From QCD string breaking to quarkonium spectrum

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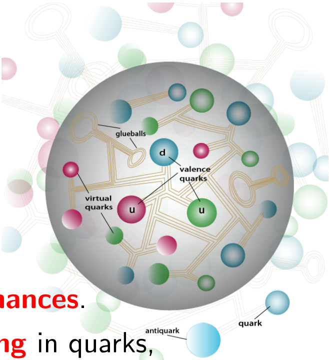
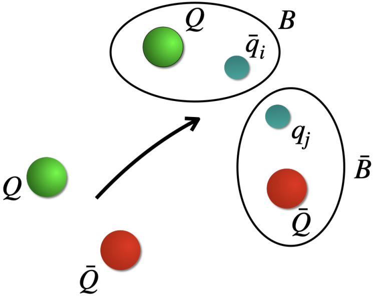
Motivation



- Interest in **quarkonium states** and **resonances**.
- Study the phenomenum of **string breaking** in quarks, analyzing its implication in the study of the spectrum.
- Looking for **exotic states**: tetraquarks, pentaquarks, ...
- Exploring new techniques in Lattice QCD for **improving signal to noise ratio**.

String breaking



- String breaking occurs when distance between two quarks ($Q\bar{Q}$) increases.
- In this case it is more convenient for the system to produce couple meson-meson ($B\bar{B}$).
- The potential describing the system before string breaking occur is (Cornell potential)

$$V(r) = A + \frac{B}{r} + \sigma r$$

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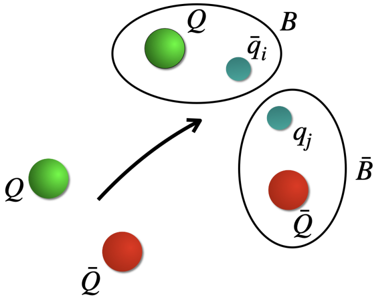
E. Eichten et al. (1975)
E. Eichten et al. (1978)
C. Bernard et al. (2001)

String breaking



J. Bulava et al. (2019)
V. Koch et al. (2019)

- At the end, we have 2 mesons $B = Q\bar{q}$, $\bar{B} = \bar{Q}q$ with mass E_B.
- Condition for string breaking:

$$V(r) - 2E_B > 0.$$

- Just Wilson loops are not enough to get string breaking: quarkonium and meson-meson operators are needed. *Alternatively, see F. Gliozzi, A. Rago (2005)*
- An interested approach is to study the matrix correlator: *Bali et al. (2005)*

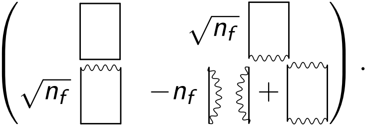
$$C(t) = \begin{pmatrix} C_{QQ}(t) & C_{QB}(t) \\ C_{BQ}(t) & C_{BB}(t) \end{pmatrix}$$

Computation

Matrix of correlators:

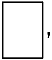
$$C(t) = \begin{pmatrix} C_{QQ}(t) & C_{QB}(t) \\ C_{BQ}(t) & C_{BB}(t) \end{pmatrix}$$

$$= e^{-2m_Q t}$$



We concentrate on the first element and get the potential:

$$C_{QQ}(t) = e^{-2m_Q t}$$



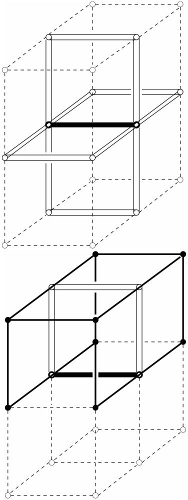
$$V_{QQ}(r) = \lim_{t\to\infty} \frac{1}{a} \log\left(\frac{C_{QQ}(t)}{C_{QQ}(t+a)}\right).$$

Lattice setup

- 79 configurations generated with $n_f = 2$ $O(a)$ improved Wilson fermion action (CLS ensembles).
- Lattice volume: $64x32x32x32$.
- $m_\pi = 330\ MeV$.
- Lattice spacing: $a = 0.0755(11)\ fm$.

On smearing techniques

The improving of the signal pass through the use of the right gauge configurations. We explore different methods:



- APE smearing (smoothing over nearest gauge links). It depends on 1 parameter α: $\alpha = 0.5$ or 0.7
  M. Albanese et al. (1987)

- HYP smearing (smoothing on hypercubes). It depends on 3 parameters $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$.
  $\boldsymbol{\alpha} = (0.75, 0.6, 0.3)$
  A. Hasenfratz et al. (2002)
  - ➔ HYP2 $\Rightarrow$ improved choice of parameters.
    $\boldsymbol{\alpha} = (1.0, 1.0, 0.5)$
    M. Donnellan et al. (2011)

On smearing techniques

$$W(r,t)_{lm} = \left\langle tr\left\{ V_t^\dagger(r,0) U_r(t,0)^{(l)} V_0(r,0) U_0^\dagger(t,0)^{(m)} \right\}\right\rangle$$

where

$$U_r(t,0)^{(l)} = (\mathcal{S}_{sm})^{n_l} U_r(t,0)$$

$$\hat{W}(r,t)\mathbf{v} = \lambda(r,t)\hat{W}(r,t_0)\mathbf{v}$$

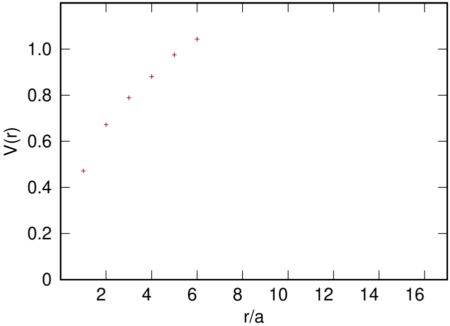
$$V(r) = \lim_{t\to\infty}\frac{1}{a}\log\left(\frac{\lambda(r,t)}{\lambda(r,t+a)}\right)$$

- sHYP $\Rightarrow$ HYP smearing in the spatial direction of the links. $\alpha_2 = 0.6, \alpha_3 = 0.3$
- Solving the GEVP problem. $\Rightarrow$ The matrix given by different smearing levels is used to get the generalized eigenvalues.

M. Donnellan et al. (2011)
M. Della Morte et al. (2004)

Potentials

No smearing

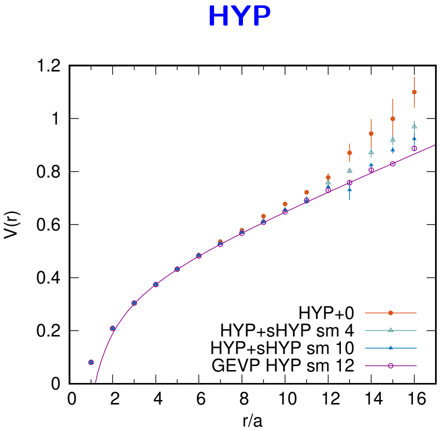


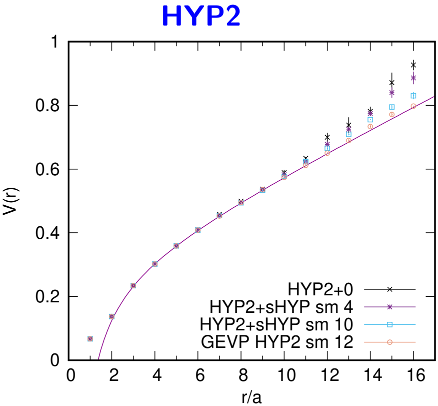
Only at small r the potential can be obtained and at large t the data gets noisy.

$$V_{QQ}(r) = \lim_{t\to\infty} \frac{1}{a} \log\left(\frac{C_{QQ}(t)}{C_{QQ}(t+a)}\right) = -2m_Q + \frac{1}{a}V(r)$$

Potentials: different smearing levels

The more sHYP is applied, the more the signal gets in agreement with the theoretical expectation for large r/a.

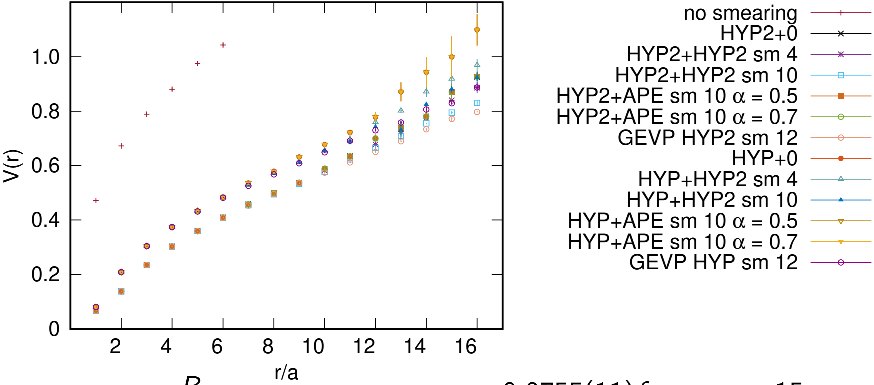




GEVP procedure seems to give better results.

Different smearing choices

Smearing shifts the potential by a constant factor. Low-r region not well described with our parametrization and smearing choices. The potential with HYP2 is below the potential with HYP smearing.



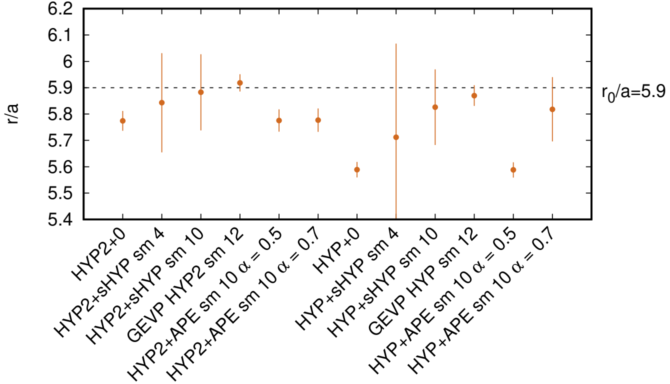
$$V(r) \propto \frac{B}{r} + \sigma r$$

$$a = 0.0755(11) fm \Rightarrow r_c \simeq 15a$$

Bali et al. (2005)

Sommer parameter

The more smearing is applied in the spatial direction, the more the Sommer parameter approaches to $r_0/a = 5.9$ (reference value taken from *P. Fritzsch et al. (2012)*).



$$F(r) = V'(r)$$

$$r_0^2 F(r_0) = 1.65$$

R. Sommer (1994)

Summary

- Different smearing techniques of gauge links are studied, APE, HYP, HYP2 and GEVP.
- Smearing improves the signal for large distances but it is not good for short distances.
- The GEVP procedure gives the best signal to noise ratio and agreement with the theory.

For more info, look here *M.Catillo, M. Marinković, P. Bicudo, N. Cardoso, arXiv:2005.05723v2 (2020).*

Going further...

The potential that one gets from the heavy quark-antiquark system can be plugged in the Schrödinger equation (Born-Oppenheimer approximation):

$$\left(- \frac{1}{2}\mu^{-1}\left(\partial_r^2 + \frac{2}{r}\partial_r - \frac{\mathbf{L}^2}{r^2}\right) + V(\mathbf{r}) + 2m_M - E \right)\psi(\mathbf{r}) = 0$$

and one can determine e.g. for bottomonium bound states and resonances (*P. Bicudo et al. (2020)*), including potential tetraquark resonances.

⇒See also L. Mueller and M. Wagner talks for similar studies.

Outlook

- Obtain remaining elements of the correlator matrix (renormalize the potential, include mixing effects) to get the info on the string breaking.
- Use the Born-Oppenheimer approximation approach
  - ➔ to get the spectrum with increased precision (cross-check), and look for exotic bound states.
  - ➔ Repeat calculation for different gauge ensembles (continuum limit for string breaking with dynamical quarks still missing in the literature).
- Systematical study of the noise-reduction techniques (different stochastic methods, distillation and so on).
- Exploring the static potentials with C^* boundary conditions.

Thank you!