$B_{(s)} - \bar{B}_{(s)}$ mixing on domain-wall lattices

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RBC/UKQCD and JLQCD

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Outline

Motivation

- $B_{(s)} - \bar{B}_{(s)}$ mixing gives access to CKM matrix elements $|V_{ts}|$ and $|V_{td}|$
  - $\Rightarrow$ test whether the CKM matrix is indeed unitary
- Data produced on RBC-UKQCD and JLQCD ensembles using Grid and Hadrons.

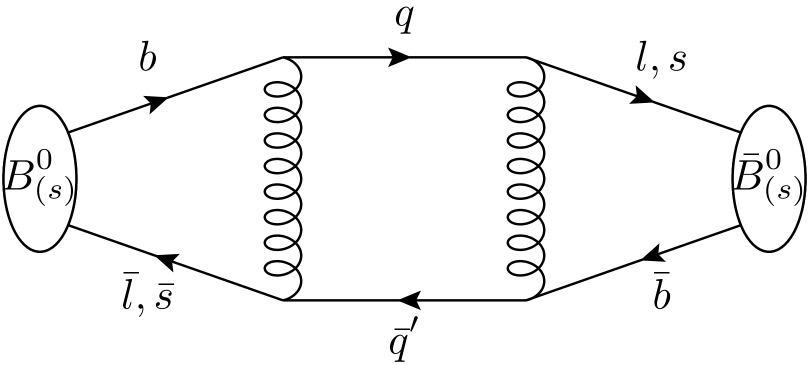
[github.com/paboyle/Grid]

Hadrons

[github.com/aportelli/Hadrons]

Related RBC/UKQCD and JLQCD talks:

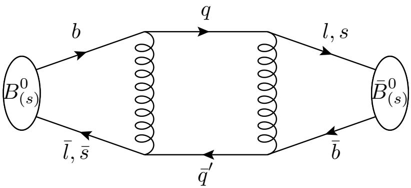
- Semileptonic $B_s \to K$ and $B_s \to D_s$ decays **[Wed 16:00, Tobias Tsang]**
- $B->D^{(*)}\ell\nu$ form factors from relativistic lattice QCD **[Wed 16:40, Takashi Kaneko]**
- Semileptonic $B \to \pi\ell\nu$ decays **[Wed 17:40, Ryan Hill]**


q
b
l, s
$B^0_{(s)}$
$\bar{B}^0_{(s)}$
$\bar{l}, \bar{s}$
$\bar{b}$
$\bar{q}'$

neutral meson mixing

W bosons heavier than anything we simulate on the lattice → treat mixing operator as point-like.

[arxiv 1812.08791]



$$C_3^{\mathcal{O}}(t, \Delta T) = \sum_{i,j} \frac{(M_{\rm snk})_i (M_{\rm src})_j}{4E_i E_j} \langle i|\mathcal{O}|j\rangle e^{-(E_j - E_i)(t - \Delta T/2)} e^{-(E_j + E_i)\Delta T/2}$$

with $(M_{\rm src/snk})_i \in \{\langle P|i\rangle, \langle A_4|i\rangle\}$
For $\rm src = snk = P$:

$$C_3^{\mathcal{O}}(t, \Delta T; PP) \approx \frac{P_0^2}{4E_0^2} \langle gr|\mathcal{O}|gr\rangle e^{-E_0 \Delta T} \Bigg[1 + 2\frac{P_1 E_0}{P_0 E_1} \frac{\langle gr|\mathcal{O}|ex\rangle}{\langle gr|\mathcal{O}|gr\rangle} e^{-\Delta E \Delta T/2} \cosh\left[\Delta E\left(t - \Delta T/2\right)\right] + \left(\frac{P_1 E_0}{P_0 E_1}\right)^2 \frac{\langle ex|\mathcal{O}|ex\rangle}{\langle gr|\mathcal{O}|gr\rangle} e^{-\Delta E \Delta T} \Bigg]$$

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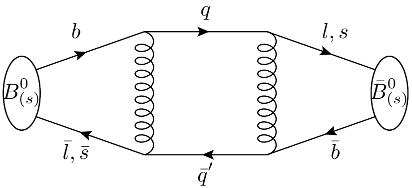

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$$+ 2 \frac{P_1 E_0}{P_0 E_1} \frac{\langle gr|\mathcal{O}|ex\rangle}{\langle gr|\mathcal{O}|gr\rangle} e^{-\Delta E \Delta T/2} \cosh\left[\Delta E (t - \Delta T/2)\right]$$
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Different fit strategies to extract $\langle gr|\mathcal{O}|gr\rangle$

- 1-state fit to a ratio of 3pt and 2pt functions [arxiv 1812.08791]
- extract P_0, A_0, E_0 and P_1, A_1, E_1 from a 2-state fit to heavy-light or heavy-strange 2pt functions (A and P simultaneously)
  - 1-state fit to 3pt functions, for each ΔT
  - 2-state fit to 3pt functions, for each ΔT
  - fit in ΔT direction for $C_3(t = \Delta T/2, \Delta T)$
  - could also include $\langle ex|\mathcal{O}|ex\rangle$
- extract all parameters in a simultaneous fit to 2pt and 3pt functions

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  - fit in ΔT direction for $C_3(t = \Delta T/2, \Delta T)$
  - could also include $\langle ex|\mathcal{O}|ex\rangle \Rightarrow$ cannot be resolved at our level of statistics
- extract all parameters in a simultaneous fit to 2pt and 3pt functions

  $\Rightarrow$ Correlation matrix becomes large and fits are less stable

Lattice setup

- RBC-UKQCD's 2+1 flavour domain wall fermions [arxiv 1411.7017]
  - pion masses from $m_\pi = 139$ MeV to $m_\pi = 430$ MeV
  - several heavy-quark masses from below m_c to $0.5 m_b$, using a stout-smeared action ($\rho = 0.1, N = 3$) with $M_5 = 1.0, L_s = 12$ and Moebius-scale $= 2$ [arxiv:1812.08791]
- JLQCD's 2+1 flavour domain wall fermions [arxiv 1711.11235]
  - pion masses from $m_\pi = 226$ MeV to $m_\pi = 310$ MeV
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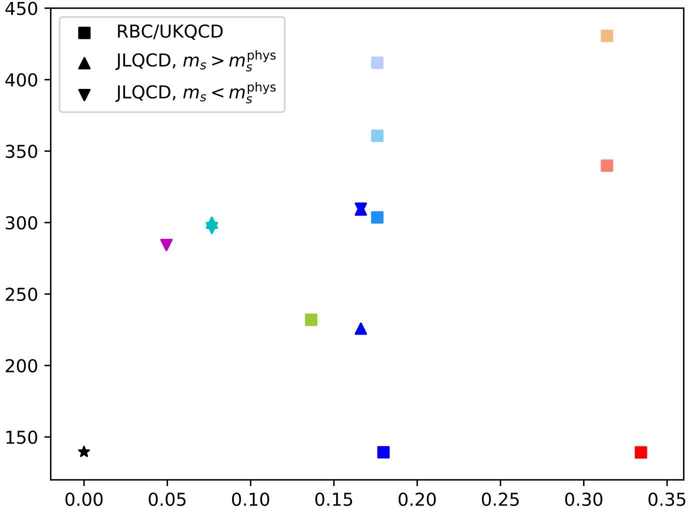
first glance at one RBC-UKQCD ensemble (F1M) [arxiv 1701.02644]:

- $m_\pi = 232 MeV$
- solve on every 2nd timeslice
- $16 \leq \Delta T \leq 48$
- m_s tuned to be near physical value
- 5 heavy masses $0.32 \leq m_h \leq 0.68$

Lattice setup

	L/a	T/a	a^{-1} [GeV]	m_π [MeV]	$m_\pi L$	hits $\times N_{\rm conf}$
RBC-UKQCD						
C0	48	96	1.7295(38)	139.2	3.86	48 $\times$ 90
C1	24	64	1.7848(50)	339.8	4.57	32 $\times$ 100
C2	24	64	1.7848(50)	430.6	5.79	32 $\times$ 101
M0	64	128	2.3586(70)	139.3	3.78	64 $\times$ 82
M1	32	64	2.3833(86)	303.6	4.08	32 $\times$ 83
M2	32	64	2.3833(86)	360.7	4.84	32 $\times$ 76
M3	32	64	2.3833(86)	411.8	5.51	32 $\times$ 81
F1M	48	96	2.708(10)	232.0	4.11	48 $\times$ 72
JLQCD						
C1L	48	96	2.453(4)	225.8	4.4	tbd
C2a	32	64	2.453(4)	309.7	4.0	16 $\times$ 100
C2b	32	64	2.453(4)	309.1	4.0	16 $\times$ 100
M1a	48	96	3.610(9)	296.2	3.9	24 $\times$ 50
M1b	48	96	3.610(9)	299.9	3.9	24 $\times$ 50
F1	64	128	4.496(9)	284.3	4.0	32 $\times$ 50

Landscape plot of our ensembles



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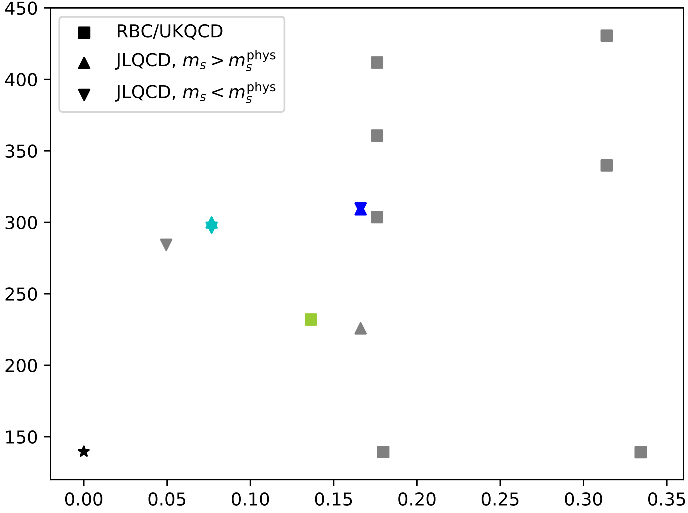
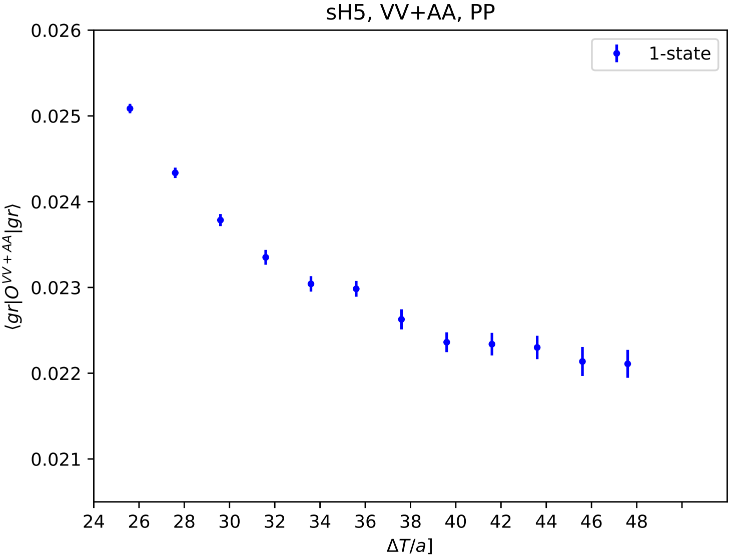


comparison of fit strategies



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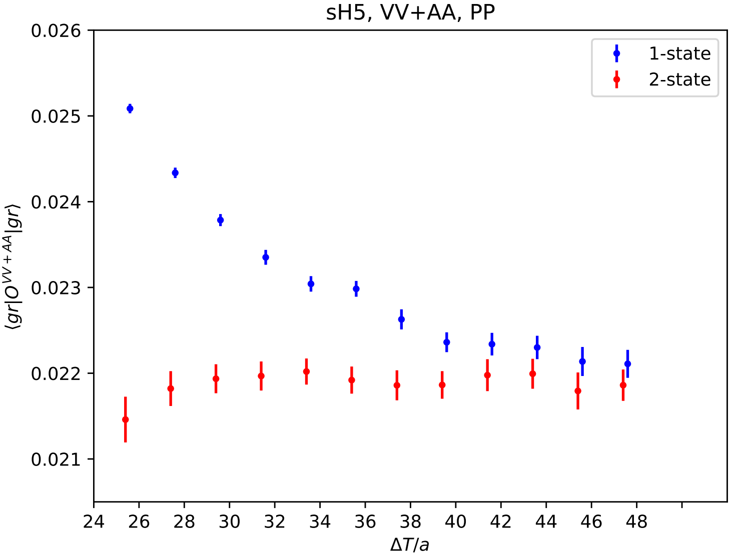

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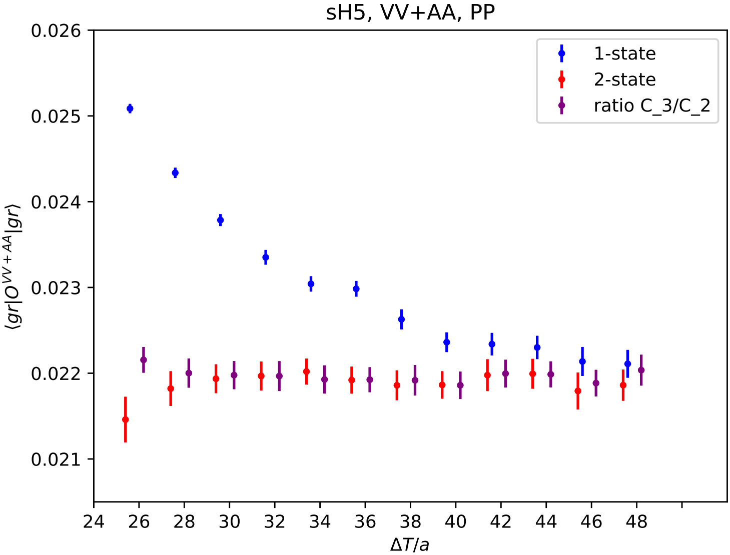
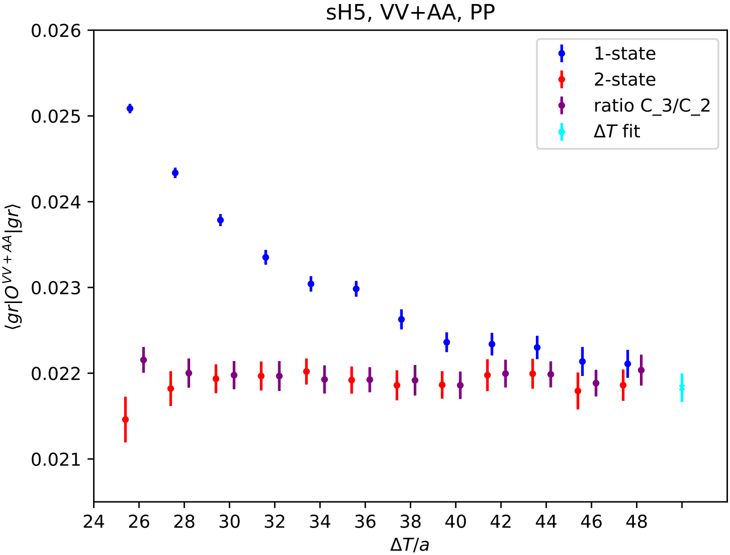
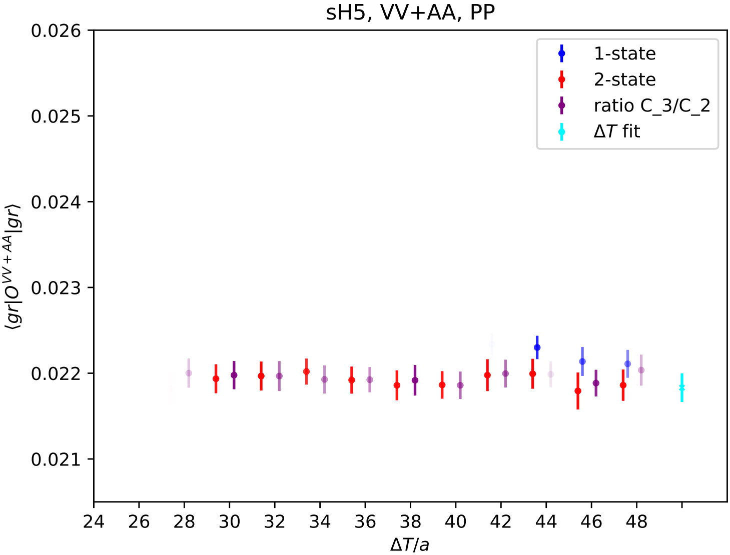
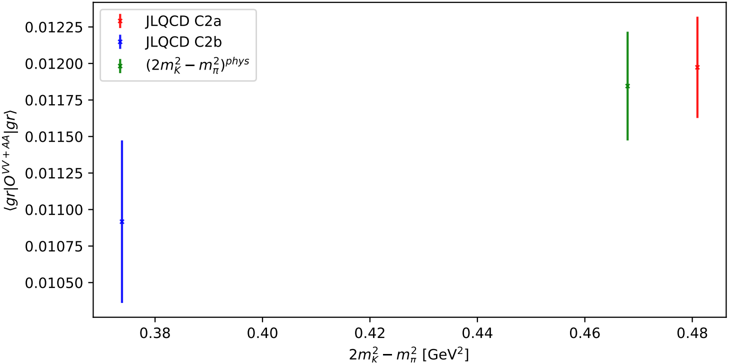



sH5, VV+AA, PP
0.026
0.025
0.024
0.023
0.022
0.021
$\langle gr|O^{VV+AA}|gr\rangle$
24 26 28 30 32 34 36 38 40 42 44 46 48
$\Delta T/a$
1-state
2-state
ratio C_3/C_2
ΔT fit

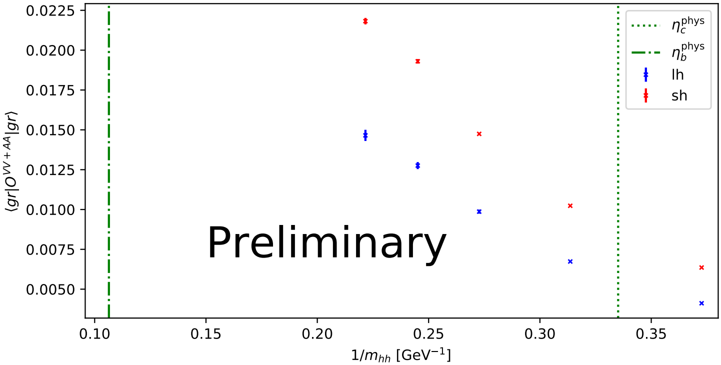
comparison of fit strategies



strange mass interpolation in JLQCD ensembles

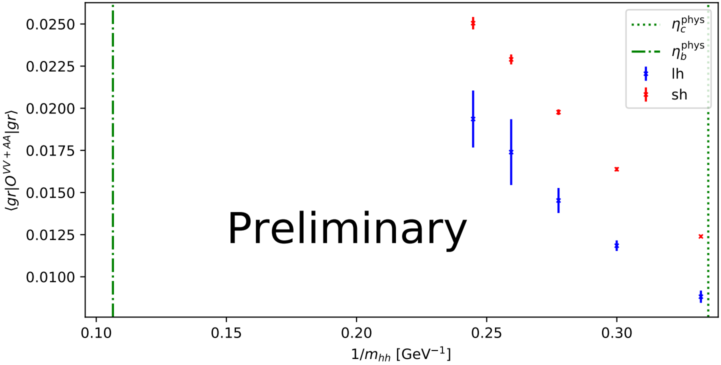


Interpolation in $2m_K^2 - m_\pi^2$ on JLQCD ensembles. The RBC/UKQCD ensembles are tuned to be near physical strange quark mass, so we don't have to do this step.



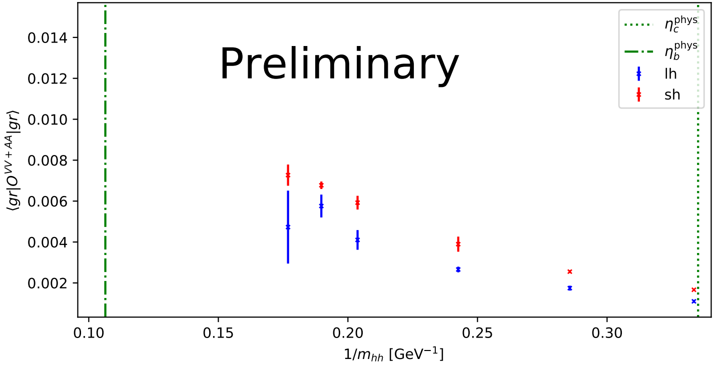
RBC-UKQCD F1M ensemble

for each meson, we chose one representative fit of the 2-state fits (red data points on earlier slides). The green vertical lines are the physical masses of η_c and η_b.



JLQCD C2 ensemble

The JLQCD ensembles are not tuned to have a near-physical strange mass (like the RBC-UKQCD ones do), so we do a linear interpolation between each pair of ensembles in $2m_K^2 - m_\pi^2$



JLQCD M1 ensemble

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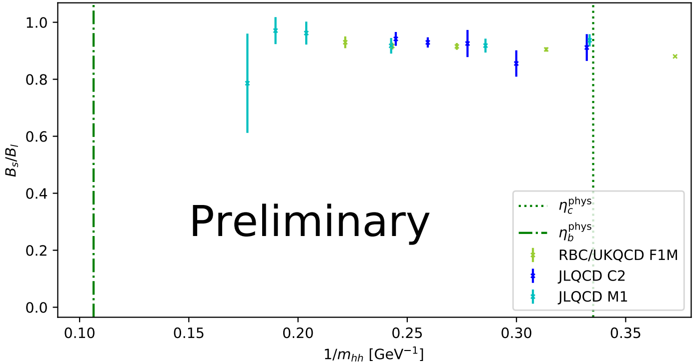

with bag parameters

$$B_P = \frac{\langle gr|O|gr\rangle}{8/3 f_P^2 m_P^2}$$

The standard-model operator

$$O_1 = O^{VV+AA} \,,$$

forms a full basis with four other operators

$$\begin{aligned} O_2 &= O^{VV-AA} \\ O_3 &= O^{SS-PP} \\ O_4 &= O^{SS+PP} \\ O_5 &= O^{TT} \,. \end{aligned}$$

This set of operators has a block-structure, meaning that O_2, O_3 as well as O_4, O_5 mix. O_1 is linearly independent from the others. **[arxiv 1708.05552]**

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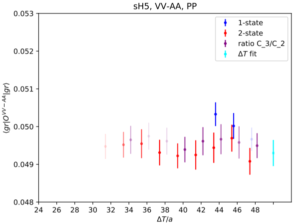
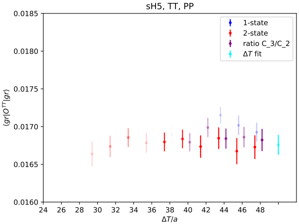
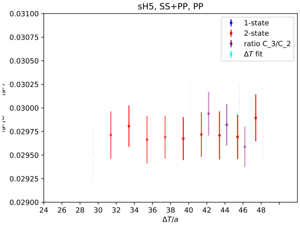
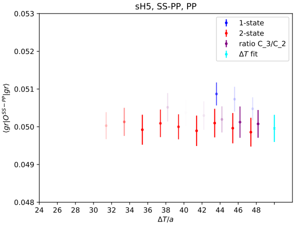
This set of operators has a block-structure, meaning that O_2, O_3 as well as O_4, O_5 mix. O_1 is linearly independent from the others. [arxiv 1708.05552]
We need to do a non-perturbative renormalisation (NPR) for those operators as well which we have not done yet as part of this work but are planning to implement as a next step! [arxiv 1812.08791]

non-SM operators

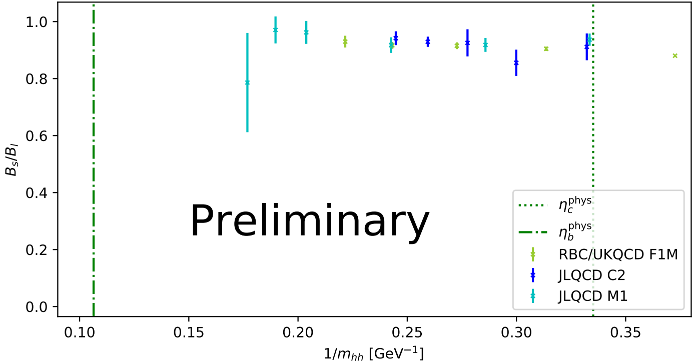








Heavy-mass dependence shown earlier:



The JLQCD F1 ensemble can get us much closer to the physical η_b mass.

possible heavy quark masses

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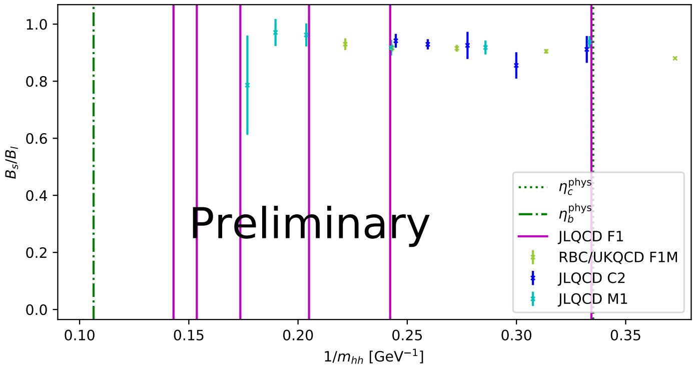

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Conclusions & Outlook

Conclusions:

- We can extract bag parameter matrix elements $\langle gr|\mathcal{O}|gr\rangle$
  - $\Rightarrow$ Consistently using a variety of methods

Outlook:

- want to go closer to physical m_b mass (JLQCD ensemble F1)
- We have done measurements already on a number of ensembles, and we will repeat this analysis on these.
  - $\Rightarrow$ Continuum limit, chiral limit, ...

Thank you!

Backup