Quantum dissipation of quarkonium in quark-gluon plasma: Lindblad equation approach

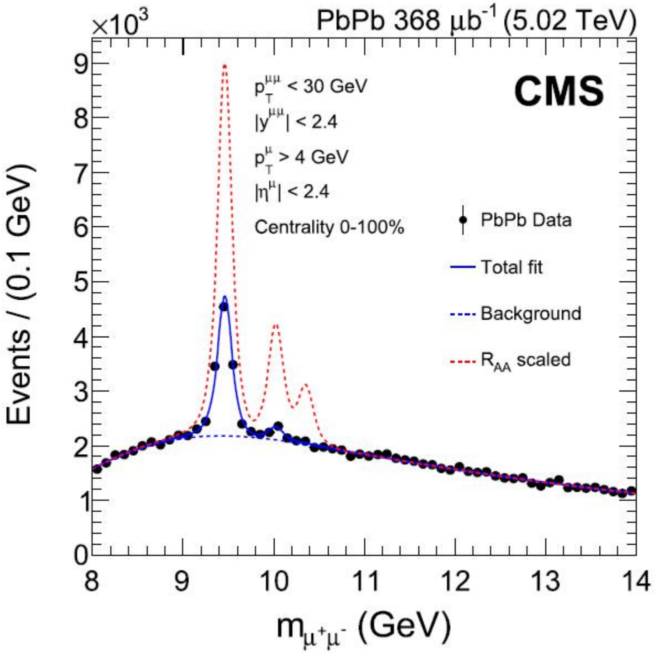
Takahiro Miura(Osaka)

with
Y.Akamatsu, M.Asakawa(Osaka)
and A.Rothkopf(Stavanger)

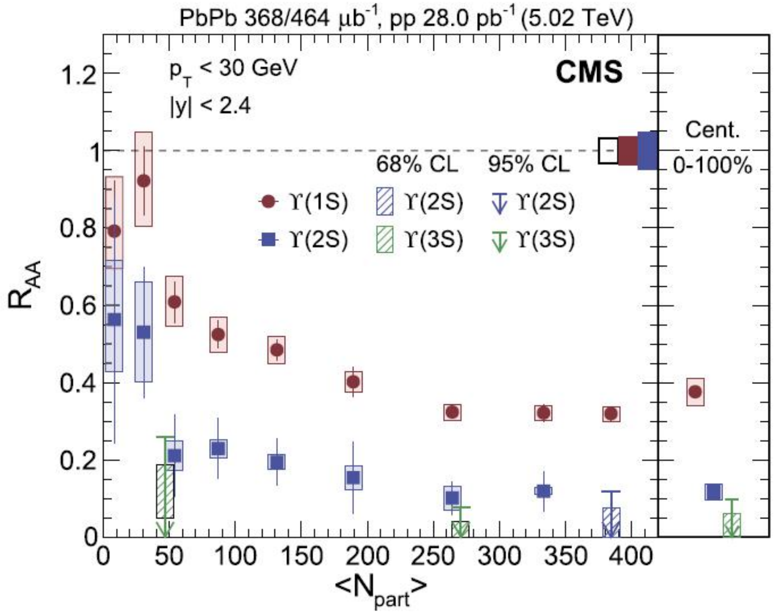
Data from heavy ion collisions

[CMS collaboration(19)]

Spectral for upsilon



RAA for upsilon

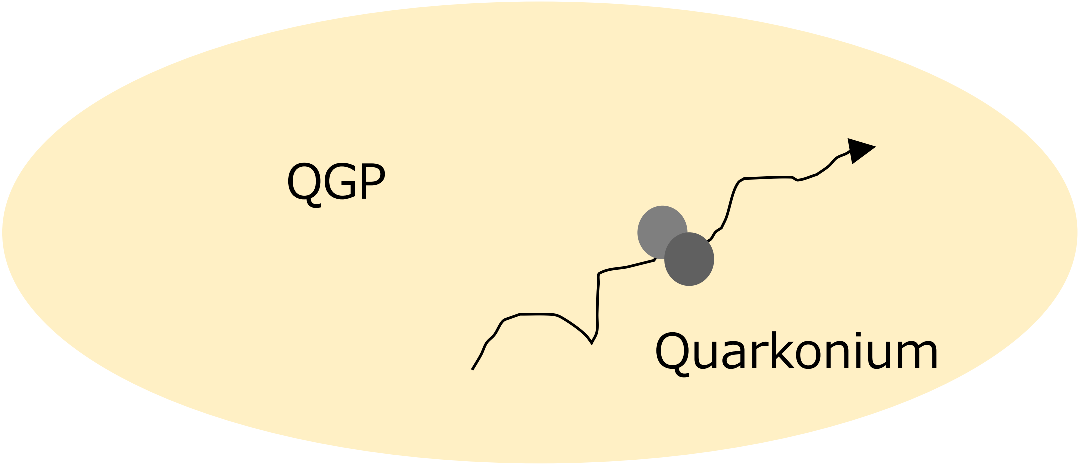


Comparing pp collision data, yields are relatively suppressed

Data reflect interactions between a quarkonium and QGP

We need to understand the dynamics of a quarkonium in QGP

Quarkonium in QGP



How to describe?

- Two interacting Brownian particles

  → Langevin eq. $\frac{dp}{dt} = -\eta p - \nabla V(x) + \xi(t)$

  White noise

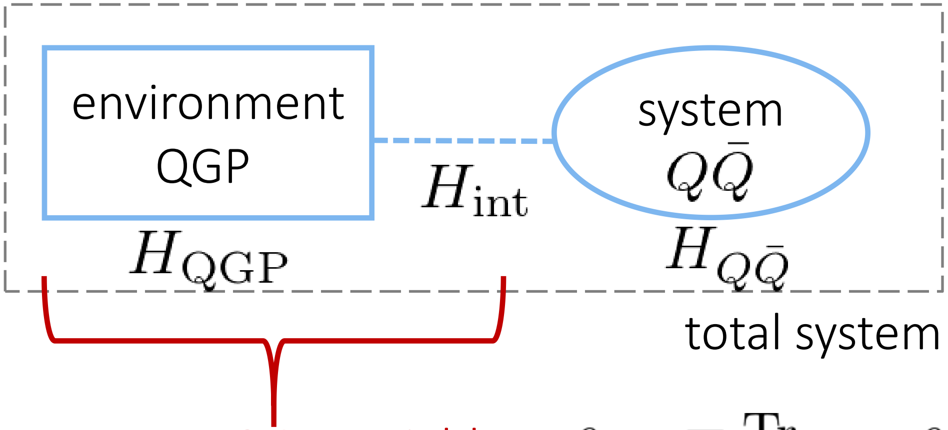
- Quantum mechanical bound state

  Quantum Brownian motion

Open quantum system

We would like to describe a quarkonium in quantum way

→ open quantum system approach



von Neumann eq.

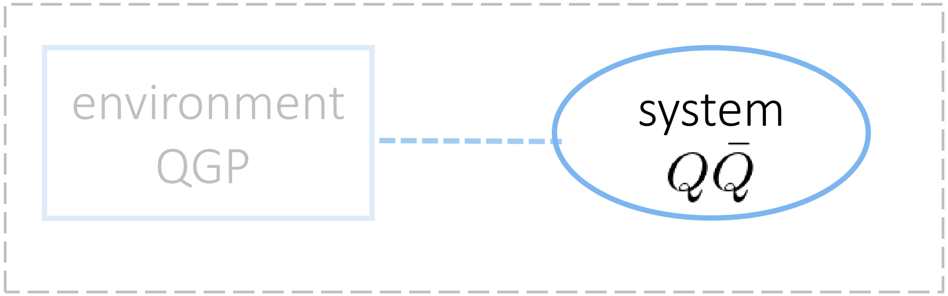
$$\frac{d}{dt}\rho_{\mathrm{total}} = -i[H_{\mathrm{total}}, \rho_{\mathrm{total}}]$$

$$H_{\mathrm{total}} = H_{\mathrm{QGP}} + H_{\mathrm{Q\bar{Q}}} + H_{\mathrm{int}}$$

trace out QGP variables

$$\rho_{Q\bar{Q}} = \mathrm{Tr}_{\mathrm{QGP}}\ \rho_{total}$$

reduced density matrix



master eq. for only quarkonium

$$\frac{d}{dt}\rho_{Q\bar{Q}} = \hat{\mathcal{L}}\rho_{Q\bar{Q}}$$

Liouville operator

information of interactions

Lindblad master equation

- Positivity of density matrix

We would like to interpret quarkonium state as a mixed state

→ Is positivity satisfied? $^{\forall}\left|\alpha\right\rangle,\ \left\langle\alpha\right|\rho_{Q\bar{Q}}\left|\alpha\right\rangle \geq 0$

- Lindblad form [Lindblad(76)]

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[H'_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \int dk\{2L_k\rho_{Q\bar{Q}}L_k^\dagger - L_k^\dagger L_k\rho_{Q\bar{Q}} - \rho_{Q\bar{Q}}L_k^\dagger L_k\}$$

L : Lindblad operator

→ interactinf forces

Important properties

$$\mathrm{Tr}[\rho_{Q\bar{Q}}] \equiv 1$$

$$\rho_{Q\bar{Q}} = \rho_{Q\bar{Q}}^\dagger$$

$$^{\forall}\left|\alpha\right\rangle,\ \left\langle\alpha\right|\rho_{Q\bar{Q}}\left|\alpha\right\rangle \geq 0$$

Quantum State Diffusion(QSD) method

- Stochastic unravelling

[Gisin, Persival (92)]

Lindblad master eq.

via QSD

⇔

equivalent

nonlinear
stochastic Schrödinger eq.

density matrix
ensemble

$$\rho_{Q\bar{Q}}(x,y,t)$$
$$= \langle \psi(x,t)\psi^*(y,t)\rangle$$

wave function
individual

nonlinear stochastic Schrödinger eq. form

$$|d\psi\rangle = -iH'_{Q\bar{Q}}\,|\psi(t)\rangle\,dt + \int d\vec{k}\big(2\langle L^\dagger_{\vec{k}}\rangle_\psi L_{\vec{k}} - L^\dagger_{\vec{k}}L_{\vec{k}} - \langle L^\dagger_{\vec{k}}\rangle_\psi\langle L_{\vec{k}}\rangle_\psi\big)\,|\psi(t)\rangle\,dt$$
$$+ \int d\vec{k}\big(L_{\vec{k}} - \langle L_{\vec{k}}\rangle_\psi\big)\,|\psi(t)\rangle\,d\xi_{\vec{k}}$$

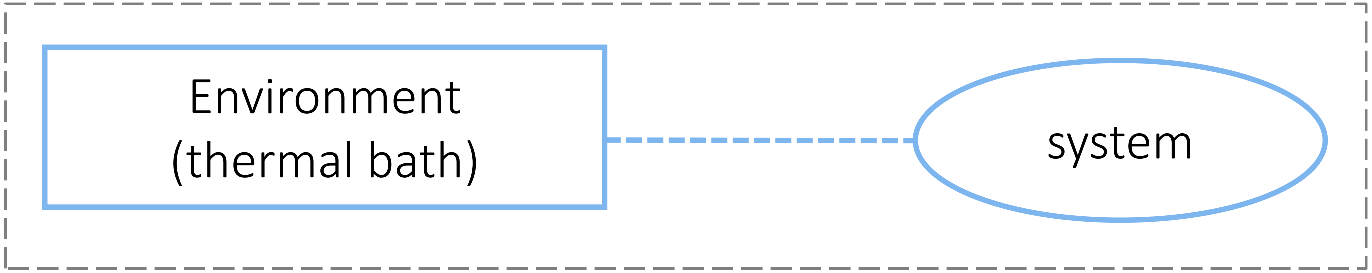
$\langle\ \ \rangle_\psi$ expectation value
with respect to wave function
→ nonlinearity

Apply QSD method to Lindblad master equation

Caldeira Leggett model for quarkonium?

Caldeira Legette model [Caldeira-Leggett(83)]

- Prototype of quantum Brownian particle with potential V(x)



- quantum Brownian particle

  ← localized wave packet

  smaller than QGP correlation length $l_{\mathrm{corr}} \sim m_{\mathrm{D}}^{-1}$

In our case, NOT the case → improve the model based on QCD

[Akamatsu(15)]

Lindblad operator for quarkonium in QGP

Reduction to relative motion

$L_{\vec{k},a}$ for a quarkonium ($\vec{x}_Q, \vec{x}_{\bar{Q}}$) $\longrightarrow$ $L^{\text{relative}}_{\vec{k},a}$

H $\qquad$ H^{relative}

[Akamatsu(15)]

trace out center-of-mass motion
under its constant momentum

Result (CM momentum=0)

Q part $\qquad$ $\bar{Q}$ part

$$L^{\text{relative}}_{\vec{k},a} = \sqrt{\frac{D(\vec{k})}{2}}\left[\underline{1} - \underline{\frac{\vec{k}\cdot\hat{\vec{p}}}{4MT}}\right]\mathrm{e}^{i\vec{k}\cdot\hat{\vec{r}}/2}(t^a \otimes 1) - \sqrt{\frac{D(\vec{k})}{2}}\left[\underline{1} + \underline{\frac{\vec{k}\cdot\hat{\vec{p}}}{4MT}}\right]\mathrm{e}^{-i\vec{k}\cdot\hat{\vec{r}}/2}(1 \otimes t^{a*})$$

fluctuation $\qquad$ dissipative term
(heavy quark recoil) $\qquad$ momentun transfer

$H^{\text{relative}} \in V(x)$

Solve Lindblad eq. for relative motion with this Lindblad operator(NOT model)

$$\frac{d}{dt}\rho_{Q\bar{Q}} = -i[H'_{Q\bar{Q}}, \rho_{Q\bar{Q}}] + \int dk\{2L_k\rho_{Q\bar{Q}}L_k^\dagger - L_k^\dagger L_k\rho_{Q\bar{Q}} - \rho_{Q\bar{Q}}L_k^\dagger L_k\}$$

NUMERICAL ANALYSIS

QSD simulation for quarkonium relative motion

For simplicity, in one spatial dimension, with heavy quark color ignored

Parameter setups in terms of heavy quark mass *M*

Δx	Δt	N_x	T	γ	$l_{\rm corr}$	α	m_D	$r_{\rm c}$
$1/M$	$0.1M(\Delta x)^2$	254	$0.1M$	T/π	$1/T$	0.3	T	$1/M$

Noise correlation function

$$D(r) = \gamma \exp(-r^2/l_{\rm corr}^2)$$

- ➢ More realistic setup (Bjorken expanding QGP)

QGP temperature decreases in time

$$T(t) = T_0\left(\frac{t_0}{t+t_0}\right)^{1/3} \qquad T_0/M_b = 0.1,\ M_b t_0 = 20$$

QSD simulation for quarkonium relative motion

For simplicity, in one spatial dimension, with heavy quark color ignored

Outline of the numerical calculation

Initial
Wave function

Time evolution
event by event

Eigenstate
projection

Occupations

QSD method

$$H = \frac{p^2}{M} - \frac{\alpha}{r}\mathrm{e}^{-m_{\mathrm{D}} r}$$

$$N_i = \int dx dy\, \phi_i^*(x)\rho(x,y)\phi_i(y)$$

- ➢ More realistic setup (Bjorken expanding QGP)

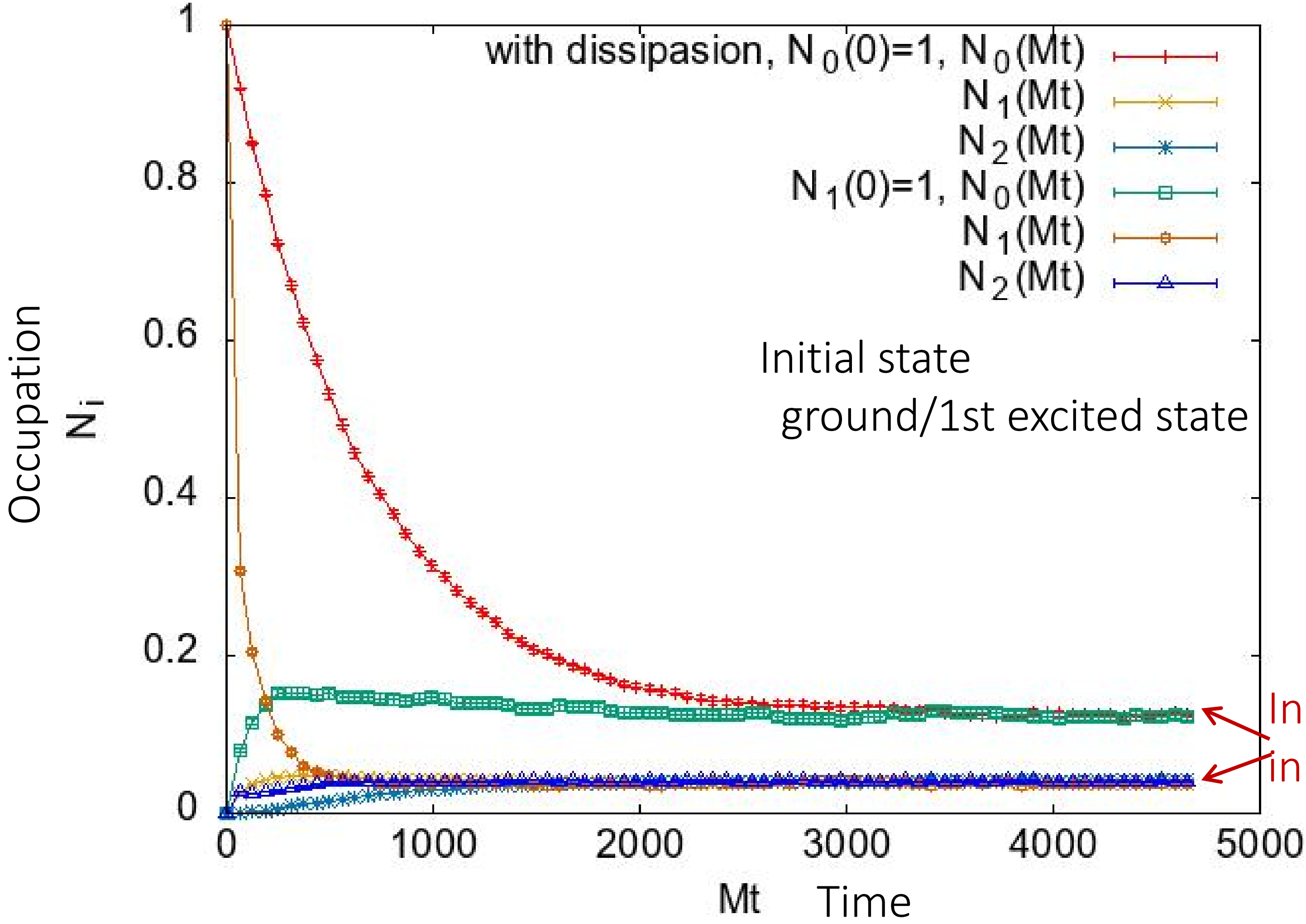
Eigenstate
projection

vacuum eigenstate $\phi_i(x)$

$$H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r \qquad \sigma = 0.01 M_b^2$$

Results - equilibration

- Time evolution of occupation number of eigenstates $H = \frac{p^2}{M} - \frac{\alpha}{r}\mathrm{e}^{-m_\mathrm{D} r}$



Initial state
ground/1st excited state

Time

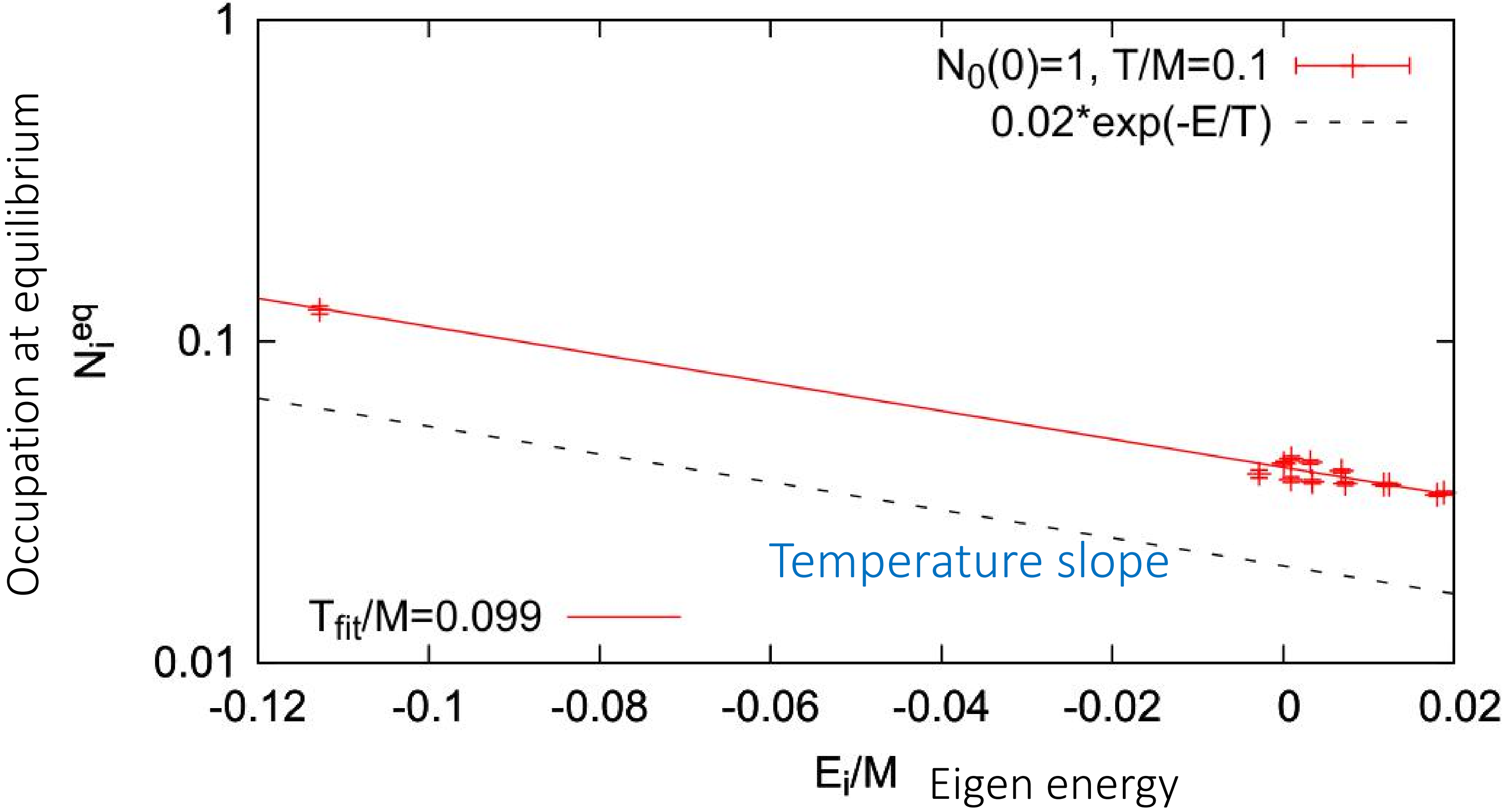
Independent of initial conditions

Eigenstate occupation respectively approaches the static value

Results - equilibration

- Eigenstate steady distribution at Mt=4650

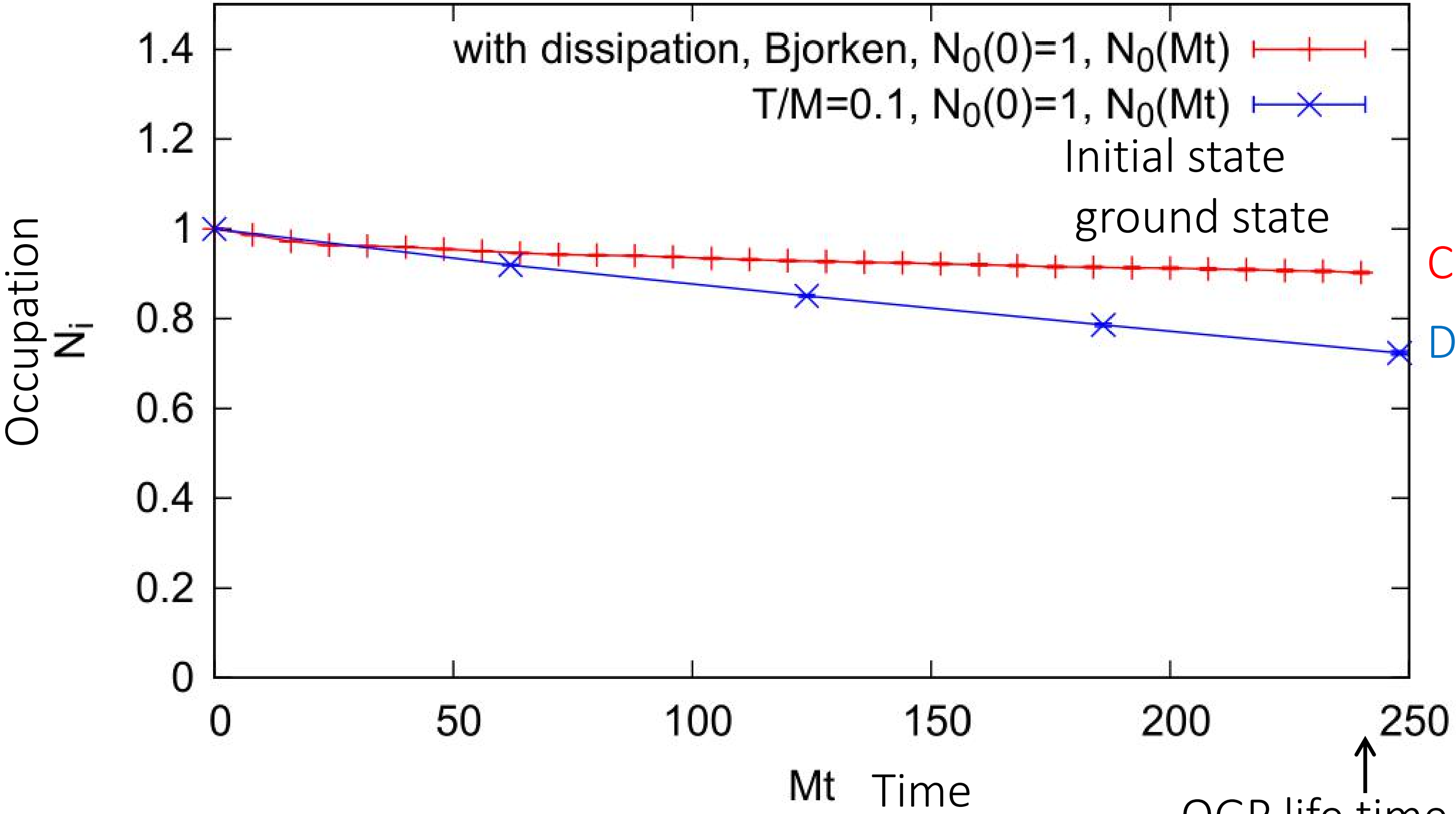
$$H = \frac{p^2}{M} - \frac{\alpha}{r} \mathrm{e}^{-m_{\mathrm{D}} r}$$



Eigenstate distribution approches the Boltzmann distribution

Results - In Bjorken expanding QGP

- Time evolution of occupation number of eigenstates $H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r$



Initial state
ground state

Cornell

Debye screening

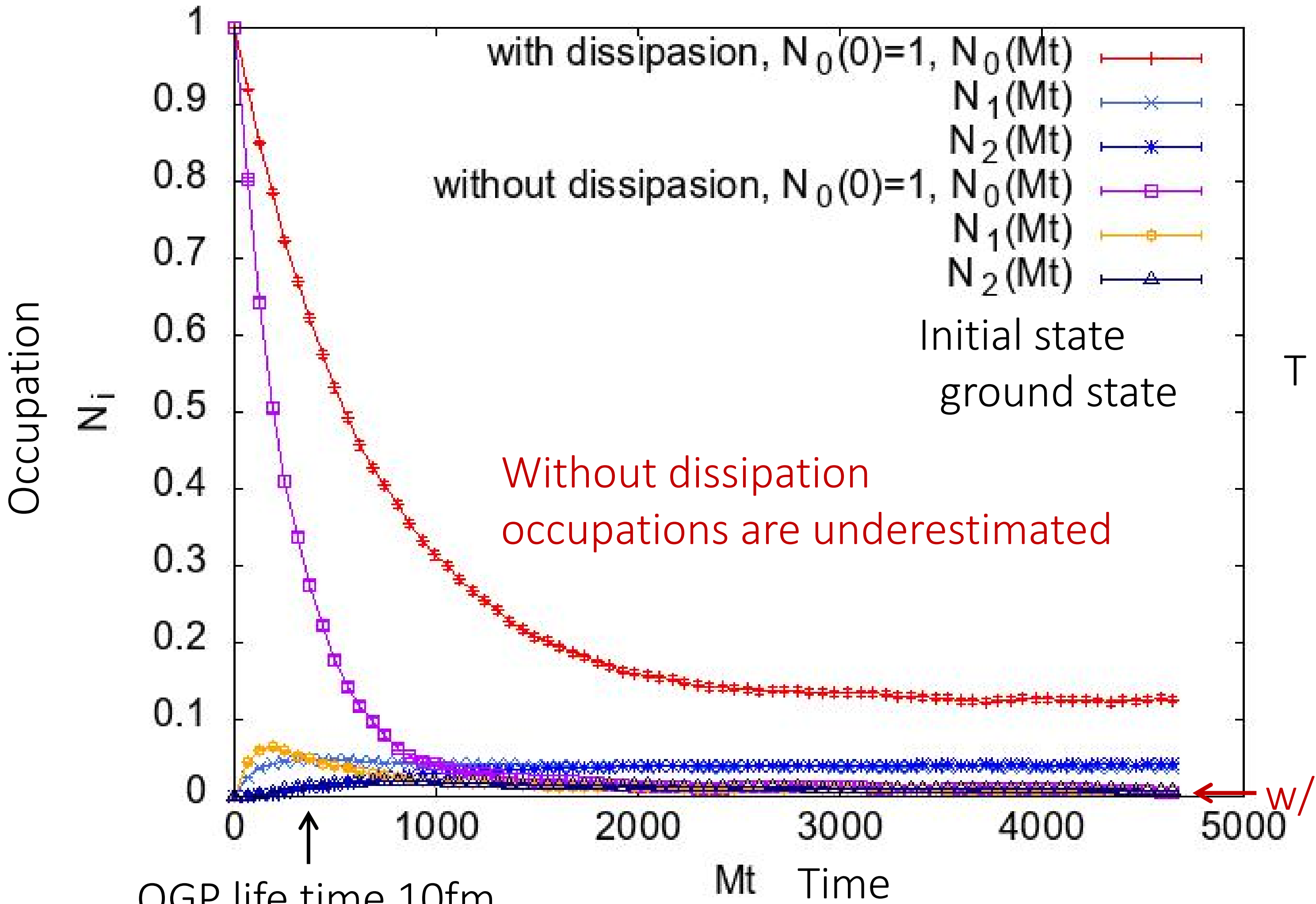
Time

QGP life time 10fm

Evolution becomes milder in Bjorken expanding QGP

Results - Dissipative effect

- Time evolution of occupation number of eigenstates $H = \frac{p^2}{M} - \frac{\alpha}{r}\mathrm{e}^{-m_\mathrm{D} r}$



Initial state
ground state

T ≃ 400MeV

Without dissipation
occupations are underestimated

← w/o dissipation

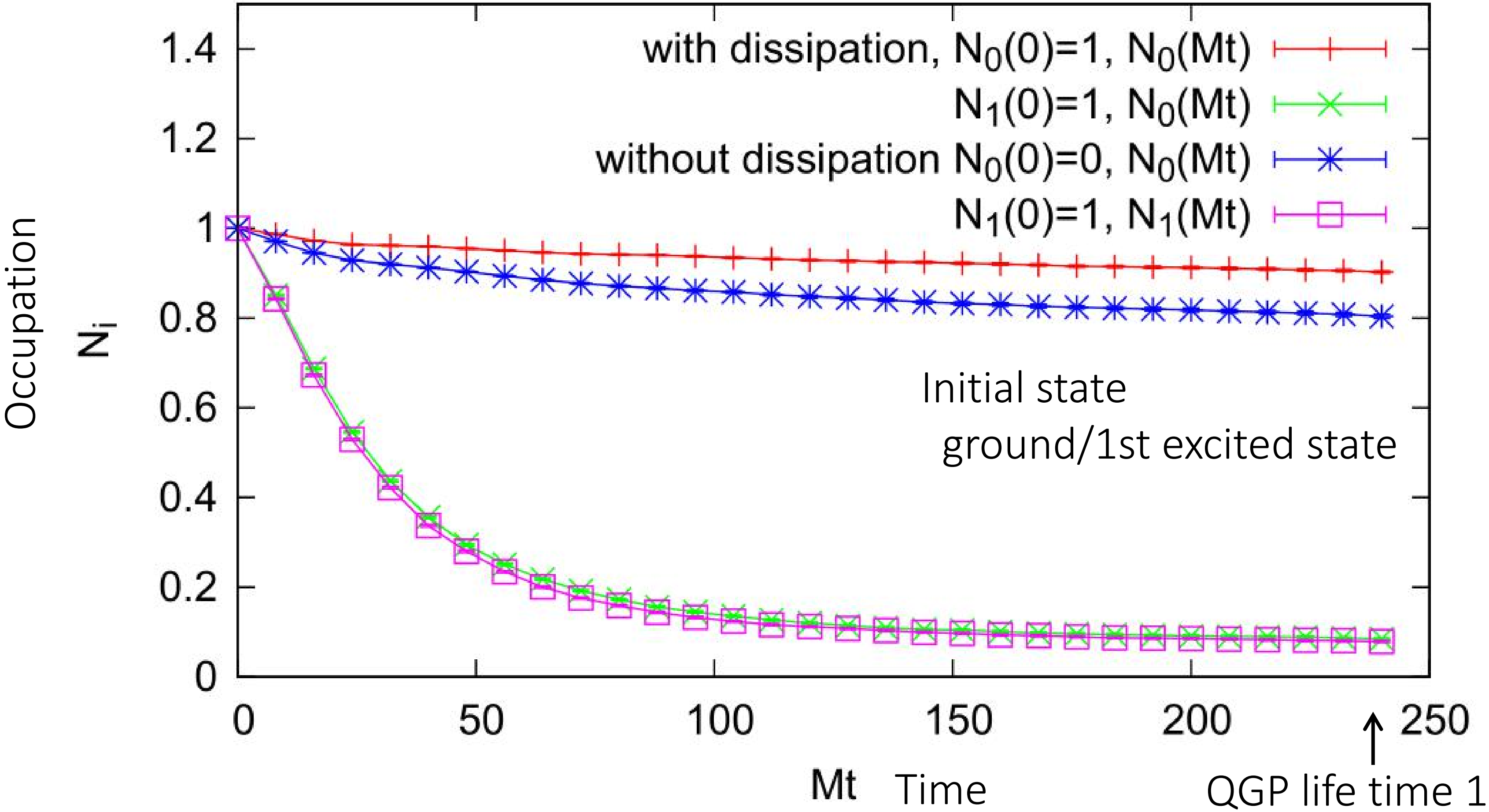
Time

QGP life time 10fm

Dissipation is effective and important to be considered in QGP life time

Results - In Bjorken expanding QGP

- Time evolution of occupation number of eigenstates $H = \frac{p^2}{M_b} - \frac{\alpha}{r} + \sigma r$



Initial state
ground/1st excited state

Time

↑
QGP life time 10fm

Dissipative effect is not negligible in short lived QGP

Summary

- Quarkonium is treated as a open quantum system
- Simulation for Lindblad master equation in Quantum State Diffusion method

  - We numerically confirm a quarkonium is thermalized under dissipation

  - We simulate effects of dissipation

    Localized bound state is affected more, non negligible effect

Outlook

3D analysis

Color effect

Comparison with experimental results

BACK UP

Nonlinear stochastic Schoroedinger equation

Nonlinear stochastic Schoroedinger eq. in QSD

$$
\begin{aligned}
d\psi(x) =& - idt\left[-\frac{\nabla^2}{M}+V(x)\right]\psi(x) - dt\Big[D(0)-D(x)\Big]\psi(x) \\
&+\frac{2dt}{\langle\psi|\,\psi\rangle}\int dy\Big[D\Big(\frac{x-y}{2}\Big)-D\Big(\frac{x+y}{2}\Big)\Big][\psi^\dagger(y)\psi(y)]\psi(x) \\
&+\Big[d\xi\Big(\frac{x}{2}\Big)-d\xi\Big(\frac{-x}{2}\Big)\Big]\psi(x)+\mathcal{O}(T/M),
\end{aligned}
$$

Noise correlation of complex noise

$$
\langle d\xi(x)d\xi^*(y)\rangle = D(x-y)dt, \quad \langle d\xi(x)d\xi(y)\rangle = 0,
$$

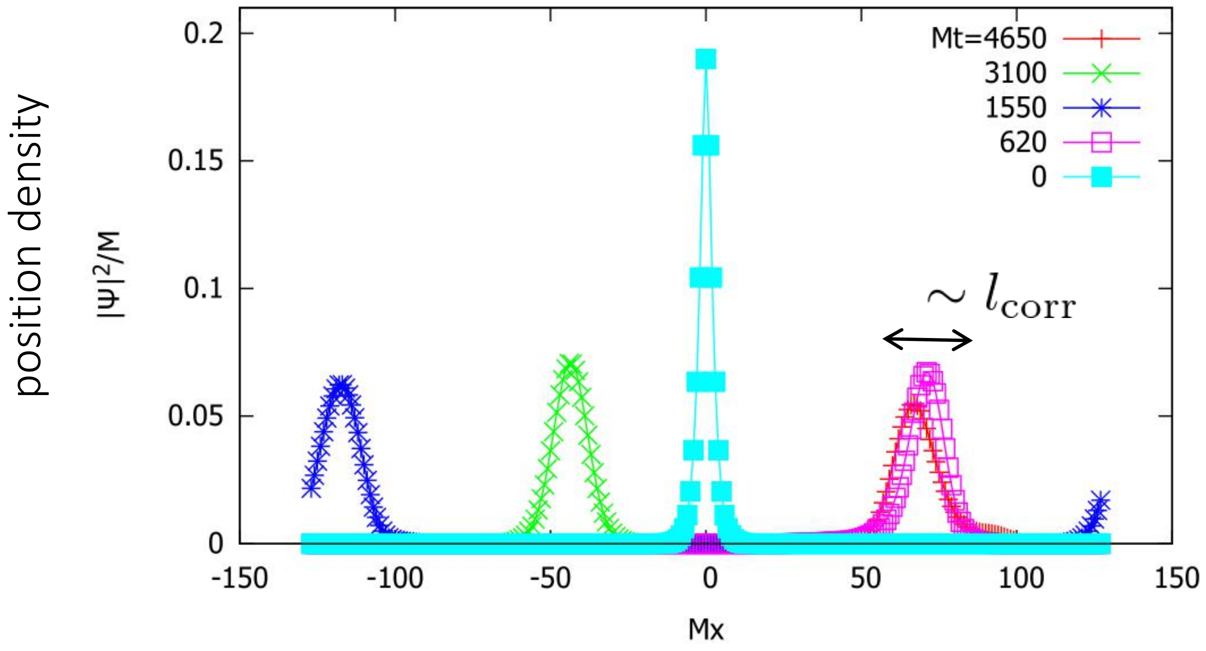
Density matrix

$$
\rho_{Q\bar{Q}}(x,y) = \Big\langle \frac{\psi(x)\psi^*(y)}{||\psi||^2} \Big\rangle
$$

Results - localized wave function

- Time evolution of wave function via QSD eq. in one sampling

$$H = \frac{p^2}{M} - \frac{\alpha}{r}\mathrm{e}^{-m_{\mathrm{D}} r}$$



Initial condition
ground state

Wave function is typically solitonic from nonlinearlity

Each sample shows similar behavior