Anomalous Casimir Effect in Axion Electrodynamics

Axion Electrodynamics

Our theoretical framework is the axion electrodynamics described by the Maxwell-Chern-Simons (MCS) equation. It contains a Chern-Simons-like term $\frac{1}{4}\theta(x)F_{\mu\nu}\tilde{F}^{\mu\nu}$ in the Lagrangian, with the background axion field $\theta(x) = b_\mu x^\mu$.

This model successfully accounts for several novel effects induced by quantum anomaly, attracting great attention in high energy nuclear experiments. This theory also plays a significant role in the study of Lorentz-violating extension of the Standard Model.

Recent development of condensed matter physics materials like topological insulator and Weyl semimetal provides the experimental realization of anomalous chiral phenomena.

$$\boldsymbol{\nabla}\cdot\boldsymbol{E} = -\boldsymbol{b}\cdot\boldsymbol{B},$$

$$\boldsymbol{\nabla}\times\boldsymbol{B} - \frac{\partial\boldsymbol{E}}{\partial t} = b_0\boldsymbol{B} + \boldsymbol{b}\times\boldsymbol{E},$$

$$\boldsymbol{\nabla}\cdot\boldsymbol{B} = 0,$$

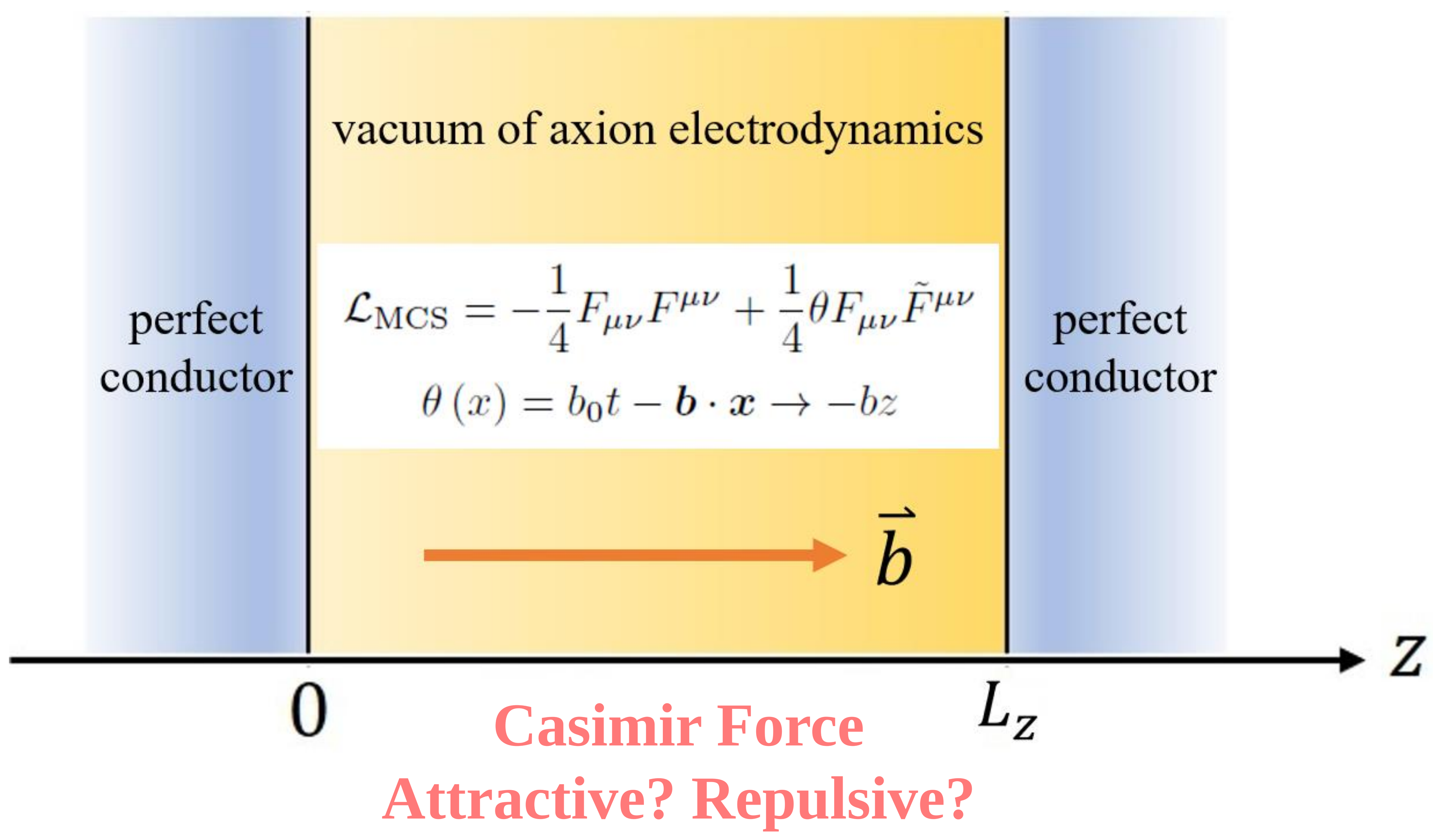
$$\boldsymbol{\nabla}\times\boldsymbol{E} + \frac{\partial\boldsymbol{B}}{\partial t} = 0.$$

Witten Effect ($-\boldsymbol{b}\cdot\boldsymbol{B}$)

Chiral Magnetic Effect ($b_0\boldsymbol{B}$)

Anomalous Hall Effect ($\boldsymbol{b}\times\boldsymbol{E}$)

Influence on Casimir effect?



Casimir Effect

The Casimir effect refers to a force resulting from the quantum fluctuation in the vacuum restricted by a certain boundary.

In its **original** study (H. B. G. Casimir, 1948), an **attractive force** emerges from the QED vacuum between two parallel plates of perfect conductor/ideal metal. In our setup, the more non-trivial vacuum of axion electrodynamics causes anomalous modifications.

Especially, as hinted by recent work (Q.-D. Jiang and F. Wilczek, 2019), **chiral media** with intrinsic parity symmetry breaking may **flip the sign of the Casimir force**, circumventing a previous "no-go" theorem (O. Kenneth, I. Klich, A. Mann, and M. Revzen, 2002). We specify our setup to a pure spacelike $b_\mu = (0,0,0,b)$ to pursue such possibility of a repulsive Casimir force.

Vacuum Energy

Axion electrodynamics Lagrangian with ghost c and gauge fixing:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}bA_\nu\tilde{F}^{z\nu} + \frac{1}{2\xi}(\partial_\mu A^\mu)^2 + \frac{1}{2}\partial_\mu\bar{c}\partial^\mu c$$

Vacuum energy density ε calculated from generating functional Z:

$$VT\varepsilon = i\log Z = -\frac{i}{2}\sum_\pm\sum_k \log\left[k_0^2 - \omega_\pm^2(\boldsymbol{k})\right]$$

$V = L_x L_y L_z$: volume of system. T: time interval in path integral.

$\omega\sim\boldsymbol{k}$: photon dispersion relation determined by on-shell condition:

$$\omega_\pm^2 = k_x^2 + k_y^2 + \left(\sqrt{k_z^2 + \frac{b^2}{4}} \pm \frac{b}{2}\right)^2$$

Adopt Dirichlet boundary condition, and the limits $L_x, L_y, T \to \infty$:

$$k \to (k_0, k_x, k_y, n\pi/L_z), \qquad \frac{1}{VT}\sum_k \to \frac{1}{L_z}\sum_{n=0}^{\infty}\int\frac{dk_0 dk_x dk_y}{(2\pi)^3}$$

Vacuum energy per unit area (Casimir energy):

$$\mathcal{E} \equiv \frac{\varepsilon V}{L_x L_y} = \sum_\pm\sum_{n=0}^{\infty}\int\frac{dk_x dk_y}{(2\pi)^2}\frac{1}{2}\omega_\pm(\boldsymbol{k})$$

Alternative Method: Scattering Formula

Scattering formula for Casimir energy in chiral media developed by "*Q.-D. Jiang and F. Wilczek, Phys. Rev. B99, 125403 (2019)*":

$$\mathcal{E} = \int_0^\infty\frac{d\zeta}{2\pi}\int_{-\infty}^{\infty}\frac{dk_x dk_y}{(2\pi)^2}\log\det\left(\mathbb{I} - R_1U_{12}R_2U_{21}\right) \qquad \zeta = -i\omega$$

Translation matrix U_{ij} and reflection matrix R_i:

$$U_{12} = \begin{pmatrix} e^{ik_z^+L_z} & 0 \\ 0 & e^{ik_z^-L_z}\end{pmatrix}, U_{21} = \begin{pmatrix} e^{ik_z^-L_z} & 0 \\ 0 & e^{ik_z^+L_z}\end{pmatrix}, R_1 = R_2 = \begin{pmatrix} 0 & -1 \\ -1 & 0\end{pmatrix}.$$

Employ our dispersion relation: $k_z^\pm \equiv i\sqrt{\zeta^2 + \boldsymbol{k}_\perp^2 \pm ib\sqrt{\zeta^2 + \boldsymbol{k}_\perp^2}}$.

Casimir Force

Define $\bar{b} \equiv bL_z/2\pi$. Rescale $k_{x,y} \to \tilde{k}_{x,y} \equiv (L_z/\pi\mu_\pm)k_{x,y}$.

$$\mathcal{E} = \frac{\pi^3}{L_z^3}\sum_\pm\sum_{n=0}^{\infty}\left(\sqrt{n^2+\bar{b}^2}\pm\bar{b}\right)^3\int^{\tilde{\Lambda}_\pm}\frac{d\tilde{k}_x\tilde{k}_y}{(2\pi)^2}\frac{1}{2}\sqrt{1+\tilde{k}_x^2+\tilde{k}_y^2}$$

Sum over helicity $\pm$: $\sum_{n=0}^{\infty}\left[(n^2+\bar{b}^2)^{\frac{3}{2}} + 3\bar{b}^2(n^2+\bar{b}^2)^{\frac{1}{2}}\right]$

Regulated by $\Theta(\Lambda_\pm - \omega_\pm(\boldsymbol{k}))$: $-\frac{1}{6\pi}\left(1-\tilde{\Lambda}_\pm^3\right)$

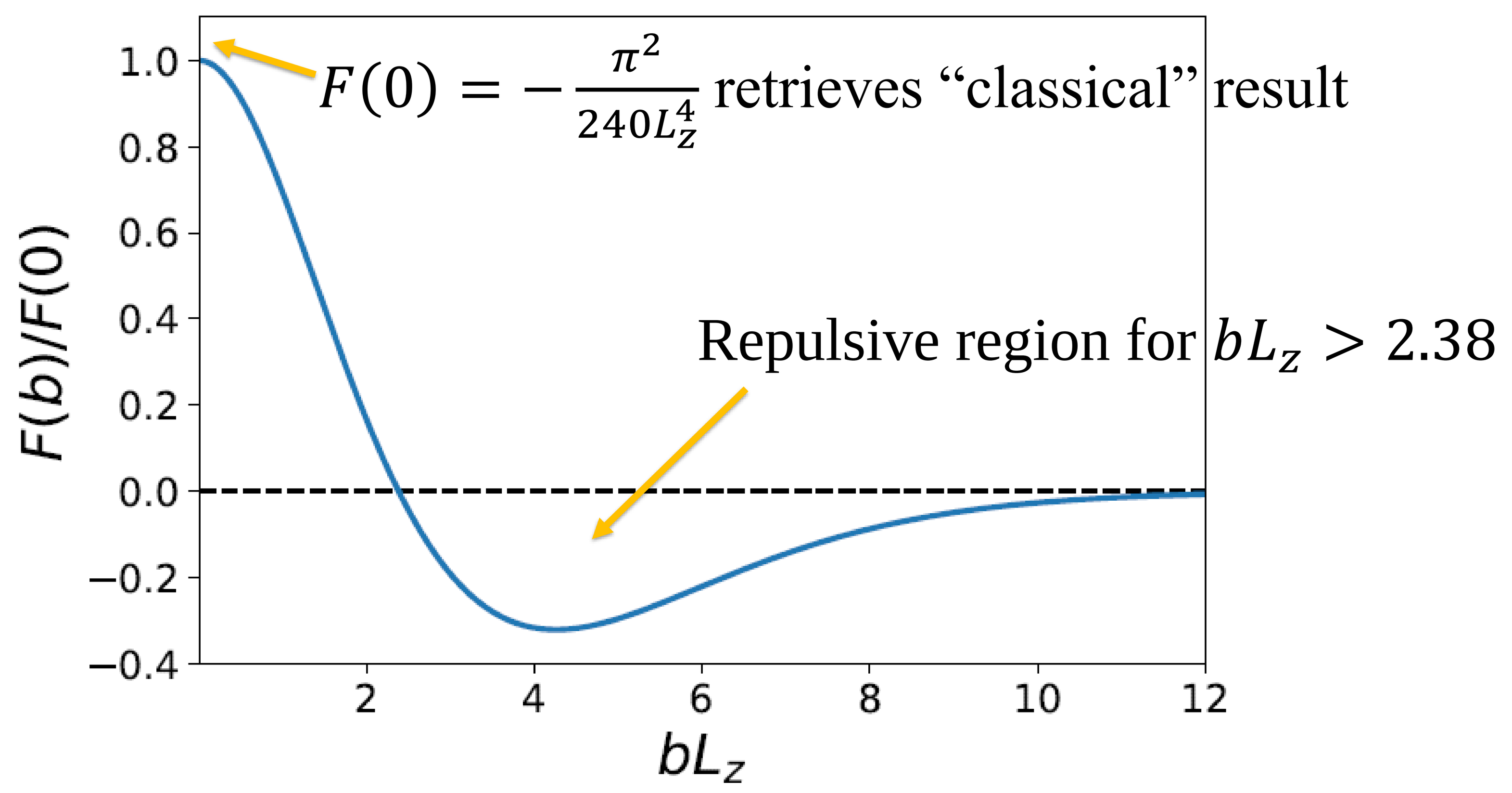
The problem reduces to the following summation:

$$\sum_{n=-\infty}^{\infty}(n^2+\bar{b}^2)^{-s} = \frac{\sqrt{\pi}\bar{b}^{1-2s}}{\Gamma(s)}\left[\Gamma\left(s-\frac{1}{2}\right) + 4\sum_{m=1}^{\infty}\frac{K_{\frac{1}{2}-s}(2\pi m\bar{b})}{(\pi m\bar{b})^{\frac{1}{2}-s}}\right]$$

Subtracting a divergent portion in energy density $-\frac{5b^4}{512\pi^3}\Gamma(0)$, which is infinite but independent on L_z thus irrelevant to force, we obtain the "regular" part of Casimir energy:

$$\mathcal{E}_{\rm reg} = \frac{b^4L_z}{16\pi^2}\sum_{m=1}^{\infty}\left[\frac{K_1(mbL_z)}{mbL_z} - \frac{K_2(mbL_z)}{(mbL_z)^2}\right]$$

Casimir Force: $F(b) = -\frac{\partial\mathcal{E}_{\rm reg}}{\partial L_z} = -\frac{b^4}{16\pi^2}\sum_{m=1}^{\infty}\left[\frac{3K_2(mbL_z)}{(mbL_z)^2} - K_0(mbL_z)\right]$



Kenji Fukushima, Shota Imaki, and Zebin Qiu, arXiv:1906.08975