Pseudoscalar Transition Form Factors $P \to \gamma^*\gamma^*$ from Twisted Mass Lattice QCD

Sebastian Burri

Flavor Singlet Project for ETMC

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Project overview

Goal

Computing $\mathcal{F}_{P\to\gamma^*\gamma^*}$, $P = \pi_0, \eta, \eta'$ to determine the corresponding contributions to HLbL in the muon $g-2$.

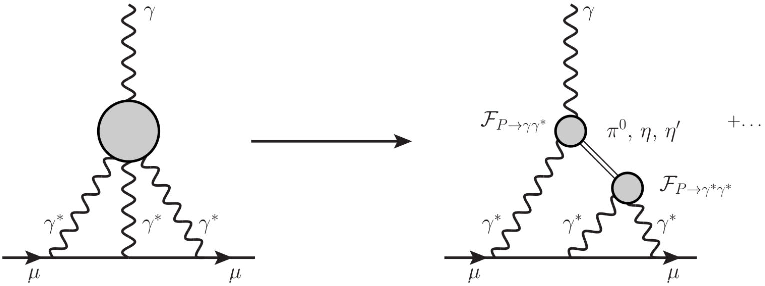
Using

- Twisted mass clover improved lattice QCD at maximal twist
- Four dynamical flavors ($N_f = 2+1+1$) [C. Alexandrou et al., Phys. Rev. D98, 054518 (2018)]
- Analysis on

ensemble	$L^3 \cdot T/a^4$	m_π [MeV]	a [fm]	$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$	κ_{crit}
cA30.32	$32^3 \cdot 64$	260	0.0927	0.0030	0.1408	0.1521	0.1400645
cB072.64	$64^3 \cdot 128$	135	0.0800	0.00072	0.1247	0.1315	0.1394265

- Two ensembles at the physical point

Hadronic light-by-light



Anomalous muon magnetic moment

- Extracted from the muon electromagnetic vertex function
- Hadronic contributions: HVP, HLbL and higher order electroweak corrections
- Motivation: Dispersive approach to HLbL [G. Colangelo, M. Hoferichter, M. Procura and P. Stoffer, JHEP 91 (2014)]

Following [M. Knecht, A. Nyffeler, Phys. Rev. D65, 073034 (2002)], the transition form factors are defined via the matrix element

$$M_{\mu\nu}(p,q_1) = i\int d^4x\, e^{iq_1x}\,\langle 0|T\{j_\mu(x)j_\nu(0)\}|P(p)\rangle$$
$$= \varepsilon_{\mu\nu\alpha\beta}\, q_1^\alpha q_2^\beta \mathcal{F}_{P\gamma^*\gamma^*}(q_1^2,q_2^2).$$

Defining

$$\tilde{A}_{\mu\nu}(\tau) = \langle 0|T\{j_\mu(\vec{q}_1,\tau)j_\nu(\vec{p}-\vec{q}_1,0)\}|P(p)\rangle\,,$$

the matrix element is recovered by integration:

$$M^E_{\mu\nu} = \int_{-\infty}^{\infty} d\tau\, e^{\omega_1\tau}\,\tilde{A}_{\mu\nu}(\tau), \quad i^{n_0}M^E_{\mu\nu}(p,q_1) = M_{\mu\nu}(p,q_1).$$

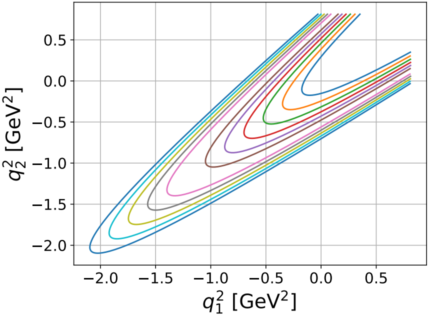
On the lattice, starting from the amplitude

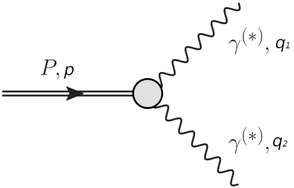
$$C_{\mu\nu}(\tau,t_P) = a^6\sum_{\vec{x},\vec{z}}\langle j_\mu(\vec{x},t_i)j_\nu(\vec{0},t_f)P^\dagger(\vec{z},t_0)e^{i\vec{p}\vec{z}}e^{-i\vec{x}\vec{q}_1}\rangle,$$

one constructs

$$\tilde{A}_{\mu\nu}(\tau) = \frac{2E_P}{Z_P}\lim_{t_P\to\infty} e^{E_P(t_f-t_0)}C_{\mu\nu}(\tau,t_P).$$

Kinematics





Pseudoscalar at rest, i.e. $\vec{p} = \vec{0}$

$$\Rightarrow q_1^2 = \omega_1^2 - \vec{q}_1^2$$
$$q_2^2 = (m_P - \omega_1)^2 - \vec{q}_1^2$$

Threshold for hadron production

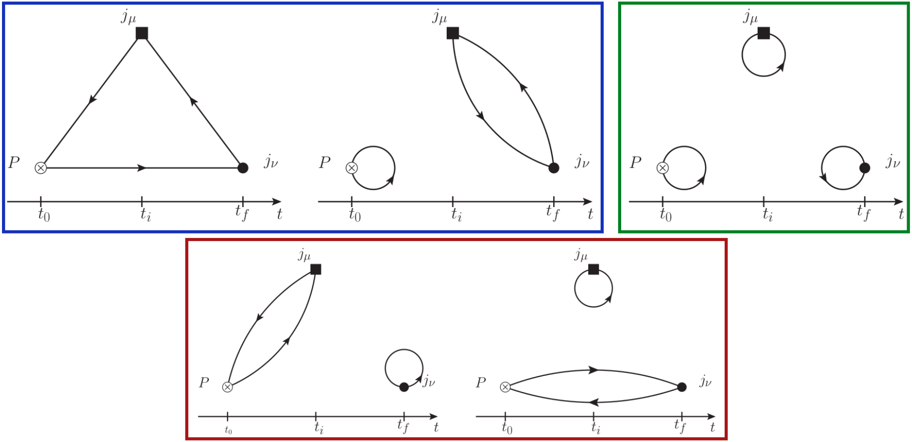
$$\Rightarrow q_{1,2}^2 < m_V^2 = \min(m_\rho^2, 4m_P^2)$$
$$-\sqrt{m_V^2 + \vec{q}_1^2} + m_P < \omega_1 < \sqrt{m_v^2 + \vec{q}_1^2}$$

Define

$$\tilde{A}(\tau) = i m_P^{-1} \varepsilon_{ijk} \frac{\vec{q}_1^{\,i}}{\vec{q}_1^{\,2}} \tilde{A}_{jk}(\tau)$$

Considered diagrams

The amplitude $C_{\mu\nu}$ contains connected, pseudoscalar disconnected, doubly disconnected and vector current disconnected diagrams.



Current operators and isospin combinations - pion

- Restricted to light contributions in the electromagnetic current, i.e.

$$j_\mu(x) = \frac{2}{3}\bar{u}\gamma_\mu u(x) - \frac{1}{3}\bar{d}\gamma_\mu d(x) = \frac{1}{6}j_\mu^{(0,0)} + \frac{1}{2}j_\mu^{(1,0)}.$$

- Decomposition into definite isospin yields

$$\begin{aligned} C_{\mu\nu} = \; & \frac{1}{12}\left\langle \pi_0\, j_\mu^{(0,0)}\, j_\nu^{(1,0)} \right\rangle + \frac{1}{12}\left\langle \pi_0\, j_\mu^{(1,0)}\, j_\nu^{(0,0)} \right\rangle \\ & + \frac{1}{4}\left\langle \pi_0\, j_\mu^{(0,0)}\, j_\nu^{(0,0)} \right\rangle + \frac{1}{36}\left\langle \pi_0\, j_\mu^{(1,0)}\, j_\nu^{(1,0)} \right\rangle \end{aligned}$$

  for the amplitude.
- Avoid $\mathcal{O}(a)$ lattice artefact due to parity. [R. Frezzotti, G. Martinelli, M. Papinutto, G. C. Rossi, JHEP 04 (2006) 038]

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From π_0 to $\pi_\pm$

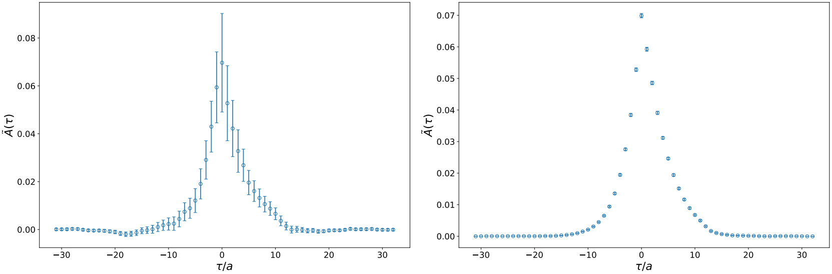
- Isospin rotation $\pi_0 \to \pi_+ + \pi_-$.
- Difference between neutral and charged pion form factors is a lattice artefact of order $\mathcal{O}(a^2)$.
- No disconnected contribution to $\pi_\pm$ amplitude.

The amplitude transforms as

$$\left\langle \pi_0 j_\nu^{(1,0)} j_\mu^{(0,0)} \right\rangle \to \left\langle \pi_- j_\nu^{(1,+)} j_\mu^{(0,0)} \right\rangle + \left\langle \pi_+ j_\nu^{(1,-)} j_\mu^{(0,0)} \right\rangle$$

under the same isospin rotation.

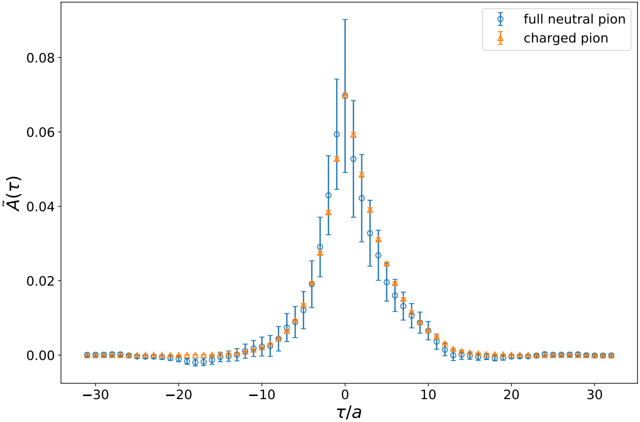
260 MeV ensemble



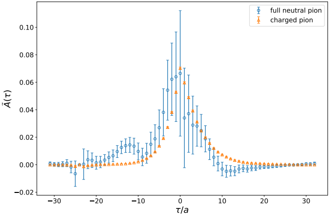
(a) neutral (b) charged

$\tilde{A}(\tau)$ for source-sink separation $t_{seq} = 12$ and $\vec{q}^2 = 1^*$. Signal for the charged pion is clearer.

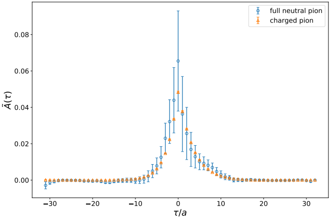
$^* t_{seq}/a = 12, \ \left(\frac{La}{2\pi}\right)^2 \vec{q}^2 = 1$



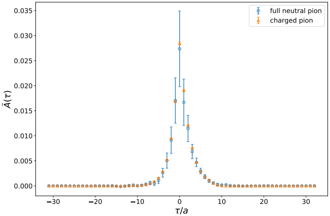
$\tilde{A}(\tau)$ for charged and neutral pion agree within errors, difference is a lattice artefact.



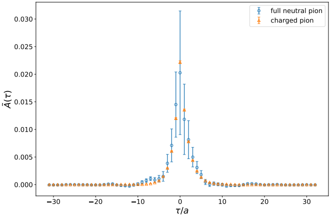
(a) $t_{seq} = 20, \vec{q}^2 = 1$



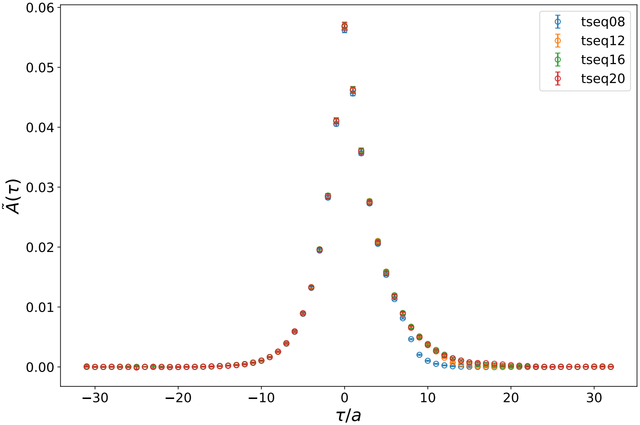
(b) $t_{seq} = 16, \vec{q}^2 = 3$



(c) $t_{seq} = 8, \vec{q}^2 = 8$

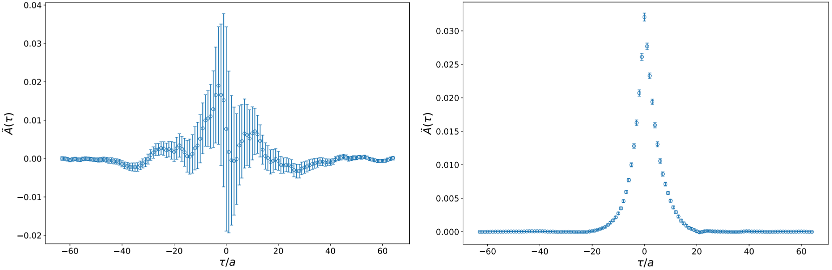


(d) $t_{seq} = 12, \vec{q}^2 = 12$



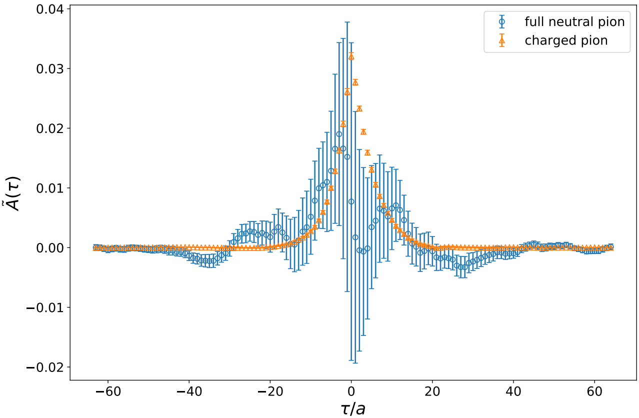
Close agreement for the charged amplitude for different source-sink separations t_{seq}, thus the integrated amplitude also will not depend on t_{seq}.

Physical point ensemble

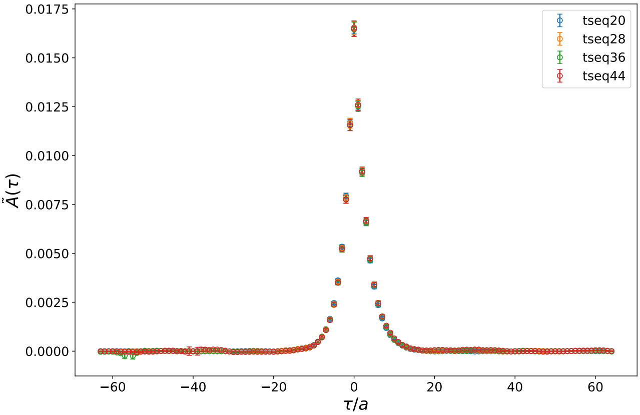


(a) neutral (b) charged

$\tilde{A}(\tau)$ for $t_{seq} = 20$ and $\vec{q}^2 = 12$. With current statistics, $\tilde{A}(\tau)$ for the neutral pion has huge errors. Charged signal is much clearer.



$\tilde{A}(\tau)$ still agree within errors.



Again close agreement for the charged amplitude for different source-sink separations t_{seq}.

Current operators and isospin combinations - eta/eta'

- Also consider strange contributions in the electromagnetic current, i.e.

$$j_\mu(x) = \underbrace{\frac{2}{3}\bar{u}\gamma_\mu u(x) - \frac{1}{3}\bar{d}\gamma_\mu d(x)}_{=j^l_\mu(x)} - \underbrace{\frac{1}{3}\bar{s}\gamma_\mu s(x)}_{=j^s_\mu(x)} = \frac{1}{6}j^{l,(0,0)}_\mu + \frac{1}{2}j^{l,(1,0)}_\mu + j^s_\mu.$$

- Same decomposition into definite isospin for the light contribution

$$\begin{aligned} C_{\mu\nu} = & \frac{1}{12}\left\langle \eta\, j^{l,(0,0)}_\mu\, j^{l,(1,0)}_\nu \right\rangle + \frac{1}{12}\left\langle \eta\, j^{l,(1,0)}_\mu\, j^{l,(0,0)}_\nu \right\rangle \\ & + \frac{1}{4}\left\langle \eta\, j^{l,(0,0)}_\mu\, j^{l,(0,0)}_\nu \right\rangle + \frac{1}{36}\left\langle \eta\, j^{l,(1,0)}_\mu\, j^{l,(1,0)}_\nu \right\rangle \end{aligned}$$

- ... but other part needed to avoid $\mathcal{O}(a)$ artefact.
- Osterwalder-Seiler calculations are used for introducing the strange quark. [R. Frezzotti, G. C. Rossi, JHEP 10 (2004) 070]

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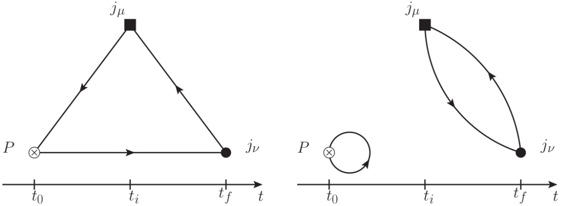
Mixing & Diagrams

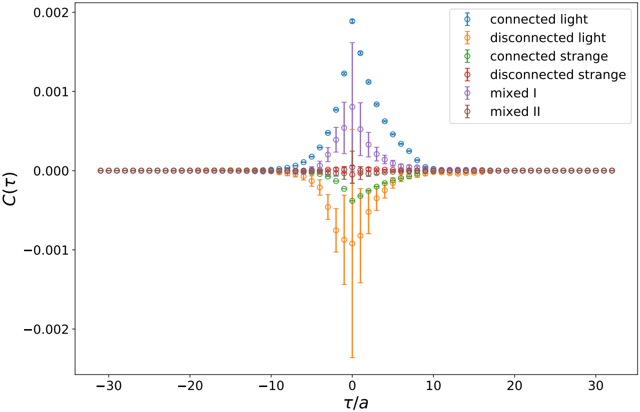
- Mixing not considered at the moment:

$$\eta \approx \eta_8 \propto \bar{u}u + \bar{d}d - 2\bar{s}s$$
$$\eta' \approx \eta_1 \propto \bar{u}u + \bar{d}d + \bar{s}s$$

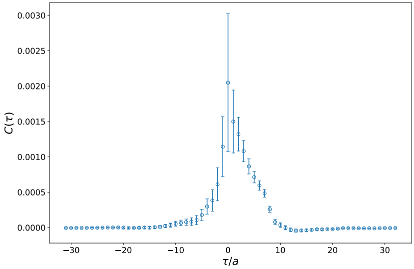
- Disconnected contribution can mix light and strange quarks



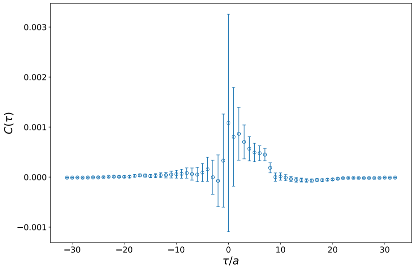
260 MeV ensemble



Contributions to η amplitude $C(\tau)$ for $t_{seq} = 8, \vec{q}^2 = 4$.



(a) η

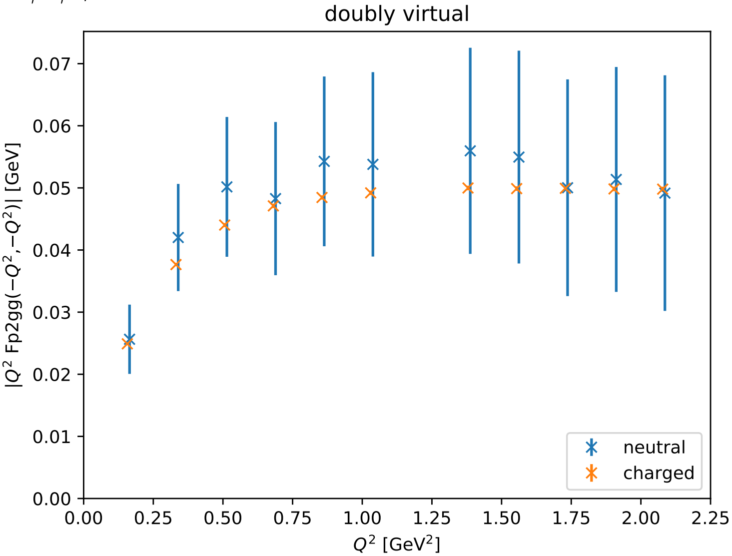


(b) η'

Amplitudes $C(\tau)$ for $t_{seq} = 8, \vec{q}^2 = 1$.

Integration: first look, very preliminary

For $\pi \to \gamma^*\gamma^*$, 260 MeV ensemble.



Summary

- We are able to extract a signal for the pion amplitude $\tilde{A}(\tau)$ directly at the physical point.
- For η (and η'), while numerically more challenging, we can still measure a signal.
- First integration results look promising.

Next steps

- Finishing data production and analysis on

ensemble	$L^3 \cdot T/a^4$	m_π [MeV]	a [fm]	$a\mu_l$	$a\mu_\sigma$	$a\mu_\delta$	κ_{crit}
cC062.80	$80^3 \cdot 160$	135	0.069	0.00062	0.1060	0.1135	0.1387510

- Continue integration of $\tilde{A}(\tau)$ for the pion and η
- Increasing statistics for η, looking at the η decay constant
- Mixing for η/η'
- Exploring different kinematic frames

Ensembles

ensemble	$L^3 \cdot T/a^4$	β	c_{SW}	M_π [MeV]	a [fm]	$a\mu_\ell$	$a\mu_\sigma$	$a\mu_\delta$	κ_{crit}
cA53.24	$24^3 \cdot 48$	1.726	1.74	350	0.0927	0.0053	0.1408	0.1521	0.1400645
cA40.24	$24^3 \cdot 48$	1.726	1.74	301	0.0927	0.0040	0.1408	0.1521	0.1400645
cA30.32	$32^3 \cdot 64$	1.726	1.74	260	0.0927	0.0030	0.1408	0.1521	0.1400645
cA12.48	$48^3 \cdot 96$	1.726	1.74	165	0.0927	0.0012	0.1408	0.1521	0.1400645
cB25.48	$48^3 \cdot 96$	1.778	1.69	255	0.0800	0.0025	0.1247	0.1315	0.1394267
cB140.64	$64^3 \cdot 128$	1.778	1.69	190	0.0800	0.00140	0.1247	0.1315	0.1394267
cB072.64	$64^3 \cdot 128$	1.778	1.69	135	0.0800	0.00072	0.1247	0.1315	0.1394265
cC062.80	$80^3 \cdot 160$	1.836	1.645	135	0.069	0.00062	0.1060	0.1135	0.1387510

Table: Parameter values for the gauge configurations available through (and under production by) ETMC with $N_f = 2 + 1 + 1$ Wilson clover twisted mass quark flavours. For each ensemble we provide the volume, the gauge coupling β, the clover coefficient c_{SW}, the pion mass M_π and the lattice spacing a in physical units, the bare twisted mass values $a\mu_\ell$, $a\mu_\sigma$, $a\mu_\delta$, and the hopping parameter κ_{crit}.

Clover improved tmLQCD action [C. Alexandrou et al., Phys. Rev. D98, 054518 (2018)]

$$S = S_g + S^l_{tm} + S^h_{tm}$$

Iwasaki improved gauge action for S_g:

$$S_g = \frac{\beta}{3} \sum_x \left(b_0 \sum_{\substack{\mu,\nu=1 \\ 1\leq\mu<\nu}}^{4} \left\{ 1 - \operatorname{Re}\operatorname{Tr}\left(U^{1\times1}_{x,\mu,\nu}\right) \right\} + b_1 \sum_{\substack{\mu,\nu=1 \\ \mu\neq\nu}}^{4} \left\{ 1 - \operatorname{Re}\operatorname{Tr}\left(U^{1\times2}_{x,\mu,\nu}\right) \right\} \right)$$

Light up and down doublet:

$$S^l_{tm} = \sum_x \bar{\chi}_l(x) \left[D_W(U) + \frac{i}{4} c_{SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_l + i\mu_l \tau^3 \gamma^5 \right] \chi_l(x)$$

Heavy quark action, non-degenerate strange and charm quarks*:

$$S^h_{tm} = \sum_x \bar{\chi}_h(x) \left[D_W(U) + \frac{i}{4} c_{SW} \sigma^{\mu\nu} \mathcal{F}^{\mu\nu}(U) + m_h - \mu_\delta \tau_1 + i\mu_\sigma \tau^3 \gamma^5 \right] \chi_h(x)$$

*In practise, we use $m_h = m_l$, this constitutes an additional $\mathcal{O}(a^2)$ lattice artefact which is small for the strange quark and practically vanishes for the charm quark.

Extraction of the form factors

We follow the conventions from [A. Gérardin, H. B. Meyer and A. Nyffeler, Phys. Rev. D94, 074507 (2016)]

- The relevant hadronic quantity is the rank-four hadronic vacuum polarization tensor

$$\Pi_{\mu\nu\lambda\rho}(q_1, q_2, q_3) = \int d^4x_1 d^4x_2 d^4x_3 \Big[e^{i(q_1x_1+q_2x_2+q_3x_3)} \cdot \langle 0| T\{j_\mu(x_1) j_\nu(x_2) j_\lambda(x_3) j_\rho(0)\}|0\rangle \Big]$$

- Each pseudoscalar pole $P \in \{\pi_0, \eta, \eta'\}$ contributes to the amplitude via one-particle-reducible single pseudoscalar exchanges

$$\Pi^{(P)}_{\mu\nu\lambda\rho}(q_1, q_2, q_3) = \Big[i \frac{\mathcal{F}_{P\gamma^*\gamma^*}(q_1^2, q_2^2)\mathcal{F}_{P\gamma^*\gamma^*}(q_3^2, (q_1+q_2+q_3)^2)}{(q_1+q_2)^2 + M_P^2} \cdot \epsilon_{\mu\nu\alpha\beta} q_1^\alpha q_2^\beta \epsilon_{\lambda\rho\gamma\delta} q_3^\gamma (q_1+q_2)^\delta \Big] + \text{(crosses)}$$