Hadronic light-by-light scattering contribution to $(g-2)_\mu$: results near the $\mathrm{SU}_f(3)$ point

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Introduction

- There is a 3.5-4 σ discrepency between theory and experiment on the anomalous magnetic moment of the muon (a_μ)
- **Hadronic Light-by-Light** (HLbL) scattering ($\mathcal{O}(\alpha_{\mathrm{QED}}^3)$) plays an important role in the theoretical uncertainty determination
- Mainz's position-space approach : QED in the continuum & infinite volume + QCD part on the lattice [N. Asmussen et al., LATTICE '19]

$$a_\mu^{\mathrm{hlbl}} \equiv \frac{(g-2)_\mu}{2} = \frac{m_\mu e^6}{3} \int \mathrm{d}^4 y \int \mathrm{d}^4 x \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) \quad (1)$$

$$\begin{aligned} i\hat{\Pi}_{\rho;\mu\nu\lambda\sigma} &= -\int \mathrm{d}^4 z z_\rho \tilde{\Pi}_{\mu\nu\sigma\lambda} \\ \tilde{\Pi}_{\mu\nu\sigma\lambda}(x,y,z) &\equiv \Big\langle j_\mu(x) j_\nu(y) j_\sigma(z) j_\lambda(0) \Big\rangle_{\mathrm{QCD}} \end{aligned} \quad (2)$$

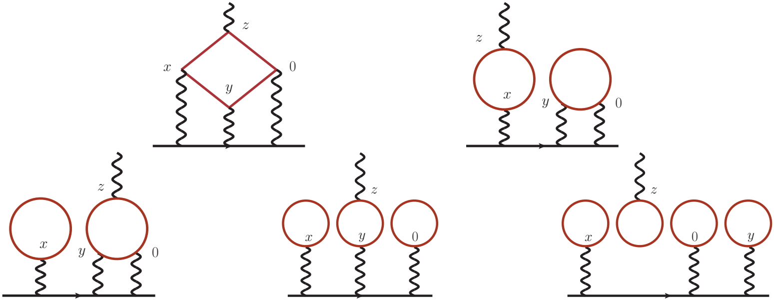
j_μ's are electromagnetic currents. In $N_f = 3$,

$$j_\mu(x) = \frac{2}{3}(\bar{u}\gamma_\mu u)(x) - \frac{1}{3}(\bar{d}\gamma_\mu d)(x) - \frac{1}{3}(\bar{s}\gamma_\mu s)(x) \quad (3)$$

Goal : to control the long-ranged QED effects

Content of the talk :

- Optimization of the computational cost
- Our dedicated study at $\mathrm{SU}_f(3)$ in order to understand how to correct the finite size effects (FSEs) [EHC et al., arXiv:2006.16224[hep-lat]]
- Some preliminary results for the $\mathrm{SU}_f(3)$-suppressed diagrams (also N_c-suppressed)



Computational strategy

- Making use of the restored $\mathcal{O}(4)$-symmetry of the QED kernel, we write a_μ^{hlbl} as

$$a_\mu^{\mathrm{hlbl}} = \int_0^\infty \mathrm{d}|y| f(|y|) \tag{4}$$

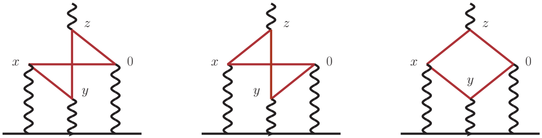
- Change of variables in the integrand $\Rightarrow$ only compute the "easy" diagrams for each case.
- Point sources on a line or a 2-D grid to increase number of samples per $|y|$
- Single-propagator trace available in position-space computed via the One-End-Trick

  [K. Jansen, C. Michael, C. Urbach, EPJC '08 ; L. Giusti *et al.*, EPJC '19]

Computational strategy

Examples of computed diagrams

- $a_\mu^{\rm hlbl,conn}$: **Method 1** (direct) all 3 diagrams vs. **Method 2** (change of variables) only the left-most



- Factorized expression for disconnected contribution, *e.g.* $a_\mu^{\rm hlbl,3+1}$

$$
\begin{aligned}
&a_\mu^{\rm hlbl,3+1} = \\
&\frac{2me^6}{3}\sum_{i,j} c_i c_j^3 \Big\{ \Big\langle \int_{xy} \left(\mathcal{L}_{[\rho,\sigma]\nu\mu\lambda}(y,x) + \mathcal{L}_{[\rho,\sigma]\mu\nu\lambda}(x,y) + \mathcal{L}_{[\rho,\sigma]\lambda\nu\mu}(-x,y-x)\right) T^i_\mu(x) \int_z z_\rho R^j_{\lambda\nu\sigma}(0,y,z) \Big\rangle \\
&\qquad - \Big\langle \int_{xy} \mathcal{L}_{[\rho,\sigma]\lambda\nu\mu}(-x,y-x) x_\rho T^i_\mu(x) \int_z R^j_{\lambda\nu\sigma}(0,y,z) \Big\rangle \\
&\qquad + \Big\langle \int_z z_\rho T^i_\sigma(z) \int_{xy} \mathcal{L}_{[\rho,\sigma]\mu\nu\lambda}(x,y) R^j_{\mu\nu\lambda}(x,y,0) \Big\rangle \Big\}
\end{aligned} \tag{5}
$$

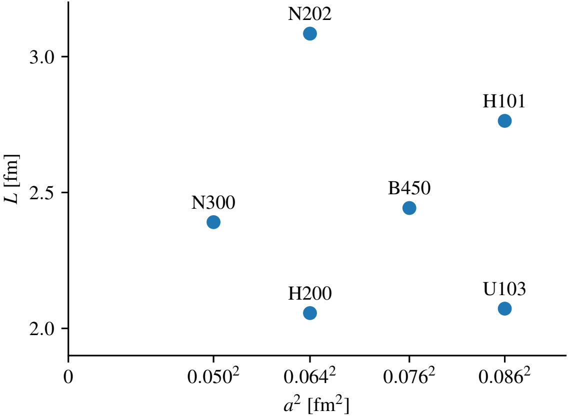
c_i : prefactor for the quark i

$T^i_\mu(x) \equiv \mathrm{Im}\big(\mathrm{tr}[\gamma_\mu S^i(x,x)]\big)$, the single-propagator trace

$R^j_{\lambda\nu\sigma}(0,y,z) \equiv \mathrm{Im}\Big(\mathrm{tr}[\gamma_\lambda S^j(0,y)\gamma_\nu S^j(y,z)\gamma_\sigma S^j(z,0)]\Big)$, the triangle diagram

FSE correction at $SU_f(3)$
Lattice landscape

- We use ensembles generated by the CLS consortium
- $m_\pi \approx 420$ MeV

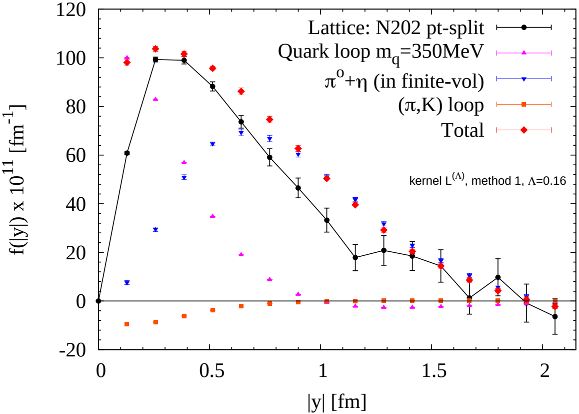


FSE correction at $SU_f(3)$

- Advantages of our $SU_f(3)$ ensembles : cheap to invert with large $m_\pi L$ (4.4 to 6.4)
- Only 2 non-vanishing Wick contraction classes : fully-connected and $(2+2)$-disconnected
- Meson spectrum study for the ensemble H101 using distillation method [M. Peardon et al., PRD '09 ; PRD '11]
  $\Rightarrow$ the FSEs can be reasonably modelled by PS meson contributions

J^{PC}	octet		singlet	
0^{-+}	π	418(1)(0)	η'	865(32)(61)
0^{++}	a_0	856(27)(12)	f_0	678(20)(10)
1^{++}	a_1	1264(7)(9)	f_1	1427(32)(50)
1^{--}	ρ	853(2)(1)	ω	884(5)(3)
1^{+-}	b_1	1329(9)(3)	h_1	1330(16)(30)

- At long distances, pseudoscalar (PS) meson pole contributions dominate
- Parametrize the pion transition form factor (TFF) with Vector Meson Dominance (VMD) model [A. Gérardin, H.B. Meyer & A. Nyffeler, PRD '19]
- Matching QCD diagram to the relevant pion exchange diagrams using Partially-Quenched ChPT



PS pole exchange describes well the tail of $f(|y|)$, but there is still some deficit (might come from the scalar meson exchange).

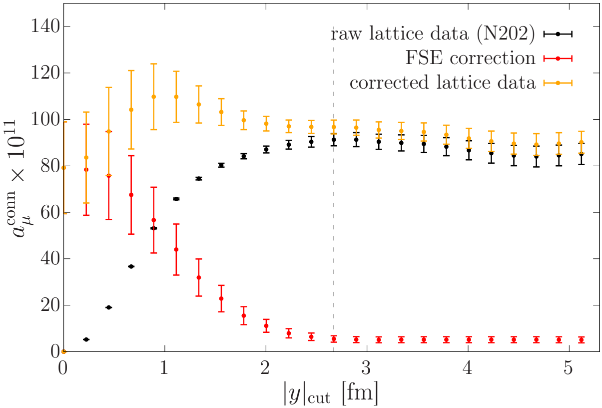
Our procedure to correct the FSE of the lattice data

- $a_\mu = a_\mu^{\rm data} + a_\mu^{\rm tail} + a_\mu^{\rm FSE}$ where

  $a_\mu^{\rm data}$: integrated lattice data up to $|y| = |y|_{\rm cut}$

  $a_\mu^{\rm FSE}$: volume correction with PS-exchange up to $|y|_{\rm cut}$

  $a_\mu^{\rm tail}$: modelling the integrand using PS-exchange for $|y| \geq |y|_{\rm cut}$
- 25% systematics to a_μ^{FSE} and a_μ^{tail}, added to the statistical error in quadrature
- $|y|_{\rm cut}$ is chosen such that the total error is minimized



Results

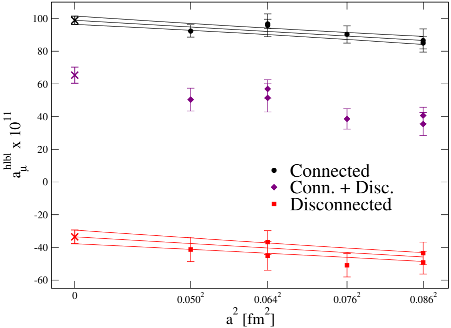
At $SU_f(3)$

- Extensive use of 4 renormalized unimproved local currents [A. Gérardin, T. Harris & H. B. Meyer, PRD '19]
- Continuum extrapolation of the connected and disconnected contribution änsatze

$$a_\mu^{\rm conn}(a) = a_\mu^{\rm conn}(0) + Aa^n, \quad a_\mu^{\rm disc}(a) = a_\mu^{\rm disc}(0) + Ba^m \qquad (6)$$

- Final result quoted from a linear fit in a^2 for $a_\mu^{\rm conn}$ and $a_\mu^{\rm disc}$ with the same slope

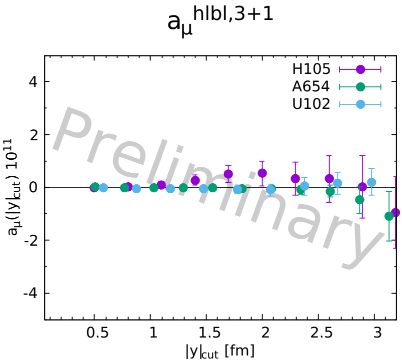
$$a_\mu^{\rm hlbl} = 65.4(4.9)_{\rm stat.+FSE}(6.6)_{\rm fit\ sys.} \times 10^{-11} \qquad (7)$$

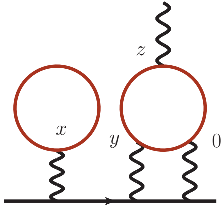


Results

Preliminary lattice results for $(3+1)$

Label	size	bdy. cond.	a (fm)	m_π (MeV)
H105	$32^3 \times 96$	open	0.08636(98)(40)	280
U102	$24^3 \times 128$	open	0.08636(98)(40)	350
A654	$24^3 \times 48$	periodic	~ 0.09929	~ 330

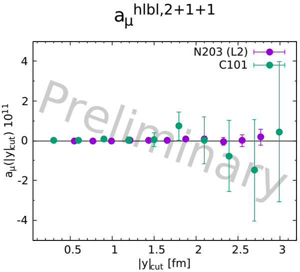


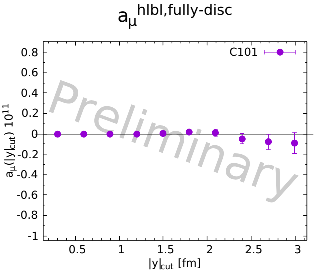


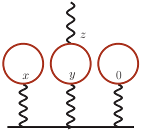
Results

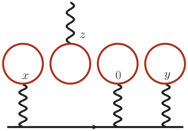
Preliminary lattice results $(2+1+1)$ and the fully disconnected

Label	size	bdy. cond.	a (fm)	m_π (MeV)
C101	$48^3 \times 96$	open	0.08636(98)(40)	220
N203	$48^3 \times 128$	open	0.06426(74)(17)	340







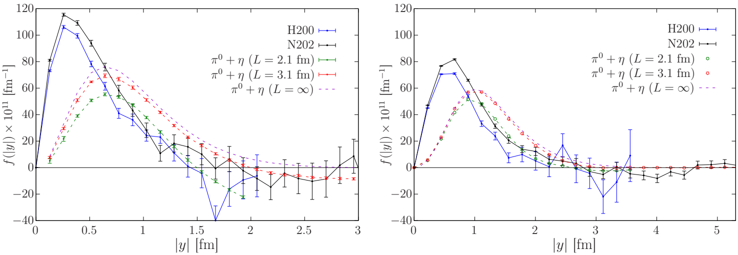


Summary and outlook

- Change of variables in the integrand provides an efficient way to calculate a_μ^{hlbl} by avoiding diagrams expensive to compute
- We develop a systematic way of correcting the FSE's based on PS-pole exchange, which seems to give satisfactory results at $\mathrm{SU}_f(3)$ with $m_\pi \approx 420$ MeV.
- Our preliminary results for the $\mathrm{SU}_f(3)$-suppressed topologies appear to be small as expected.
- As the data is expected to be much noisier as we approach physical pion mass, a better understanding of the integrand might play a crucial role.

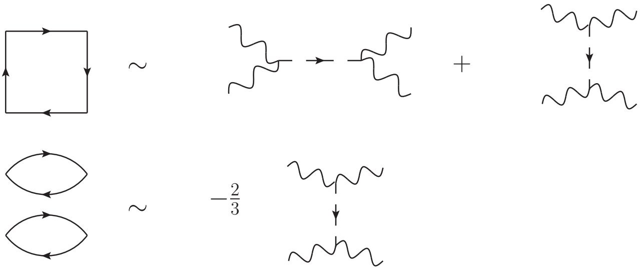
Back-up slides

$SU_f(3)$ FSE : VMD prediction vs. direct lattice comparison



$a_\mu^{\text{conn.}}$ intergrand for Method 1 (left) and Method 2 (right)

$\mathrm{SU}_f(3)$: diagram matching between different contractions



Mapping between different QCD Wick-contractions (left) and pseudo-scalar-meson exchange channels (right).