Complex Langevin analysis of the spontaneous breaking of 10D rotational symmetry in the Euclidean IKKT matrix model (arXiv:2002.07410)

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Asia-Pacific Symposium for Lattice Field Theory

Aug. 5th 2020, 14:00-14:20

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1. Introduction

Difficulties in simulating complex partition functions.

$$Z = \int dA \exp(-S_0 + i\Gamma), \quad Z_0 = \int dA e^{-S_0}$$

Sign problem:
The reweighting $\langle \mathcal{O} \rangle = \frac{\langle \mathcal{O} e^{i\Gamma} \rangle_0}{\langle e^{i\Gamma} \rangle_0}$ requires configs. $\exp[O(N^2)]$

$\langle * \rangle_0$ = (V.E.V. for the phase-quenched partition function Z_0)

2. Euclidean type IIB matrix model

Promising candidate for nonperturbative string theory

[N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, hep-th/9612115] (a.k.a. "IKKT model")

$$Z = \int dA d\psi e^{-(S_b + S_f)}$$

$$S_b = -\frac{N}{4} \mathrm{tr}[A_\mu, A_\nu]^2, \quad S_f = N \mathrm{tr} \bar{\psi}_\alpha (\Gamma^\mu)_{\alpha\beta} [A_\mu, \psi_\beta]$$

- ***Euclidean*** case after Wick rotation $A_0 \to iA_{10}, \Gamma^0 \to -i\Gamma_{10}$.

⇒Path integral is finite without cutoff.

[W. Krauth, H. Nicolai and M. Staudacher, hep-th/9803117, P. Austing and J.F. Wheater, hep-th/0103059]

- A_μ, Ψ_α ⇒ $N \times N$ Hemitian traceless matrices. ($\mu=1,2,\dots,10$, $\alpha,\beta=1,2,\dots,16$)
- Eigenvalues of A_μ : spacetime coordinate⇒ $\mathcal{N}=2$ SUSY

2. Euclidean type IIB matrix model

SETSUDAI

$$Z = \int dA de^{-S_b} \underbrace{\left(\int d\psi e^{-S_f} \right)}_{= \mathrm{Pf}\mathcal{M} = |\mathrm{Pf}\mathcal{M}| e^{i\Gamma} = e^{-S_{f,eff}}} = \int dA \underbrace{e^{-S}}_{e^{-(S_b + S_{f,eff})}}$$

- $\mathrm{Pf}\mathcal{M}$'s *complex phase Γ* contributes to the Spontaneous Symmetry Breaking (SSB) of SO(10).

- Result of Gaussian Expansion Method (GEM)
  [T.Aoyama, J.Nishimura, and T.Okubo, arXiv:1007.0883、J.Nishimura, T.Okubo and F.Sugino, arXiv:1108.1293]

SSB **SO(10)→SO(3).**
Dynamical compactification to 3-dim spacetime.

3. Complex Langevin Method

Complex Langevin Method (CLM)

⇒Solve the complex version of the Langevin equation.

[Parisi, Phys.Lett. 131B (1983) 393, Klauder, Phys.Rev. A29 (1984) 2036]

$$\frac{d(A_\mu)_{ij}}{dt} = -\boxed{\frac{\partial S}{\partial (A_\mu)_{ji}}} + \eta_{\mu,ij}(t) = -\left\{ \frac{\partial S_b}{\partial (A_\mu)_{ji}} - \underbrace{\frac{1}{2}\mathrm{Tr}\left(\frac{\partial \mathcal{M}}{\partial (A_\mu)_{ji}} \mathcal{M}^{-1} \right)}_{=\partial \log \mathrm{Pf}\mathcal{M} / \partial (A_\mu)_{ji}} \right\} + \eta_{\mu,ij}(t)$$

drift term

- A_μ : Hermitian→general complex traceless matrices.
- η_μ: Hermitian-matrix white noise obeying the probability distribution $\exp\left(-\frac{1}{4}\int \mathrm{tr}\eta^2(t)dt \right)$

3. Complex Langevin Method

SETSUDAI

CLM does not work when it encounters these problems:

(1) Excursion problem: A_μ is too far from Hermitian

⇒Gauge Cooling minimizes the Hermitian norm

$$\mathcal{N} = \frac{-1}{10N} \sum_{\mu=1}^{10} \mathrm{tr}[(A_\mu - (A_\mu)^\dagger)^2]$$

(2) Singular drift problem:

The drift term $dS/d(A_\mu)_{ji}$ diverges due to $\mathcal{M}$'s near-zero eigenvalues.

We trust CLM when the distribution p(u) of the drift norm

$$u = \sqrt{\frac{1}{10N^3} \sum_{\mu=1}^{10} \sum_{i,j=1}^{N} \left| \frac{\partial S}{\partial (A_\mu)_{ji}} \right|^2}$$

falls exponentially as $p(u) \propto e^{-au}$.

[K. Nagata, J. Nishimura and S. Shimasaki, arXiv:1606.07627]

Look at the drift term ⇒ Get the drift of CLM!!

3. Complex Langevin Method

Mass deformation [Y. Ito and J. Nishimura, arXiv:1609.04501]

- SO(D) symmetry breaking term $\Delta S_{\mathrm{b}} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu \mathrm{tr}(A_\mu)^2$

Order parameters for SSB of SO(10): $\lambda_\mu = \mathrm{Re}\left\{\frac{1}{N}\mathrm{tr}(A_\mu)^2\right\}$

- Fermionic mass term: $\Delta S_{\mathrm{f}} = N m_{\mathrm{f}} \mathrm{tr}\left(\bar{\psi}_\alpha (i\Gamma_8 \Gamma_9^\dagger \Gamma_{10})_{\alpha\beta} \psi_\beta\right)$

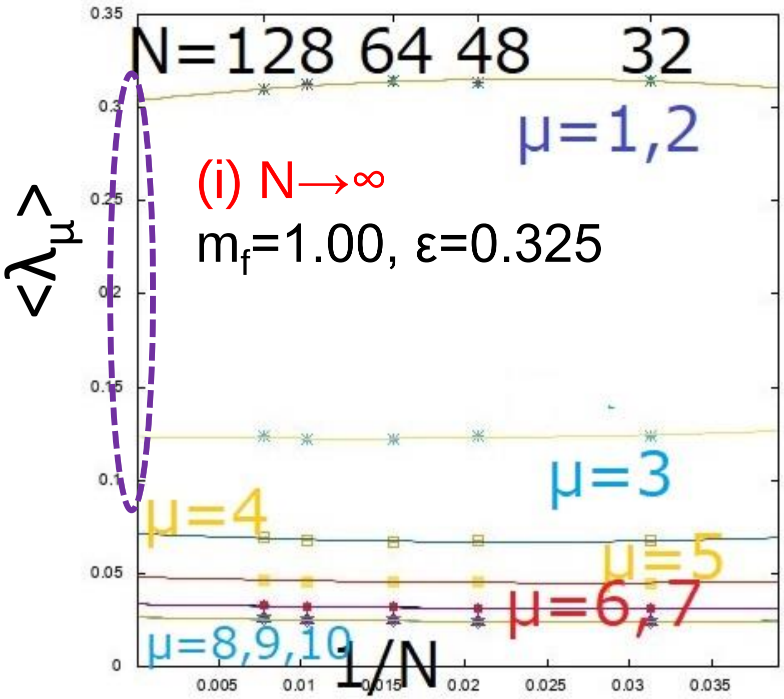
Avoids the singular eigenvalue distribution of $\mathscr{M}$.

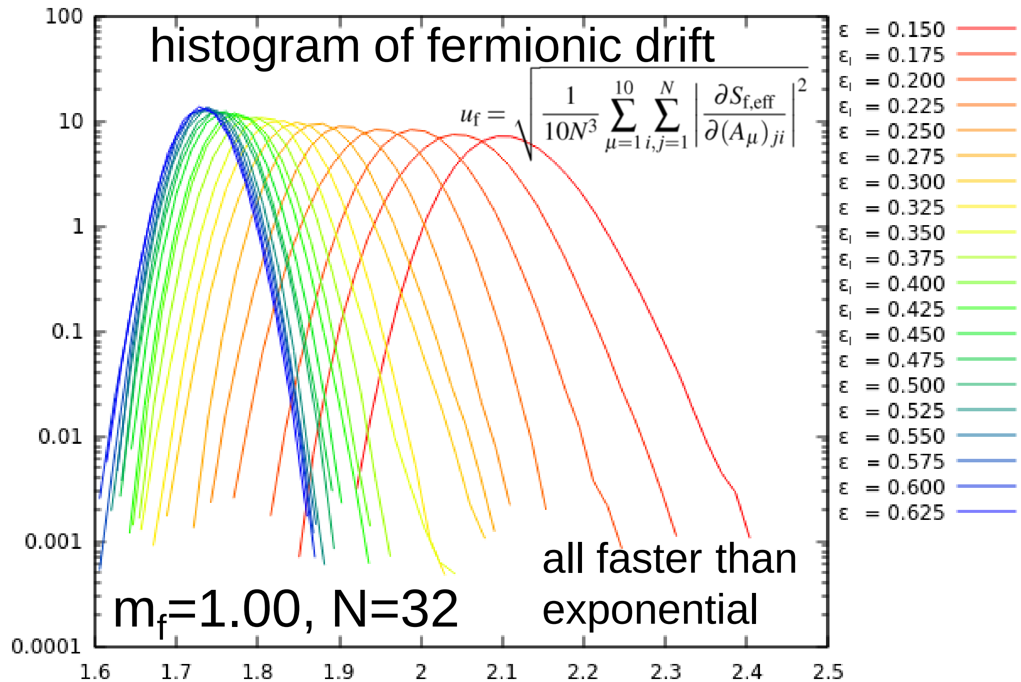
Extrapolation (i) $N\to\infty$ $\Rightarrow$ (ii) $\varepsilon_m \to 0$ $\Rightarrow$ (iii) $m_f \to 0$.

4. Result

$$\Delta S_b = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu \mathrm{tr}(A_\mu)^2 \qquad \Delta S_f = N m_f \mathrm{tr}\left(\bar{\Psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\Psi_\beta\right) \quad \rho_\mu(\varepsilon, m_f) = \frac{\langle\lambda_\mu\rangle_{\varepsilon,m_f}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_f}}$$

m_μ=(0.5,0.5,1,2,4,8,8,8,8,8)



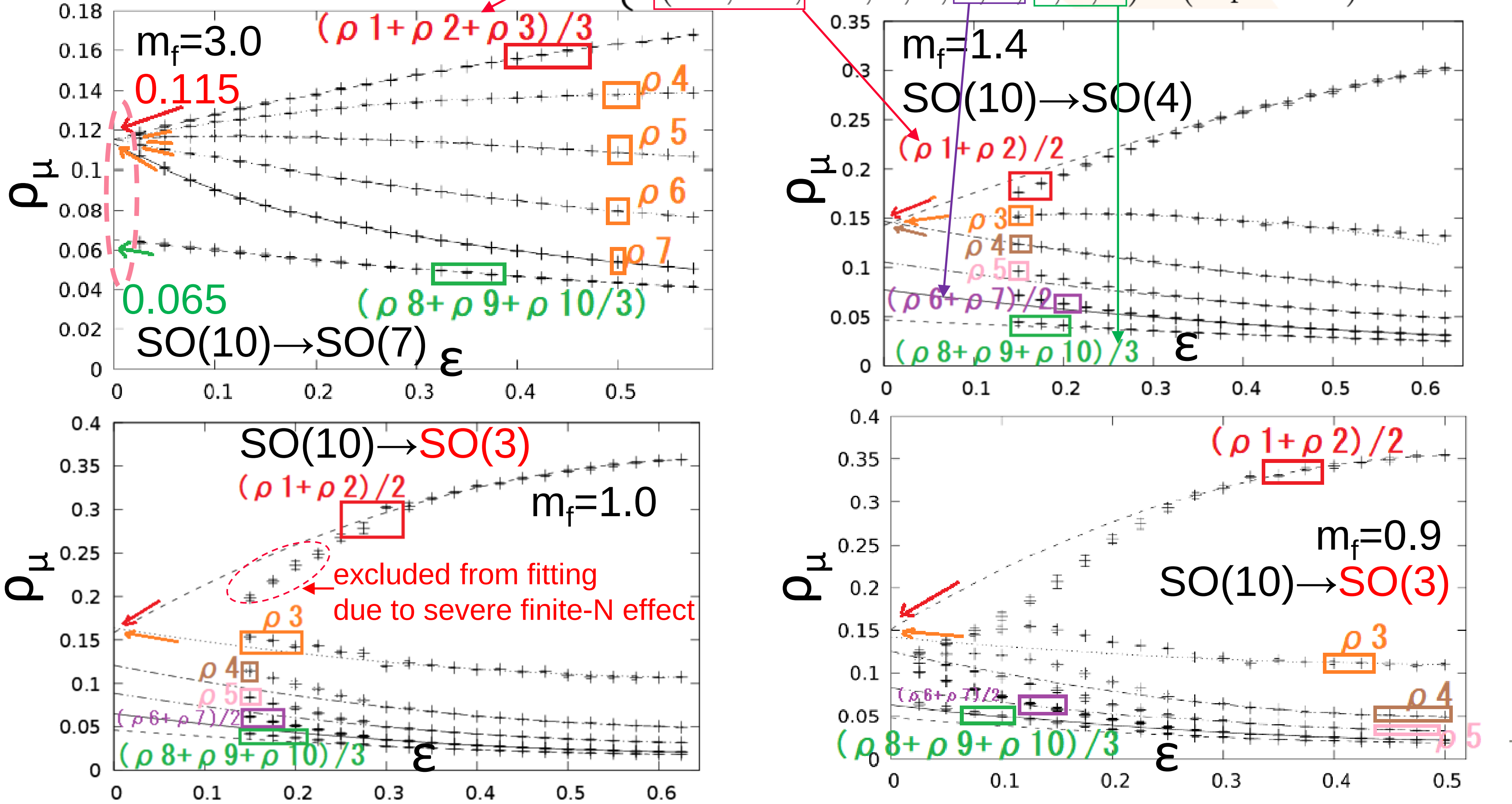


4. Result

$$\Delta S_{\rm b} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu {\rm tr}(A_\mu)^2 \qquad \Delta S_{\rm f} = N m_{\rm f} {\rm tr}\left(\bar{\Psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\Psi_\beta\right) \qquad \rho_\mu(\varepsilon, m_{\rm f}) = \frac{\langle\lambda_\mu\rangle_{\varepsilon,m_{\rm f}}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_{\rm f}}}$$

(ii) ε→0 after N→∞

$$m_\mu = \begin{cases} (0.5, 0.5, 0.5, 1, 2, 4, 8, 8, 8, 8) & (m_{\rm f} = 3.0) \\ (0.5, 0.5, 1, 2, 4, 8, 8, 8, 8) & (m_{\rm f} < 3.0) \end{cases}$$

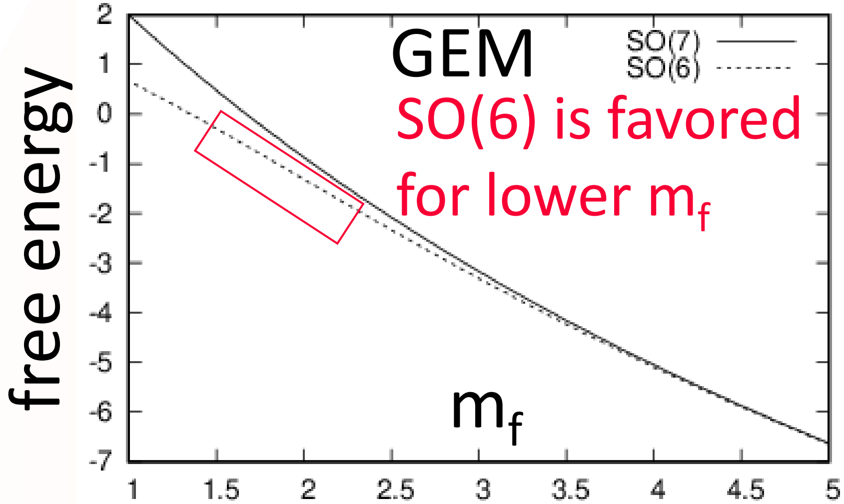


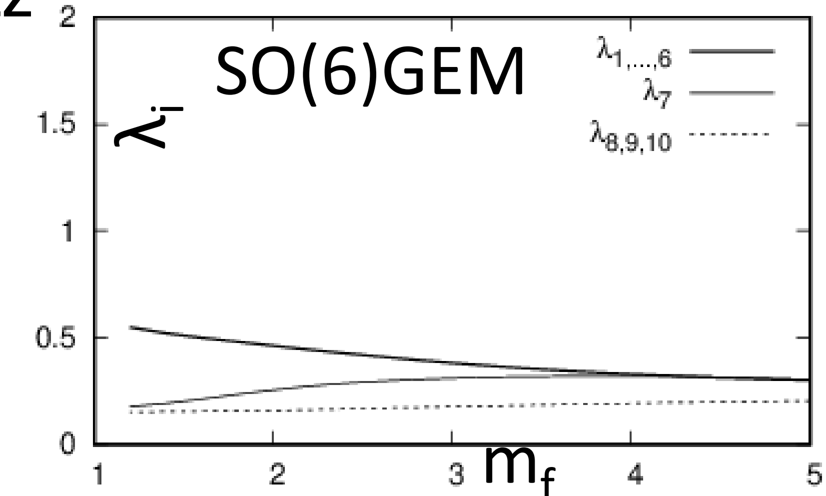
4. Result

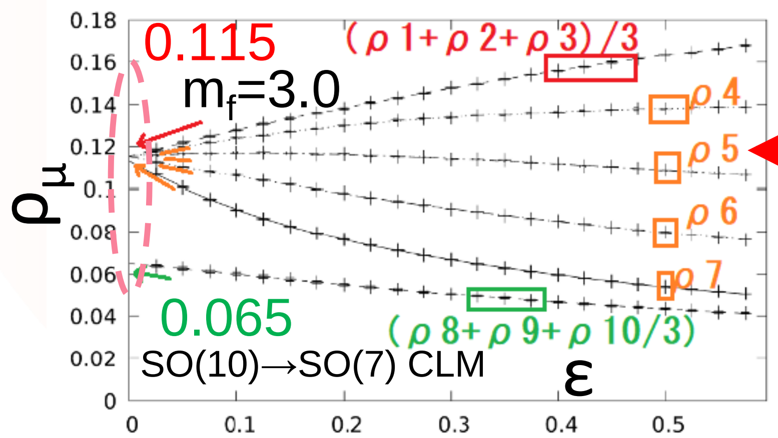
$$\Delta S_{\rm f} = N m_{\rm f} {\rm tr}\left(\bar{\psi}_\alpha (i\Gamma_8\Gamma_9^\dagger\Gamma_{10})_{\alpha\beta}\psi_\beta\right)$$ (no $\Delta S_{\rm b} = N\frac{\varepsilon}{2}\sum_{\mu=1}^{10} m_\mu {\rm tr}(A_\mu)^2$ term)

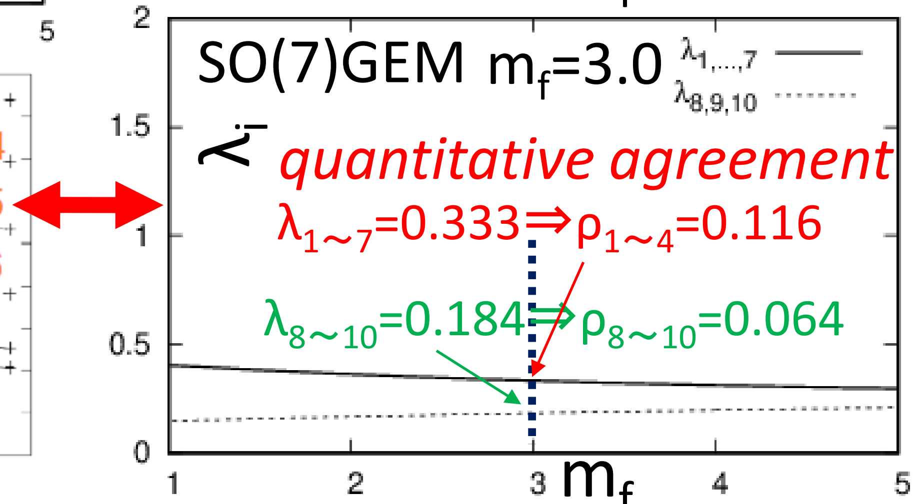
GEM result for $m_f>0$ at 3 loop solutions of SO(6), SO(7) ansatz

$$\rho_\mu(\varepsilon, m_{\rm f}) = \frac{\langle\lambda_\mu\rangle_{\varepsilon,m_{\rm f}}}{\sum_{\nu=1}^{10}\langle\lambda_\nu\rangle_{\varepsilon,m_{\rm f}}}$$









5. Summary

Dynamical compactification of the spacetime in the Euclidean IKKT model.

"Complex Langevin Method"
⇒trend of SSB SO(10)→SO(3).

Future works

Application of CLM to the Lorentzian version of the type IIB matrix model

[K.N. Anagnostopoulos, T. Azuma, K. Hatakeyama, M. Hirasawa, Y. Ito, J. Nishimura, S.K. Papadoudis and A. Tsuchiya, work in progress]

their talks