Relativistic treatment of dark thermalization

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Dark matter

Albada et al. ApJ (1985)

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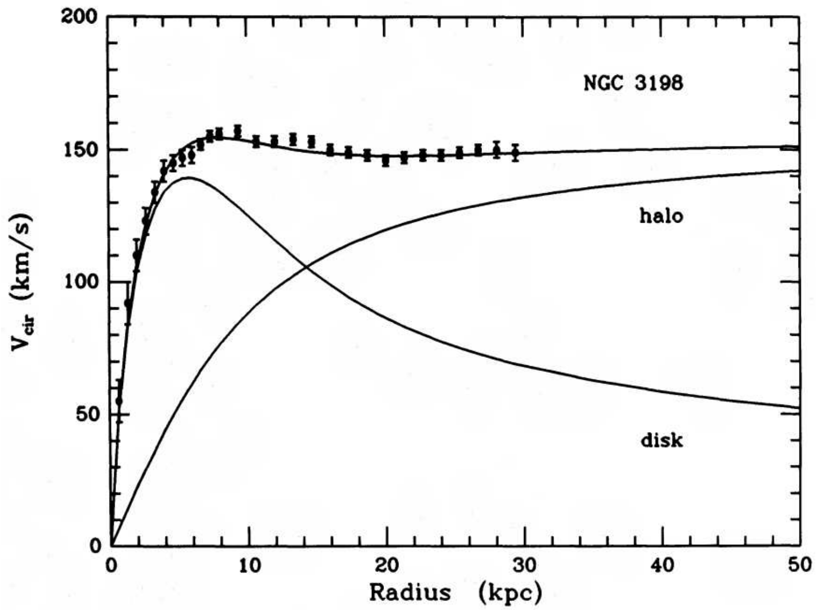
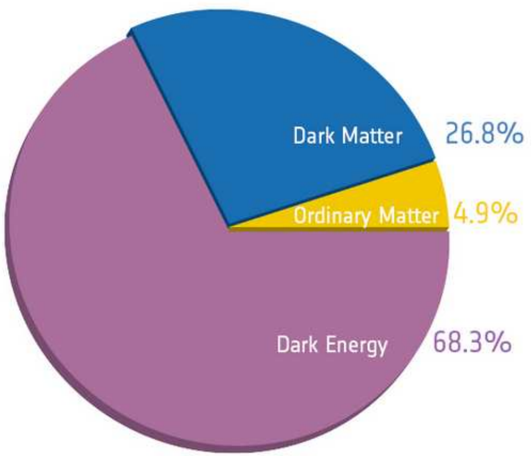
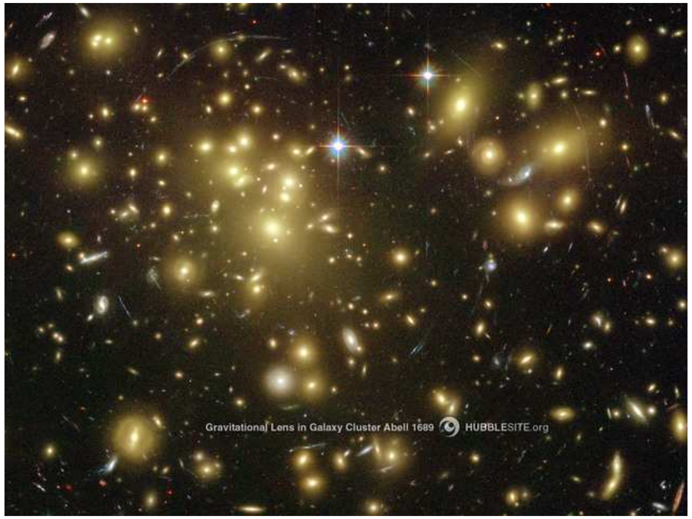
There is a lot of evidence of dark matter.

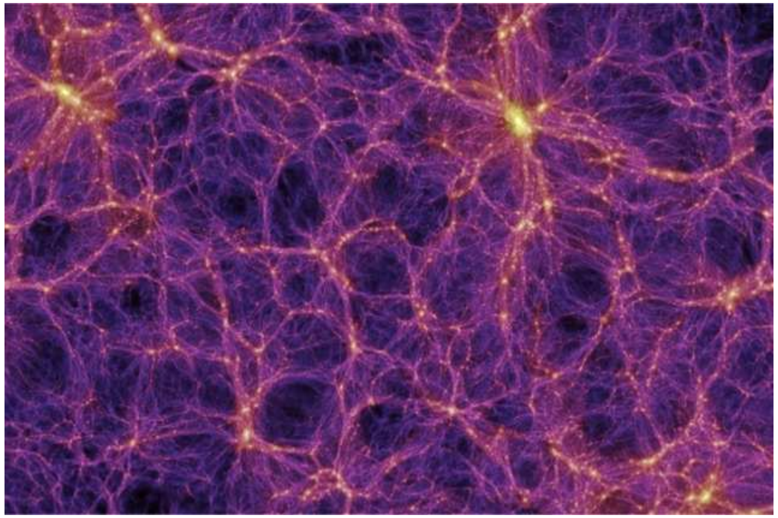
- Rotation curves of spiral galaxies
- CMB observations
- Gravitational lensing
- Structure formation of the universe
- Collision of bullet cluster

Existence of DM is crucial.









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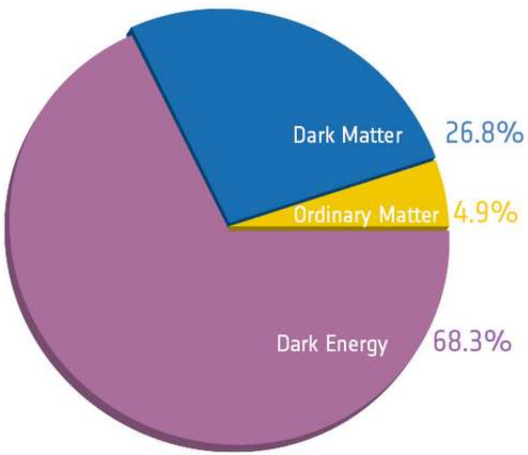

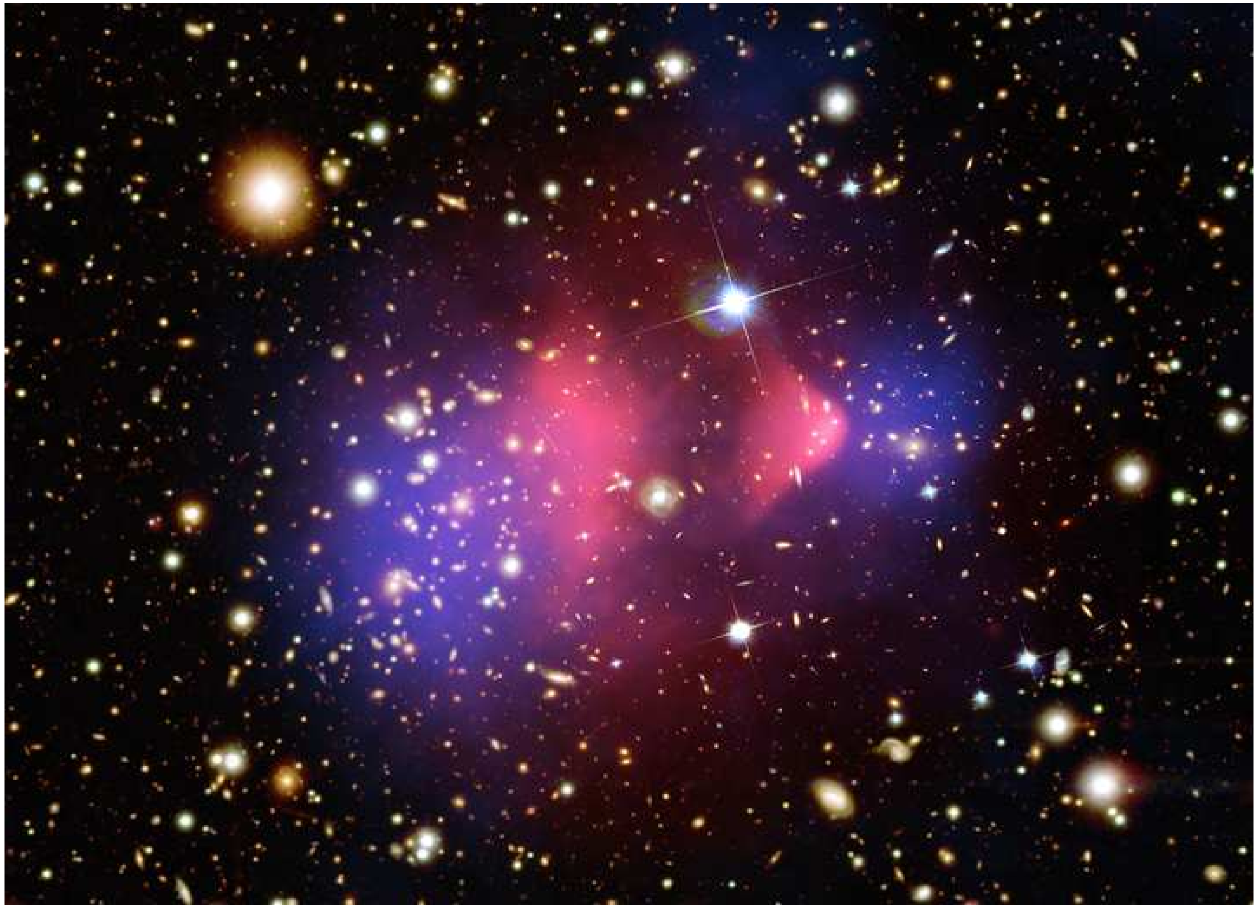

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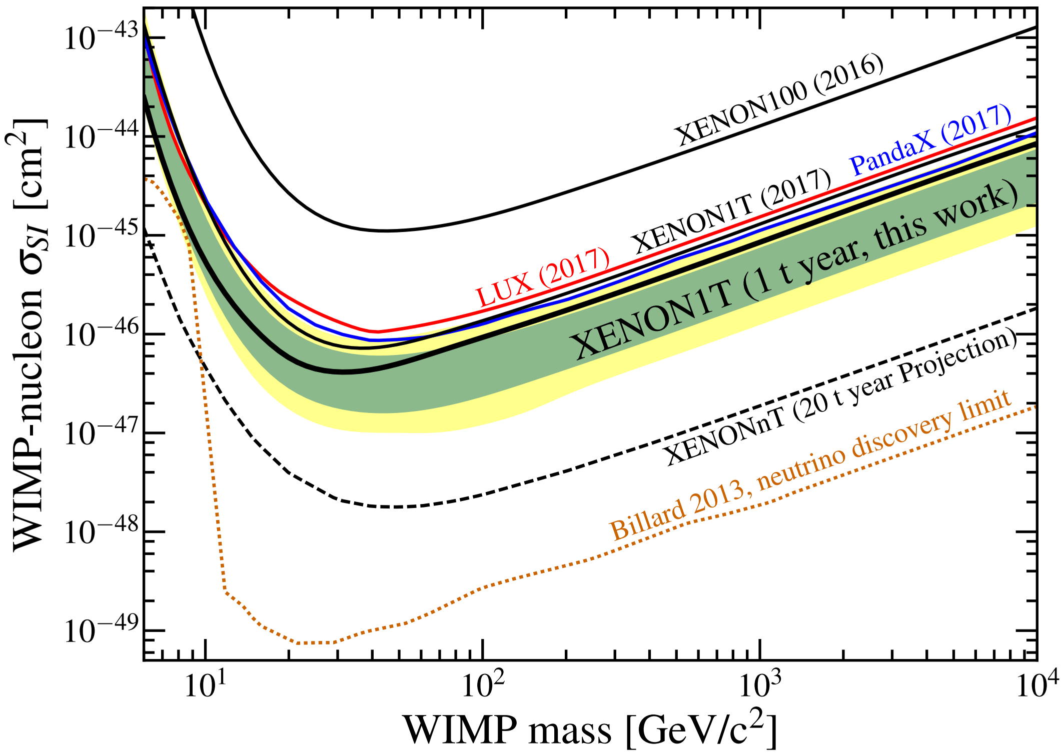
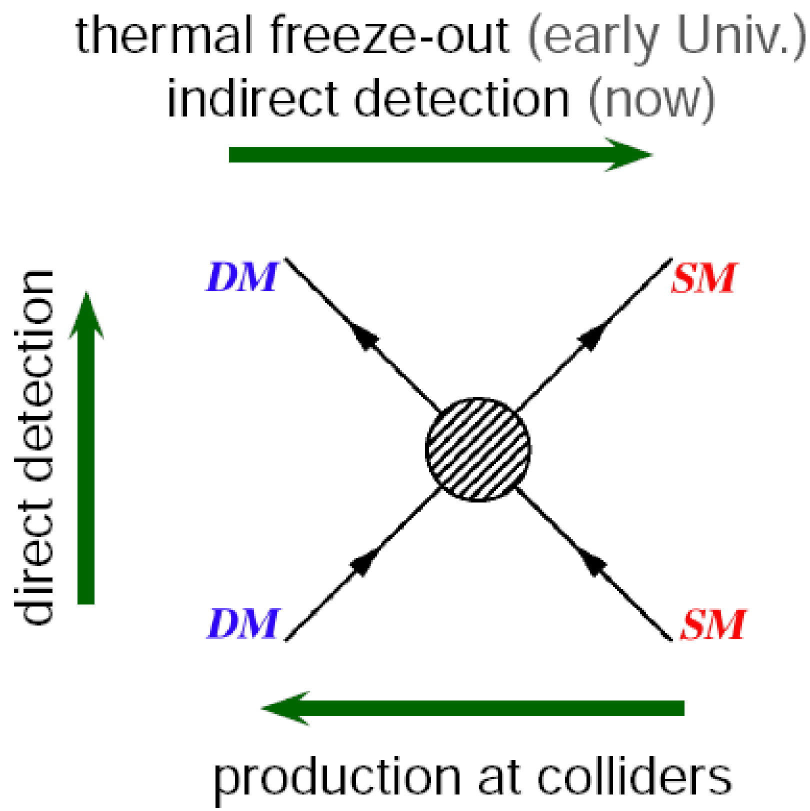
WIMP search status

Direct detection XENON1T, PRL (2018)



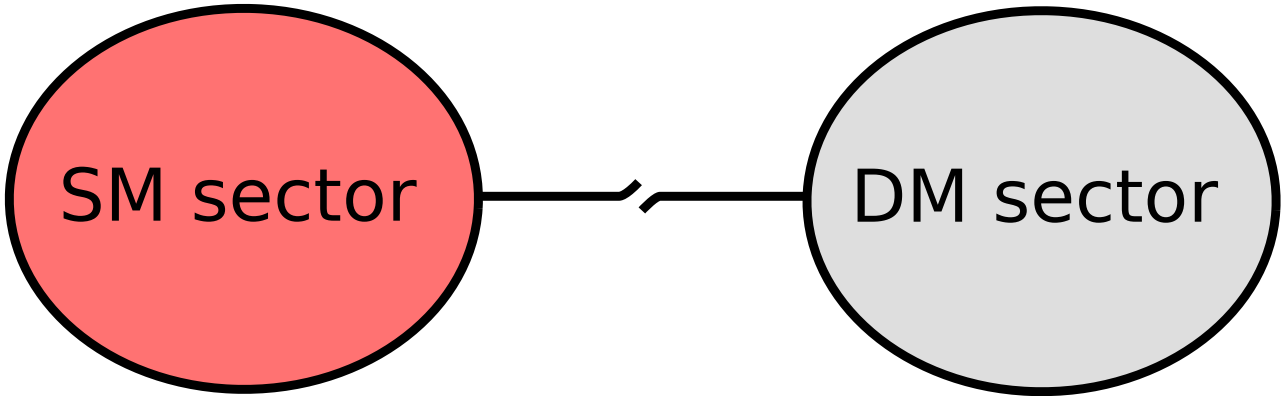


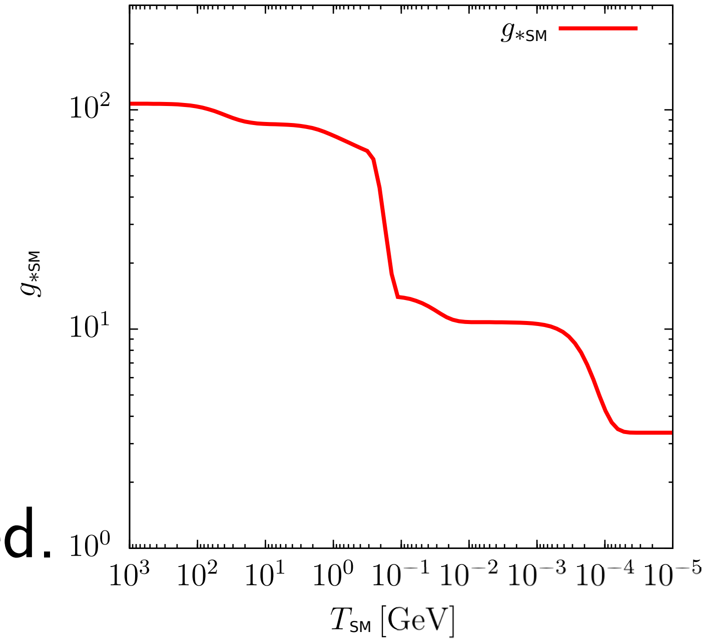
- Experimental bounds are stronger and stronger.
- Interactions between DM and SM are very weak?
  $\rightarrow$ non-WIMP DM? $\rightarrow$ FIMP, SIMP etc

In this talk, we will consider a DM model decoupled from SM sector.

Setup

- Dark sector is never thermalized with the SM
  $\Rightarrow$ couplings between SM and dark sector are small enough.





But both sectors are independently thermalized.

- Total energy density is dominated by the SM.

$$H^2 = \frac{8\pi}{3m_{\rm pl}^2}(\rho_{\rm SM} + \rho_{\rm DM}) \approx \frac{8\pi\rho_{\rm SM}}{3m_{\rm pl}^2}, \qquad \rho_{\rm SM} = \frac{\pi^2 g_{*\rm SM}}{30} T_{\rm SM}^4 \gg \rho_{\rm DM}$$

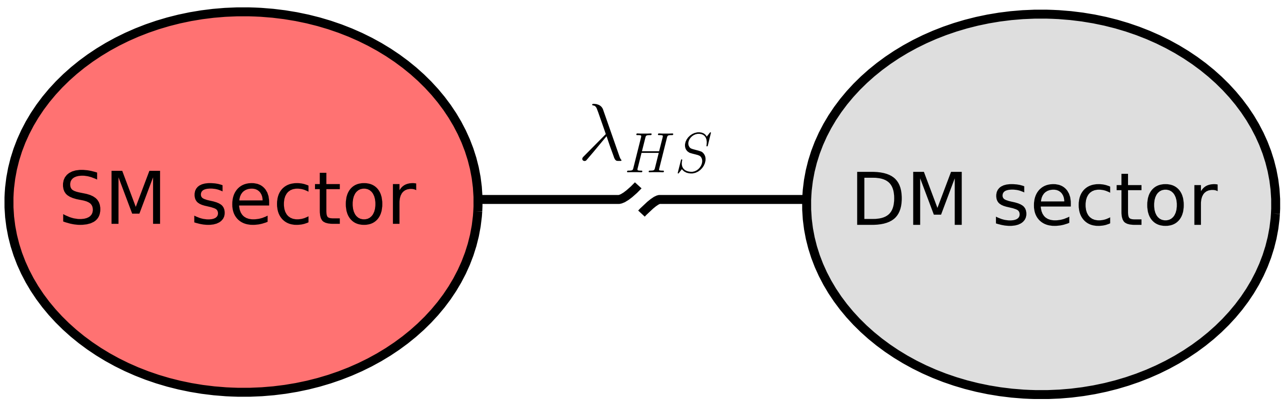
$\Rightarrow T_{\rm SM} \gg T$ (temperature in dark sector) or $g_{*\rm SM} \gg g_{*\rm DM}$

The simplest dark matter model

- SM + singlet scalar S (DM) $\Rightarrow$ simplest
  DM is stabilized by $\mathbb{Z}_2$: $S \to -S$

$$\mathcal{V} = \mu_H^2 |H|^2 + \frac{\lambda_H}{4}|H|^4 + \frac{\lambda_{HS}}{2} S^2 |H|^2 + \frac{m^2}{2} S^2 + \frac{\lambda}{4!} S^4$$

  - New parameters: $(\lambda_{HS},\, m,\, \lambda)$
  - $\lambda_{HS} \ll 1$ (not to be thermalized), but λ is not too small



- Bose-Einstein distribution: $f(\boldsymbol{p}) = \left(e^{(E(\boldsymbol{p})-\mu)/T} - 1\right)^{-1}$

$\rightarrow$ Investigate effect of BE dist. and parameter space for DM relic

Reaction rates

Our definition of reaction rate:

$$\Gamma_{a\to b} = \int \left(\prod_{i\in a} \frac{d^3 p_i}{(2\pi)^3 2E_i} f(\boldsymbol{p}_i) \right) \left(\prod_{j\in b} \frac{d^3 p_j}{(2\pi)^3 2E_j} (1 + f(\boldsymbol{p}_j)) \right) |\mathcal{M}_{a\to b}|^2 (2\pi)^4 \delta^4 (p_a - p_b)$$

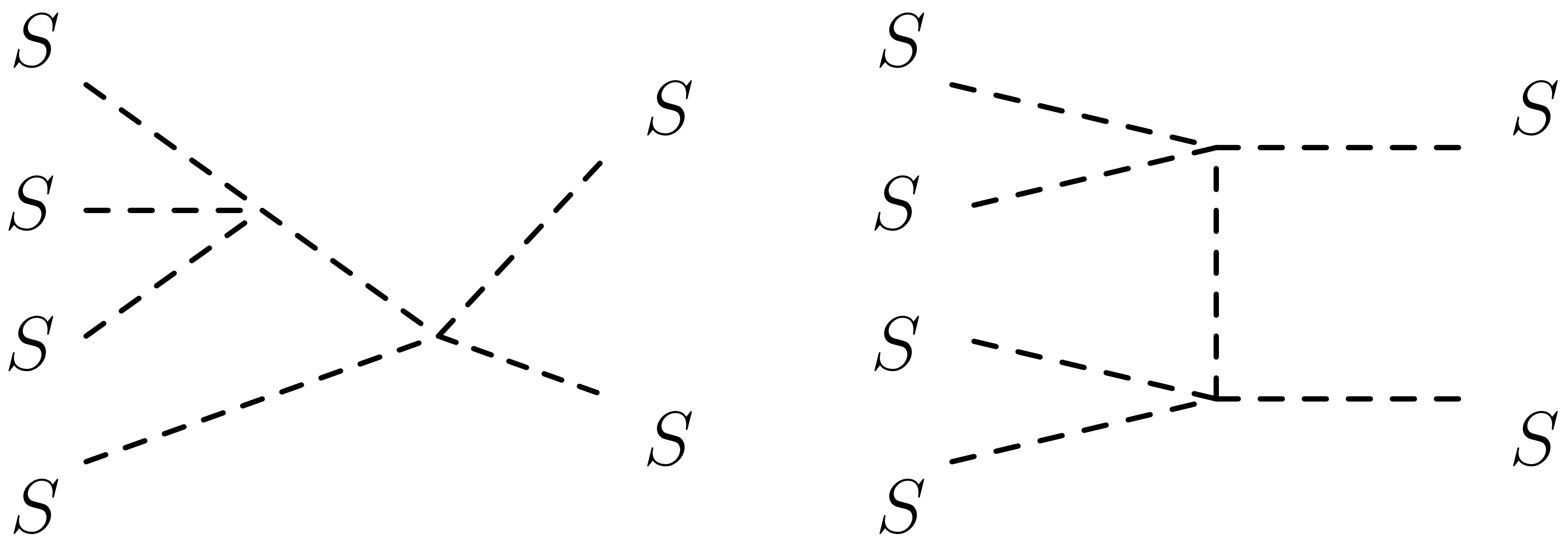
- Reaction rate has mass dimension 4
- We compare $\Gamma_{a\to b}$ with $3Hn$
  $\Gamma_{a\to b} > 3Hn \Rightarrow$ coupled
  $\Gamma_{a\to b} < 3Hn \Rightarrow$ decoupled

Integrated Boltzmann equation:

$$\frac{dn}{dt} + 3Hn = 2\left(\Gamma_{2\to 4} - \Gamma_{4\to 2}\right)$$

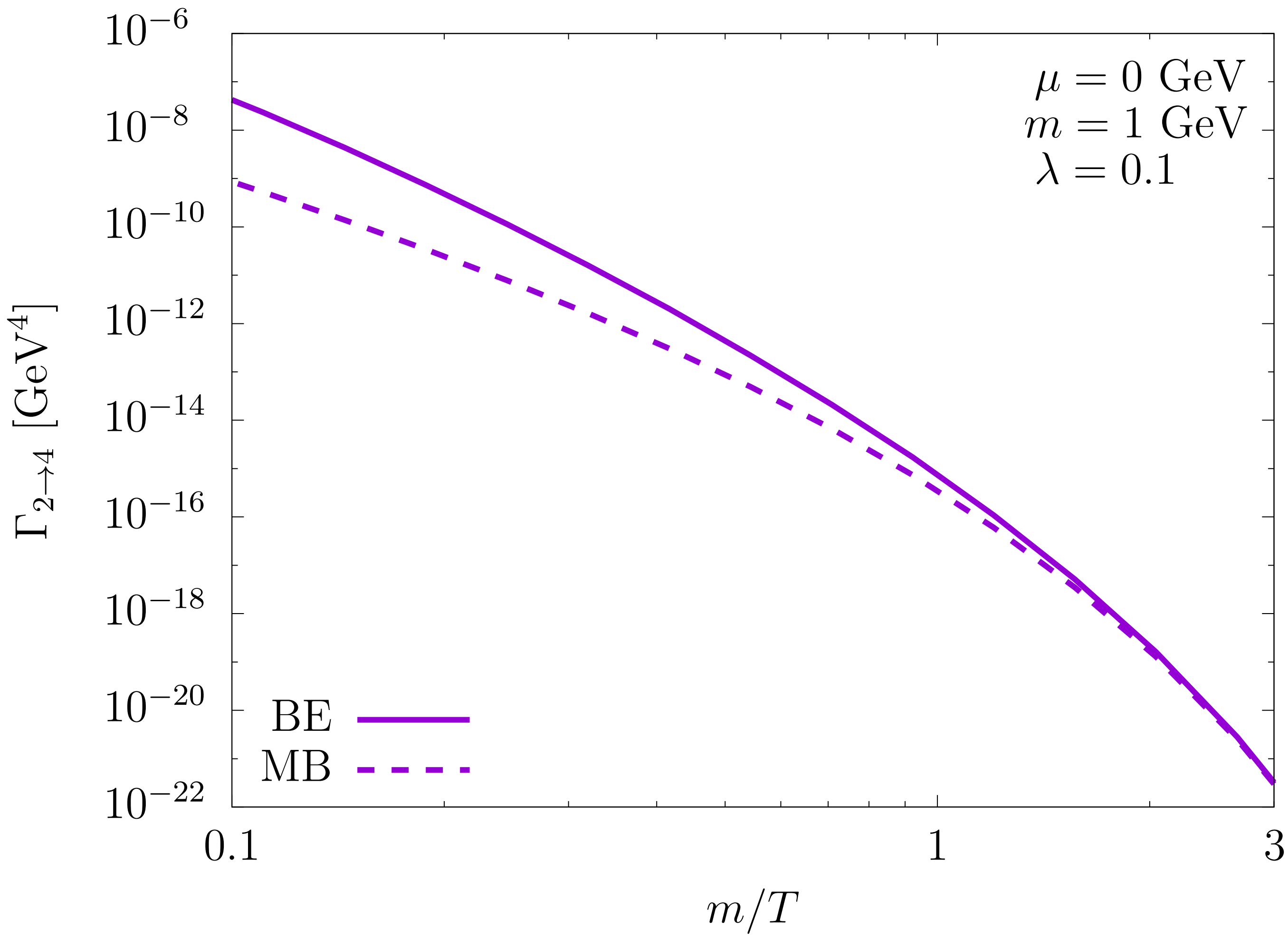
$\Rightarrow$ DM number density is obtained.

Number changing process with BE effect (Inelastic scattering)



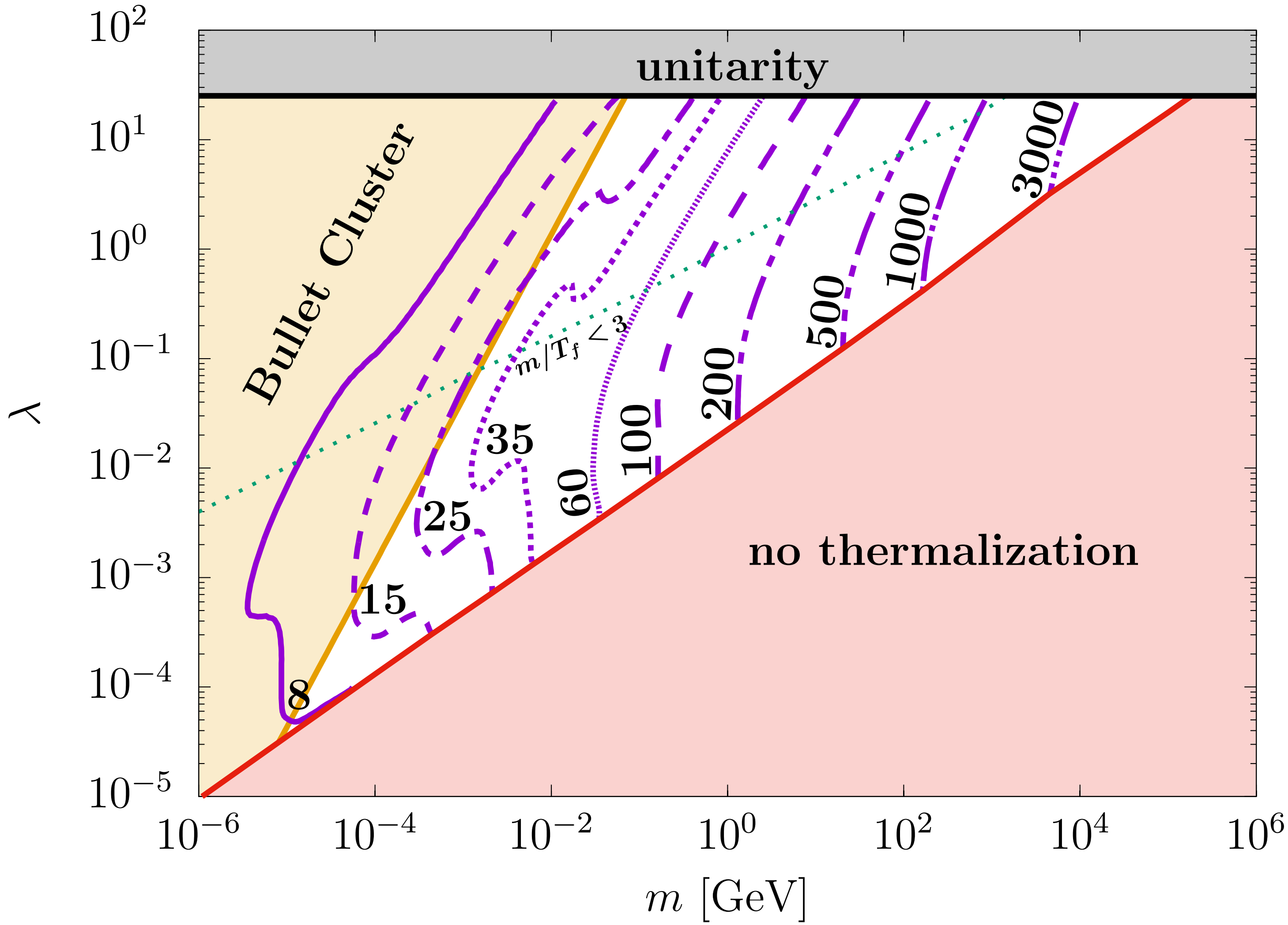
- General formula is too complicated.
- We numerically evaluate using CalcHEP.
  - · CalcHEP can treat $1 \to n$ and $2 \to n$ processes. $(\Gamma_{4\to2} = \Gamma_{2\to4} e^{2\mu/T})$
  - · $f(\boldsymbol{p})$ can be included by editing the source code.

Bose-Einstein dist.vs Maxwell Boltzmann dist.



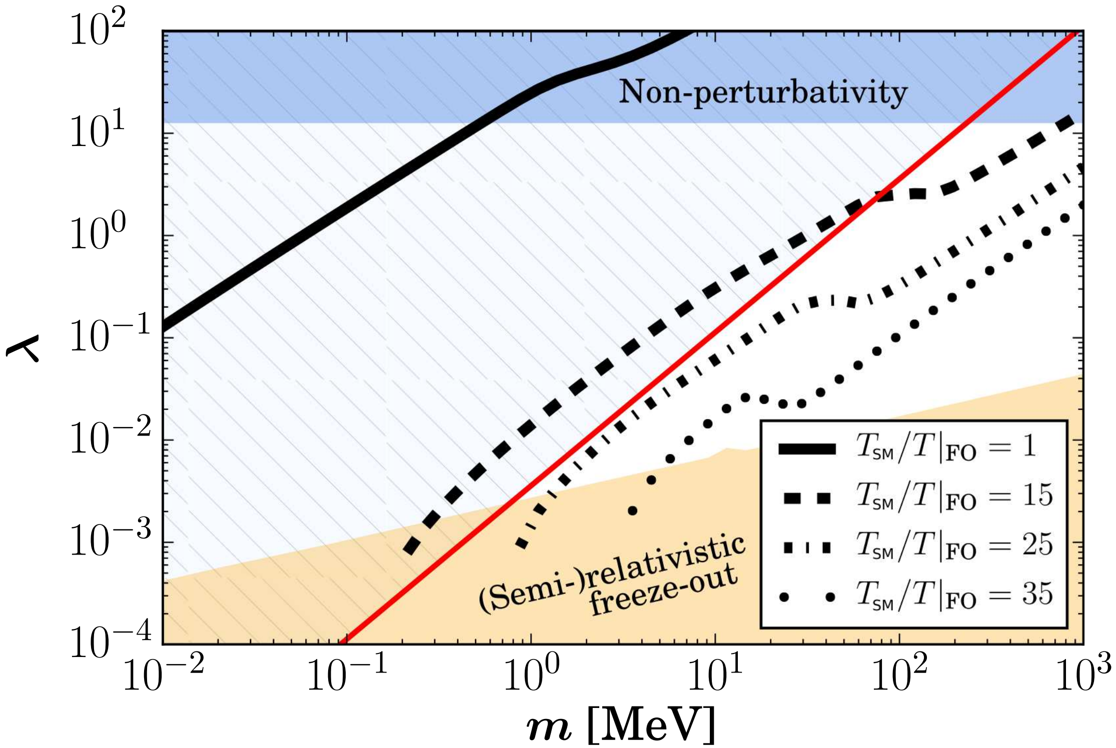
- A few orders of magnitude enhancement for $\Gamma_{2\to4}$ at high temperature
  $\Rightarrow$ BE dist. is important in relativistic regime.
  $\Gamma_{2\to4} \propto ff(1+f)(1+f)(1+f)(1+f) \sim f^6$

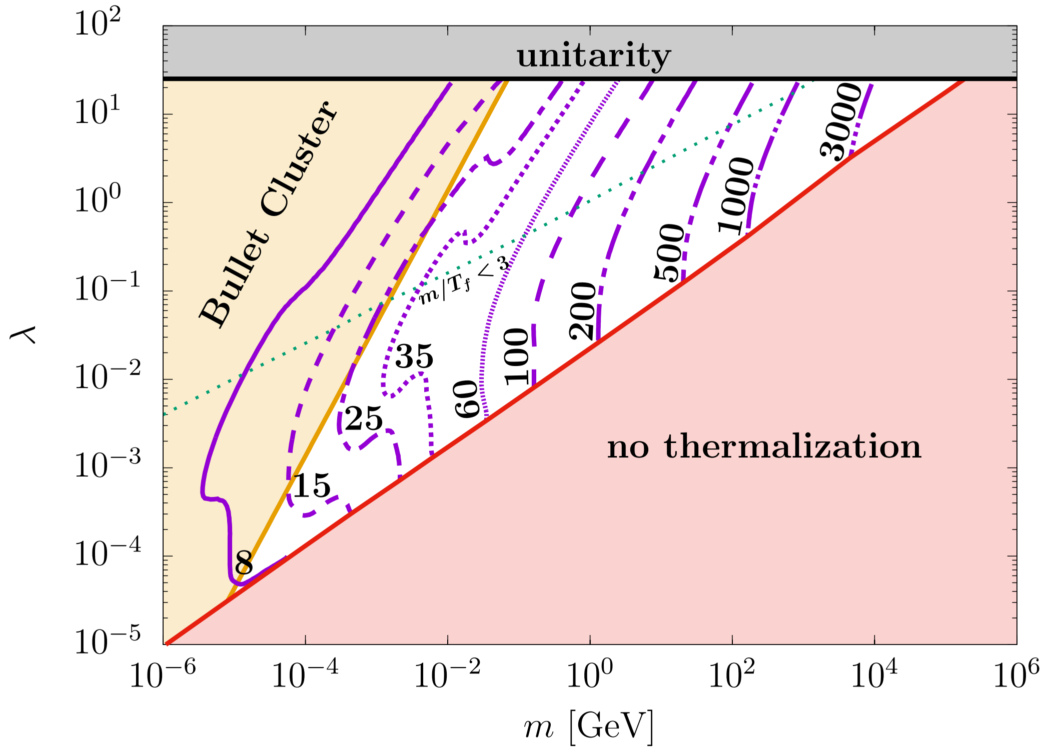
Parameter space



- Temperature ratio at dark freeze-out $T_{\rm SM}/T = 8, 15, 25, \cdots$
- $\sigma_{\rm self}/m \lesssim 1$ [cm^2/g], (self-interaction: $SS \to SS$)

Comparison with previous works





Bernal and Chu, JCAP 1601, 006 (2016)

- Possible parameter space for dark matter relic abundance is extended.
  Temperature raito: $8 \lesssim T_{\rm SM}/T \lesssim 5000$
  DM mass: $10\ {\rm keV} \lesssim m \lesssim 100\ {\rm TeV}$
  Self-coupling: $10^{-4} \lesssim \lambda \lesssim 4\pi$

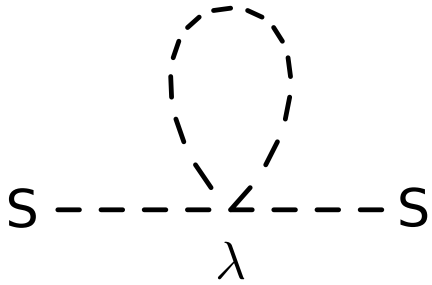
Summary

1. In dark thermalization, the reaction rate of $SS \to SSSS$ is enhanced by the BE distribution with a few orders of magnitude.
2. This effect is important for (semi-)relativistic dark freeze-out and evaluating DM relic abundance.
3. We extended parameter space of the previous work reproduing the correct DM relic abundance.

Backup

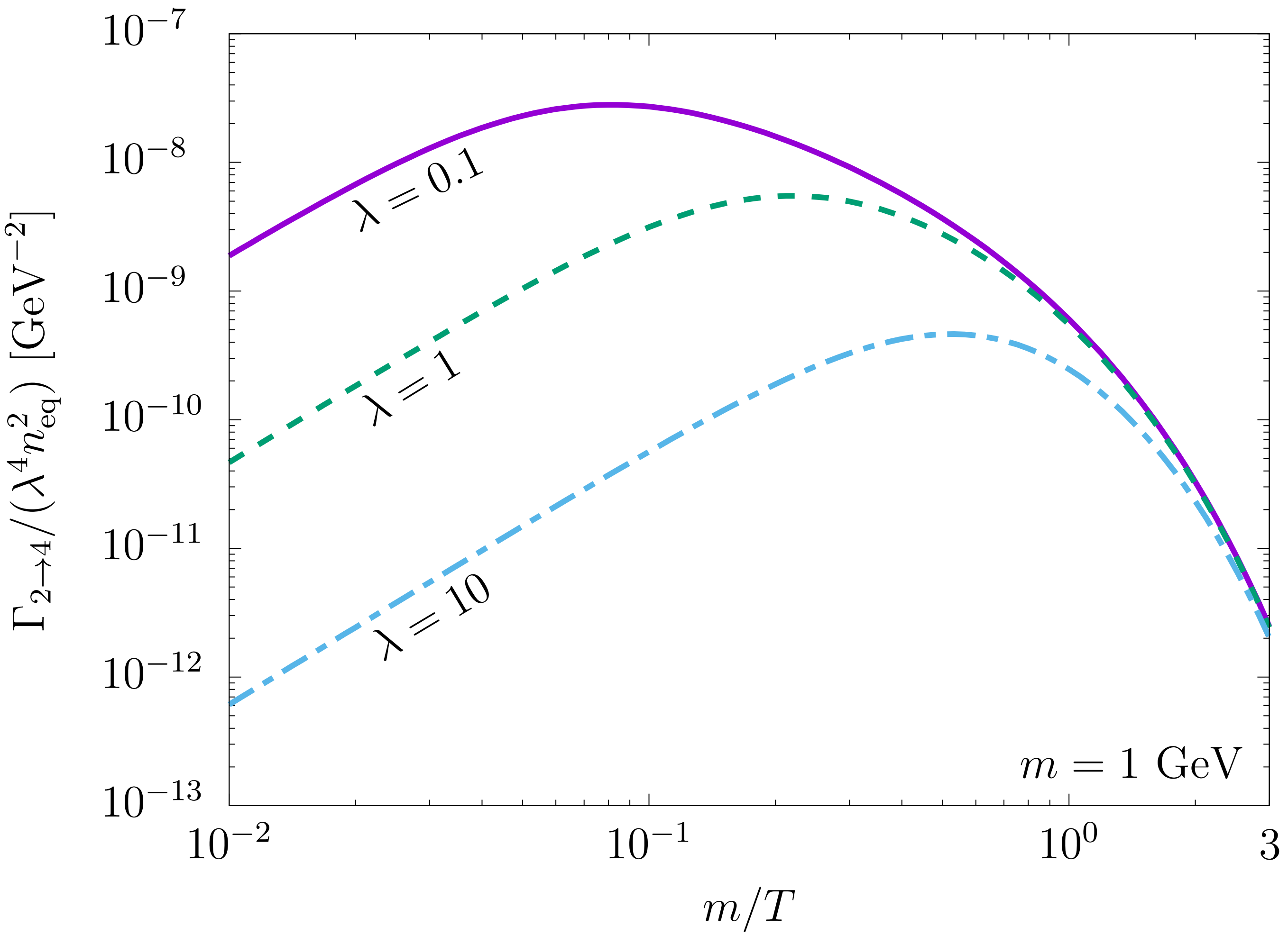
Chemical eq. and kinetic eq. in dark sector

- If $SS \leftrightarrow SS$ (elastic scattering) happens fast enough,
  $\Rightarrow$ kinetic eq. $\Rightarrow T$ and μ can be defined
- If $SSSS \leftrightarrow SS$ (inelastic scattering) happens fast enough,
  $\Rightarrow$ chemical eq. $\Rightarrow \mu \approx 0$ $(4\mu = 2\mu)$
- If both are satisfied at the same time, $\Rightarrow$ thermal eq.
  Note: Usually, $\Gamma_{2\to 2} \gg \Gamma_{4\to 2}$
  $\Gamma_{4\to 2} = \Gamma_{2\to 4}\, e^{2\mu/T}$ in general.
- Thermal mass is included $m^2 \to m^2 + \dfrac{\lambda T^2}{24}$
  $\Rightarrow$ regularize reaction rates at relativistic regime



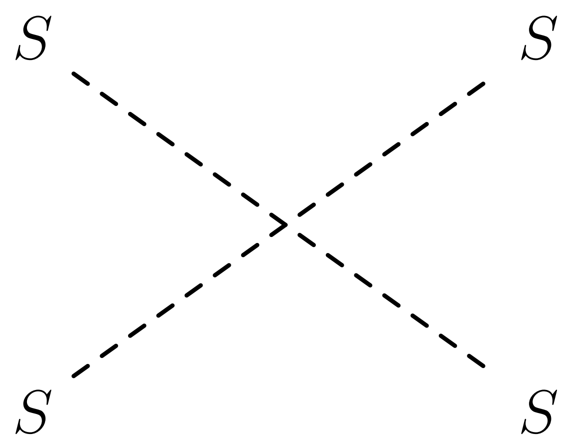
$$f(\boldsymbol{p}) = \left(e^{\left(\sqrt{\boldsymbol{p}^2 + m^2 + \lambda T^2/24} - \mu\right)/T} - 1\right)^{-1} \to \left(e^{\sqrt{\lambda/24}} - 1\right)^{-1}$$

Thermal mass effect



- $\dfrac{\Gamma_{2\to4}}{\lambda^4 n_{\rm eq}^2} \sim \langle \sigma_{2\to4} v \rangle / \lambda^4$ in non-relativistic limit

Elastic scattering process with BE effect



General formula

$$\Gamma_{2\to 2} = \frac{4T}{\pi^4}\int_m^\infty dE E^3\sqrt{E^2-m^2}\int_0^\infty \frac{d\eta \sinh\eta}{e^{2\frac{E\cosh\eta-\mu}{T}}-1}\log\left(\frac{\sinh\left(\frac{E\cosh\eta+\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}\right)}{\sinh\left(\frac{E\cosh\eta-\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}\right)}\right)$$

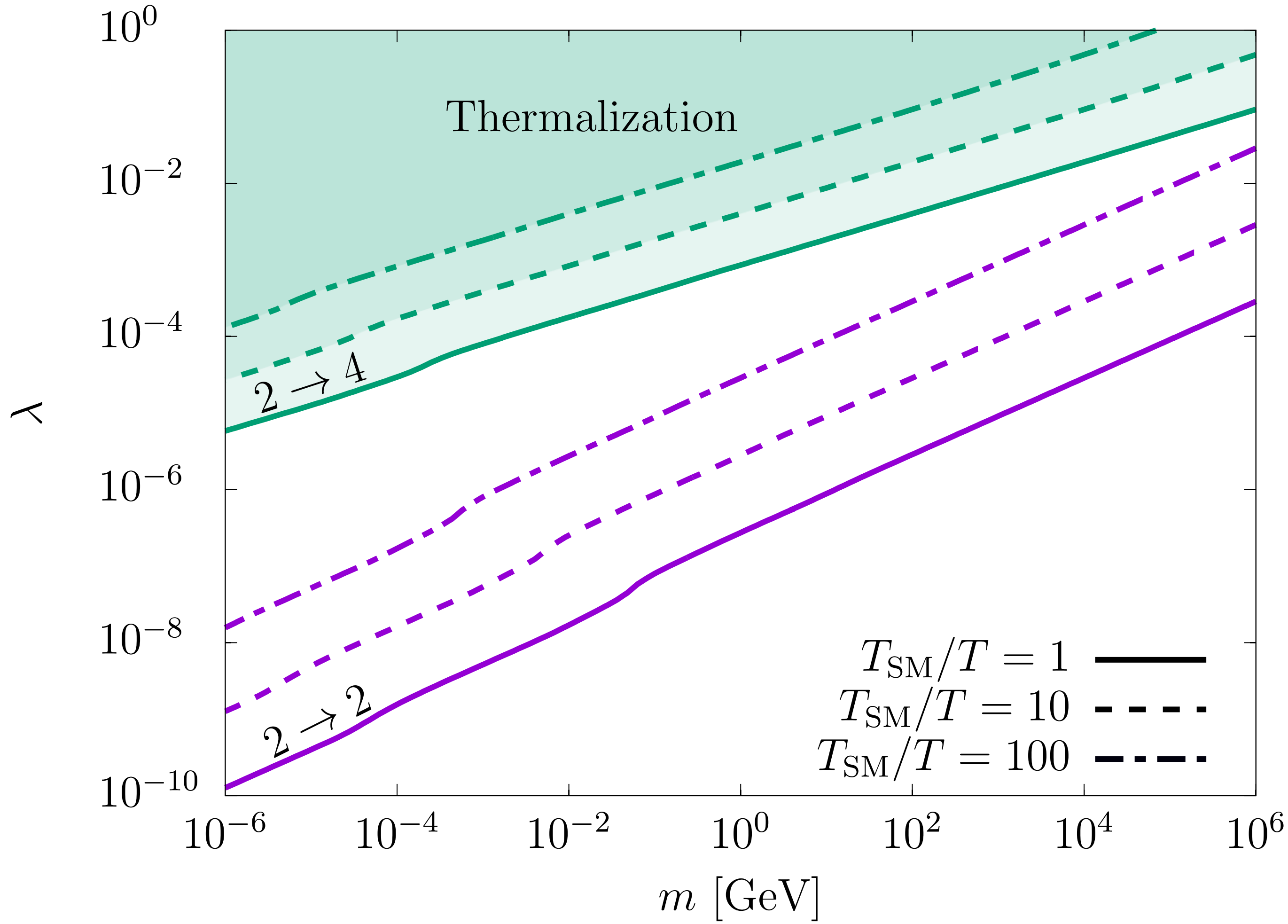
$$\times\,\sigma_{\mathrm{CM}}^{2\to 2}(E,\eta)$$

$$\Rightarrow \langle\sigma_{2\to 2}v\rangle n^2 \quad \text{in non-relativistic limit}$$

where $\sigma_{\mathrm{CM}}^{2\to 2}$ is the cross section at centre of mass frame:

$$\sigma_{\mathrm{CM}}^{2\to 2}(E,\eta) = \frac{1}{2!2!}\frac{\lambda^2 T\left(1-e^{-2\frac{E\cosh\eta-\mu}{T}}\right)^{-1}}{64\pi E^2\sqrt{E^2-m^2}\sinh\eta}\log\left(\frac{\sinh\left(\frac{E\cosh\eta+\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}\right)}{\sinh\left(\frac{E\cosh\eta-\sqrt{E^2-m^2}\sinh\eta-\mu}{2T}\right)}\right)$$

Parameter space for dark thermalization



- Compare $\Gamma_{2\to2}$, $\Gamma_{2\to4}$ and $3Hn$.
- $\Gamma_{2\to4}/(Hn)$ is maximized at $T \sim 5m/\sqrt{\lambda}$.
- $\Gamma_{2\to2} \propto \lambda^2$ and $\Gamma_{2\to4} \propto \lambda^4$