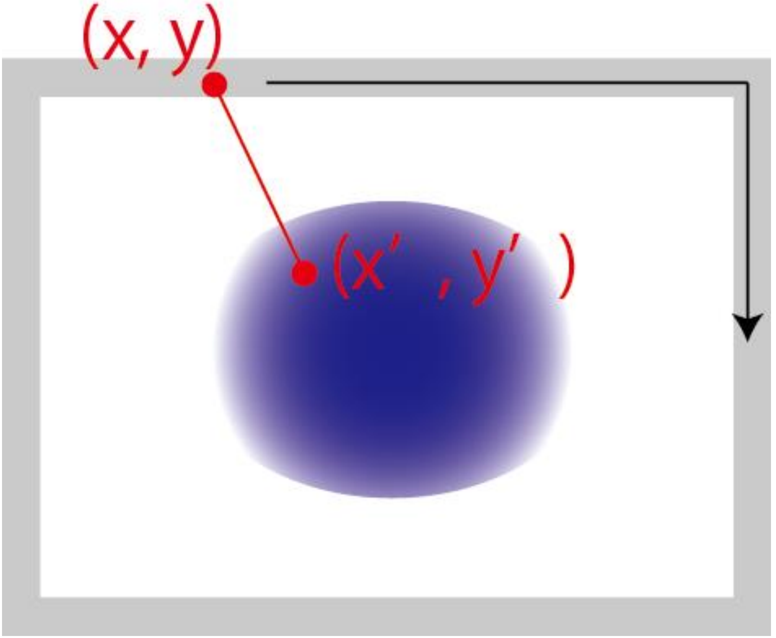
$$\varphi_B(\pm\infty) = 0$$

Same as free space

φ_B and φ_E must be separately calculated

Free Space 2D Poisson Equation

1. Find potential at the boundary using the Green function*



$$\varphi(x,y) = \frac{1}{2\pi}\int \rho(x',y') \ln \frac{1}{\sqrt{(x-x')^2+(y-y')^2}} dx'dy'$$

2. Modify the differential equation

$$\frac{u_{0,j} - 2u_{1,j} + u_{2,j}}{\Delta x^2} + \frac{u_{1,j-1} - 2u_{1,j} + u_{1,j+1}}{\Delta y^2} = f_{1,j}$$

Obtained by the Green function

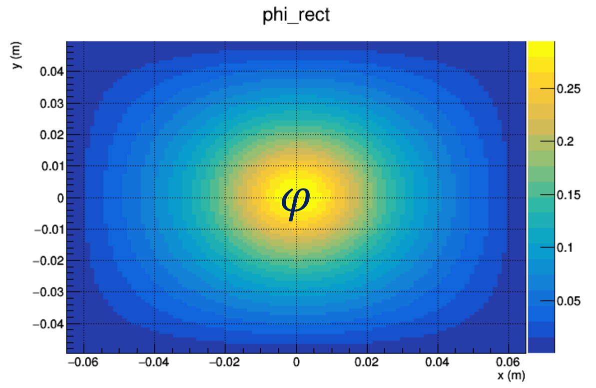
$$\frac{-2u_{1,j} + u_{2,j}}{\Delta x^2} + \frac{u_{1,j-1} - 2u_{1,j} + u_{1,j+1}}{\Delta y^2} = f_{1,j} - \frac{u_{0,j}}{\Delta x^2}$$

Modifying charge distribution near the boundary

* Calculating whole region using the Green function costs a lot since it needs $O(n^2)$ operations

In case of J-PARC Main Ring (1)

1. Same Boundary Condition $\varphi_E(\pm X/2, y) = \varphi_E(x, \pm Y/2) = \text{Const.}$ $\varphi_B(\pm X/2, y) = \varphi_B(x, \pm Y/2) = \text{Const.}$

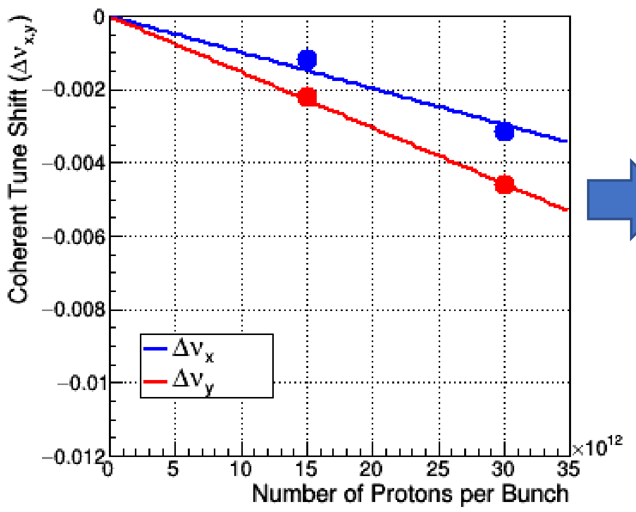


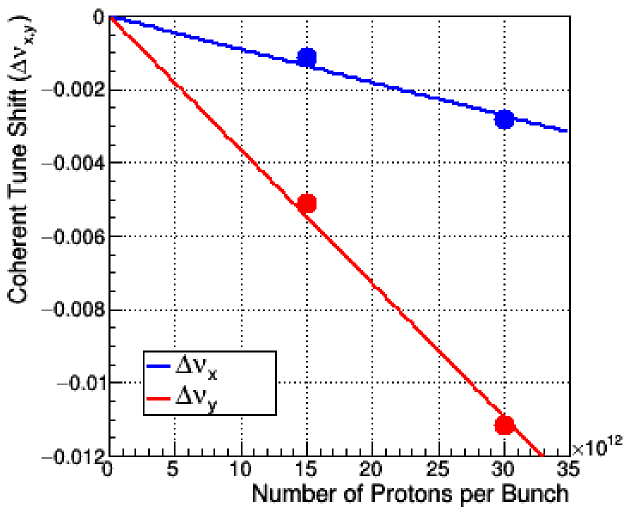
- Comparison with measurement of coherent tune shift

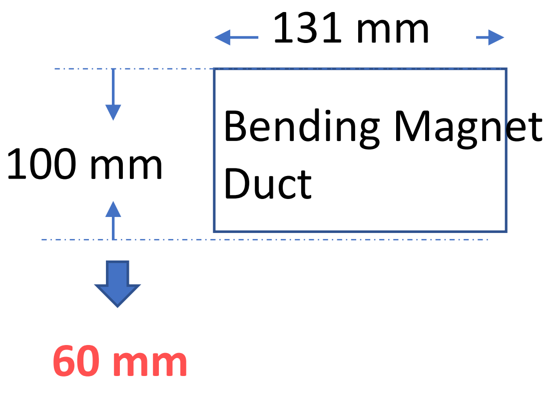
Simulation Results

BM duct height 100mm
(real value)

BM duct height 60mm
(good agreement with measurement)





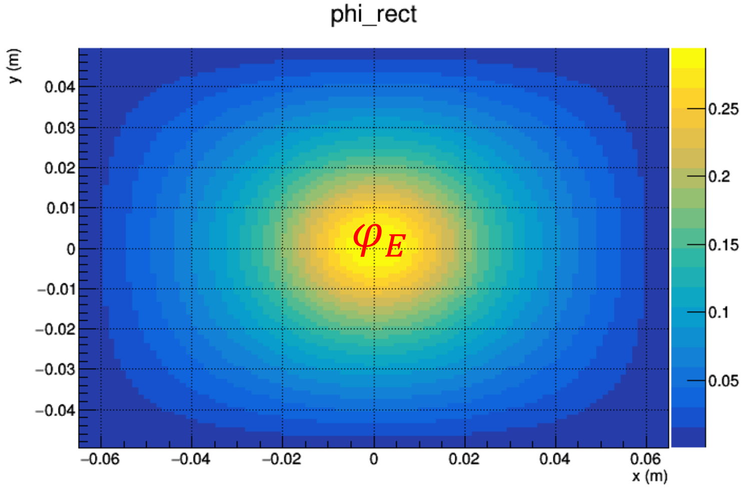


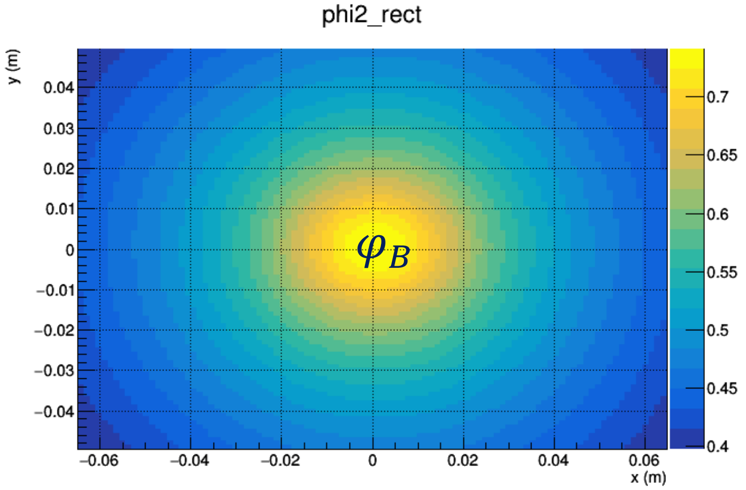
Need to make duct height unreasonably smaller

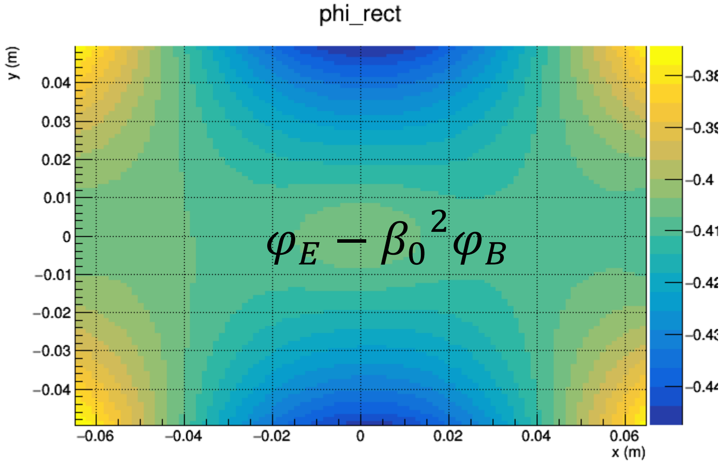
In case of J-PARC Main Ring (2)

2. $\varphi_E(\pm X/2, y) = \varphi_E(x, \pm Y/2) = \text{Const.}$ $\varphi_B(\pm\infty) = 0$

<u>Potential Distribution in a BM Duct</u>





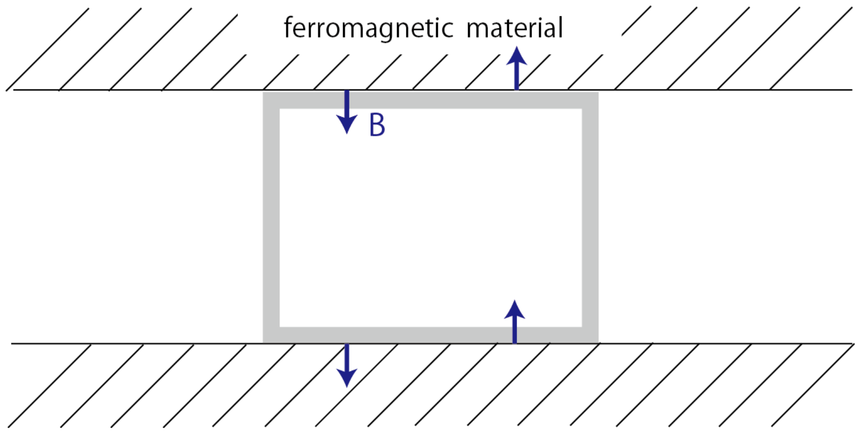


Incoherent tune shifts to vertical direction are so large and cause huge beam loss within a few turns…

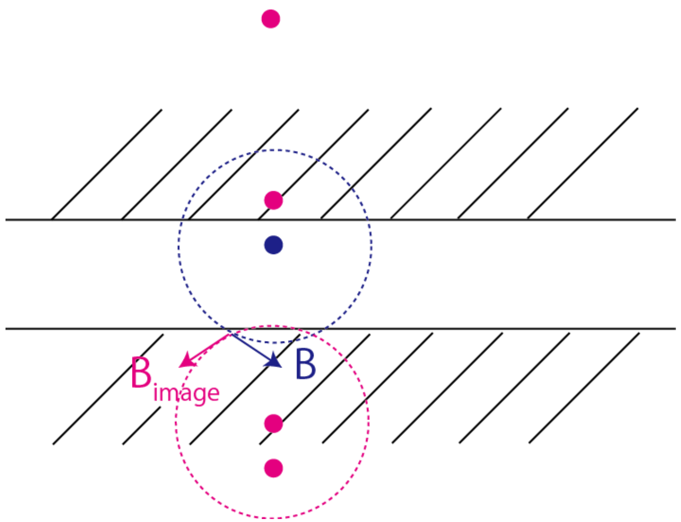
Need to consider other physics

One Possibility

Boundary with Magnetic Material



- B at the surface of ferromagnetic material must be perpendicular to the surface ($\delta \ll w$)
- **The cores of the BMs touch with all horizontal surface of the duct. (J-PARC MR)**



➢ Use image current density so that the tangential components of B can be eliminated

$$i_{image}(x, y) = \sum [i(x, (-1)^n y + nY)]$$

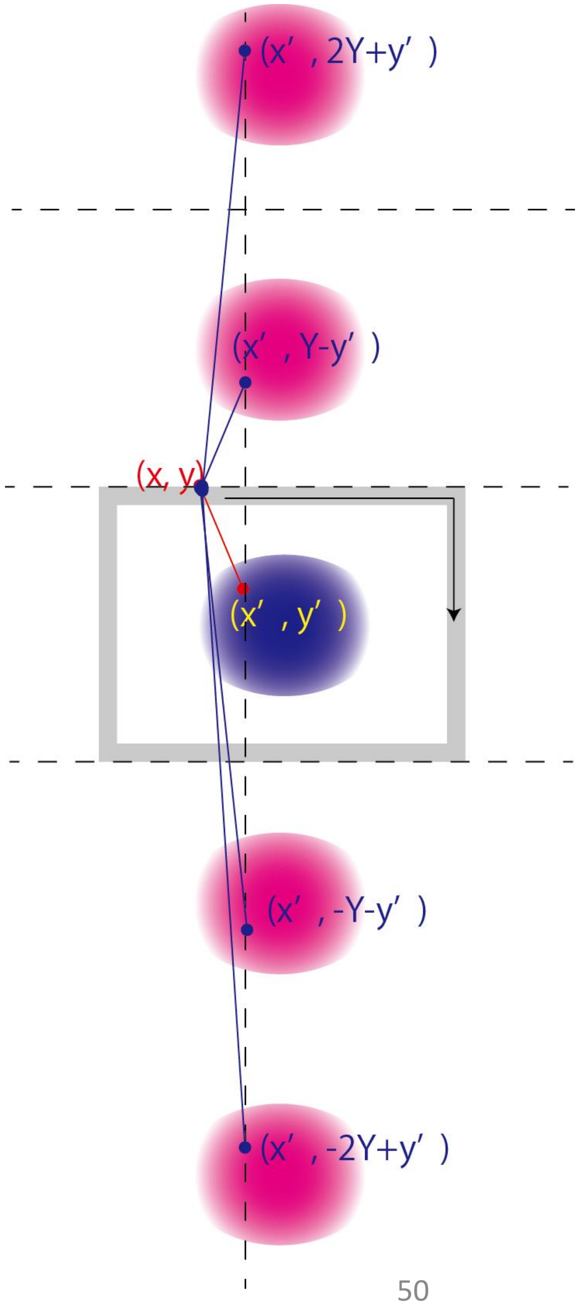
Finding Potential with Image Current

1. Find potential including **image current** at the boundary using the Green function*

$$\varphi(x,y) = \frac{1}{2\pi}\int(\rho(x',y') + \rho_{image}(x',y'))\ln\frac{1}{\sqrt{(x-x')^2+(y-y')^2}}dx'dy'$$

$$\rho_{image}(x,y) = \sum[\rho(x,(-1)^n y + nY)]$$

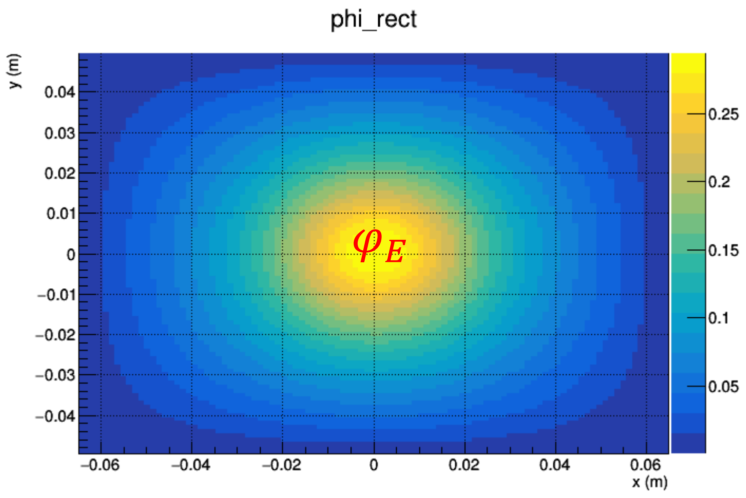
2. Modify the differential equation (same as for free space)

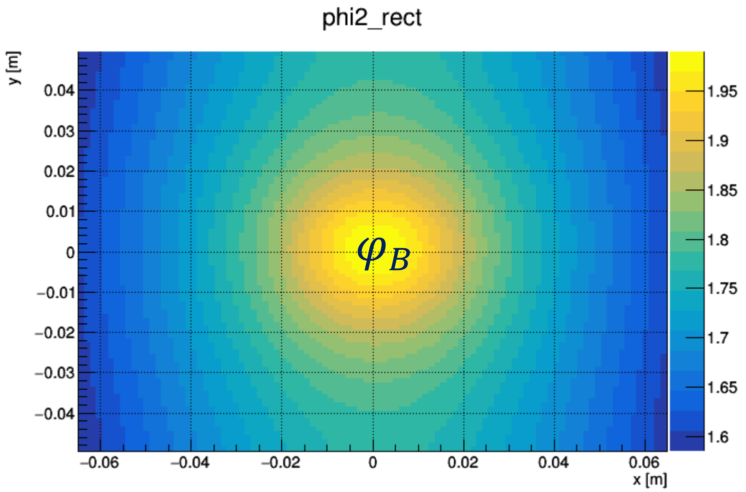


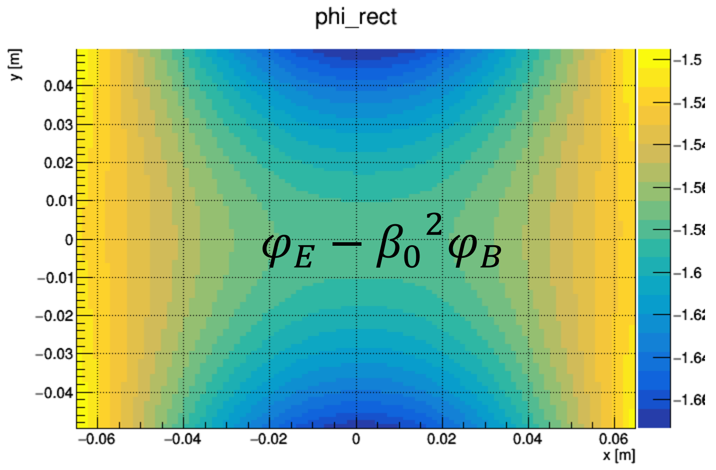
In case of J-PARC Main Ring (3)

2. $\varphi_E(\pm X/2, y) = \varphi_E(x, \pm Y/2) = \text{Const.}$ $B_x(x, \pm Y/2) = B_y(\pm X/2, y) = 0$

<u>Potential Distribution in a BM Duct</u>







Incoherent tune shifts to vertical direction are **still** so large and cause huge beam loss within a few turns…

The potential in the BM duct can not be simply modeled.
May need combinations of phenomena shown here ??

Summary

- Particle tracking simulation code with space charge effect has been developed for GPU usage.
- Not only single particle mechanics but also space charge calculation can be accelerated using FFT.
- For the simulation of J-PARC MR, the developed code with GPU is at least 10 times faster than our official(?) code with CPU.
- Currently working on the calculation of SC in thin rectangular ducts.

Backup

Canonical Valuables

x : horizontal coordinate of a plane perpendicular to the reference orbit

y : vertical coordinate of a plane perpendicular to the reference orbit

$\sigma = s - \beta_0 t$

p_x : momentum of x direction normalized by the reference momentum p_0

p_y : momentum of y direction normalized by the reference momentum p_0

$p_\sigma = \frac{E-E_0}{\beta_0 p_0 c}$: canonical conjugate of σ

$\delta = \frac{p-p_0}{p_0} \approx p_\sigma - \frac{1}{2{\gamma_0}^2}{p_\sigma}^2,$

Implemented Components

Ex2 Acceleration

- Cavity

V_{RF} : Amplitude of RF voltage, f_{RF} : RF frequency, n : n-th order harmonics, φ_s : Synchrotron phase

$$x(s) = x(0) \quad y(s) = y(0) \quad \sigma(s) = \sigma(0)$$

$$p_x(s) = p_x(0) \quad p_y(s) = p_y(0) \quad p_\sigma(s) = p_\sigma(0) + \frac{eV_{RF}}{m\gamma\beta c^2}\left(\sin\left(\frac{2\pi f_{RF}}{\beta c}\sigma(0) + \varphi_s\right) - \sin\varphi_s\right)$$

- Adiabatic Damping (turn by turn)

$$p_{x,n+1} = p_{x,n}\left(1 - \frac{\Delta E}{m\gamma_n{\beta_n}^2}\right)$$

$$p_{y,n+1} = p_{y,n}\left(1 - \frac{\Delta E}{m\gamma_n{\beta_n}^2}\right)$$

$$p_{\sigma,n+1} = p_{\sigma,n}\left(1 - \frac{\Delta E\left(1 + \frac{1}{{\gamma_n}^2}\right)}{m\gamma_n{\beta_n}^2}\right)$$