$B_c \to J/\psi$ Semileptonic Form Factors and $R(J/\psi)$ Using the Heavy-HISQ Method

2007.06596, 2007.06597

Judd Harrison, University of Glasgow

with Christine Davies and Andrew Lytle

APLAT 2020

HPQCD

Outline

- $b \to c\ell\bar{\nu}_\ell$ semileptonic decays and experimental motivation
- $B_c \to J/\psi\ell\bar{\nu}_\ell$
- Extracting form factors with heavy-HISQ
- Results and Discussion
- Angular LFUV Observables

$b \to c\ell\bar{\nu}_\ell$ semileptonic decays - V_{cb}

- Tension between inclusive and exclusive determinations of V_{cb}.
- Exclusive favour $B \to D^*\ell\bar{\nu}_\ell$, extrapolate data to zero recoil and compare to lattice.
- Recently (e.g. M.Bordone et al. 1908.09398) shown that using more general parameterisations when extrapolating reduces tension.
- Desirable to use full kinematic range.
- Complementary determinations possible using $B_s \to D_s^{(*)}$ [1], $B_c \to J/\psi$.
- LHCb working on $B_c \to J/\psi$

[1] LHCb 2001.03225

$b \to c\ell\bar{\nu}_\ell$ semileptonic decays - $R(D^{(*)})$, $R(J/\psi)$

Early experimental results show slight tension with model results for $R(J/\psi)$

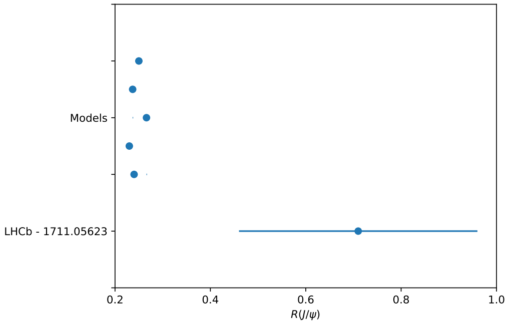
$$R(J/\psi) = \frac{\Gamma(B_c \to J/\psi\tau\bar{\nu}_\tau)}{\Gamma(B_c \to J/\psi\mu\bar{\nu}_\mu)}$$

Deviations from lepton flavour universality seen in branching ratios

$$R(D^{(*)}) = \frac{\Gamma(B \to D^{(*)}\tau\bar{\nu}_\tau)}{\Gamma(B \to D^{(*)}\mu\bar{\nu}_\mu)}$$

	SM	Exp
$R(D)$	0.299(3)	0.340(30)
$R(D^*)$	0.258(5)	0.295(14)

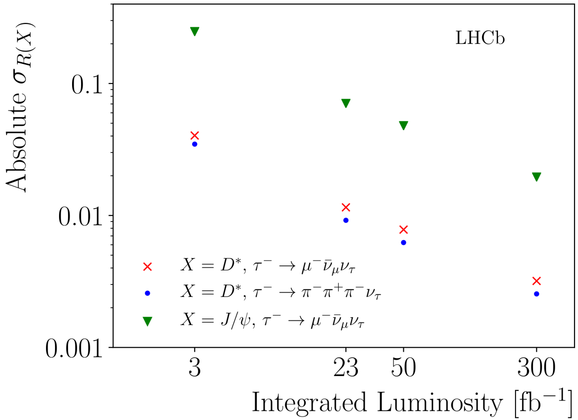
[HFLAV 1909.12524]



- large spread in model results for $R(J/\psi)$ in range $0.23 - 0.28$
- Model calculations include limited treatment of systematic uncertainties
- Experimental error recieves a large contribution from lack of knowledge about form factors

$b \to c\ell\bar{\nu}_\ell$ semileptonic decays - $R(D^{(*)})$, $R(J/\psi)$

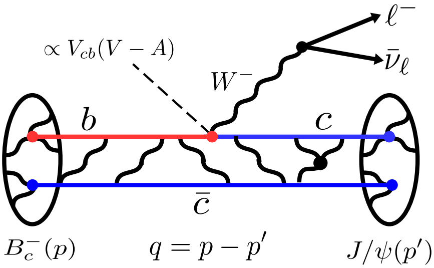
Projected uncertainties[2] in $R(D^*)$ and $R(J/\psi)$.



- Expect factor of ≈ 10 reduction in experimental uncertainty of $R(J/\psi)$

[2] LHCb 1808.08865v4

$B_c \to J/\psi$ Differential Rate and Form Factors



After integrating over angular variables, get differential rate w.r.t. q^2

$$\frac{d\Gamma}{dq^2} = \mathcal{N} \times \left[\left(|H_-|^2 + |H_0|^2 + |H_+|^2 \right) + \frac{m_\ell^2}{2q^2} \left(|H_-|^2 + |H_0|^2 + |H_+|^2 + 3|H_t|^2 \right) \right]$$

where

$$\mathcal{N} = \frac{G_F^2}{(2\pi)^3} |\eta_{EW}|^2 |V_{cb}|^2 \frac{(q^2 - m_\ell^2)^2 |\vec{p'}|}{12 M_{B_c}^2 q^2}$$

$B_c \to J/\psi$ Differential Rate and Form Factors

Helicity amplitudes are functions of form factors:

$$
\begin{aligned}
H_\pm(q^2) =& f_\pm^{A_1} A_1(q^2) + f_\pm^{V} V(q^2),\\
H_0(q^2) =& f_0^{A_2} A_2(q^2) + f_0^{A_1} A_1(q^2),\\
H_t(q^2) =& f_t A_0(q^2)
\end{aligned}
$$

where $f = f(q^2, M_{B_c}, M_{J/\psi})$

Form factors are in turn related to matrix elements:

$$
\begin{aligned}
\langle J/\psi(p',\lambda)|\bar{c}\gamma^\mu b|B_c^-(p)\rangle& \\
=& V(q^2) \times \mathrm{Kin}_V^\mu\\
\langle J/\psi(p',\lambda)|\bar{c}\gamma^\mu\gamma^5 b|B_c^-(p)\rangle& \\
=& A_0(q^2) \times \mathrm{Kin}_{A_0}^\mu\\
&+ A_1(q^2) \times \mathrm{Kin}_{A_1}^\mu\\
&+ A_2(q^2) \times \mathrm{Kin}_{A_2}^\mu
\end{aligned}
$$

- Need to compute matrix elements for 4 combinations of current operator and J/ψ interpolating operator in order to extract form factors.

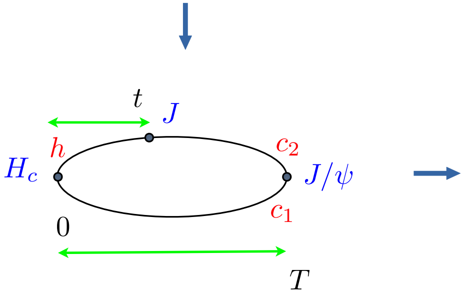
Heavy Quarks on the Lattice

To simulate precisely need $am_q < 1$, but most lattices have lattice spacing $a > 1/m_b$, on these lattices cannot simulate physical b quarks directly.

- Instead, use unphysically light heavy masses am_h on multiple lattices with $a > 1/m_b$
- fit data to polynomial in Λ_{QCD}/m_h, motivated by HQET, and extrapolate to $m_h = m_b$
- use Highly Improved Staggered Quarks (HISQ) $\rightarrow$ very small discretisation errors, crucial for calculations involving heavy quarks.

Extract matrix elements in standard way, for multiple heavy quarks with masses m_h:

$$C_{3\text{pt}}(T,t,0) = \langle 0|\bar{c}\gamma^{\nu} c(T)\, \bar{c}\Gamma h(t)\, \bar{h}\gamma^5 c(0)|0\rangle$$



Fit computed 3pt correlation function to:

$$C_{3\text{pt}}(T,t,0) = \sum_{n,m} \Big(A^n B^m J^{nm} e^{-(T-t)E_n - tM_m} + (-1)^{T+t} A^n_o B^m J^{nm}_{oe} e^{-(T-t)E^o_n - tM_m} + (-1)^t A^n B^m_o J^{nm}_{eo} e^{-(T-t)E_n - tM^o_m} + (-1)^T A^n_o B^m_o J^{nm}_{oo} e^{-(T-t)E^o_n - tM^o_m} \Big)$$

z-space

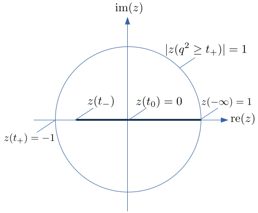
In order to fit q^2 dependence we change variables to $z(q^2)$

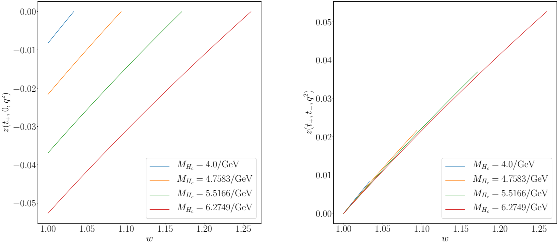
$$z(t_+, t_0, q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

$$t_- = (M_{H_c} - M_{J/\psi})^2$$

$$t_+ = (M_H + M_{D*})^2$$

$$t_0 = t_-$$





left: $z(t_+, 0, q^2)$ against w, right: $z(t_+, t_-, q^2)$ against w

Heavy-HISQ Form Factor Fit Function

The fit form we use for the form factors is then given by

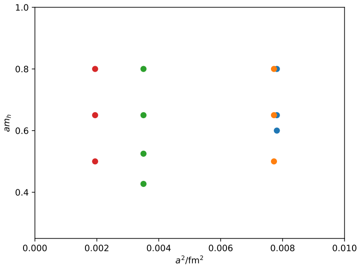
$$F(q^2) = P(q^2) \sum_{n,i,j,k}^{3} a_{ijk} \left(\frac{\Lambda_{QCD}}{M_{\eta_h}}\right)^i \left(\frac{am_c}{\pi}\right)^{2j} \left(\frac{am_h}{\pi}\right)^{2k} z^n(q^2)(1+\delta_n)$$

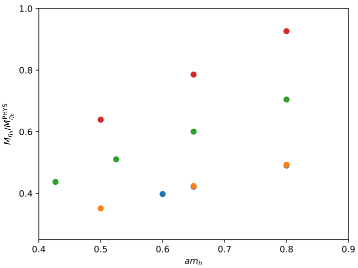
$P(q^2)$ incorporates subthreshold $\bar{c}h$ resonances, δ_n captures sea and valence quark mass mistuning effects. Physical continuum form factors given by

$$F^{\mathrm{phys}}(q^2) = P(q^2) \sum_{n,i}^{3} a_{i00} \left(\frac{\Lambda_{QCD}}{M_{\eta_b}^{\mathrm{phys}}}\right)^i z(q^2)^n \tag{1}$$

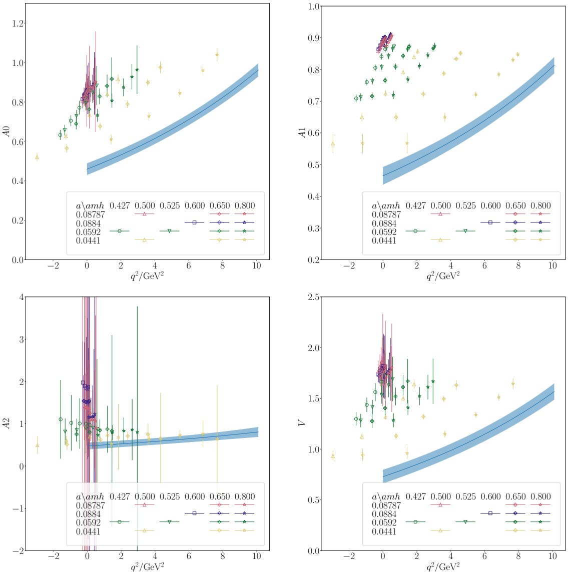
Lattice Details

Set	ω_0/a	$N_x \times N_t$	am_{l0}	am_{s0}	am_{c0}	n_{configs}
1	1.9006(20)	32×96	0.0074	0.037	0.440	980
2	2.896(6)	48×144	0.0048	0.024	0.286	500
3	3.892(12)	64×192	0.00316	0.0158	0.188	374
4	1.9518(7)	64×96	0.0012	0.0363	0.432	300

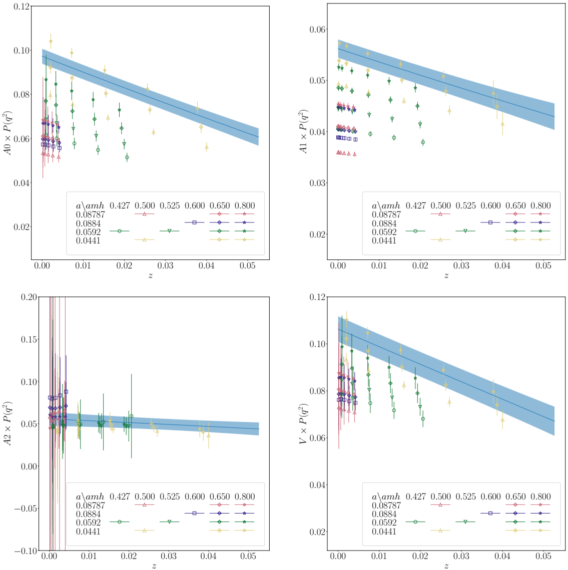




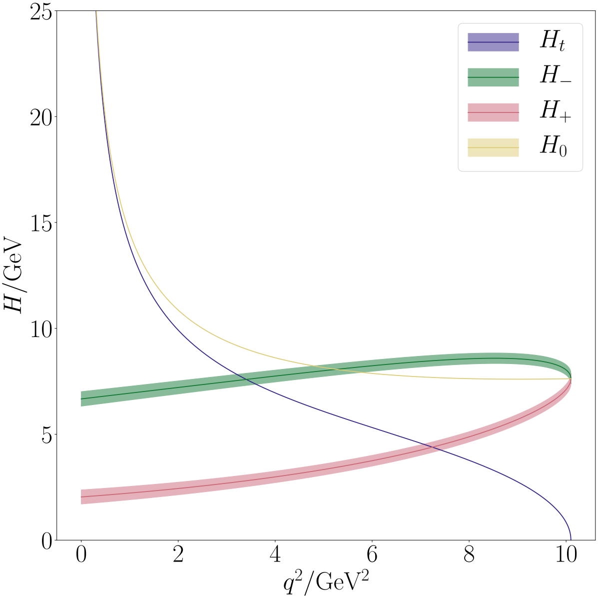
Results - q^2


A0
A1
A2
V
q^2/GeV^2
a\amh 0.427 0.500 0.525 0.600 0.650 0.800
0.08787
0.0884
0.0592
0.0441

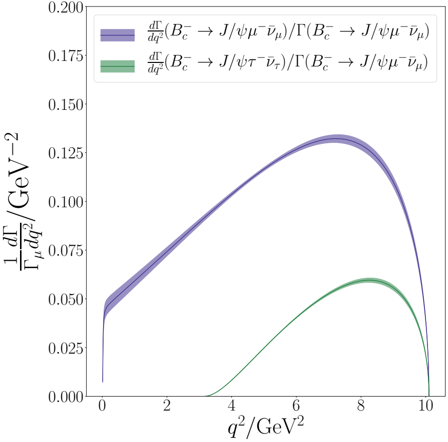
Results: z-space, Poles Removed



Helicity Amplitudes



Differential Decay Rates and $R(J/\psi)$



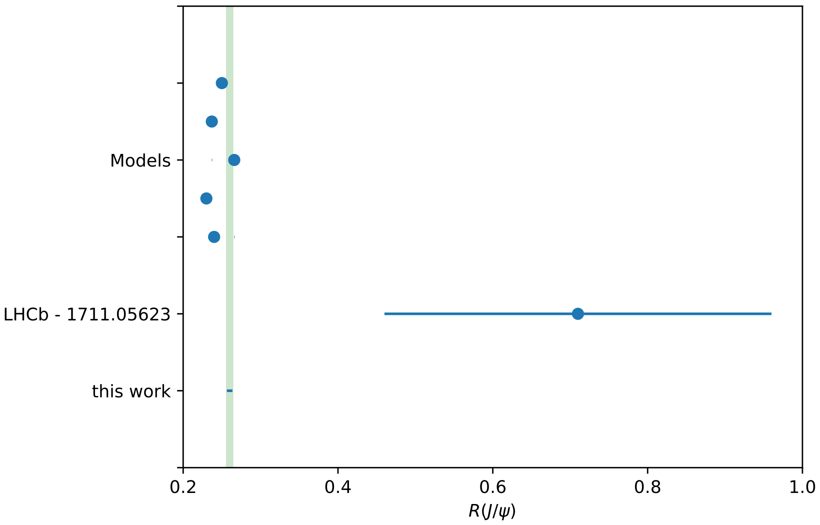
$\Gamma_e/|\eta_{\mathrm{EW}} V_{cb}|^2 =$
$11.52(80) \times 10^{-12}\mathrm{GeV}$

$\Gamma_\mu/|\eta_{\mathrm{EW}} V_{cb}|^2 =$
$11.47(79) \times 10^{-12}\mathrm{GeV}$

$\Gamma_\tau/|\eta_{\mathrm{EW}} V_{cb}|^2 =$
$2.99(19) \times 10^{-12}\mathrm{GeV}$

$R(J/\psi) = 0.2601(36)$

Our Result in the Context of Model values and Experiment



Error Budget

	$\Gamma/\lvert\eta_{\mathrm{EW}} V_{cb}\rvert^2$		
Source	$\ell = \mu$	$\ell = \tau$	$R(J/\psi)$
m_h dependence	2.4	2.2	0.6
$a \to 0$	3.9	3.6	0.8
$\delta_n \to 0$	3.5	3.3	0.3
lattice spacing determination	1.2	1.2	0.1
Statistics	3.5	3.1	1.0
Other	1.4	1.3	0.0
Total(%)	6.9	6.4	1.4

- Dominant statistical uncertainty may be reduced by increasing number of configurations on finest lattices

Other Useful Quantities

- Can insert values of V_{cb} and η_{EW} to find $\Gamma = 2.96(20)_{\mathrm{latt}}(20)_{V_{cb}} \times 10^{10} s^{-1}$
- Then using the measured value of the B_c lifetime we can compute the branching fraction $\mathrm{Br}(B_c^- \to J/\psi \mu^- \bar{\nu}_\mu) = 0.0151(10)_{\mathrm{latt}}(10)_{V_{cb}}(3)_{\tau_{B_c}}$

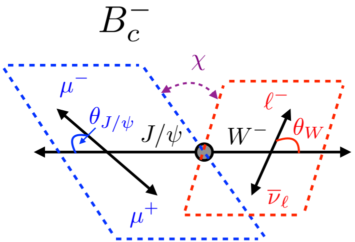
B_c branching fractions in the PDG are given in modified form $B(\bar{b} \to B_c^+)\Gamma_i/\Gamma$, Where $B(\bar{b} \to B_c^+)$ is the probability for a b to hadronise as a B_c.

- Use branching fraction together with the experimental value of the modified branching fraction to give $B(\bar{b} \to B_c^+) = 0.00576(57)_{\mathrm{latt}+V_{cb}}(66)_{\mathrm{exp}}$

compare to probability[3] for $B(\bar{b} \to B^+) = 0.407(8)$

[3] P. Zyla et al.(Particle Data Group), Prog. Theor. Exp. Phys. , 083C01 (2020)

Angular Observables



$$\mathcal{O}^\ell = \mathcal{N}^\ell(q^2)/\mathcal{D}^\ell(q^2)$$

$$\langle \mathcal{O}^\ell \rangle = \frac{\int_{m_\ell}^{q^2_{\max}} \mathcal{N}^\ell(q^2) dq^2}{\int_{m_\ell}^{q^2_{\max}} \mathcal{D}^\ell(q^2) dq^2}$$

$$R(\mathcal{O}) = \frac{\langle \mathcal{O}^\tau \rangle}{\frac{1}{2}\langle \mathcal{O}^\mu \rangle + \frac{1}{2}\langle \mathcal{O}^e \rangle}$$

$$\frac{d^2\Gamma}{dq^2 d\cos(\theta_W)} = a_{\theta_W}(q^2) + b_{\theta_W}(q^2)\cos(\theta_W) + c_{\theta_W}(q^2)\cos^2(\theta_W)$$

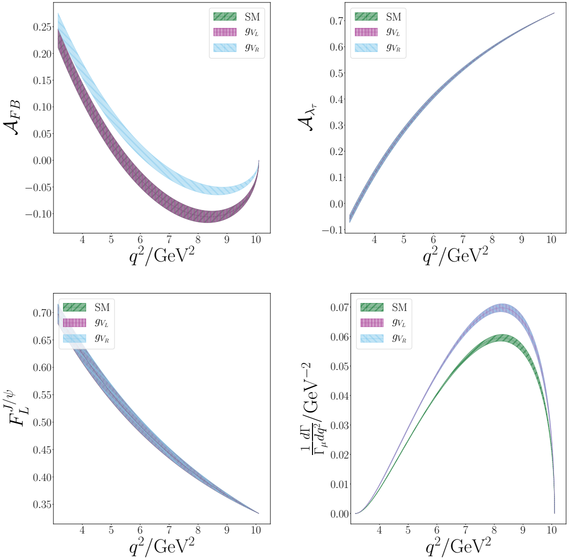
the observables are defined as

$$\mathcal{A}_{FB}(q^2) = -\frac{b_{\theta_W}(q^2)}{d\Gamma/dq^2}$$

$$\mathcal{A}_{\lambda_\ell}(q^2) = \frac{d\Gamma^{\lambda_\ell=-1/2}/dq^2 - d\Gamma^{\lambda_\ell=+1/2}/dq^2}{d\Gamma/dq^2}$$

$$F_L^{J/\psi}(q^2) = \frac{d\Gamma^{\lambda_{J/\psi}=0}/dq^2}{d\Gamma/dq^2}$$

Angular Observables



Consider NP modifications to the left and right handed $\bar{c}b$ vector couplings, which resolve the tension seen in $R(D)$ and $R(D^*)$[a]

$g_{V_L} = 0.07 - i\,0.16$

$g_{V_R} = -0.01 - i\,0.39$

[a] Becirevic et al. 1907.02257

Integrated Angular Observables and Ratios

	SM	g_{V_R}	g_{V_L}
$\langle \mathcal{A}_{FB} \rangle$	-0.064(12)	-0.0153(92)	-0.064(12)
$\langle \mathcal{A}_{\lambda_\tau} \rangle$	0.5296(59)	0.5295(59)	0.5296(59)
$\langle F_L^{J/\psi} \rangle$	0.4337(82)	0.4343(82)	0.4337(82)
$R(\mathcal{A}_{FB})$	0.281(35)	0.067(36)	0.281(35)
$R(\mathcal{A}_{\lambda_\ell})$	0.5325(58)	0.5324(58)	0.5325(58)
$R(F_L^{J/\psi})$	0.891(10)	0.892(10)	0.891(10)

- g_{V_L} just rescales the helicity amplitudes equally and does not affect these ratios.

- The combination $|H_+|^2 + |H_-|^2 = |H_A|^2 + |H_V|^2$ also rescales with $|H_0|^2$ and $|H_t|^2$ when g_{V_R} is purely imaginary. As such $\mathcal{A}_{\lambda_\ell}$ and $F_L^{J/\psi}$ are insensitive to g_{V_R}.

- $\mathcal{A}_{FB}$ and $R(\mathcal{A}_{FB})$ are sensitive to NP appearing through the right handed coupling.

Conclusions and Outlook

- Computed $B_c \to J/\psi$ semileptonic form factors across full q^2 range with high precision, the first time these quantities have been computed using lattice QCD - form factors available as ancillary files at https://arxiv.org/abs/2007.06957
- These can be fed into experiments to improve the systematics coming from form factor uncertainty, as well as used to normalise other B_c processes through determination of $B(b \to B_c^+)$.
- Computed $R(J/\psi)$ with an uncertainty of $\approx 1.4\%$
- Demonstrated efficacy of heavy-HISQ for computing heavy $P \to V$ form factors.
- Very similar $B_s \to D_s^*$ computation is complete with analysis in an advanced stage, near completion.

Thanks!