Applicability of the complex Langevin method for QCD at finite density

Shoichiro Tsutsui[A], Yuta Ito[B], Hideo Matsufuru[C], Jun Nishimura[C,D], Shinji Shimasaki[E], Asato Tsuchiya[F]

([A] RIKEN, [B] Tokuyama National College of Technology, [C] KEK, [D] Sokendai Univ., [E] Keio Univ., [F] Shizuoka Univ.)

Abstract: We examine the applicability of the complex Langevin method (CLM), which is a promising approach to overcome the sign problem for QCD at finite density. We performed numerical simulations on a 24^3 x 12 lattice with four-flavor staggered fermions around the phase transition line in the (T-μ)-plane. As a result, we found that the region where the CLM fails is the chiral symmetry broken phase. Since the origin of the failure is the singular drift problem, this feature of the CLM is understood by a generalized version of the Banks-Casher relation which relates Dirac zero modes to the chiral condensate.

QCD Partition function

$$Z = \int dU \det M[U,\mu] e^{-S_g}$$

$\det(M(\mu)^\dagger) = \det M(-\mu)$ … complex number!

Complex Langevin Method (CLM)

[Parisi '83]
[Klauder '84]

Complexification:

$$U_{x\mu} \in SU(3) \to \mathcal{U}_{x\mu} \in SL(3,C)$$

Solve the Langevin equation:

$$\mathcal{U}_{x\mu}(t+\epsilon) = \exp\left[i\left(-\epsilon \mathcal{D}_{x\mu} S[\mathcal{U}] + \sqrt{\epsilon}\eta_{x\mu}\right)\right]\mathcal{U}_{x\mu}(t)$$

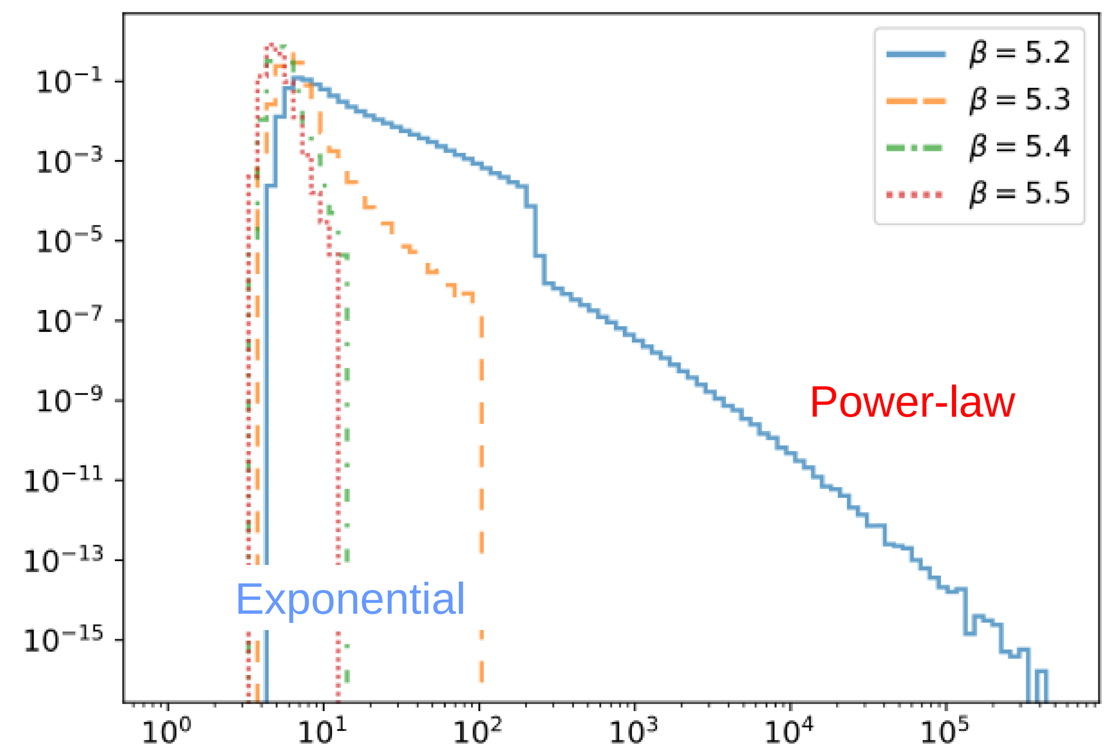
Drift term

Reliability of CLM

[Nagata, Nishimura, Shimasaki '15]

The histogram of the drift term shows

- ◆ exponential fall-off: CLM = path integral
- ◆ Power-law fall-off: CLM ≠ path integral



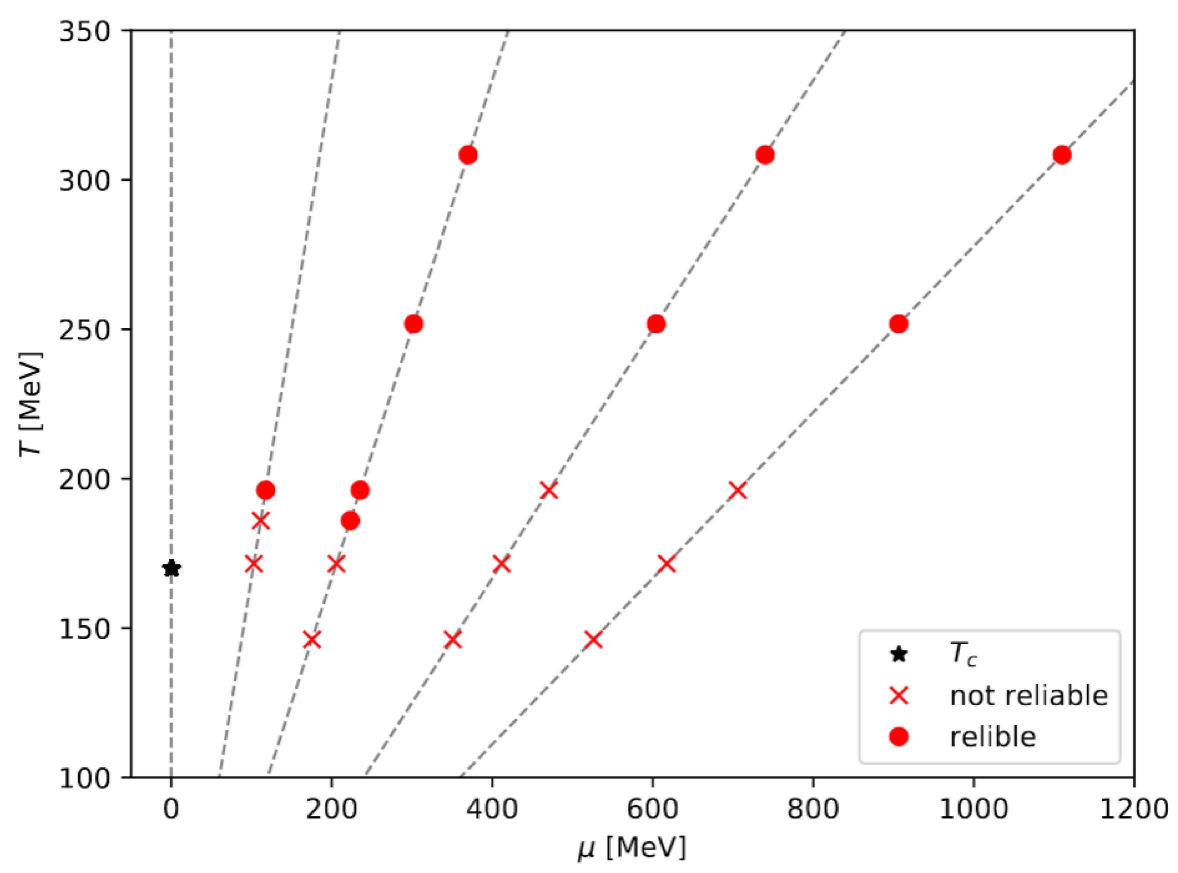
e.g.) At μ = 0.1,

CLM is reliable for β=5.3,5.4,5.5

Singular drift problem occurs for β=5.2

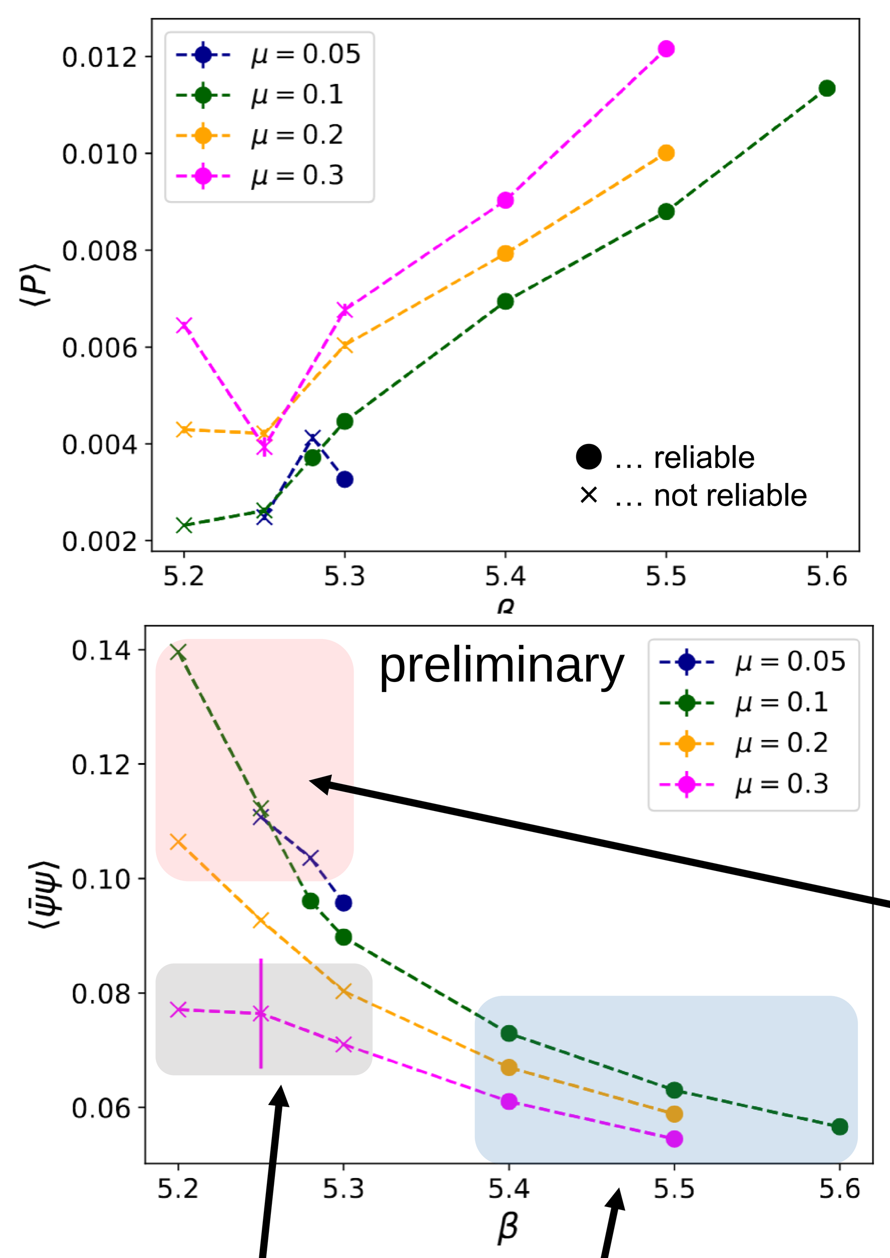
Setup

- ➢ $N_f = 4$ (staggered fermions)
- ➢ 24^3 x 12 lattice
- ➢ μa = 0.05, 0.1, 0.2, 0.3 (μ/T = 0.6 - 3.6)
- ➢ ma = 0.01



c.f.) 1811.07647 (Proc. of Lat.2018) for 20^3 x 12 lattice

Polyakov loop and Chiral condensate



Singular drift problem

The drift term becomes large due to the zero eigenvalues of the Dirac operator.

$$v_{x,\nu} = D_{x,\nu} S[\mathcal{U}] \propto \mathrm{tr}\left(M[\mathcal{U}]^{-1}\mathcal{D}_{x,\nu} M[\mathcal{U}]\right)$$

Generalized Banks-Casher relation

$$\lim_{m\to+0}\langle\bar{q}q\rangle = \pi \lim_{r\to+0} r \lim_{m\to+0} \hat{\rho}^{(\mathrm{CL})}(r)$$

- In chiral symmetry broken phase, the singular drift problem should occur.
- Our numerical results are consistent to this observation.

Details:
In CLM, the distribution of the Dirac eigenvalue λ is defined by

$$\rho^{(\mathrm{CL})}(x,y) = \left\langle \frac{1}{n}\sum_{i=1}^{n} \delta(x - \mathrm{Re}\,\lambda_i)\delta(y - \mathrm{Im}\,\lambda_i)\right\rangle$$

We also introduce integrated distribution:

$$\hat{\rho}^{(\mathrm{CL})}(r) = \int_0^\pi d\theta \sin\theta \, \tilde{\rho}^{(\mathrm{CL})}(r,\theta)$$

[Splittorff '14][Nagata, Nishimura, Shimasaki '16]

Comments

- Another reason for the breakdown of the CLM is the excursion problem (Complexified link variables deviate from SU(3) subspace).
- As long as β is sufficiently large (or lattice spacing is small enough), the unitarity norm can be well controlled even if μ is large.

Summary

- ➢ We studied the applicability of the CLM on 24^3 x 12 lattice with four-flavor staggered fermions around the phase transition line.
- ➢ The Banks-Casher relation tells us that the singular drift problem should occur in the chiral symmetry broken phase.
- ➢ In symmetry restored phase, CLM is applicable as long as β is large enough.

Outlook

- ◆ Can we find the phase transition line relying on the CLM ? Namely, can we determine the curvature of the line?
- ◆ Compute the Dirac spectrum in order to discuss the appearance of the singular drift problem directly.