Magnetic Polarisability with the Background Field Method

R. Bignell W. Kamleh D. Leinweber

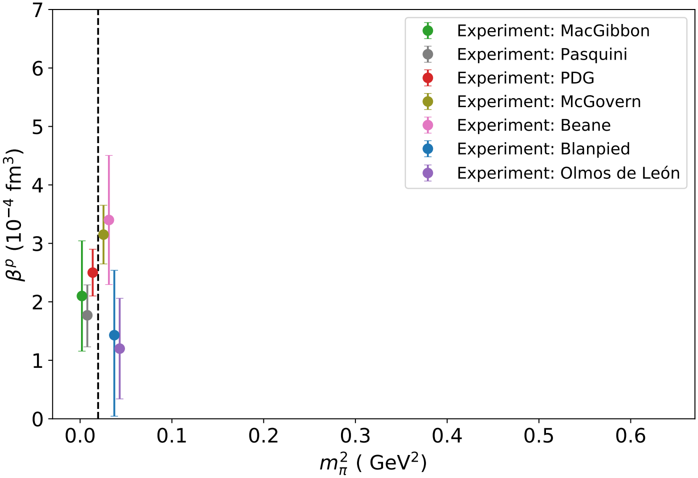
The Special Research Centre for the Subatomic Structure of Matter
University of Adelaide

Asia-Pacific Symposium for Lattice Field Theory
August 6, 2020

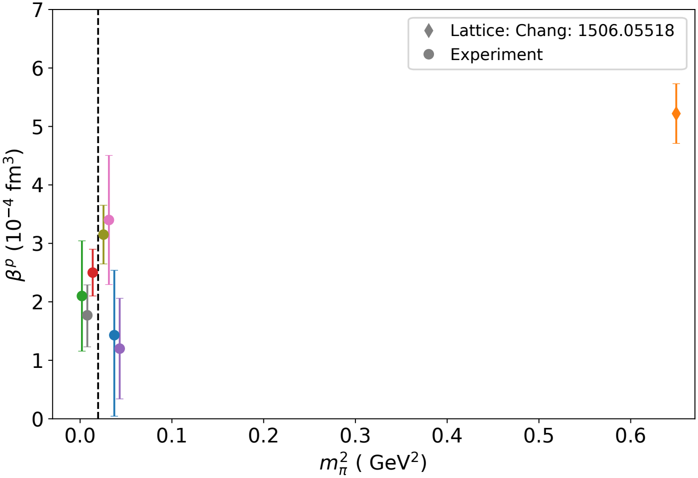
β^p Status

- The magnetic polarisability is a fundamental property of a system of charged particles
  - Describes the response to an external magnetic field
  - Provides a description of hadron structure
  - Experimentally measured in Compton scattering experiments
- What is the magnetic polarisability of the proton and neutron?

β^p Status



β^p Status

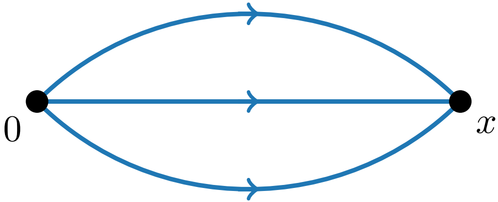


Energy & Two-point Correlation Function

- Directly calculate hadron energies in an external magnetic field
- The energy of a baryon in an external magnetic field is

$$E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

- Evaluate two point correlation functions $G(\vec{p}, t) \propto \sum_\alpha e^{-E_\alpha\, t}$



Two point correlation function quark-flow diagram for a baryon

Background Field Method

- Introduce a background field on the lattice

$$\text{Continuum:} D_\mu \rightarrow D_\mu^{QCD} + iqe\,A_\mu \qquad \text{Lattice:} U_\mu(x) \rightarrow e^{i\,a\,qe\,A_\mu(x)}\,U_\mu(x)$$

- Choose $\vec{A}$ appropriately to generate constant magnetic field $\vec{B} = +B\,\hat{z}$
- Periodic spatial boundary conditions impose a quantisation for a uniform field

$$a^2\,qe\,B^2 = \frac{2\,\pi\,k}{N_x\,N_y}$$

- $k_d = 0, 1, 2, \ldots$ for the field strength experienced by the d quark

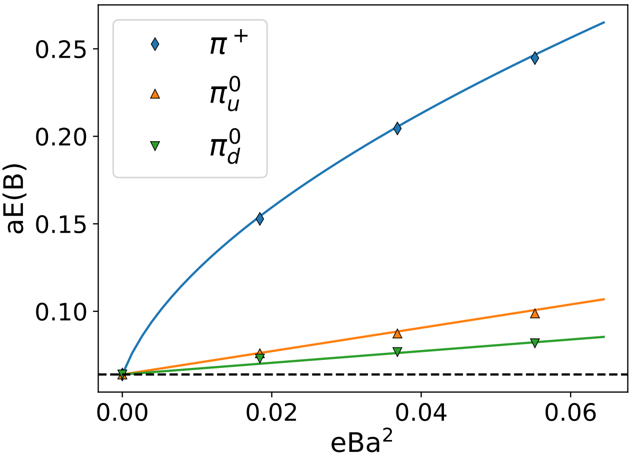
Wilson Term Mass Renormalisation

- Wilson term causes unphysical quark mass renormalisation in background magnetic field
- In free-field limit this change is

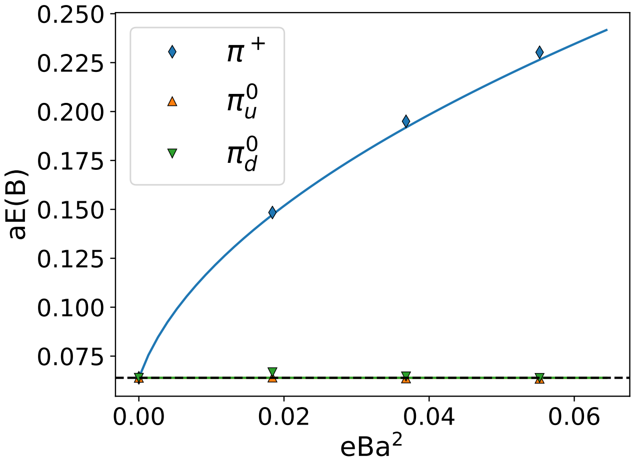
$$m_{[w]}(B) = m(0) + \frac{a}{2}|qe\,B|$$

- Observe through investigation of QCD-free (connected) neutral pion energies
- First discussed by Bali *et al.* 1510.03899, 1707.05600

Wilson Term Mass Renormalisation



Clover term


0.250
0.225
0.200
0.175
0.150
0.125
0.100
0.075
aE(B)
0.00
0.02
0.04
0.06
eBa²
π⁺
π⁰_u
π⁰_d

Clover term

- Careful examination of the clover term

$$a\, c_{cl} \sum_{\mu<\nu} \sigma_{\mu\nu} F_{\mu\nu}$$

- reveals it cancels the Landau shift induced by the Wilson term in the free-field limit
- This condition is modified by inclusion of QCD
  - Allow QCD and electromagnetic field strengths to have different clover coefficients

$$c_{cl} \to C_{SW}\, F^{QCD}_{\mu\nu} + C_{EM}\, F^{EM}_{\mu\nu}$$

  - and set C_{EM} such that Wilson Landau shift is cancelled

$$C_{EM} = C^{Tree}_{EM}$$

- 1910.14244

Quark Operators

- Standard lattice QCD interpolators are inefficient at isolating energy eigenstates in a background magnetic field
- Quarks are charged!
  - Quarks experience Landau type effects
  - QCD causes quarks to hadronise for composite Landau energy
- Competing effects, introduce a quark projection operator that includes QCD and QED

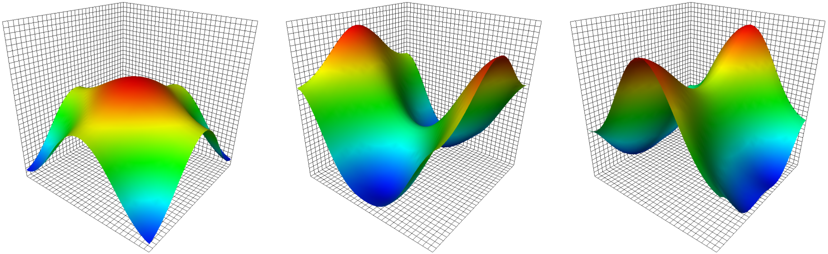


Figure: **Left:** Mode for the lowest quantised magnetic field strength $k_d = 1$. **Right:** Two degenerate eigenmodes of second quantised field strength $k_d = 2$.

$SU(3) \times U(1)$ Projection Operator

- Two-dimensional lattice Laplacian operator

$$\Delta_{\vec{x},\vec{x}'} = 4\,\delta_{\vec{x},\vec{x}'} - \sum_{\mu=1,2} U_\mu\left(\vec{x}\right)\,\delta_{\vec{x}+\hat{\mu},\vec{x}'} + U_\mu^\dagger\left(\vec{x}-\hat{\mu}\right)\,\delta_{\vec{x}-\hat{\mu},\vec{x}'},$$

- Use low-lying eigenmodes of the 2D Laplacian $\left(\psi_{i\,\vec{B}}\right)$ to project the propagator

$$P_n\left(\vec{x},t;\vec{x}',t'\right) = \sum_{i=1}^{n} \left\langle \vec{x},t \,\middle|\, \psi_{i,\vec{B}} \right\rangle \left\langle \psi_{i\,\vec{B}} \,\middle|\, \vec{x}',t' \right\rangle \delta_{zz'}\,\delta_{tt'}$$

- Projected propagator is

$$S_n(\vec{x},t;\vec{0},0) = \sum_{\vec{x}'} P_n(\vec{x},t;\vec{x}',t)\,S(\vec{x}',t;\vec{0},0)$$

- Also project hadronic level Landau effects for proton- using lattice Landau levels

Energy Shifts

- Recall the energy-field relation for an external magnetic field

$$E(B) = M + \vec{\mu} \cdot \vec{B} + \frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^3\right)$$

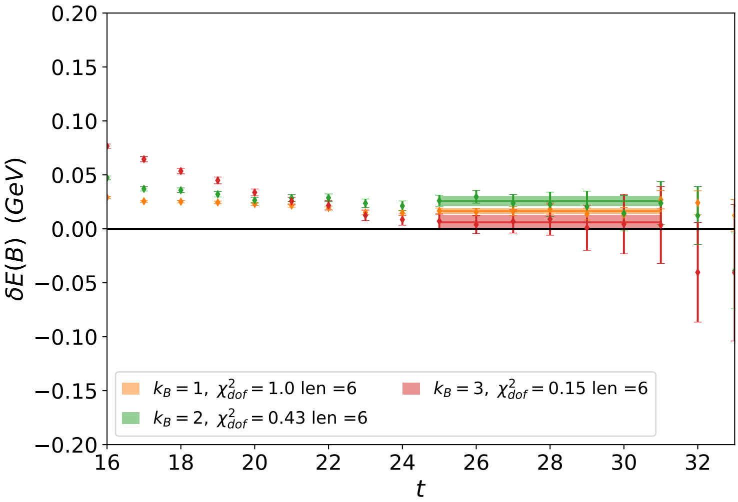
- Construct energy difference $E(B) - m$ for aligned and anti-aligned spin-field orientations and combine

$$\left(\frac{G(+s,+B) + G(-s,-B)}{G(+s,0) + G(-s,0)}\right) \left(\frac{G(+s,-B) + G(-s,+B)}{G(+s,0) + G(-s,0)}\right) = e^{-(2\,\delta\,E)\,t}$$

- Extract effective energy shift in standard manner
- Hence determine β using

$$\delta\,E(B,t) = +\frac{|qe\,B|}{2\,M} - \frac{4\,\pi}{2}\,\beta\,B^2 + \mathcal{O}\left(B^4\right)$$

Proton Energy Shift



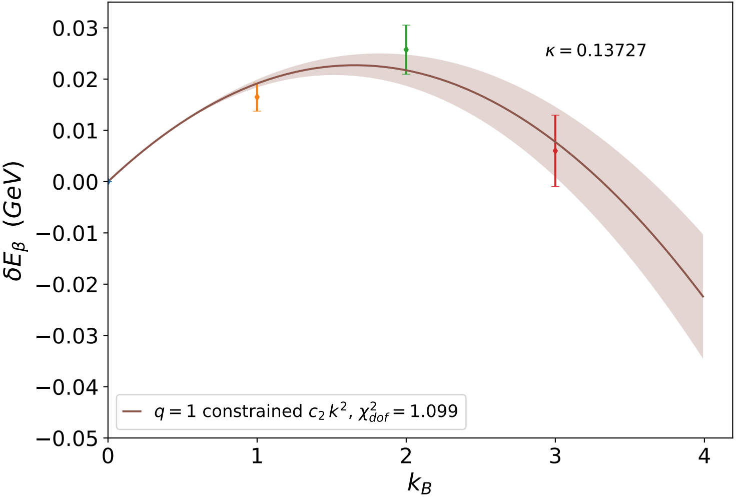
Polarisability Fit

- Fit to these energy shifts $\delta E(B, t)$

$$\delta E(B, t) - \frac{|qe\,B|}{2\,M} = \frac{-4\,\pi}{2}\,\beta\,B^2 = c_2\,k^2$$

- where k is the field quanta from background magnetic field quantisation condition

Polarisability Fit

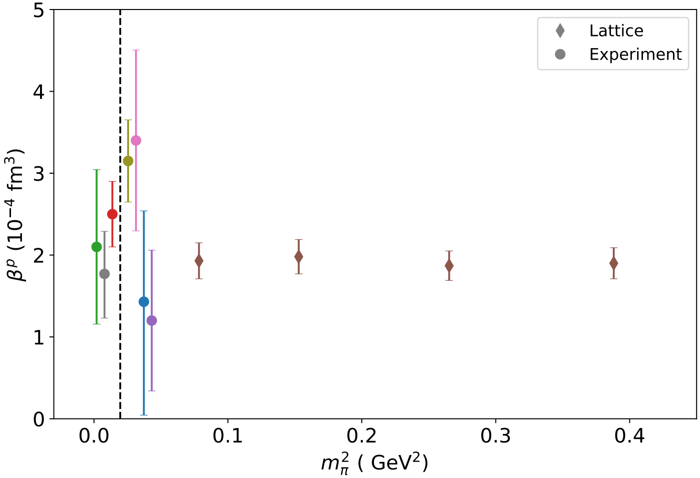


Ensemble Details

κ_{ud}	m_π (MeV)	Number of Sources	Number of configurations
0.13700	702	5	399
0.13727	570	4	400
0.13754	411	6	450
0.13770	296	7	400

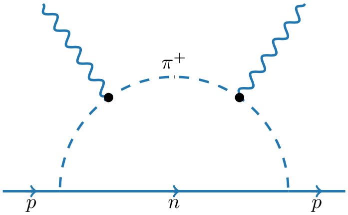
- Available through the International Lattice Data Grid and PACS-CS Collaboration: Phys. Rev. D79 (2009) 034503
- Lattice Volume: $32^3 \times 64$
- $2 + 1$ flavour dynamical-fermion QCD
- Physical lattice spacing $a = 0.0907$ fm
- Electroquenched - "sea" quarks experience no background magnetic field

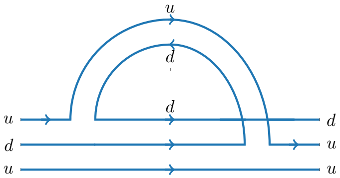
Lattice Results

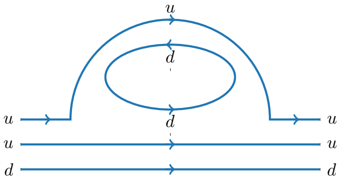


Making Contact With Experiment

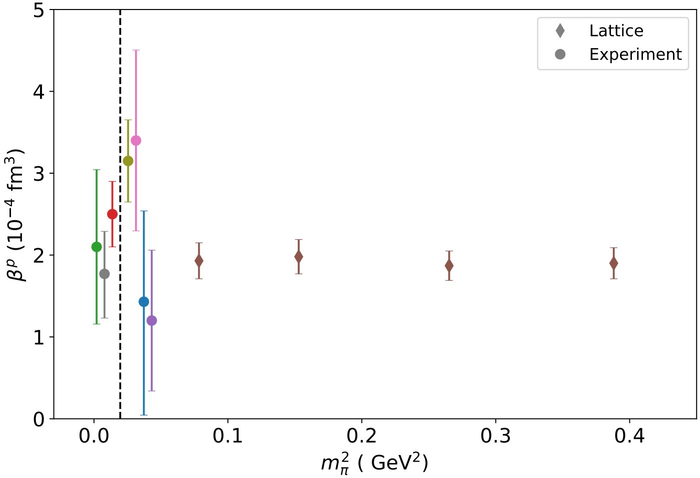
- Use a chiral effective field theory analysis to
  1. Account for finite volume effects
  2. Model sea-quark-loop contributions to β using techniques of partially quenched χPT



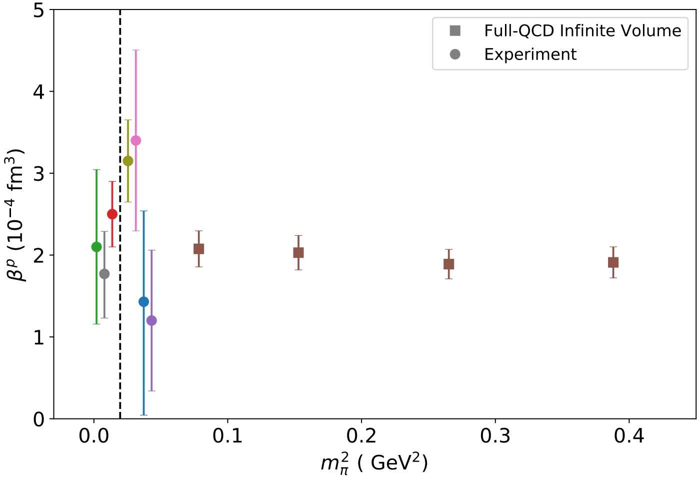




Making Contact With Experiment



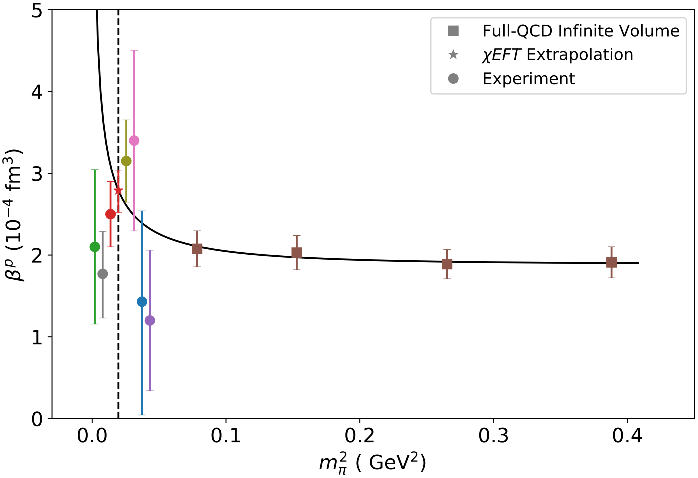
Making Contact With Experiment



Making Contact With Experiment

- Use a chiral effective field theory analysis to
  1. Account for finite volume effects
  2. Model sea-quark-loop contributions to β using techniques of partially quenched χPT
  3. Perform a chiral extrapolation to the physical point
- Use the techniques of
  - J. M. M. Hall, D. B. Leinweber, and R. D. Young, Phys. Rev. D89, 054511 (2014), arXiv:1312.5781 [hep-lat]
  - R. Bignell, J. Hall, W. Kamleh, D. Leinweber, and M. Burkardt, Phys. Rev. D98, 034504 (2018), arXiv:1804.06574 [hep-lat]

Making Contact With Experiment



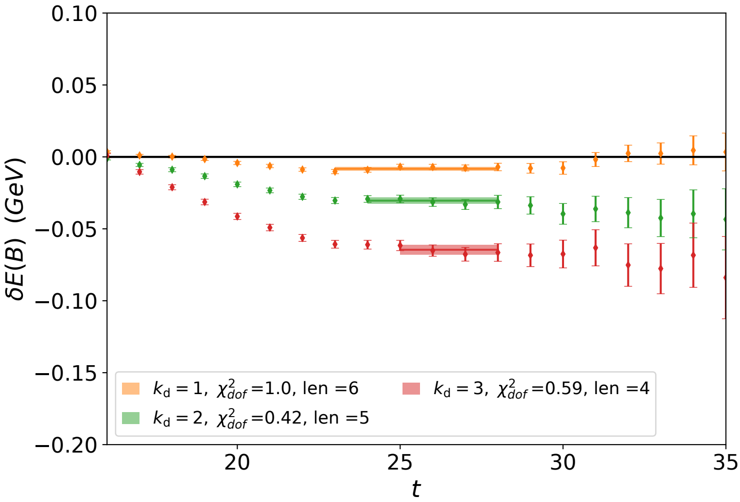
Neutron Energy Shift

- Identical process for neutron correlation functions
- A different fit function used

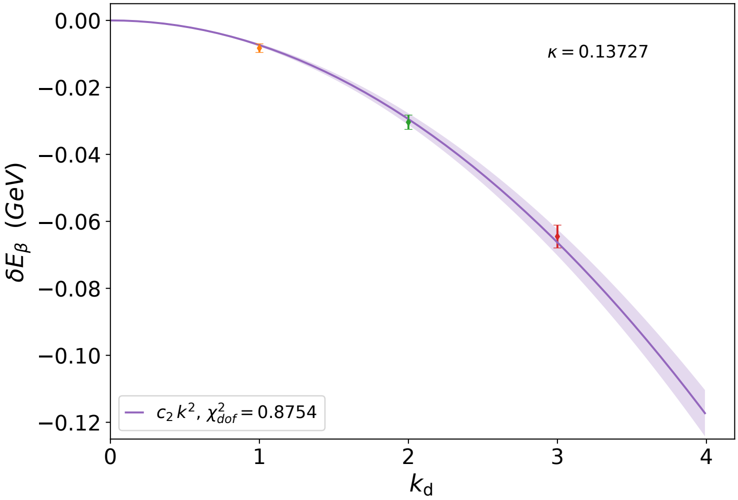
$$\delta E(B,t) = \frac{-4\pi}{2}\,\beta\, B^2 = c_2\, k^2$$

- No Hadronic U1 Landau wavefunction projection

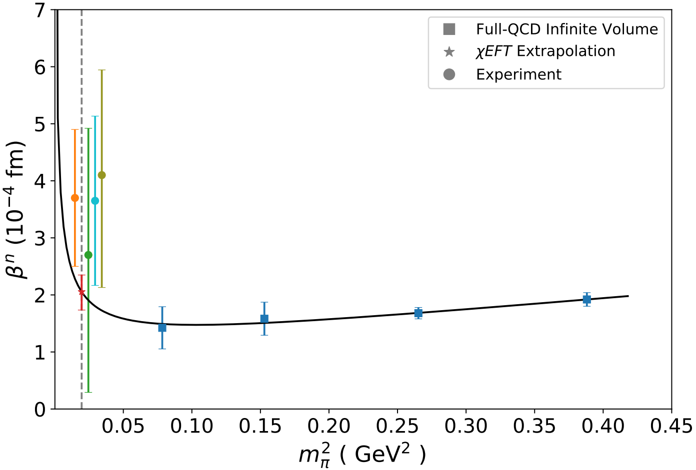
Neutron Energy Shift



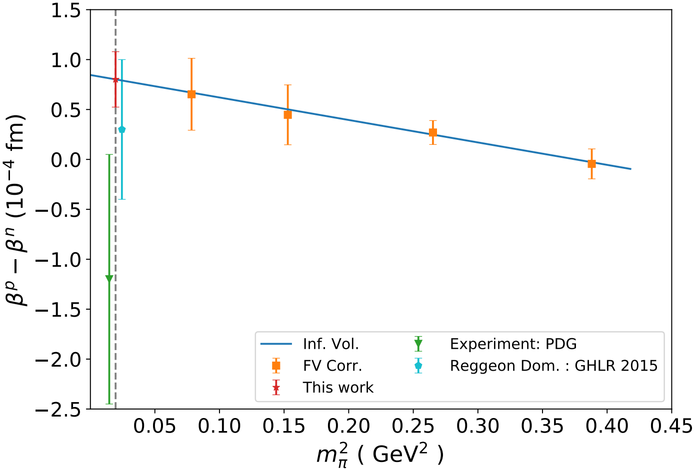
Polarisability Fit



Neutron Extrapolation



$\beta^p - \beta^n = 0.80\,(28)\,(4) \times 10^{-4}\ \text{fm}^3$



Summary

- Removed the additive mass renormalisation due to the Wilson term in a background magnetic field
- Calculated the magnetic polarisability of the proton and neutron using lattice QCD and background field method
- Specialised projection techniques have been used to account Landau effects
  - Enabling energy shift plateaus
- Chiral effective field theory analysis has been performed to connect lattice results to experiment.
- Techniques are applicable to further elements of the hadronic spectrum

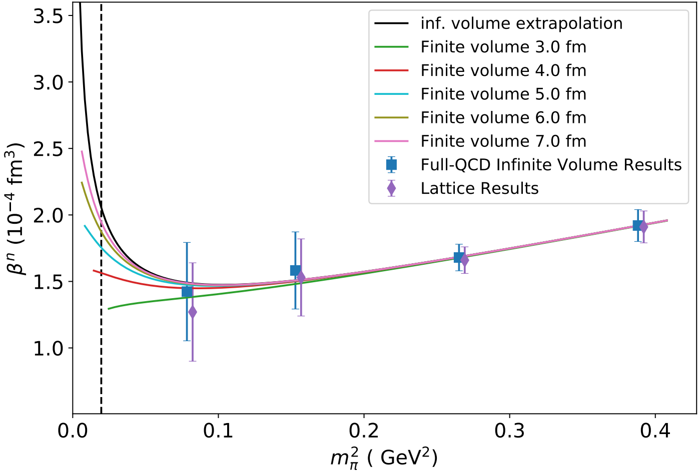

inf. volume extrapolation
Finite volume 3.0 fm
Finite volume 4.0 fm
Finite volume 5.0 fm
Finite volume 6.0 fm
Finite volume 7.0 fm
Full-QCD Infinite Volume Results
Lattice Results
β^n (10^{-4} fm^3)
3.5
3.0
2.5
2.0
1.5
1.0
0.0
0.1
0.2
0.3
0.4
m_π^2 (GeV^2)

Table: Magnetic polarisability values for the neutron and proton at each quark mass. The numbers in parantheses describe statistical (systematic) uncertainties. The value at $m_\pi = 0.140$ GeV is the result of our chiral extrapolation.

m_π (GeV)	$\beta^n \left(\text{fm}^3 \times 10^{-4}\right)$	${}^{n}\chi^2_{dof}$	$\beta^p \left(\text{fm}^3 \times 10^{-4}\right)$	${}^{p}\chi^2_{dof}$
0.702	1.91(12)	0.85	1.91(19)	0.96
0.570	1.68(10)	0.88	1.89(18)	1.10
0.411	1.58(29)	0.74	2.03(21)	0.67
0.296	1.42(37)	0.91	2.08(22)	0.33
0.140	2.06(26)(20)		2.79(22)(18)	

BONUS SLIDES

Review of Nucleon Polarisabilities

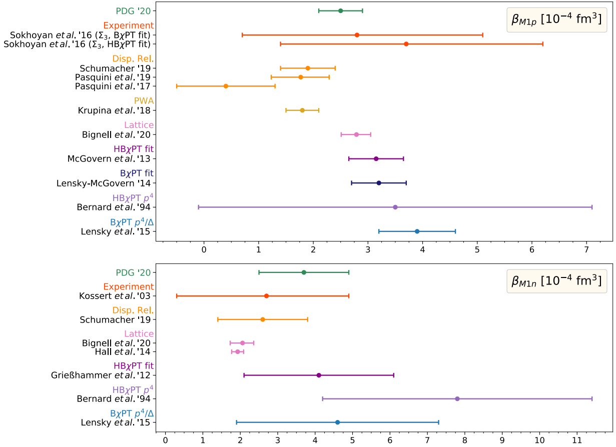


Figure: Magnetic dipole polarizability β_{M1} of the nucleon. Figure from 2006.16124