Lattice study of rotating gluodynamics

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In collaboration with

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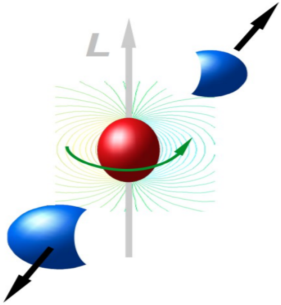
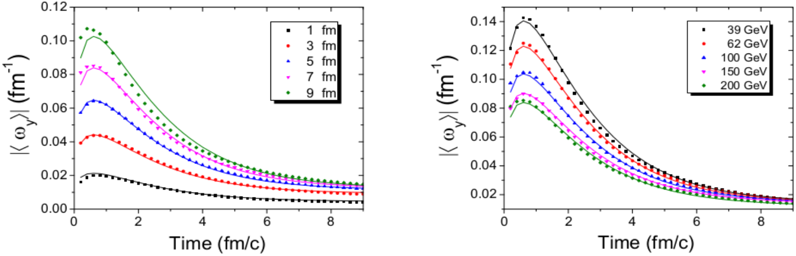
Rotation of QGP in heavy ion collisions



- QGP is created with non-zero angular momentum in non-central collisions

Rotation of QGP in heavy ion collisions



Hydrodynamic simulations (arxiv:1602.06580)

- Au-Au: *left* $\sqrt{s} = 200$ GeV, *right* $b = 7$ fm,
- $\Omega \sim 20$ MeV ($v \sim c$ at distances 7 fm)
- Relativistic rotation of QGP

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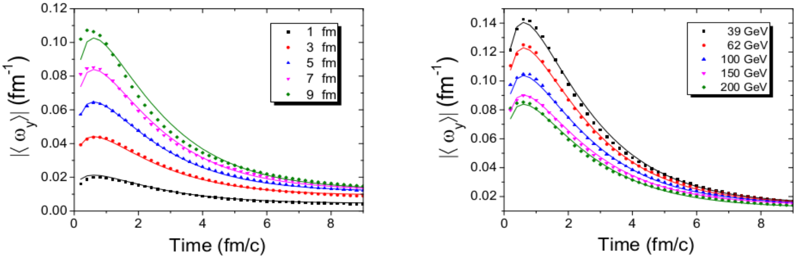

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How relativistic rotation influences QCD?

Study of rotating QGP

- Rotating QGP at thermodynamic equilibrium
  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field

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  - At the equilibrium the system rotates with some Ω
  - The study is conducted in **the reference frame which rotates with QCD matter**
  - QCD in external gravitational field
- **Boundary conditions are very important!**

Recent works

- Arata Yamamoto, Yuji Hirono, Phys.Rev.Lett. 111 (2013) 081601
- S. Ebihara, K. Fukushima, K. Mameda, Phys. Lett. B 764 (2017) 94–99
- M.N. Chernodub, Shinya Gongyo, Phys.Rev.D 95 (2017) 9, 096006
- M.N. Chernodub, Shinya Gongyo, JHEP 01 (2017) 136
- Hui Zhang, Defu Hou, Jinfeng Liao, e-Print: 1812.11787 [hep-ph]
- Yin Jiang, Jinfeng Liao, Phys.Rev.Lett. 117 (2016) 19, 192302

...

Common features

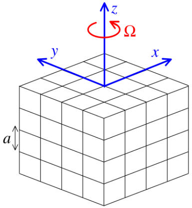
- The studies are carried out in NJL (chiral transition)
- Critical temperature of the chiral phase transition drops with angular velocity
- Explanation: polarization of the chiral condensate (Phys.Rev.Lett. 117 (2016) 19, 192302)
- **Confinement/deconfinement transition was not considered**

Details of the simulations

- Gluodynamics is studied at thermodynamic equilibrium in external gravitational field
- The metric tensor

$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & \Omega y & -\Omega x & 0 \\ \Omega y & -1 & 0 & 0 \\ -\Omega x & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

- Geometry of the system: $N_t \times N_z \times N_x \times N_y = N_t \times N_z \times N_s^2$



Details of the simulations

- Partition function ($\hat{H}$ is conserved)

$$Z = \mathrm{Tr}\,\exp\left[-\beta \hat{H}\right]$$

- Euclidean action

$$S_G = -\frac{1}{2g_{YM}^2}\int d^4x\,\sqrt{g_E}\,g_E^{\mu\nu}\,g_E^{\alpha\beta}F_{\mu\alpha}^{(a)}F_{\nu\beta(a)}$$

$$S_G = \frac{1}{2g_{YM}^2}\int d^4x \mathrm{Tr}\big[(1-r^2\Omega^2)F_{xy}^aF_{xy}^a + (1-y^2\Omega^2)F_{xz}^aF_{xz}^a +$$

$$+(1-x^2\Omega^2)F_{yz}^aF_{yz}^a + +F_{x\tau}^aF_{x\tau}^a + F_{y\tau}^aF_{y\tau}^a + F_{z\tau}^aF_{z\tau}^a -$$

$$-2iy\Omega(F_{xy}^aF_{y\tau}^a + F_{xz}^aF_{z\tau}^a) + 2ix\Omega(F_{yx}^aF_{x\tau}^a + F_{yz}^aF_{z\tau}^a) - 2xy\Omega^2F_{xz}F_{zy}\big]$$

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- *Ehrenfest–Tolman effect:* **In gravitational field the temperature is not constant in space at thermal equilibrium**

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- We use the designation $T = T(r=0) = 1/\beta$

Details of the simulations

Boundary conditions

- **Periodic b.c.:**
  - $U_{x,\mu} = U_{x+N_i,\mu}$
  - Not appropriate for the field of velocities of rotating body
- **Dirichlet b.c.:**
  - $U_{x,\mu}\big|_{x\in\Gamma} = 1, \quad A_\mu\big|_{x\in\Gamma} = 0$
  - Violate Z_3 symmetry
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Sign problem

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$$+(1 - x^2\Omega^2) F^a_{yz} F^a_{yz} + + F^a_{x\tau} F^a_{x\tau} + F^a_{y\tau} F^a_{y\tau} + F^a_{z\tau} F^a_{z\tau} -$$

$$-2iy\Omega(F^a_{xy} F^a_{y\tau} + F^a_{xz} F^a_{z\tau}) + 2ix\Omega(F^a_{yx} F^a_{x\tau} + F^a_{yz} F^a_{z\tau}) - 2xy\Omega^2 F_{xz} F_{zy}\big]$$

- The Euclidean action has imaginary part (**sign problem**)
- Simulations are carried out at imaginary angular velocities $\Omega \to i\Omega_I$
- The results are analytically continued to real angular velocities
- This approach works up to sufficiently large Ω ($\Omega < 50$ MeV)

Details of the simulations

The critical temperature

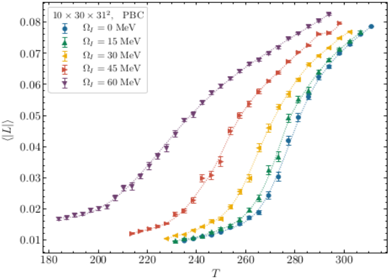
- Polyakov line

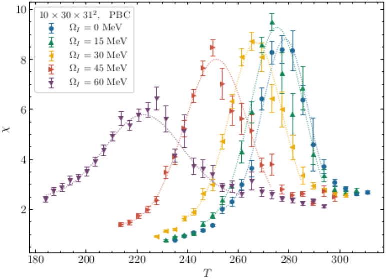
$$L = \left\langle \mathrm{Tr}\mathcal{T} \exp\left[ig \int_{[0,\beta]} A_4 \, dx^4 \right] \right\rangle$$

- Susceptibility of the Polyakov line

$$\chi = N_s^2 N_z \left(\langle |L|^2 \rangle - \langle |L| \rangle^2 \right)$$

Results of the calculation





Volume dependence of the susceptibility

- **Periodic b.c.:** $\sim V$
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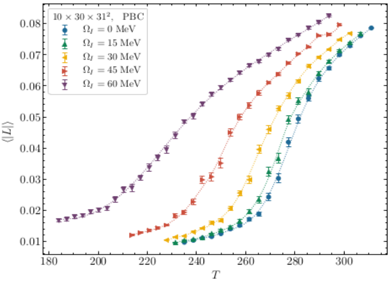

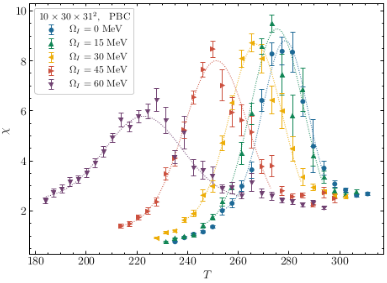

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Rotation does not modify the order of the phase transition

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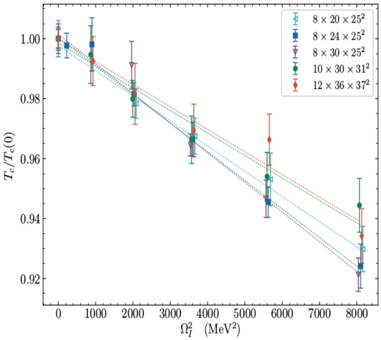
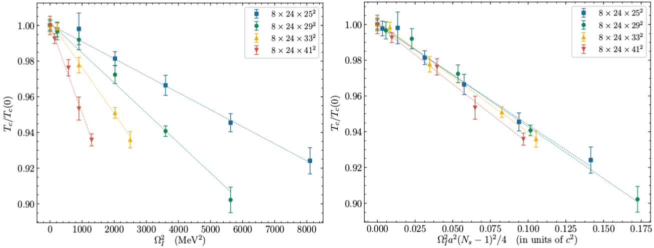


- The results can be well described by the formula ($C_2 > 0$)

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C_2\Omega_I^2 \Rightarrow \frac{T_c(\Omega)}{T_c(0)} = 1 + C_2\Omega^2$$

- **The critical temperature rises with angular velocity**
- The results weakly depend in lattice spacing and the volume in z-direction

Dependence on the transverse size



- The results can be well described by the formula

$$\frac{T_c(\Omega)}{T_c(0)} = 1 - B_2 v_I^2, \quad v_I = \Omega_I (N_s - 1) a/2, \quad C_2 = B_2 (N_s - 1)^2 a^2/4$$

- **Periodic b.c.:** $B_2 \sim 1.3$
- **Dirichlet b.c.:** $B_2 \sim 0.3$
- **Neumann b.c.:** $B_2 \sim 0.5$

Conclusion

- We have carried out lattice study of how relativistic rotation influences confinement/deconfinement transition
- Critical temperature of the confinement/deconfinement transition rises with Ω
- Critical temperature of the chiral transition drops with Ω
- One needs to include dynamical quarks to see who wins

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THANK YOU!