Determination of α_s in $N_f = 3$ QCD from current-current correlation functions in position space

based on arXiv:2003.05781 [hep-lat]

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Introduction

Physical interest

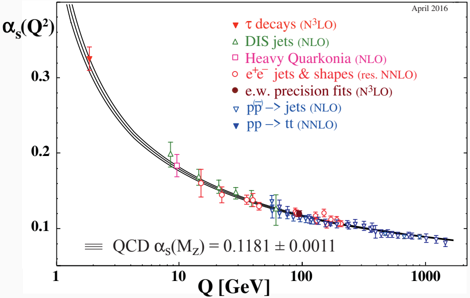
- α_s plays a key role in the understanding of QCD and in its applications to collider physics.
- The uncertainty of α_s is one of dominant sources of uncertainty in SM predictions for the partial widths $H \to bb$, $H \to gg$.
- Higher precision determinations are needed to maximize the potential of experimental measurements at the LHC, for high-precision Higgs studies at future colliders and investigate of the stability of the vacuum.
- The value of α_s yields one of the essential boundary conditions for completions of the SM at high energies.

Typical determination

- We measure a short-distance quantity $\mathcal{Q}$ at scale μ (experimentally or through lattice calculations) and then match it to a perturbative expansion in terms of α_s (typically in the $\overline{\mathrm{MS}}$ scheme):

$$\mathcal{Q}(\mu) = c_1 \alpha_{\overline{\mathrm{MS}}}(\mu) + c_2 \alpha_{\overline{\mathrm{MS}}}(\mu)^2 + \ldots$$

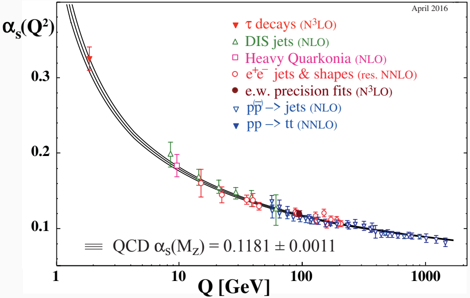
Determination of α_s



[Particle Data Group, Chin. Phys. C, 40, 100001 (2016)]

α_s is typically determined from:

- hadronic τ decays
- hadronic final states of e^+e^- annihilation
- deep inelastic lepton-nucleon scattering
- electroweak precision data
- high energy hadron collider data



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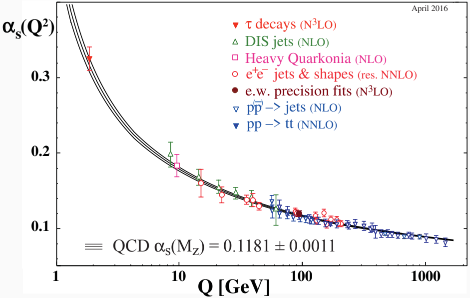

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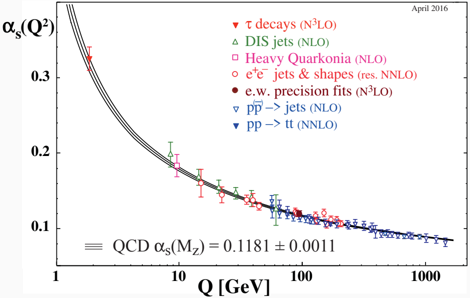

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- Combining the two estimates above, we have:
  $\alpha^{(5)}_{\overline{\rm MS}}(M_z) = 0.11806(72)$, PDG 2018 + FLAG 2019

Goals of this project

Lattice QCD approach

- Lattice QCD is a powerful tool that allows to determine α_s starting from first principles.
- Typical strategies of investigation (see e.g. FLAG Review 2019):
    1. step scaling methods;
    2. heavy quark-antiquark potential;
    3. observable in momentum space;
    4. moments of heavy quark current;
    5. eigenvalues of the Dirac operator.

Questions we want to answer to in this project

1. Can α_s be determined from current-current correlation functions in position space?
2. If yes, what kind of precision can be achieved?

Strategy

Lattice setup

$N_f = 3$ ensembles

β	name	$\kappa_l = \kappa_s$	m_π [MeV]	t_0/a^2	# conf.
3.46	B450	0.136890	419	3.663(11)	320
3.46	rqcd30	0.136959	320	3.913(15)	280
3.46	X450	0.136994	264	3.994(10)	280
3.55	B250	0.136700	709	4.312(8)	84
3.55	N202	0.137000	412	5.165(14)	177
3.55	X250	0.137050	348	5.283(27)	182
3.55	X251	0.137100	269	5.483(26)	177
3.7	N303	0.136800	641	7.743(23)	99
3.7	N300	0.137000	423	8.576(21)	197
3.85	N500	0.13672514	599	12.912(75)	100
3.85	J500	0.136852	410	14.045(38)	120

- Consortium of several groups to generate "large volume" ensembles
  CLS: Coordinated Lattice Simulations
  [Berlin, Dublin, Geneva, Madrid, Mainz, Milan, Münster, Odense, Regensburg, Rome, Valencia, Wuppertal, Zeuthen (NIC)]
- S_G: tree-level Symanzik improved action.
- S_F: Wilson $O(a)$-improved action with clover coefficient c_{SW} determined non-perturbatively.

Correlation functions in position space

- Correlation functions of flavor non-singlet bilinear quark operators in position space

$$C_\Gamma(x) = \langle \bar{\psi}^i(x)\Gamma\psi^j(x)\bar{\psi}^j(0)\Gamma\psi^i(0)\rangle$$

  - i, j, with $i \neq j$ flavor indices (no disconnected diagrams)
  - $\Gamma = \{\gamma_\mu, \gamma_\mu\gamma_5\} \equiv \{V, A\}$ (vector and axial-vector channels)
  - Renormalization constants Z_A and Z_V **[M. Dalla Brida et al., Eur.Phys.J. C79 (2019) no.1, 23, arXiv:1808.09236]**

- In particular, we investigate two types of correlation functions

$$C_V = \sum_\mu C_{\gamma_\mu}(x), \quad C_A = \sum_\mu C_{\gamma_\mu\gamma_5}(x)$$

- Perturbative formulae in the $\overline{\mathrm{MS}}$-scheme are known up to 4 loops

[K. G. Chetyrkin and A. Maier, Nucl.Phys. B844 (2011) 266-288, arXiv:1010.1145]

$$X^6 C_V = X^6 C_A = \frac{6}{\pi^4}\left[1 + c_1\alpha_{\overline{\mathrm{MS}}} + c_2\alpha^2_{\overline{\mathrm{MS}}} + c_3\alpha^3_{\overline{\mathrm{MS}}} + c_4\alpha^4_{\overline{\mathrm{MS}}}\right]$$

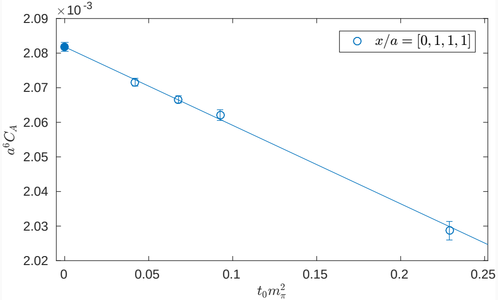
- The equality $C_V = C_A$ follows from the assumption of quarks being massless and of working in the flavour-charged sector.

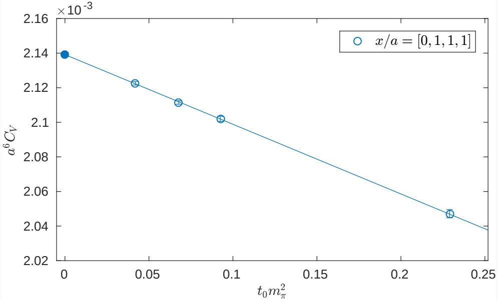
Intermediate steps

1. Average over sites that are equivalent with respect to the hypercubic symmetry (e.g. $(1,1,1,1) \sim (1,1,1,-1) \sim (1,1,-1,-1)$, etc.).
2. At fixed lattice spacing a, $C_{V,A}$ are computed at different quark masses $\Rightarrow$ chiral limit is needed to find the massless correlator.
3. Reduce discretization effects
   - tree level improvement: compare $C_{V,A}$ obtained on a unit gauge configuration and in the continuum **[K. G. Chetyrkin and A. Maier, Nucl.Phys. B844 (2011) 266-288, arXiv:1010.1145]**;
   - one loop improvement: Numerical Stochastic Perturbation Theory (NSPT) framework adapted to the QCD action in use **[F. Di Renzo and L. Scorzato, JHEP 0410 (2004) 073, arXiv:hep-lat/0410010]**.
4. Interpolate correlators at the same physical distance for every a.
5. Take the continuum limit of C_V and C_A.
6. Average axial and vector channels.
7. Use 4-loop perturbative expansion of C_V to determine $\alpha_{\overline{\mathrm{MS}}}$.
   Warning: this applies only for distances which are sufficiently small.

Results

Example of chiral extrapolation for $C_{V,A}$ at $a = 0.064$ fm



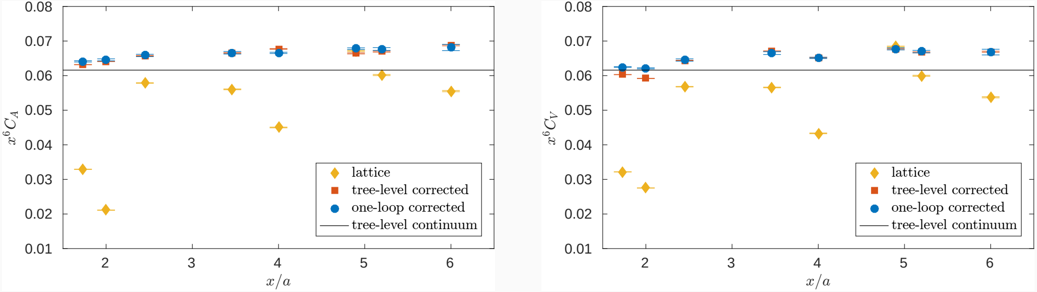


- Introduce a dimensionless variable $y = t_0 m_\pi^2$, proportional to the renormalized quark mass.
- t_0 is an artificial scale introduced in [M. Lüscher, JHEP 1008 (2010) 071, arXiv:1006.4518].
  In physical units $\sqrt{8t_0} = 0.415(4)(2)$ fm, [M. Bruno et al., PRD 95 (2017) no.7, 074504, arXiv:1608.08900].
- To obtain the chiral limit, we perform an extrapolation using a linear fit with respect to $t_0 m_\pi^2$, as suggested from Chiral Perturbation Theory:

$$C_{V,A}(x, m_\pi) = C_{V,A}(x, m_\pi = 0) + k \times t_0 m_\pi^2$$

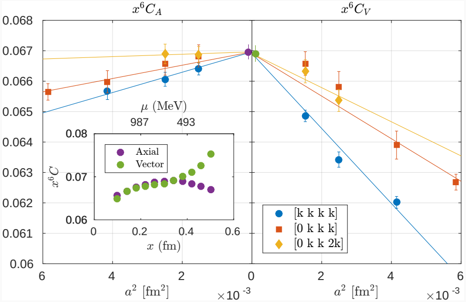
Reduction of discretization effects

Example at lattice spacing $a = 0.039$ fm



- Different kinds of points are affected by different lattice artifacts.
- Subtracting tree-level and one-loop lattice artifacts is important to reduce the size of these unwanted effects and allows a better continuum extrapolation.

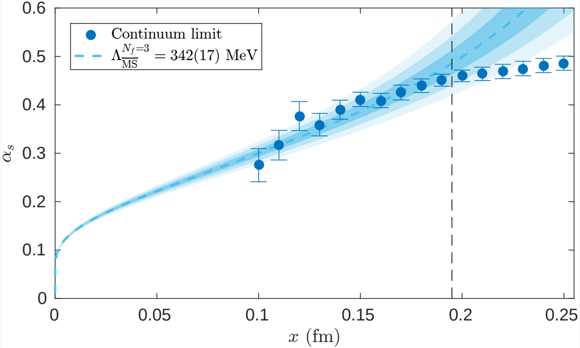
Example at the distance $X = 0.15$ fm



- X^6C is found at a fixed distance X through interpolation.
- Two interpolation ansätze: linear and quadratic in X^2. The difference of the two interpolation models is taken as systematic uncertainty.
- Interpolations are performed using points of the same type: [k k k k], [0 k k k] and [0 k k 2k].
- Combined best-fit for the continuum extrapolation.

Important remark

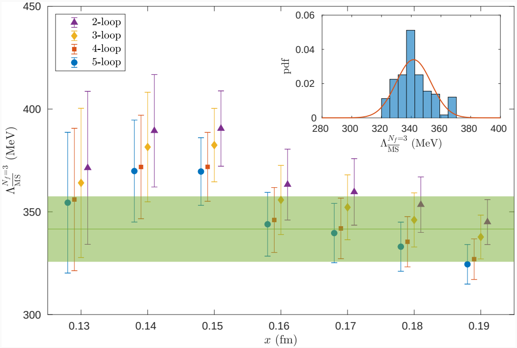
- At short distances, $C_V - C_A$ is reliably provided by the OPE [M. Shifman et al., Nuclear Physics B 147, 385 (1979)]. Using estimates from [T. Schäfer and E. V. Shuryak, Phys. Rev. Lett. 86, 3973 (2001)], the relative difference ranges from 0.03% at $x = 0.1$ fm up to 1.5% at $x = 0.3$ fm. Hence, within the statistical and systematic precision of our data, the two correlators are indistinguishable in that range of distances.



- From $X^6 C$ we can obtain α_s through PT [K. G. Chetyrkin and A. Maier, Nucl.Phys. B844 (2011) 266-288, arXiv:1010.1145].
- At distances above around 0.20 fm (scales below 1 GeV), we observe that the running of the coupling freezes, indicating the breakdown of PT.

- Using the RG equation, we convert our results for α_s and obtain:
  $$\Lambda_{\overline{\rm MS}}^{N_f=3} = 342(17)\ \text{MeV}$$
- The shaded blue band is the corresponding 5-loop perturbative running (1σ, 2σ, 3σ).
- Good agreement with the previous estimates, e.g.:
  $\Lambda_{\overline{\rm MS}}^{N_f=3} = 341(12)$ MeV [M. Bruno et al., PRL 119 (2017) no.10, 102001, arXiv:1706.03821].

Extracting the Λ parameter



Several sources of uncertainties

- statistical
- chiral limit
- NSPT
- interpolation
- errors on Z_A, Z_V
- truncation
- choice of the window of physical distances

- Decomposition of the total error:

$$\begin{aligned}\Lambda_{\overline{\rm MS}}^{N_f=3} &= 342(2.9)^{\rm lat}_{\rm stat}(5.0)_{\rm chiral}(6.5)^{\rm NSPT}_{\rm stat}(6.4)^{\rm NSPT}_{\rm infV}(0.8)_{Z_A} \\ &\quad (1.0)_{Z_V}(0.4)_{\rm interpol}(4.8)_{\rm trunc}(12)_{\rm window}\ \rm MeV \\ &= 342(17)\ \rm MeV\end{aligned}$$

Conclusions

Summary

Conclusions

- We tested a new method to extract the running coupling α_s from current-current correlation functions in position space and used it to determine $\Lambda_{\overline{\mathrm{MS}}}^{N_f=3}$.
- Using a combination of state-of-the-art simulations and novel analysis techniques, one can find a window of available scales μ and provide an estimate of $\Lambda_{\overline{\mathrm{MS}}}^{N_f=3}$ with a competitive precision (around 5%).
- Crucial steps:
  1. perturbative subtraction of hypercubic artifacts;
  2. combined continuum extrapolation using four lattice spacings and several lattice directions, which allowed us to control discretization effects at small distances in lattice units;
  3. independent evaluation of C_V and C_A, which have a common continuum limit at small distances $\Rightarrow$ characterize the quality of continuum extrapolations and gain in precision.

Future plans

- A similar strategy can be used for the determination of other quantities, such as the quark and gluon condensates.
- Test our method for $N_f = 0$ QCD, for which smaller lattice spacings can be easier reached (better contact with PT)

Thank you very much for your attention