Nucleon electromagnetic form factors at high momenta using the Feynman-Hellmann method

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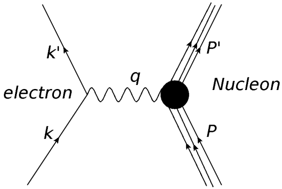
Outline

The Feynman-Hellmann approach offers an alternative for calculating matrix elements which does not require three-point functions

Two-exponential fits and a weighting method can be used to control the excited states in the form factor calculation at high momenta.

Structure of the nucleons

- How is the charge distributed inside the nucleons?
- How does this distribution change at smaller scales?
- Need to calculate vector matrix elements: $\langle P(p')| \mathcal{J}(0) |P(p)\rangle$
- matrix elements can be parameterised by two form factors: F_1, F_2



$$\langle P'|J^\nu|P\rangle = \bar{u}(p')\left[\gamma^\mu F_1(Q^2) + i\sigma^{\mu\nu}\frac{q_\nu}{2M}F_2(Q^2)\right]u(p)$$

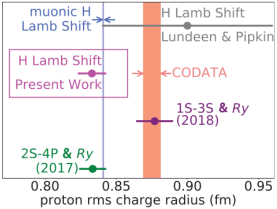
Often rewrite the matrix element in terms of G_E and G_M, as they are related to the charge and magnetisation distribution

Experimental results

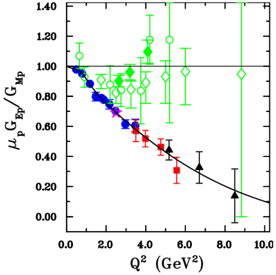
- quantity of highest interest is the momentum dependence at various Q^2 values.
- At Q^2 close to 0, this dependence gives the charge radius.
- Recoil polarisation experiments have shown ineresting large Q^2 behaviour

Ratio $\frac{G_E}{G_M}$ crossing over zero?

- Conflict with older Rosenbluth separation data
  $\rightarrow$ more data is required



[Bezginov,2019] DOI:10.1126/science.aau7807

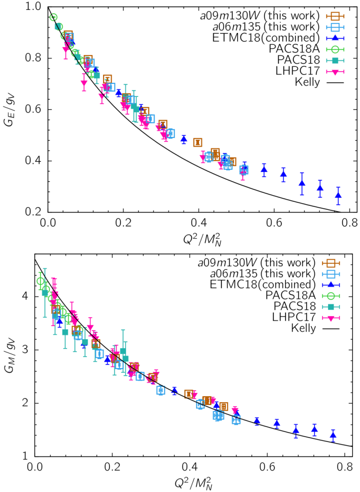


[JLab,2015] arXiv:1503.01452

Lattice results

- High quality data for low Q^2
- Fewer calculations at high Q^2 difficulties with signal-to-noise ratio and controlling excited states.
- Expensive to calculate high enough statistics at high momenta

Can the Feynman-Hellmann approach deliver improvements at high Q^2?



[Jang,2020] arXiv:1906.07217

Feynman-Hellmann theorem $\frac{\partial E_\psi}{\partial \lambda} = \langle \psi | \frac{\partial H}{\partial \lambda} | \psi \rangle$

Consider the forward case for the proton:

- Insert a new term into the Lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + \lambda \mathcal{O}(x)$$

- Calculate the energy of the proton with this new term:

$$G(\lambda, \mathbf{p}, t) \xrightarrow{t>>0} A(\lambda) e^{-E(\lambda,\mathbf{p})t}$$

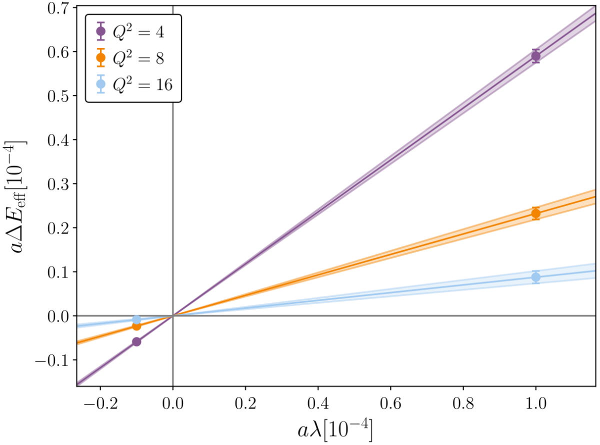
- Feynman-Hellmann theorem relates the energy shift to the matrix element:

$$\left.\frac{\partial E(\lambda, \mathbf{p})}{\partial \lambda}\right|_{\lambda=0} = \frac{1}{2E} \langle P(\mathbf{p}) | \mathcal{O}(0) | P(\mathbf{p}) \rangle$$

How does the energy shift dE behave at small λ?

Feynman-Hellmann theorem $\frac{\partial E_\psi}{\partial \lambda} = \langle \psi | \frac{\partial H}{\partial \lambda} | \psi \rangle$

- At $\lambda = 10^{-4}, -10^{-5}$ the energy shift behaves linearly with λ
- Can now calculate matrix elements by using only lattice two-point functions.



[Chambers 2017]
arXiv:1503.01452

Feynman-Hellmann theorem $\frac{\partial E_\psi}{\partial \lambda} = \langle \psi | \frac{\partial H}{\partial \lambda} | \psi \rangle$

Electromagnetic form factors at large momentum

- Insert a term with vector current into the lagrangian

$$\mathcal{L}(x) \to \mathcal{L}(x) + 2\lambda cos(\mathbf{q} \cdot \mathbf{x})\bar{q}(x)\gamma_\mu q(x)$$

- Requirement: Breit frame is necessary here.

  Since we want to access high Q^2, setting $\mathbf{p} = -\mathbf{p}'$ will satisfy this and keep noise to a minimum

- Using the temporal current: $\left.\frac{dE_p(\mathbf{p},\gamma_4)}{d\lambda}\right|_{\lambda=0} = \frac{m_p}{E_p(\mathbf{p})} G_{E,p}(Q^2)$

- Using the spatial current: $\left.\frac{dE_p(\mathbf{p},\sigma,\gamma_2)}{d\lambda}\right|_{\lambda=0} = \frac{[\mathbf{q}\times\hat{\mathbf{e}}]_2}{2E_p(\mathbf{p})} G_{M,p}(Q^2)$

Ratio of two-point functions

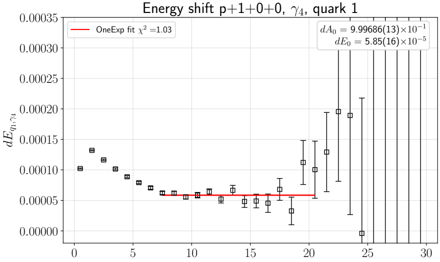
How do we get the energy shift dE?

Construct a ratio forward and backwards propagating states which gives the energy shift in the large time limit.

$$R_{E,p}(\lambda, \pm\mathbf{p}, t) \equiv \left| \frac{G^{+}(\lambda, \pm\mathbf{p}, t)}{G^{+}(0, \pm\mathbf{p}, t)} \frac{G^{-}(0, \pm\mathbf{p}, -t)}{G^{-}(\lambda, \pm\mathbf{p}, -t)} \right|^{\frac{1}{2}}$$

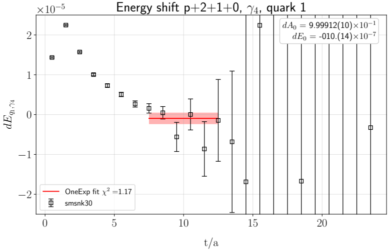
$$\xrightarrow{\text{large t}} B(\lambda) e^{\Delta E(\lambda) t}$$

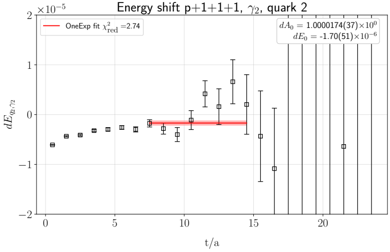
Good fit for low momentum transfer and heavy quark masses.



Ratio of two-point functions

At high momentum transfer or low quark masses, the signal-to-noise ratio is lower and choosing a suitable plateau region becomes difficult.





Need a better and more consistent method for choosing a good fitting range.

Excited states

Try to include the excited states in the fitting function to be able to include earlier timeslices in the fitting range.

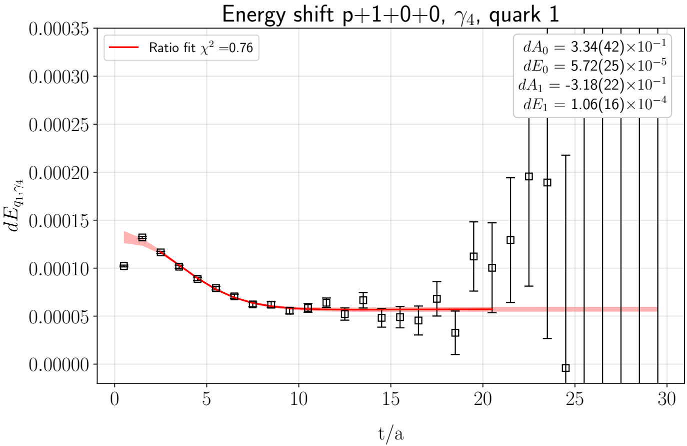
$$R_{E,p}(\lambda, \pm\mathbf{p}, t) \equiv \left| \frac{G^+(\lambda, \pm\mathbf{p}, t)}{G^+(0, \pm\mathbf{p}, t)} \frac{G^-(0, \pm\mathbf{p}, -t)}{G^-(\lambda, \pm\mathbf{p}, -t)} \right|^{\frac{1}{2}}$$

$$\xrightarrow{\text{large t}} \left| \frac{(A_0 + \Delta A_0) e^{-(E_0 + \Delta E_0)t} + (A_1 + \Delta A_1) e^{-(E_1 + \Delta E_1)t}}{(A_0 - \Delta A_0) e^{-(E_0 - \Delta E_0)t} + (A_1 - \Delta A_1) e^{-(E_1 - \Delta E_1)t}} \right|$$

Need 4 parameters from the fit to the ratio: $\Delta A_0, \Delta A_1, \Delta E_0, \Delta E_1$

Two-exponential fitting

Allows for starting the fit at earlier timeslices.



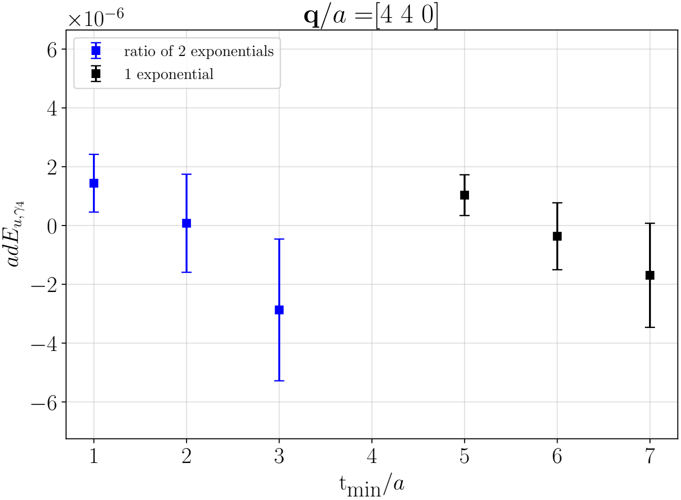
$$N_{conf} = 1500, \quad 32 \times 64, \quad m_\pi \approx 470 MeV$$

Two-exponential fitting

$\mathbf{q}/a = (4, 4, 0),\ m_\pi = 310\mathrm{MeV}$

Fit parameters still depend on the chosen time range

Can we minimize the dependence on the chosen time range?



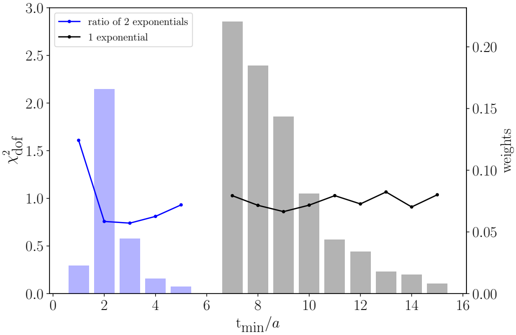
Weighted average of fits

- Include fits with several different t_{min} values and give them weighting based on [Beane 2020, ArXiv:2003.12130]

$$\bar{E}_0 = \sum_{f=1}^{N} w^f E_0^f$$

$$p_f = \frac{\Gamma(N_{dof}/2, \chi_f^2/2)}{\Gamma(N_{dof}/2)}$$

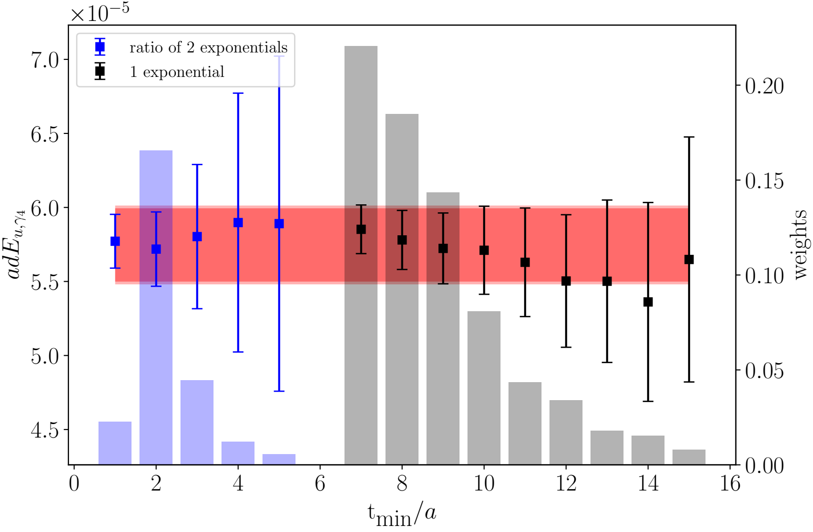
$$\tilde{w}^f = \frac{p_f \left(\delta E_0^f\right)^{-2}}{\sum_{f'=1}^{N} p_{f'} \left(\delta E_0^{f'}\right)^{-2}}$$



p_f = p-value for the fit, w^f = weighting of the fit

Weighted average of fits

$$N_{conf} = 1500, \quad 32^3 \times 64, \quad m_\pi \approx 470 MeV, \quad \frac{\mathbf{q}}{a} = (2, 0, 0)$$



Three-point functions

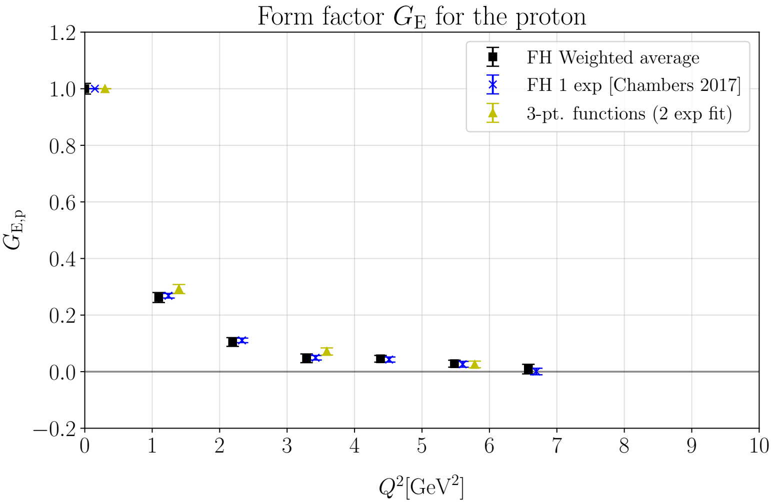
How does this method compare to the usual three-point functions?

Recalculate the electromagnetic form factors of the nucleon with three-point functions on the same lattice and using the same breit frame momenta to facilitate an exact comparison.

- UKQCD/QCDSF ensembles with $N_f = 2 + 1$
- $L^3 \times T = 32^3 \times 64, \;\; a = 0.074 \text{ fm}, \;\; m_\pi \approx 470 MeV$
- source-sink separations: $0.74 \text{ fm}, \;\; 0.96 \text{ fm}, \;\; 1.18 \text{ fm}$
- Same Breit frame momenta as the Feynman-Hellmann calculation
- Use a two-exponential fit to the three source-sink separations

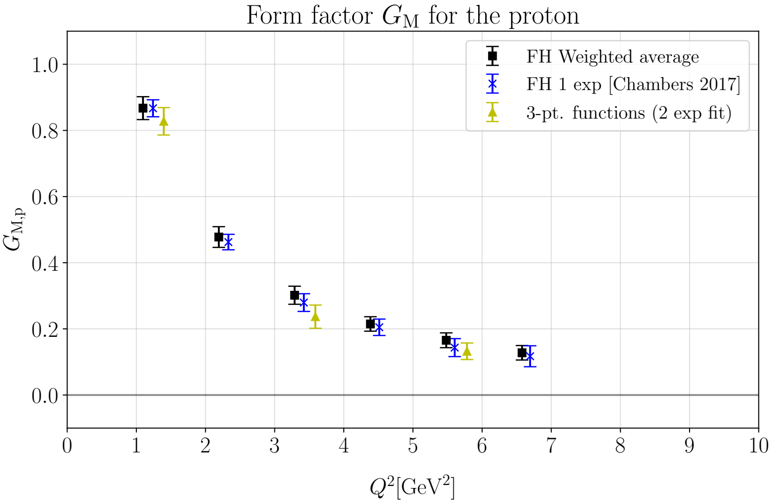
Form factors

$$N_{conf} = 1500, \quad 32^3 \times 64, \quad m_\pi \approx 470 MeV$$



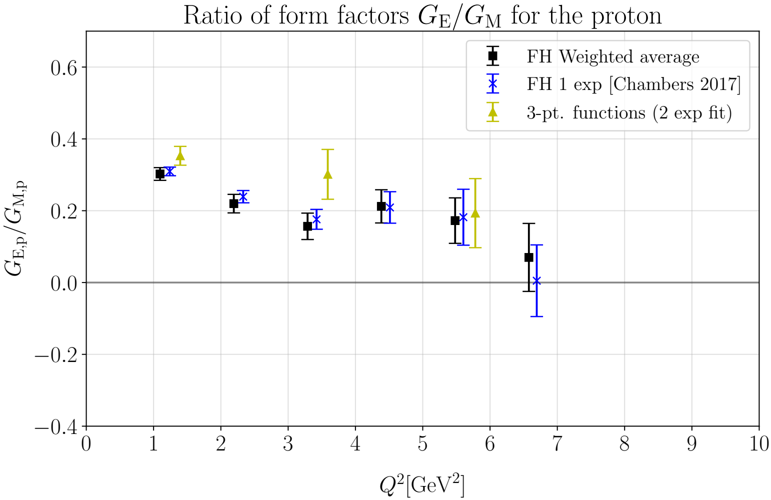
Form factors

$$N_{conf} = 1500, \quad 32^3 \times 64, \quad m_\pi \approx 470 MeV$$



Form factors

$$N_{conf} = 1500, \quad 32^3 \times 64, \quad m_\pi \approx 470 MeV$$



Conclusion

- The Feynman-Hellmann approach allows for the calculation of form factors at high momentum.
- Including the contributions from excited states in the fits makes the analysis more reliable across quark masses and momenta.

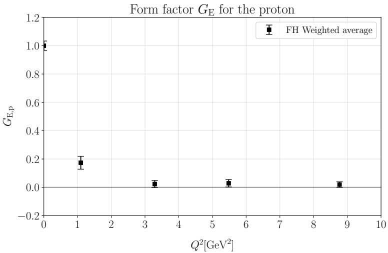
Next steps:

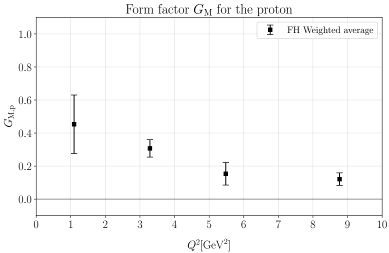
- Increase N_{conf} for lighter masses to allow for the flavour breaking expansion analysis from [Bickerton 2019]
- Include more lattice spacings and volumes in the analysis.

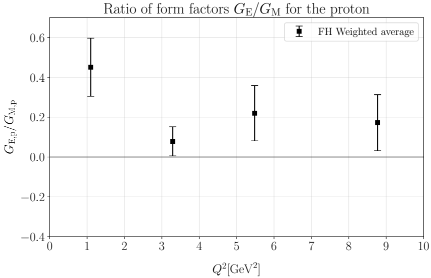
Bonus slides

Form factors

$$N_{conf} = 1500, \quad 32^3 \times 64, \quad m_\pi \approx 360\,MeV$$







Form factors

$$N_{conf} = 1500, \quad 32^3 \times 64, \quad m_\pi \approx 310 MeV$$