Parton Distribution Functions from pseudo-distributions

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Outline

1. Parton distribution functions (PDFs)
2. Pseudo PDFs
   - Reduced distribution
   - z^2 evolution and $\bar{M}S$ matching
3. Lattice Setup
4. Results
   - Bare and reduced matrix elements
   - Evolved and matched ITDs
   - Light-cone PDFs
   - Final results with quantified systematic uncertainties
5. Summary

Light cone PDF

- PDFs describe the internal structure of hadrons
- The PDFs are defined as the nucleon matrix elements of quark or gluon correlation operators along the light cone direction.
- $$q(x,\mu^2) = \int \frac{d\xi^-}{4\pi} e^{-ixP^+\xi^-} \langle P|\bar{\psi}(\xi^-)\gamma^\dagger W(\xi^-,0)\psi(0)|P\rangle$$

  where μ is the renormalization scale in a given renormalization scheme, the nucleon momentum $P_\mu = (P_0, 0, 0, P_z)$, $\xi^\pm = (t \pm z)/\sqrt{z}$ are the light cone coordinates, and the Wilson line

  $$W(\xi^-,0) = \exp\left(-ig\int_0^{\xi^-} d\eta^- A^\dagger(\eta^-)\right)$$

  is inserted to ensure gauge invariance.

PDFs from Lattice QCD

- Quasi-PDFs provide a way to determine the $x-$dependence of PDFs from lattice QCD

$$\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma^3 W(z)\psi(0)|P\rangle,$$

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$$\tilde{q}(x, \mu^2, P_3) = \int_{-1}^{1} \frac{dy}{|y|} C\left(\frac{x}{y}, \frac{\mu}{P_3}\right) q(y, \mu^2) + \mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_3^2}, \frac{M^2}{P_3^2}\right)$$

X. Ji, Phys.Rev.Lett. 110, 262002 (2013), arXiv:1306.1539 [hep-ph]

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where μ is the renormalization scale, C is a matching kernel and M is the nucleon mass and the $\mathcal{O}\left(\frac{\Lambda_{QCD}^2}{P_z^2}, \frac{M^2}{P_z^2}\right)$ terms are higher-twist corrections suppressed by the nucleon momentum.

Pseudo PDFs

- A different approach for the PDF extraction from lattice calculations was proposed based on the concept of pseudo-PDFs $\mathcal{P}(x, z_3^2)$.They generalize the light-cone PDFs $f(x)$ onto spacelike intervals like $z = (0, 0, 0, z_3^2)$.
  A. Radyushkin, Phys. Lett. B767 (2017) 314
  A. Radyushkin, Phys. Rev. D95 (2017) 056020
  A. Radyushkin, Phys. Lett. B770 (2017) 514
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  A. Radyushkin, Phys. Lett. B 788, 380 (2019),arXiv:1807.07509 [hep-ph]

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- The pseudo-PDFs are Fourier transforms of the Ioffe-time distributions

$$\mathcal{P}(x, z^2) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu \, e^{-ix\nu} \, \mathcal{M}(\nu, z^2).$$

- $\nu = (zP_3)$ is the Ioffe time

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$$\mathfrak{M}(\nu, z_3^2) = \lim_{t\to\inf} \frac{\mathcal{M}(z_3P, z_3^2; t)}{\mathcal{M}(z_3P, z_3^2; t)|_{z_3=0}} \times \frac{\mathcal{M}(z_3P, z_3^2; t)|_{z_3=0,P=0}}{\mathcal{M}(z_3P, z_3^2; t)|_{P=0}}$$

K. Orginos, A. Radyushkin, J. Karpie, and S. Zafeiropoulos, Phys. Rev. D96, 094503 (2017), arXiv:1706.05373 [hep-ph]

z^2 evolution and $\bar{M}S$ matching

- The reduced-ITDs $\mathcal{M}(\nu, z_3^2)$ need to be evolved to a common scale and then scheme converted to $\overline{MS}$-scheme$\rightarrow Q(\nu, \mu^2)$

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$$\mathcal{M}(\nu, z^2) = Q(\nu, \mu^2) +) \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2 \mu^2 \frac{e^{2\gamma_E+1}}{4} \right) B(u) + L(u) \right] Q(u\nu, \mu^2)$$

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$$\text{where } B(u) = \left[\frac{1+u^2}{1-u} \right]_+, \qquad L(u) = \left[4\frac{ln(1-u)}{1-u} + 2(1-u) \right]$$

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$$\int_0^1 [f(u)]_+ Q(u\nu) = \int_0^1 f(u)(Q(u\nu) - Q(\nu))$$

PDF reconstruction

$$Q(\nu, \mu^2) = \int_{-1}^{1} dx\, e^{i\nu x} q(x, \mu^2), \tag{1}$$

where the antiquark distribution for positive x is $\bar{q}(x) = -q(-x)$. Decomposing into real and imaginary parts and using this property, one obtains

$$\begin{aligned} \mathrm{Re}\, Q(\nu, \mu^2) &= \int_0^1 dx \cos(\nu x)\big(q(x, \mu^2) - \bar{q}(x, \mu^2)\big) \\ &= \int_0^1 dx \cos(\nu x) q_v(x, \mu^2), \end{aligned} \tag{2}$$

$$\begin{aligned} \mathrm{Im}\, Q(\nu, \mu^2) &= \int_0^1 dx \sin(\nu x)\big(q(x, \mu^2) + \bar{q}(x, \mu^2)\big) \\ &= \int_0^1 dx \sin(\nu x) q_{v2s}(x, \mu^2), \end{aligned} \tag{3}$$

where: $q_\nu = q - \bar{q}$,$q_{\nu 2s} = q_\nu + 2\bar{q} = q + \bar{q}$

PDF reconstruction

- Naive Fourier transforms
  - Inverse problem
  - We will use two additional ways of handling the inverse problem.
- We will apply the Backus-Gilbert (BG) method J. Karpie, K. Orginos, A. Rothkopf, S. Zafeiropoulos, JHEP 04 (2019) 057
- The other reconstruction technique that we will use is to assume a functional form of a fitting ansatz for the light-cone PDF.

$$q(x) = Nx^a(1-x)^b, \tag{4}$$

Lattice Setup

- We use gauge field configurations generated by the Extended Twisted Mass Collaboration (ETMC)
- Physical value of the pion mass ($m_\pi = 130.4(4)$ MeV) and the nucleon mass ($m_N = 932(4)$ MeV)
- The lattice spacing is $a = 0.0938(2)(3)$ fm
- The lattice volume is $48^3 \times 96$
- $L \approx 4.5$ fm

Bare and reduced matrix elements

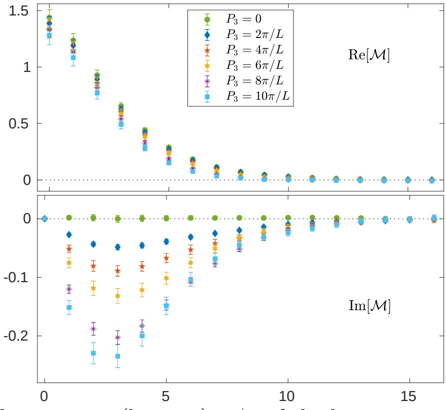


Figure 1: Real (top) and imaginary (bottom) part of the bare matrix elements ($\mathcal{M}(\nu, z^2)$ at fixed P_3) for the unpolarized PDF. Shown are all nucleon boosts: $P_3 = 0$ (green circles), $P_3 = 2\pi/L$ (blue rhombuses), $P_3 = 4\pi/L$ (red 5-stars), $P_3 = 6\pi/L$ (yellow 6-stars), $P_3 = 8\pi/L$ (purple asterisks), $P_3 = 10\pi/L$ (cyan squares).

Bare and reduced matrix elements

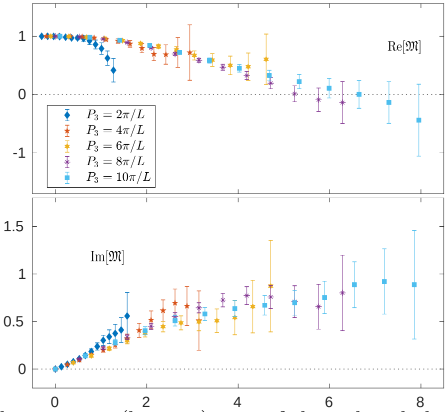


Figure 2: Real (top) and imaginary (bottom) part of the reduced elements ($\mathfrak{M}(\nu, z^2)$ at fixed P_3) for the unpolarized PDF. Symbols are the same as used in Fig. 1.

Evolved and matched ITDs

- All matrix elements are evolved to the scale corresponding to $\mu = 2$ GeV
- $\alpha_s/\pi \approx 0.129$

Evolved and matched ITDs

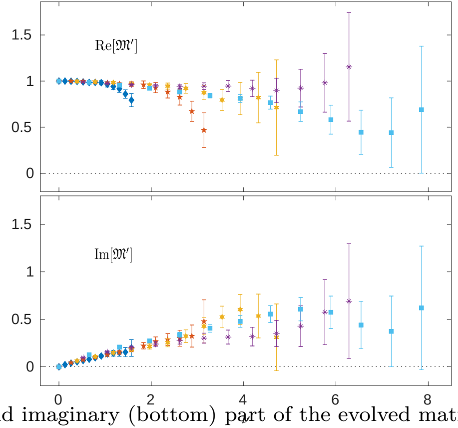


Figure 3: Real (top) and imaginary (bottom) part of the evolved matrix elements ($\mathfrak{M}'(\nu, z^2, \mu^2)$ at fixed P_3) for the unpolarized PDF. The scale after evolution is $1/z = \mu e^{\gamma_E + 1/2}/2 \approx 2.9$ GeV and corresponds to the $\overline{MS}$ scale $\mu = 2$ GeV.

Evolved and matched ITDs

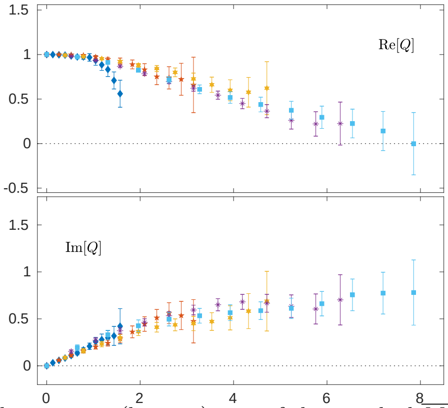


Figure 4: Real (top) and imaginary (bottom) part of the matched $\overline{MS}(\mu = 2$ GeV) matrix elements ($Q(\nu, z^2, \mu^2)$ at fixed P_3) for the unpolarized PDF. Symbols are the same as used in Fig. 1.

Evolved and matched ITDs

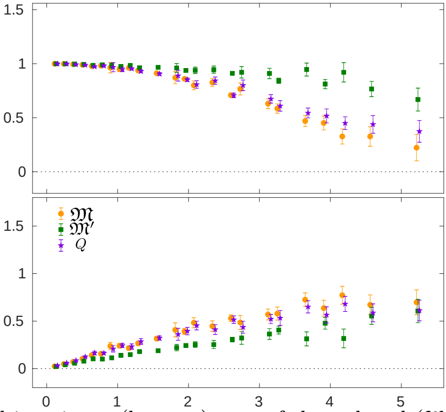


Figure 5: Real (top) and imaginary (bottom) part of the reduced ($\mathfrak{M}(\nu, z^2)$; orange circles), evolved ($\mathfrak{M}'(\nu, \mu^2)$; green squares) and matched ($Q(\nu, \mu^2)$; purple stars) Ioffe time distributions. The matrix elements corresponding to the same Ioffe time ν coming from different combinations of (P_3, z) were averaged, keeping only the ones at $z/a \leq 8$.

Light-cone PDFs

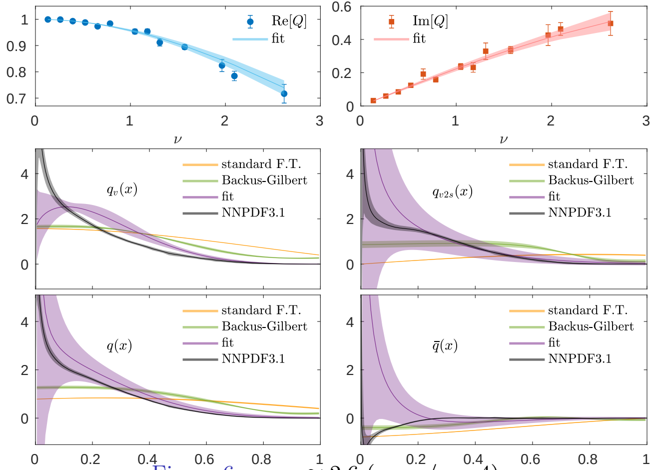


Figure 6: $\nu_{max} \approx 2.6\ (z_{max}/a = 4)$

Light-cone PDFs

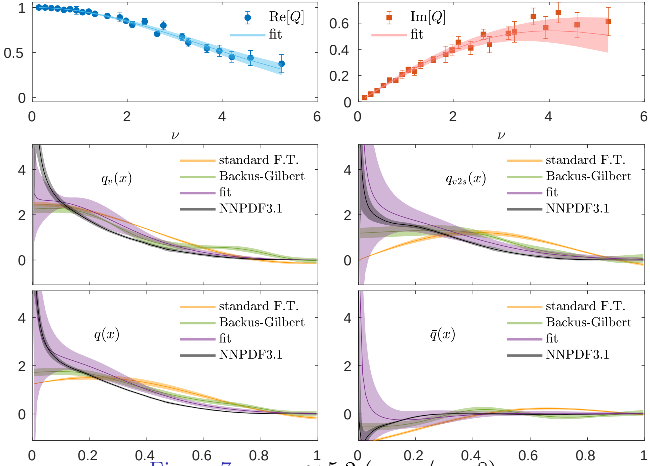


Figure 7: $\nu_{max} \approx 5.2\ (z_{max}/a = 8)_x$

Light-cone PDFs

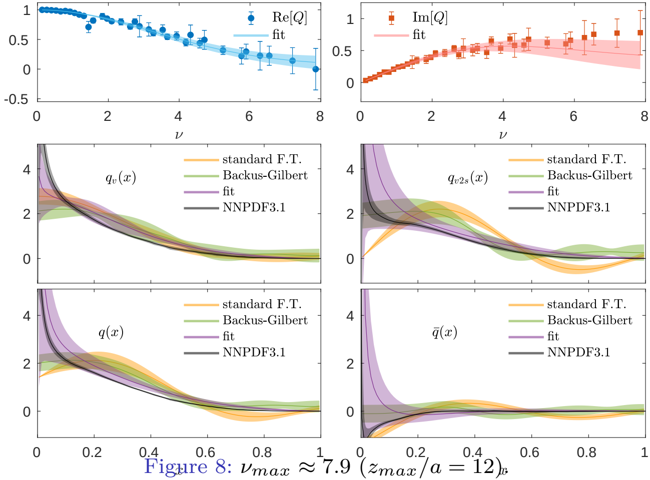


Figure 8: $\nu_{max} \approx 7.9$ $(z_{max}/a = 12)$.

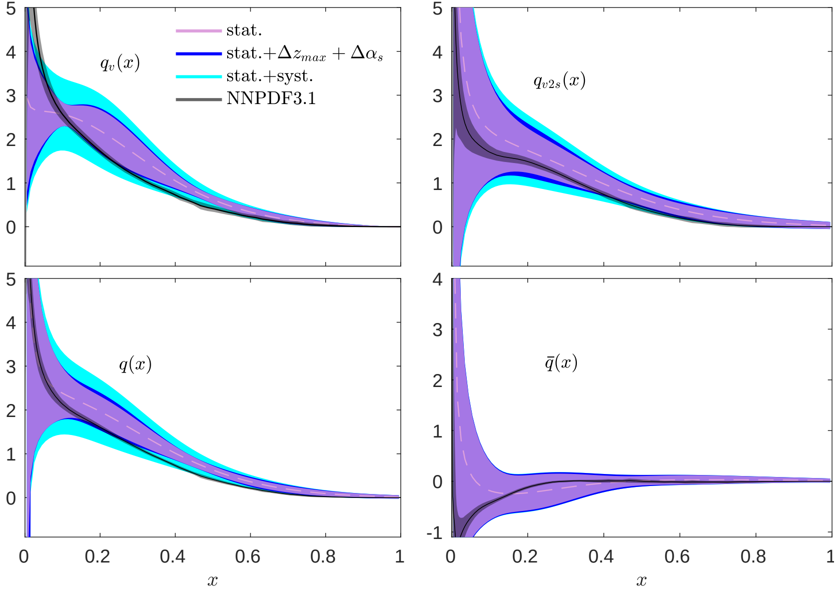
- The uncertainty related to the range of Ioffe times to be taken in the reconstruction procedure, Δz_{max}.

$$\Delta z_{max}(x) = \frac{|q_{z_{max}/a=12}(x) - q_{z_{max}/a=4}(x)|}{2}. \tag{5}$$

- The uncertainty from the choice of α_s, $\Delta\alpha_s(x)$:

$$\Delta\alpha_s(x) = |q_{\alpha_s/\pi=0.129}(x) - q_{\alpha_s/\pi=0.1}(x)|. \tag{6}$$

- Estimated systematics:
  - Discretization effects: assume 20%
  - Finite Volume Effects: assume 5%
  - Excited states: assume 10% ETMC, Phys. Rev. D 99 (2019) 114504
  - Matching (truncation effects and HTE): assume 20%


stat.
stat.+$\Delta z_{max} + \Delta\alpha_s$
stat.+syst.
NNPDF3.1
$q_v(x)$
$q_{v2s}(x)$
$q(x)$
$\bar{q}(x)$
x
x

Summary

- We used the pseudo-distribution approach to calculate the x-dependence of the unpolarized parton distribution functions of the nucleon
- We reconstructed the x-distributions using 3 methods
- The naive Fourier transform does not lead to robust results.
- The Backus-Gilbert methods offers one solution to the inverse problem by assuming that the reconstructed distribution should have minimal variance with respect to the statistical variation of the data.
- Functional fit to data regulates the inverse problem and leads, in practice, to well-behaved and robust PDFs
- Fully robust lattice-extracted PDFs need to have all relevant sources of systematic effects quantified
- Extend the work to polarized PDFs, GPDs and transverse-momentum-dependent parton distributions (TMDs), as well as to singlet distributions.

Thank You!

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