Lattice Calculation of the Pion Light-Cone Distribution Amplitude

THE HOPE COLLABORATION

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Outline

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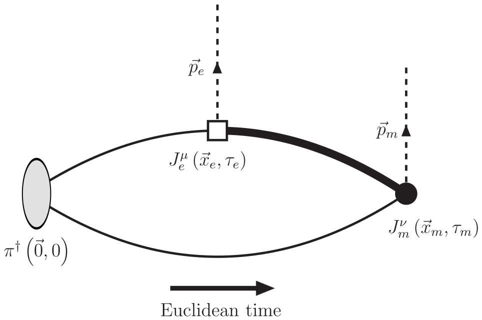
where

$$U^{\mu\nu}(p,q) \equiv \int d^4z\, e^{iq\cdot z} \left\langle 0 \left| \mathcal{T} \left[J^\mu \left(\frac{z}{2}\right) J^\nu \left(-\frac{z}{2}\right)\right] \right| \pi(\mathbf{p}) \right\rangle$$

$$J^\mu \equiv \bar{\Psi}\gamma^\mu\gamma^5\psi + \bar{\psi}\gamma^\mu\gamma^5\Psi$$

and where Ψ is an artifically heavy valence quark

Correlator on Lattice



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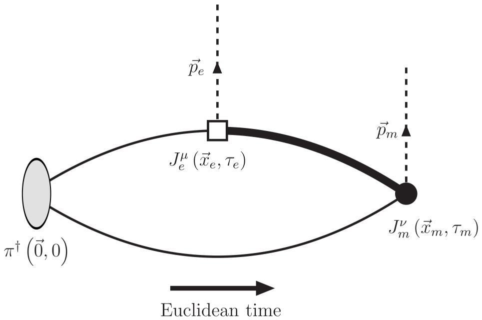

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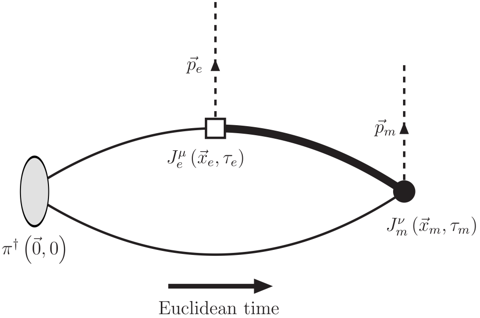

$$\tau = \tau_m - \tau_e,\ p = p_e + p_m,\ q = (p_m - p_e)/2$$

- Fix $\tau_e, \mathbf{p}_e$ and allow $\tau_m, \mathbf{p}_m$ to vary
- Fourier transforming over the spatial components gives

$$R^{\mu\nu}(\tau; \mathbf{p}, \mathbf{q}) = \int d^3\mathbf{z}\, e^{i\mathbf{q}\cdot\mathbf{z}} \left\langle 0 \middle| \mathcal{T}\left[J^\mu\left(\frac{z}{2}\right) J^\nu\left(-\frac{z}{2}\right)\right] \middle| \pi(\mathbf{p}) \right\rangle$$

Lattice Details

- Heavy quark mass requires fine lattice spacing
  - Quark masses: $m_{\Psi}^{(0)} \approx 1.6, 2.5$ GeV
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- Cost of dynamical gauge configurations $\sim a^{-7}$ or worse
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- $m_\pi \sim 560$ MeV and $L \sim 1.92$ fm, so $m_\pi L \gtrsim 5$
- Wilson-clover fermions with c_{SW} set non-perturbatively

Ensembles Used

a (fm)	$L^3 \times T$	N_{cfg}	N_{src}	Light Props	Heavy Props
0.060	$32^3 \times 64$	450	6	2700	108,000
0.048	$40^3 \times 80$	250	2	500	20,000
0.041	$48^3 \times 96$	341	2	682	27,280

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$$\begin{aligned}\text{Re}[U(\mathbf{p}, q)] &= \text{Re}\left[\int_{-\infty}^{\infty} d\tau \, R(\tau; \mathbf{p}, \mathbf{q}) e^{-iq_4\tau}\right] \\ &\propto \int_0^{\infty} d\tau \, [R(\tau; \mathbf{p}, \mathbf{q}) - R(-\tau; \mathbf{p}, \mathbf{q})] \sin(q_4\tau)\end{aligned}$$

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- τ_e is fixed during calculations, and $C^{\mu\nu}(\tau_e, \tau_e + \tau; \mathbf{p}, \mathbf{q})$ is poorly correlated with $C^{\mu\nu}(\tau_e, \tau_e - \tau; \mathbf{p}, \mathbf{q})$

Reducing Noise

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- γ_5-hermiticity tells us, configuration by configuration,

$$C_3^{\mu\nu}(\tau_e, \tau_m, \mathbf{p}_e, \mathbf{p}_m)^* = C_3^{\nu\mu}(\tau_m, \tau_e, -\mathbf{p}_m, -\mathbf{p}_e)$$

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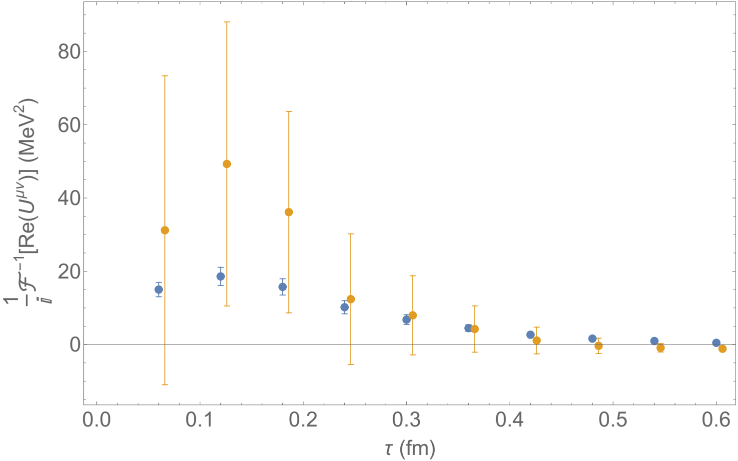
- This lets us compute

$$R(\tau; \mathbf{p}, \mathbf{q}) - R(-\tau; \mathbf{p}, \mathbf{q}) = R(\tau; \mathbf{p}, \mathbf{q}) + R(\tau; -\mathbf{p}, \mathbf{q})$$

The RHS is the correlated difference of two quantities measured at the same τ_e, τ_m, so correlated errors will cancel.

Reducing Noise



Measurement of $R(\tau;\mathbf{p},\mathbf{q}) + R(\tau,-\mathbf{p},\mathbf{q})$ (blue) and $R(\tau;\mathbf{p},\mathbf{q}) - R(-\tau;\mathbf{p},\mathbf{q})$ (earth)

Extraction of Parameters

$$U^{[\mu\nu]} = \frac{2if_\pi \varepsilon_{\mu\nu\rho\lambda} q^\rho p^\lambda}{\tilde{Q}^2} \left[\mathcal{C}_W^{(0)} + \langle \xi^2 \rangle \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} \mathcal{C}_W^{(2)} + \cdots + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\tilde{Q}} \right) \right]$$

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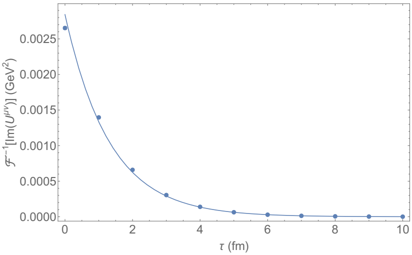
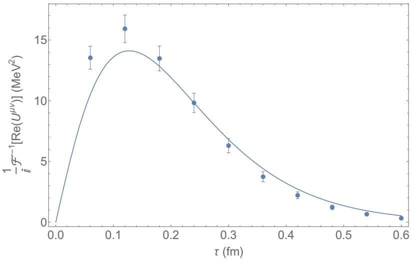
- f_π, m_Ψ fit from imaginary part of $U^{[\mu\nu]}$, while $\langle \xi^2 \rangle$ determined by real part
- Comparing continuum OPE to discretized lattice data will mean $\langle \xi^2 \rangle$ is $\mathcal{O}(a)$-contaminated
    - Need to extrapolate this away at end of calculation

Extraction of Parameters

$$a = 0.06\ \text{fm},\ \mathbf{p}_e = (0, 0, -1),\ \mathbf{p}_m = (1, 0, 1)$$





$$f_\pi = 153 \pm 2\ \text{MeV}$$

$$m_\Psi = 2.42 \pm 0.01\ \text{GeV}$$

$$\langle \xi^2 \rangle = 0.21 \pm 0.01$$

(Errors are statistical and exclude $\mathcal{O}\left(a, \frac{\Lambda_{QCD}}{m_\Psi}\right)$ corrections)

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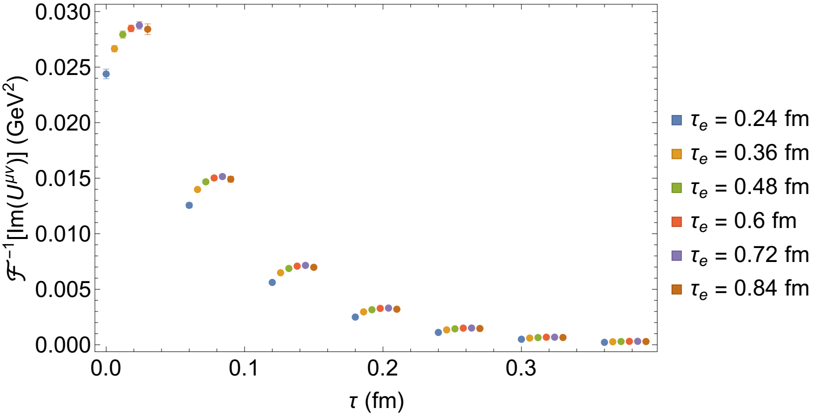
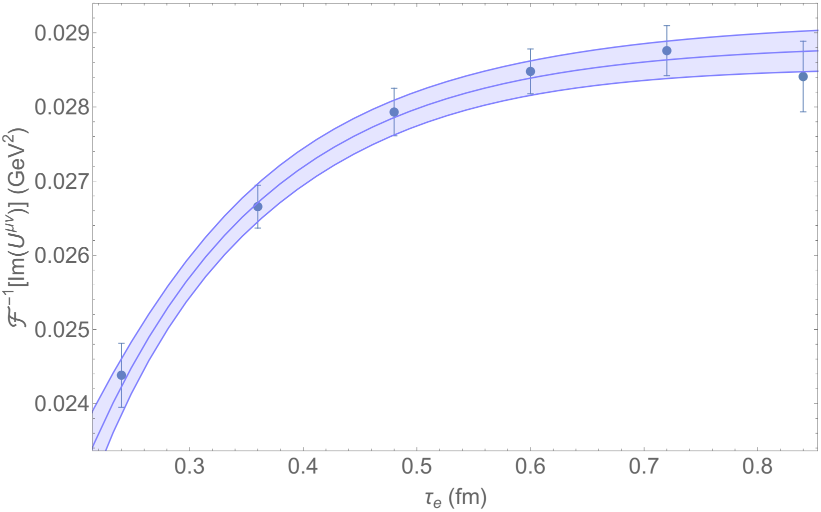
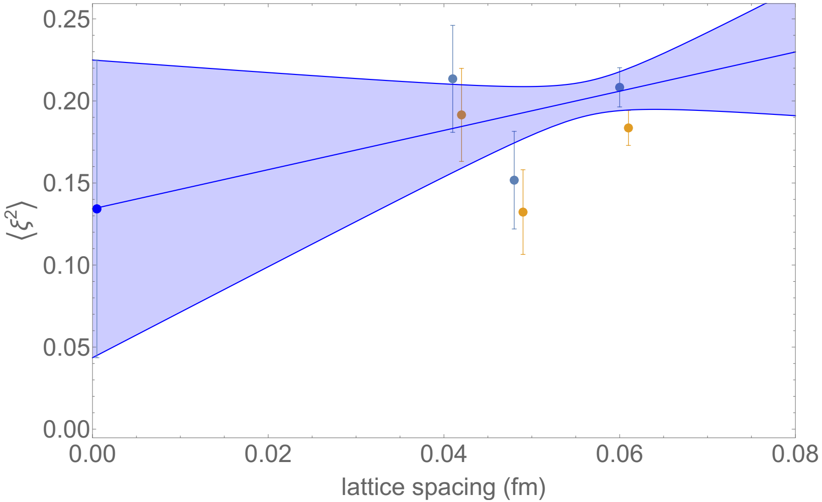
Excited States ($a = 0.060$ fm)



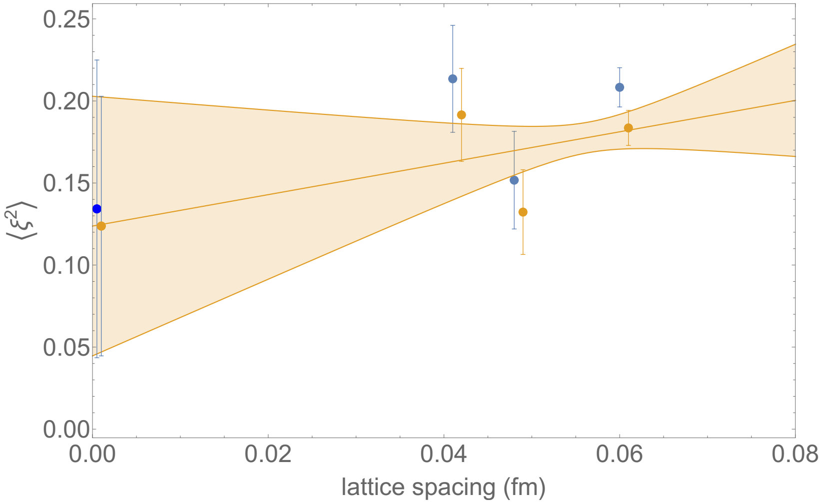
Excited States ($a = 0.060$ fm, $\tau = 0$)



Preliminary Extrapolation ($m_\psi^{(0)} = 1.6$ GeV)



Preliminary Extrapolation ($m_\Psi^{(0)} = 2.5$ GeV)



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- Currently in the process of increasing statistics, moving to finer lattices to facilitate continuum extrapolation
- Investigating potential of this method for computation of $\langle \xi^4 \rangle$

References

- W. Detmold and C.-J. D. Lin, "Deep-Inelastic Scattering and the Operator Product Expansion for Lattice QCD," arXiv:hep-lat/0507007v2
- W. Detmold, I. Kanamori, C.-J. David Lin, S. Mondal, Y. Zhao, "Moments of Pion Distribution Amplitude Using Operator Product Expansion on the Lattice," arXiv:hep-lat/1810.12194

Higher-Twist Effects

$$U^{[\mu\nu]} = \frac{2if_\pi \varepsilon_{\mu\nu\rho\lambda} q^\rho p^\lambda}{\tilde{Q}^2} \left[\mathcal{C}_W^{(0)} + \langle \xi^2 \rangle \frac{6(p\cdot q)^2 - p^2 q^2}{6(\tilde{Q}^2)^2} \mathcal{C}_W^{(2)} + \cdots + \mathcal{O}\left(\frac{\Lambda_{\mathrm{QCD}}}{\tilde{Q}} \right) \right]$$

- Twist-2 OPE is only valid up to $\frac{\Lambda_{\mathrm{QCD}}}{\tilde{Q}}$ corrections
- Higher-twist effects suppressed as m_Ψ^{-1}
- Cannot take $m_\Psi \to \infty$ on lattice due to $\mathcal{O}(am_\Psi)$ effects
- Must do combined fit to lattice spacing and m_Ψ with preliminary fit form:

$$\langle \xi^2 \rangle_{\mathrm{data}} = \langle \xi^2 \rangle_{\mathrm{cont.}} + \frac{A}{m_\Psi} + Ba + Cam_\Psi$$

(a = lattice spacing, $\langle \xi^2 \rangle_{\mathrm{cont.}}$, A, B, C = fit parameters)

Preliminary Extrapolation (Combined)