Integrable Open Spin Chains of Flavored ABJM Theory

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JW, 1711.##### submit/20066579
Motivations

• As a kind of weak/strong duality, AdS/CFT correspondence allows us to study the strongly coupled gauge theories using weakly coupled gravity theories. (AdS/QCD, AdS/CMT)

• But this also makes the non-trivial check of this duality hard!

• Also it is still hard to compute quantities when coupling is intermediate(~O(1)).

• New tools in field theory: supersymmetric localization, integrable structure, bootstrap...
Motivations

• Rich integrable structures were found in both side of the holographic duality between 4d $\mathcal{N}=4$ SYM and type IIB string theory on $\text{AdS}_5*\text{S}^5$.

  [Minaham, Zarembo, 02] [Bena et al, 03]

  collection of reviews: [Beiserts et al, 10]

• Using this remarkable structure, people can compute non-trivial quantities like cusp anomalous dimension for a large range of the ‘t Hooft coupling in the planar limit.
Motivations

• Integrable structures were also found on both side of gauge-gravity duality between 3d $\mathcal{N}=6$ $U(N)_k \ast U(M)_{-k}$ Chern-Simons-matter theory (ABJ(M) theory) and type IIA string theory on AdS$_4 \ast$CP$^3$.
  
  [Minaham, Zarembo, 08] [Bak, Rey, 08] ...

• We want to search for theories with less supersymmetries while they still have such integrable structure.
Three roads in 4d case

• Beta-/gamma-deformation
  \[\text{Roiban, 03}\][\text{Berenstein, Cherkis, 04}\][\text{Beisert, Roiban, 05}]  

• Orbifold \[\text{Wang, Y.-S. Wu, 03}\][\text{Ideguchi, 04}\][\text{Beisert, Roiban, 05}]  

• Adding flavors (+ orentifolding)
  \D7’s+O7 \[\text{Chen, Wang, Y.-S. Wu, 04}\]*2  
  \D7’s\[\text{Erler, Mann, 05}\]
Three roads in 3d

• Beta-/gamma-deformed planar ABJM theories
two loop, scalar sector: [He, JW, 13]
all-loop and full sector (asymptotic Bethe ansatz), Y-system for finite size effects for composite operators: [Chen, JW, 16]
Finite-size effects for Dyonic Gaint magnons: [Chen, JW, 16]

• Planar orbifold ABJM theories: from two-loop to all-loop

[Bai, Chen, Ding, Li, JW, 16]

• Planar flavored ABJM theory: two-loop scalar sector [This talk]
ABJM theory

- Aharony-Bergman-Jafferis-Maldacena theory is a three-dimensional Chern-Simons-matter theory with $\mathcal{N}=6$ supersymmetries. The gauge group is $U(N)\times U(N)$ with Chern-Simons levels $k$ and $-k$.
- The matter fields are scalars, $Y^I$ and fermions $\Psi_i$ in the bi-fundamental representation of the gauge group. Here $I=1, \ldots, 4$ and the R-symmetry group is $SU(4)$.
Properties of ABJM theory

• 1/k is the coupling constant.

• The theory has a large N (planar or ‘t Hooft) limit:

\[ N \to \infty, k \to \infty, \lambda \equiv \frac{N}{k} \text{ fixed} \]

• This theory is found to be the LEFT for N M2-branes putting on the tip of the orbifold \( \mathbb{C}^4/\mathbb{Z}_k \).
Adding flavors

• One can add flavor which is (anti-)fundamental representation of either $U(N)$.

[Hohenegger, Kirsch, 09][Gaiotto, Jafferis, 09]
[Hikida, Li, Takayanagi, 09]

• This is dual to adding D6 branes to IIA theory on $AdS_4 \ast CP^3$.

• The maximal supersymmetry one can get is 3d $\mathcal{N}=3$. 
ADM

• We compute the anomalous dimension matrix (ADM) of

\[ \hat{O} = Y_i^\dagger X_{i_1A_1} X_{i_2A_2} \ldots X_{i_{2L-1}A_{2L-1}} X_{i_{2L}A_{2L}}^\dagger Y^{i'} \]

• We focus the ‘t Hooft limit, with k, N to infinity, \( \lambda = N/k \) fixed and \( N_f << N \). This is dual to probe limit of D6 brane in the gravity side.
Two loop diagrams
Two loop Hamiltonian

\[ \mathcal{H} = \mathcal{H}_l + \mathcal{H}_r + \mathcal{H}_{\text{bulk}} + \alpha \mathbb{I}, \]

\[ (\mathcal{H}_l)_{i, j_1 B_1, i_2 A_2}^{j, i_1 A_1, j_2 B_2} = \lambda^2 \left[ (\delta_{A_1}^{B_2} - \delta_{B_1}^{A_2}) \cdot \delta_{i_2}^{j_2} \delta_{j_1}^{i_1} \delta_{i_1}^{j_1} + \delta_{B_1}^{A_2} \cdot \delta_{i_2}^{i_1} \delta_{j_1}^{j_2} \right], \]

\[ (\mathcal{H}_r)_{j_2 L-1 B_{2 L-1}, i_2 L A_{2 L}, j'}^{i_2 L-1 A_{2 L-1}, j_2 L B_{2 L}, i'} = \lambda^2 \left[ (\delta_{B_{2 L-1}}^{A_{2 L}} - \delta_{B_{2 L-1}}^{A_{2 L}}) \cdot \delta_{j'}^{j_2 L} \delta_{j_2 L-1}^{i_2 L} \delta_{j_2 L-1}^{i_2 L} \cdot \delta_{j_2 L-1}^{i_2 L} \delta_{j_2 L-1}^{i_2 L} \right], \]

\[ \mathcal{H}_{\text{bulk}} = \lambda^2 \sum_{l=1}^{2L-2} \left( \mathbb{I}_{l, l+1} - P_{l, l+2} + \frac{1}{2} P_{l, l+2} K_{l, l+1} + \frac{1}{2} K_{l, l+1} P_{l, l+2} \right), \]
Two loop Hamiltonian

\[
(\Pi_{l,l+1})_{jB,i' A'}^{iA,j' B'} = \delta^i_j \delta^{i'}_{j'} \delta^A_B \delta^{A'}_{A'} , \quad (\mathbb{P}_{l,l+2})_{jB,j' B'}^{iA,i' A'} = \delta^i_j \delta^{i'}_{j'} \delta^A_B \delta^{A'}_{A'} , \quad (\mathbb{K}_{l,l+1})_{jB,i' A'}^{iA,j' B'} = \delta^i_j \delta^{i'}_{j'} \delta^A_{A'} \delta^{B'}_B ,
\]

- $\alpha$ was fixed to $2\lambda^2$ ($\leq$ anomalous dimension of BPS operator is zero).
Multi-body problem

• We spent a lot of time to decide whether this Hamiltonian is integrable based on algebraic Bethe ansatz, but we have not succeeded yet.

• Then we turn to coordinate Bethe ansatz, we need to compute the S-matrix of two-particle scattering and reflection matrix for one-(bulk)-particle state.
Vacuum and excitations

• Vacuum

\[ |\Omega\rangle = |Y_2^\dagger X^{11} X_{22}^\dagger \cdots X^{11} X_{22}^\dagger Y^1\rangle. \]

• Excitations

bulk A type:
\[ Y_2^\dagger (A_1 B_2) \cdots (A_2 B_2) \cdots (A_1 B_2) Y^1, \]
\[ Y_2^\dagger (A_1 B_2) \cdots (B_1^\dagger B_2) \cdots (A_1 B_2) Y^1, \]

bulk B type:
\[ Y_2^\dagger (A_1 B_2) \cdots (A_1 A_2^\dagger) \cdots (A_1 B_2) Y^1, \]
\[ Y_2^\dagger (A_1 B_2) \cdots (A_1 B_1) \cdots (A_1 B_2) Y^1, \]

boundary:
\[ Y_1^\dagger (A_1 B_2) \cdots (A_1 B_2) \cdots (A_1 B_2) Y^1, \]
\[ Y_2^\dagger (A_1 B_2) \cdots (A_1 B_2) \cdots (A_1 B_2) Y^2. \]

\[ X^{11} = A_1, \quad X^{12} = A_2, \quad X^{21} = B_1^\dagger, \quad X^{22} = B_2^\dagger. \]
Bulk S-matrix

• The two-loop S-matrix has been computed in [Ahn, Nepomechine, 09]

\[ u_i \equiv \frac{1}{2} \cot \frac{k_i}{2}, \quad u_{ij} \equiv u_i - u_j. \]

The non-zero elements of the bulk S-matrix is

\[ S_{\phi\phi}^{\phi\phi}(k_2, k_1) = \frac{u_{21} + i}{u_{21} - i}, \]

where \(\phi\) is one of \(A_2, B_1^+, A_2^+, B_1;\)
Bulk S-matrix (II)

\[ S^{A_2 B_1^\dagger}_{A_2 B_1}(k_2, k_1) = S^{B_1^\dagger A_2}_{B_1 A_2}(k_2, k_1) = S^{A_2^\dagger B_1}_{A_2 B_1}(k_2, k_1) = S^{B_1 A_2^\dagger}_{B_1 A_2}(k_2, k_1) = \frac{u_{21}}{u_{21} - i}; \]

\[ S^{B_1^\dagger A_2}_{A_2 B_1}(k_2, k_1) = S^{A_2 B_1^\dagger}_{B_1 A_2}(k_2, k_1) = S^{B_1 A_2^\dagger}_{A_2 B_1}(k_2, k_1) = S^{A_2^\dagger B_1}_{B_1 A_2}(k_2, k_1) = \frac{i}{u_{21} - i}; \]

\[ S^{A_2 B_1}_{A_2 B_1}(k_2, k_1) = S^{B_1 A_2}_{B_1 A_2}(k_2, k_1) = S^{A_2^\dagger B_1^\dagger}_{A_2^\dagger B_1^\dagger}(k_2, k_1) = S^{B_1^\dagger A_2^\dagger}_{B_1^\dagger A_2^\dagger}(k_2, k_1) = 1; \]

\[ S^{A_2 A_2^\dagger}_{A_2 A_2^\dagger}(k_2, k_1) = S^{B_1^\dagger B_1}_{B_1^\dagger B_1}(k_2, k_1) = S^{A_2^\dagger A_2^\dagger}_{A_2^\dagger A_2^\dagger}(k_2, k_1) = S^{B_1^\dagger B_1^\dagger}_{B_1^\dagger B_1^\dagger}(k_2, k_1) = \frac{u_{12}}{u_{12} - i}; \]

\[ S^{A_2 A_2^\dagger}_{A_2 A_2^\dagger}(k_2, k_1) = S^{B_1^\dagger B_1}_{A_2 A_2^\dagger}(k_2, k_1) = S^{A_2^\dagger A_2^\dagger}_{B_1^\dagger B_1^\dagger}(k_2, k_1) = S^{B_1^\dagger B_1^\dagger}_{A_2 A_2^\dagger}(k_2, k_1) = \frac{i}{u_{12} - i}. \]
YBE

• The S-matrix has been computed in [Ahn, Nepomechne, 09] and we confirm that it satisfies Yang-Baxter equation.
Action of left boundary Hamiltonian

\[ \mathcal{H}_l |1\rangle_{A_2} = \lambda^2 |1\rangle_{B_1}, \]
\[ \mathcal{H}_l |1\rangle_{B_1^\dagger} = \lambda^2 |l\rangle_{Y_1^\dagger}, \]
\[ \mathcal{H}_l |1\rangle_{A_2^\dagger} = -\lambda^2 |l\rangle_{Y_1^\dagger}, \]
\[ \mathcal{H}_l |1\rangle_{B_1} = \lambda^2 |1\rangle_{A_2}, \]
\[ \mathcal{H}_l |l\rangle_{Y_1^\dagger} = -\lambda^2 |1\rangle_{A_2^\dagger} + \lambda^2 |l\rangle_{Y_1^\dagger} + \lambda^2 |1\rangle_{B_1^\dagger}, \]
\[ \mathcal{H}_l |x\rangle = -\lambda^2 |x\rangle, \quad x \neq 1, \]
Closed sector I

• Ansatz

\[ |\psi_1(k)\rangle = \sum_{x=1}^{L} (f(x)|x\rangle A_2 + g(x)|x\rangle B_1), \]

\[ f(x) = F e^{ikx} + \tilde{F} e^{-ikx}, \]
\[ g(x) = G e^{ikx} + \tilde{G} e^{-ikx}. \]

• Eigenvalue equation

\[ \mathcal{H}|\psi_1\rangle = E(k)|\psi_1\rangle \]

• Bulk part of the chain gives

\[ E(k) = 2\lambda^2 - 2\lambda^2 \cos k. \]
Reflection

• Left boundary
  \[ F = -e^{-ik}\tilde{G}, \quad G = -e^{-ik}\tilde{F}. \]
  \[
  \begin{pmatrix}
  F \\
  G
  \end{pmatrix}
  \equiv K_l(k)
  \begin{pmatrix}
  \tilde{F} \\
  \tilde{G}
  \end{pmatrix},
  \]
  \[
  K_l(k) = \begin{pmatrix}
  0 & -e^{-ik} \\
  -e^{-ik} & 0
  \end{pmatrix}.
  \]

• The right boundary
  \[ F = -e^{-2ikL-ik}\tilde{G}, \quad G = -e^{-2ikL-ik}\tilde{F}. \]
  \[
  e^{2ikL}
  \begin{pmatrix}
  F \\
  G
  \end{pmatrix}
  \equiv K_r(k)
  \begin{pmatrix}
  \tilde{F} \\
  \tilde{G}
  \end{pmatrix},
  \]
  \[
  K_r(k) = \begin{pmatrix}
  0 & -e^{-ik} \\
  -e^{-ik} & 0
  \end{pmatrix}.
  \]
Closed sector II

\[ |\psi_2(k)\rangle = \sum_{x=1}^{L} n(x) |x\rangle_{B_1^\dagger} + \sum_{x=1}^{L} h(x) |x\rangle_{A_2^\dagger} + \beta |l\rangle_{Y_1^\dagger} + \gamma |r\rangle_{Y_2}, \]

\[ K_l(k) = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}, \quad K_r(k) = \begin{pmatrix} 0 & -e^{-2ik} \\ -e^{2ik} & 0 \end{pmatrix}, \]
Reflection matrices

\[ K_l(k) = \begin{pmatrix}
0 & 0 & 0 & -e^{-ik} \\
0 & 0 & -1 & 0 \\
0 & -1 & 0 & 0 \\
-e^{-ik} & 0 & 0 & 0 \\
\end{pmatrix}, \]

\[ K_r(k) = \begin{pmatrix}
0 & 0 & 0 & -e^{-ik} \\
0 & 0 & -e^{-2ik} & 0 \\
0 & -e^{-2ik} & 0 & 0 \\
-e^{-ik} & 0 & 0 & 0 \\
\end{pmatrix}. \]
Boundary Yang-Baxter equation

\[
\begin{aligned}
[S(k_1, k_2)]_{l_1 l_2}^{m_1 m_2} & [K_l(k_2)]_{j_2}^{l_2} [S(-k_2, k_1)]_{j_1 i_2}^{l_1 j_2} [K_l(k_1)]_{i_1}^{j_1} \\
& = [K_l(k_1)]_{l_1}^{m_1} [S(-k_1, k_2)]_{j_1 l_2}^{l_1 m_2} [K_l(k_2)]_{j_2}^{l_2} [S(-k_2, -k_1)]_{i_1 i_2}^{j_1 j_2},
\end{aligned}
\]

\[
\begin{aligned}
[S(-k_1, -k_2)]_{l_1 l_2}^{m_1 m_2} & [K_r(-k_1)]_{j_1}^{l_1} [S(-k_2, k_1)]_{j_1 i_2}^{j_1 l_2} [K_r(-k_2)]_{i_2}^{j_2} \\
& = [K_r(-k_2)]_{l_2}^{m_2} [S(-k_1, k_2)]_{l_1 j_2}^{m_1 l_2} [K_r(-k_1)]_{j_1}^{l_1} [S(k_2, k_1)]_{i_1 i_2}^{j_1 j_2},
\end{aligned}
\]
BYBE
Nonintegrability in the Veneziano limit

• Flavor-backreacted background was found in [Conde, Ramallo, 11]. The backreacted of D6 branes are taken into account. This is dual to field theory in the Veneziano limit: $N_f, N, k$ to infinity, $N_f/N, N/k$ fixed. (unquenched flavors).

• Very recently, [Giataganas, Zoubos, 17] showed that the classical string motion in this background is chaotic! This gave strong evidence that the field theory in the Veneziano limit is nonintegrable.
More general integrable boundary interactions?

- Let us consider boundary interactions with general coefficients,

\[
(\mathcal{H}_1)_{i,j_{1}A_1,j_{2}B_2}^{i_1,i_{1}A_1,i_2B_2} = (-a \delta_{B_1}^{A_1} \delta_{A_2}^{B_2} + b \delta_{A_1}^{A_2} \delta_{B_1}^{B_2}) \cdot \delta_{j_{1}}^{i_1} \delta_{i_{1}}^{j_1} \delta_{i_2}^{j_2} + c \delta_{B_1}^{A_1} \delta_{A_2}^{B_2} \cdot \delta_{i_1}^{i_1} \delta_{j_1}^{j_1} \delta_{i_2}^{i_2}.
\]

\[
(\mathcal{H}_r)_{j_{2L-1}A_2L-1,j_{2L}B_2L,i'}^{i_{2L-1}A_2L-1,j_{2L}B_2L,i'} = (-a' \delta_{A_2L}^{B_2L} \delta_{B_2L-1}^{A_2L-1} + b' \delta_{B_2L}^{B_2L} \delta_{A_2L}^{A_2L-1}) \cdot \delta_{j_{2L-1}'} \delta_{i_{2L-1}} \delta_{j_{2L}} \delta_{i_{2L}}',
\]
More general integrable boundary interactions?

• Among these boundary interactions, only two are integrable at each end of the spin chain. \([JW, 17]\)
• Left end:
  
  • 1. \(a=b=c=\lambda^2\), this is the one from flavored ABJM theory. The reflection matrix is anti-diagonal.
  
  • 2. \(a=b=c=0\). The reflect matrix is proportional to the identity matrix.
  
• Story on the right end is similar.
Conclusion and Discussions

• We proved that the planar flavored ABJM theory is integrable at two loop in the scalar sector.

• We used the coordinate Bethe ansatz, notice that the reflection matrix is anti-diagonal!

• If we consider more general coefficients of the boundary interaction, only two cases are integrable!
To do list

• Find the spectrum: using off-diagonal Bethe ansatz [Cao, Shi, Wang, Yang].

• Generalization to full sector and/or high loop.

• Integrability of the theories with both D6’s and O6-planes added?

• Integrable open spin chain from Wilson loops or giant gravitons?
To do list

• [Basso, Komatsu, Vieira, 15] introduced Hexagon program approach for 3pt functions in N=4 SYM. Similar stories for ABJM theories? [cf. talk by Changmin Ahn]

• Some Chern-Simons-matter theories are from D2 in massive IIA theory. [cf. talk by Nakwoo Kim]
• Integrable theories in this class?
Thank you very much!