To be, or not to be finite? The Higgs potential in Gauge-Higgs Unification

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Introduction

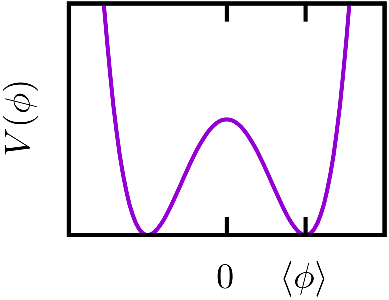
- **Higgs boson in SM**

Higgs mechanism

VEV of the Higgs, $\langle\phi\rangle$
→SSB & massive gauge bosons.

Fermion mass

Yukawa interactions are introduced.



However, SM cannot answer:

- Why "wine bottle" potential?
- What is the Yukawa interaction derived from?

SM has no principle for the Higgs boson

What is the origin of the Higgs boson? → Gauge-Higgs Unification

• Gauge-Higgs Unification (GHU)

Let us consider a gauge theory defined on $\mathbf{M}^4 \times S^1$, $(y = x^5 \in [0, 2\pi R))$

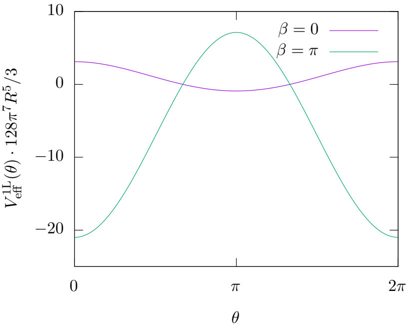
Gauge-Higgs Unification

Higgs = extra-component of gauge bosons, $A_5(x, y)$

- Yukawa interactions
  $\longleftarrow$ gauge principle.
- ~~Higgs potential~~ @ tree level
  $\longrightarrow$ generated by quant. corr.
  (Hosotani mechanism)

$$\theta := g \oint_{S^1} \mathrm{d}y\, A_5 = 2\pi R g \langle A_5 \rangle,$$

g: gauge coupling.



• Higgs potential finiteness

non-Abelian @1-loop:

- $SU(N)$ on $\mathbf{M}^{d-1} \times S^1$ [Hosotani, 1989] $\longrightarrow$ finite

Conjecture [Gersdorff, Irges, Quiros, 2002; Hosotani, 2005]

In GHU, the Higgs effective potential, $V_{\mathit{eff}}(\theta)$, is finite at all orders.

Abelian @2-loop:

$\mathbf{M}^4 \times S^1$ [Maru & Yamashita, 2006; Hosotani et al., 2007] $\longrightarrow$ finite

Finiteness is not shown in non-Abelian @2-loop level...

- **Our results**
  - $SU(\mathcal{N})$ on $\mathbf{M}^4 \times S^1$ @ one/two-loop: finite

In order to investigate the finiteness @ higher-loop levels, we increase the spacetime dimension:

- $SU(\mathcal{N})$ on $\mathbf{M}^5 \times S^1$ @ three-loop: infinite

Criteria of divergence

$V_{eff}(\theta)$ is divergent if it depends on a divergence which cannot be subtracted by any counter terms for the gauge coupling, g.

Outline

1. Introduction
2. Setup
3. Calculation method
4. Results
5. Summary

Setup

- $SU(\mathcal{N})$ **gauge theory**

$$\mathcal{L} = \mathcal{L}_{gauge} + \mathcal{L}_{fermion},$$

$$\mathcal{L}_{gauge} = -\frac{1}{4} F^a_{MN} F^{aMN},$$

$$\mathcal{L}_{fermion} = \sum_l \bar{\psi}_l \mathrm{i} \gamma^M D_M \psi_l.$$

Here, A^a_5 is shifted by its VEV, $\langle A^a_5 \rangle = \frac{\theta^a}{2\pi R g}$.

- **Boundary conditions**

$$A_M(x, y + 2\pi R) = A_M(x, y),$$

$$\psi_l(x, y + 2\pi R) = e^{i\beta_l}\psi_l(x, y), \quad \beta_l \in [0, 2\pi).$$

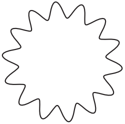
Kaluza-Klein mode expansion:

$$A_M(x, y) = \sum_{n=-\infty}^{\infty} A_M^{(n)}(x)\, e^{iny/R},$$

$$\psi_l(x, y) = \sum_{n=-\infty}^{\infty} \psi_l^{(n)}(x)\, e^{i(n/R+\beta_l/2\pi R)y}.$$

Calculation method

- **Difficulties with loop integrals**



$$= -\frac{5}{2}\frac{1}{2\pi R}\sum_{n=-\infty}^{\infty}\int_{\mathbf{M}^4}\frac{\mathrm{d}^4k}{(2\pi)^4}\,\mathrm{tr}\ln\left[k^2-\left(\frac{n}{R}+\frac{\theta^a T^a}{2\pi R}\right)^2\right]$$

T^a: adjoint rep.

1. Diagonalize matrices $\longrightarrow$ so many diagrams
2. Calculate integrals

$\longrightarrow$ Need for simple calculation!

Calculation method (2)

• Compactification by superposition

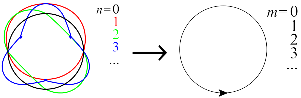
[Anber & Sulejmanpasic, 2015; Heffner & Reinhardt, 2015]

$$\frac{1}{2\pi R}\sum_{n=-\infty}^{\infty}\int_{\mathbf{M}^4}\frac{\mathrm{d}^4k}{(2\pi)^4}S\left(\frac{n}{R}+\frac{\Theta}{2\pi R}\right)=\sum_{m=-\infty}^{\infty}\mathrm{e}^{\mathrm{i}\Theta m}\int_{\mathbf{M}^5}\frac{\mathrm{d}^5k}{(2\pi)^5}\mathrm{e}^{-\mathrm{i}2\pi Rk_5m}S(k_5)$$

S: analytic function, Θ: Hermitian matrix.

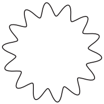
cf. Poisson resummation formula:

$$\sum_{n=-\infty}^{\infty}\delta\left(k_5-\frac{n}{R}\right)=R\sum_{m=-\infty}^{\infty}\mathrm{e}^{-\mathrm{i}2\pi Rk_5m}$$



KK decomposition → Superposition around S^1

Calculation method (3)



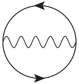
$$= -\frac{5}{2}\frac{1}{2\pi R}\sum_{n=-\infty}^{\infty}\int_{\mathbf{M}^4}\frac{\mathrm{d}^4 k}{(2\pi)^4}\,\mathrm{tr}\ln\left[k^2 - \left(\frac{n}{R} + \frac{\theta^a T^a}{2\pi R}\right)^2\right]$$

$$= -\frac{5}{2}\sum_{m=-\infty}^{\infty}\mathrm{tr}\,\mathrm{e}^{\mathrm{i}\theta^a T^a m}\int_{\mathbf{M}^5}\frac{\mathrm{d}^5 k}{(2\pi)^5}\mathrm{e}^{-\mathrm{i}2\pi R k_5 m}\ln\left[k^2 - (k_5)^2\right]$$

$$= -\frac{3\mathrm{i}}{128|m|^5\pi^7 R^5}$$

Calculation method (4)

Furthermore, at the two-loop level,



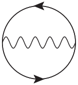
$$
= \frac{1}{2}\frac{1}{2\pi R}\sum_{n_1}\int\frac{d^4p}{(2\pi)^4}\frac{1}{2\pi R}\sum_{n_2}\int\frac{d^4k}{(2\pi)^4}\left[\frac{-i\eta_{MN}}{p^2-(\frac{n_1}{R}+\frac{\theta^c T^c}{2\pi R})^2}\right]_{ab}
$$

$$
\times(-1)\,\mathrm{tr}\left[\frac{i}{\not p+\not k-\gamma_5(\frac{n_1+n_2}{R}+\frac{\theta^c\tau_\ell^c-\beta_\ell}{2\pi R})}ig\gamma^M\tau_\ell^a\right.
$$

$$
\left.\times\frac{i}{\not k-\gamma_5(\frac{n_2}{R}+\frac{\theta^c\tau_\ell^c-\beta_\ell}{2\pi R})}ig\gamma^N\tau_\ell^b\right].
$$

Calculation method (5)

Our calculation method simplifies loop integrals greatly:



$$= -6ig^2 \sum_{m_1,m_2} \left[e^{i\theta^c T^c m_1} \right]_{ab} \mathrm{tr} \left[e^{i(\theta^c \tau^c - \beta_\ell) m_2} \tau_\ell^b \tau_\ell^a \right] \times \int \frac{d^5 p}{(2\pi)^5} \int \frac{d^5 k}{(2\pi)^5} e^{-i2\pi R(p_5 m_1 + k_5 m_2)} \times \frac{(p+k)^M k_M}{p^L p_L (p+k)^J (p+k)_J k^I k_I}.$$

Very powerful method!

Result (i): 1-loop level

[Hosotani, 1983, 1989; Davies & McLachlan, 1989]

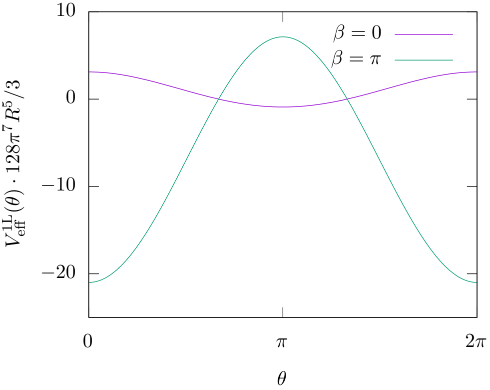
$$V_{\mathit{eff}}^{1L} = -\frac{9}{256\pi^7 R^5} \sum_{m \neq 0} \frac{1}{|m|^5} \operatorname{tr} e^{i\theta^a T^a m} + \frac{3}{64\pi^7 R^5} \sum_{l} \sum_{m \neq 0} \frac{1}{|m|^5} \operatorname{tr} e^{i(\theta^a \tau_l^a - \beta_l)m} + C$$

τ_l^a: rep. of fermions, C: constant (from $m = 0$).

finite

Consistent with the previous works.

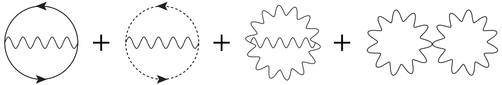
Result (i): 1-loop level (2)



Fermion: 1 adjoint rep. of SU(2).

Result (ii): 2-loop level

$$V^{2L}_{\mathit{eff}}(\theta) = \mathrm{i} \times \Bigg($$



$$\Bigg)$$

$$= -3g^2 \sum_{l} \sum_{m_1,m_2} G_l(m_1,m_2)[2F(m_1)F(m_2) - F(m_1)F(m_1 - m_2)]$$

$$+ \frac{9}{4} g^2 \sum_{m_1,m_2} G_{adj}(m_1,m_2)F(m_1)F(m_2),$$

$$F(m) = \begin{cases} \frac{1}{64\pi^5 |m|^3 R^3} & (m \neq 0) \\ 0 & (m = 0) \end{cases}$$

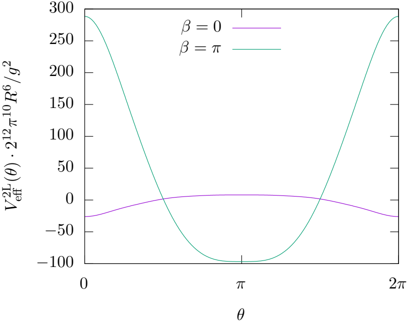
$$G_l(m_1,m_2) = [\mathrm{e}^{\mathrm{i}\theta^c T^c m_1}]_{ba} \, \mathrm{tr}[\mathrm{e}^{\mathrm{i}(\theta^c \tau^c_l - \beta_l) m_2} \tau^a_l \tau^b_l]$$

$$G_{adj}(m_1,m_2) = [\mathrm{e}^{\mathrm{i}\theta^c T^c m_1}]_{ba} \, \mathrm{tr}[\mathrm{e}^{\mathrm{i}\theta^c T^c m_2} T^a T^b]$$

finite

Consistent with [Maru & Yamashita, 2006; Hosotani et al., 2007].

Result (ii): 2-loop level (2)

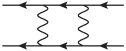
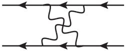


Fermion: 1 adjoint rep. of SU(2).

$$\frac{V_{eff}^{2L}}{V_{eff}^{1L}} = 8 \times 10^{-2} \times \frac{g^2}{4\pi R}.$$

Result (iii): 3-loop level (on $\mathrm{M}^5 \times S^1$)

Let us consider the 4-Fermi operators with only one fermion.
(α, γ: indices for $\bar{\psi}$; β, δ: for ψ. $\epsilon = 3 - d/2$)

[ +  $+ (crossed) \Big]_{div}$

$$= \frac{-ig^4}{768\pi^3} \frac{1}{\epsilon} [\gamma^L \gamma^N \gamma^M \tau^c \tau^a]_{\alpha\beta} [\gamma_M \gamma_N \gamma_L \tau^a \tau^c - \gamma_L \gamma_N \gamma_M \tau^c \tau^a]_{\gamma\delta} - (\alpha \leftrightarrow \gamma).$$

Corresponding counter term is

$$\mathcal{L}_{CT} = \frac{\delta_{4F}}{2} [\bar{\psi} \gamma^M \gamma^N \gamma^L \tau^a \tau^b \psi][\bar{\psi}(\gamma_M \gamma_N \gamma_L \tau^a \tau^b - \gamma_L \gamma_N \gamma_M \tau^b \tau^a)\psi],$$

where $\delta_{4F} = \dfrac{g^4}{768\pi^3} \dfrac{1}{\epsilon} + \delta_{4F}^{fin}.$

Result (iii): 3-loop level (on $\mathbf{M}^5 \times S^1$) (2)

Contribution to the Higgs effective potential from $\mathcal{L}_{CT}$ is

$$V_{CT}(\theta) = \sum_{m_1 \neq 0} \sum_{m_2 \neq 0} \frac{\delta_{4F}^{fin} \mathcal{N}}{8\pi^{16} R^{10} m_1^5 m_2^5} \times \{2\,\mathrm{tr}[\tau^a e^{i(\theta^b \tau^b - \beta) m_1}]\,\mathrm{tr}[\tau^a e^{i(\theta^b \tau^b - \beta) m_2}] + \mathrm{tr}[\tau^a e^{i(\theta^b \tau^b - \beta) m_1} \tau^a e^{i(\theta^b \tau^b - \beta) m_2}]\}.$$

$\mathcal{L}_{CT}$ is not a counter term for the gauge coupling.
$\rightarrow V_{eff}(\theta)$ is divergent.

Summary

- Using *compactification by superposition*, we simplified calculation of loop integrals in non-Abelian gauge theories.
- We estimated the Higgs effective potential up to the 2-loop level in an $SU(\mathcal{N})$ gauge theory defined on $\mathbf{M}^4 \times S^1$.
  $\rightarrow$ It is indeed finite as the conjecture says. Although the theory is nonrenormalizable, we can predict observables without uncertainty.
- Calculating the 4-Fermi operator at the 1-loop level in an $SU(\mathcal{N})$ gauge theory defined on $\mathbf{M}^5 \times S^1$, we fined that it is divergent.
  $\rightarrow V_{eff}(\theta)$ depends on UV-theories at the 3-loop level.

Backup

Brief review of GHU

Let us consider an $SU(\mathcal{N})$ gauge theory on $\mathbf{M}^4 \times S^1$.

$$A_M = (A_\mu, A_5) \leftarrow \text{scalar (Higgs)}$$

Kaluza-Klein expansion

$$A_M(x, y) = A_M(x, y + 2\pi R)$$

$$A_M(x, y) = \sum_n A_M^{(n)}(x)\, e^{iny/R}$$

Brief review of GHU (2)

- **Hosotani mechanism**

AB phase (θ): phase of the Wilson loop along S^1.

$$\theta = g \oint_{S^1} \mathrm{d}y\, A_5 = 2\pi R g \langle A_5 \rangle.$$

Because of S^1 compactification, we cannot gauge away θ;

$$\text{propagator } S_A = \frac{-\mathrm{i}\eta^{MN}}{p^2 - (\frac{n}{R} + \frac{\theta^a T^a}{2\pi R})^2}, \quad T^a\text{: adj. rep.}$$

Gauge bosons become massive. $\rightarrow$ symmetry breaking!

Wave function renormalization?

Contribution to $V_{\mathit{eff}}(\theta)$ from wave function renormalizations do not depend on θ.

$$= \frac{\mathrm{i}}{\not{p} - \gamma_5(\frac{n_2}{R} + \frac{\theta^c \tau^c_l - \beta_\ell}{2\pi R})},$$

$$= \mathrm{i}\delta_Z \left[\not{p} - \gamma_5\left(\frac{n_2}{R} + \frac{\theta^c \tau^c_l - \beta_\ell}{2\pi R}\right)\right].$$

(In addition, gauge self-energy diagrams are finite at the one-loop level thanks to the gauge symmetry.)

Regularization of loop integrals

We impose following conditions on a regularization:

- All the integrals become finite.
- Invariance under the shifts of loop momenta.
- Independence of the signs of loop momenta.
- Gauge invariance, $p_M \Pi^{MN}(p) = 0$.

Regularization of loop integrals (2)

We define

$$\Lambda^3 := \int \frac{\mathrm{d}^5 k}{(2\pi)^5} \frac{1}{k^M k_M},$$

$$\Xi(p) := \int \frac{\mathrm{d}^5 k}{(2\pi)^5} \frac{1}{(k+p/2)^M (k+p/2)_M (k-p/2)^N (k-p/2)_N}.$$

Then,

$$\int \frac{\mathrm{d}^5 p}{(2\pi)^5} \Xi(p) = (\Lambda^3)^2,$$

$$\int \frac{\mathrm{d}^5 k}{(2\pi)^5} \frac{k^A k^B}{(k+p/2)^M (k+p/2)_M (k-p/2)^N (k-p/2)_N} = \left(\frac{1+x}{5} \eta^{AB} - x \frac{p^A p^B}{p^M p_M} \right) \left[\Lambda^3 - \frac{p^M p_M}{4} \Xi(p) \right].$$

Regularization of loop integrals (3)

Gauge invariance requires

$$\Lambda^3 = 0, \qquad x = \frac{1}{4}.$$

e.g. dimensional regularization

$$\Lambda^3 = 0, \qquad x = \frac{1}{4}, \qquad \Xi(p) = -\frac{\mathrm{i}}{128\pi}\sqrt{-p^M p_M}.$$

This is consistent with above requirements.

V_{CT} in an $SU(2)$ gauge theory

With a fermion in fundamental representation, we get

$$V_{\mathrm{CT}}^{\mathcal{N}=2}(\theta) = \sum_{m_1 \neq 0} \sum_{m_2 \neq 0} \frac{3\delta_{4F}^{\mathrm{fin}} e^{-i\beta(m_1+m_2)}}{8\pi^{16} R^{10} m_1^5 m_2^5} \cos\left(\frac{m_1 + m_2}{2}\theta\right).$$