Low-lying charmonium properties from lattice QCD + quenched QED

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HPQCD Collaboration

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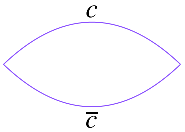
Introduction

- Lattice QCD below 1% needs a full understanding of systematics
- With the HISQ action we can study charmonium physics to very high precision
- Use $n_f = 2+1+1$ ensembles generated by MILC
- Able to achieve sub-percent tests of QCD and test systematics of previous calculations at this level
- We only calculate connected correlators



Ensembles

Set	β	w_0/a	L_s	L_t	am_l^{sea}	am_c^{sea}	am_c^{val}
1	5.80	1.1119(10)	16	48	0.013	0.838	0.888
2	5.80	1.1272(7)	24	48	0.0064	0.828	0.873
3	5.80	1.1367(5)	36	48	0.00235	0.831	0.863
4	6.00	1.3826(11)	24	64	0.0102	0.635	0.664
5	6.00	1.4029(9)	24	64	0.00507	0.628	0.650
6	6.00	1.4029(9)	32	64	0.00507	0.628	0.650
7	6.00	1.4029(9)	40	64	0.00507	0.628	0.650
8	6.00	1.4149(6)	48	64	0.00184	0.628	0.643
9	6.30	1.9006(20)	32	96	0.0074	0.440	0.450
10	6.30	1.9330(20)	48	96	0.00363	0.430	0.439
11	6.30	1.9518(7)	64	96	0.00120	0.432	0.433
12	6.72	2.8960(60)	48	144	0.00480	0.286	0.274
13	6.72	3.0170(23)	96	192	0.0008	0.260	0.260
14	7.00	3.892(12)	64	192	0.00316	0.188	0.194
15	7.28	5.243(16)	96	288	0.00223	0.1316	0.138

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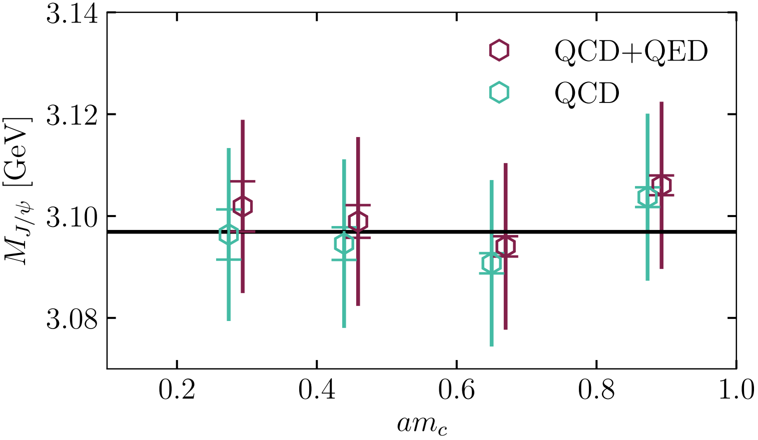
Quenched QED

- Generate A_μ fields in momentum space, then Fourier transform
- Use Feynman gauge
- QED_L $(A_\mu(k_0, \mathbf{k} = 0) = 0)$
- Multiply QCD gauge field by U(1): $\exp(ieQA_\mu(x))$

QED shifts in $M_{J/\psi}$



QCD + quenched QED fit

$$X(a^2, Q) = x\Big[1 + \sum_{i=1}^{5} c_a^{(i)} (am_c)^{2i} +$$

$$c_{m,\text{sea}} \delta_m^{\text{sea},uds} \{1 + c_{a^2,\text{sea}} (\Lambda a)^2 + c_{a^4,\text{sea}} (\Lambda a)^4\} + c_{c,\text{sea}} \delta_m^{\text{sea},c} + c_{c,\text{val}} \delta_m^{\text{val},c} +$$

$$\alpha_{\text{QED}} Q^2 \{c_{\text{QED}} + \sum_{p=1}^{3} c_{aQ}^{(p)} (am_c)^{2p} + c_{\text{val},Q} \delta_m^{\text{val},c}\}\Big]$$

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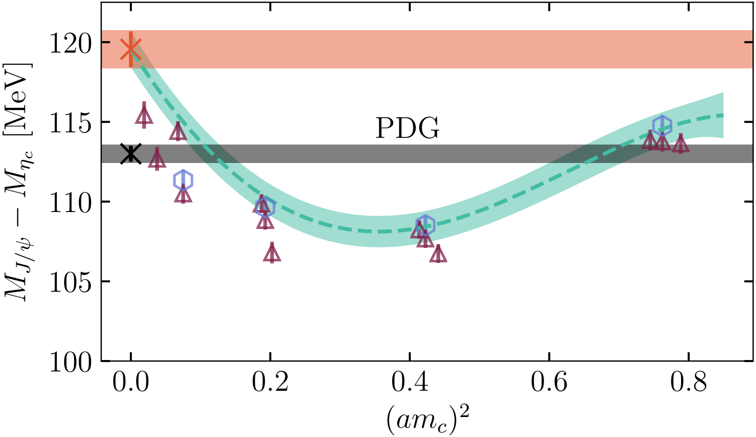
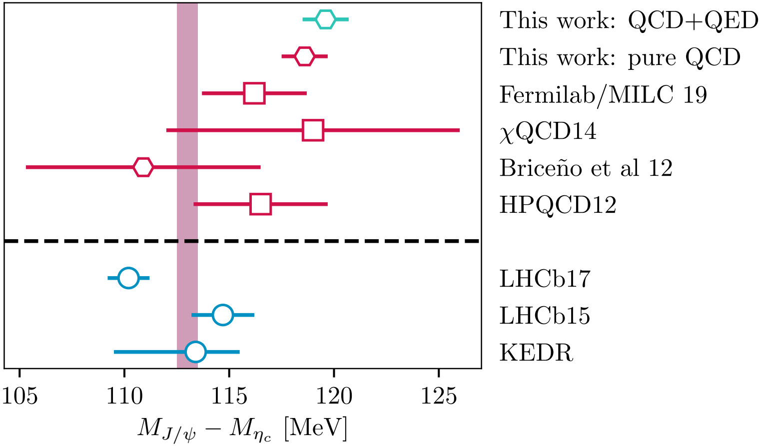
Hyperfine splitting

$M_{J/\psi} - M_{\eta_c}$



Difference from experimental average due to neglected disconnected contribution

Hyperfine splitting

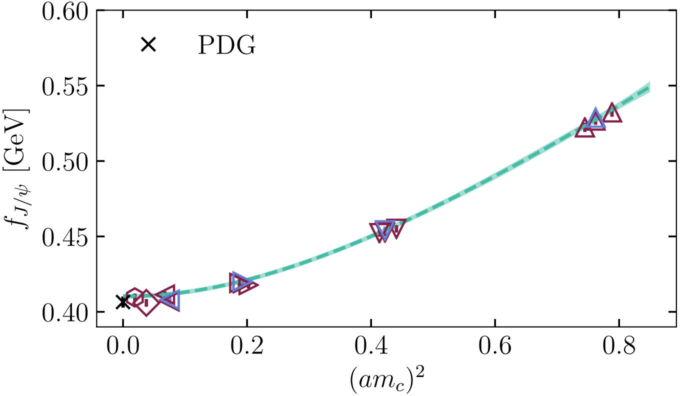


$$\Delta M_{\eta_c}^{\text{annihln}} = +7.3(1.2) \text{ MeV}$$

$f_{J/\psi}$

$$\langle 0|\overline{\psi}\gamma_\mu\psi|J/\psi\rangle = f_{J/\psi} M_{J/\psi} \epsilon_\mu$$

$$\Gamma(J/\psi \to \ell^+\ell^-) = \frac{4\pi}{3}\alpha^2_{\mathrm{QED}}(M^2_{J/\psi})Q_c^2\frac{f^2_{J/\psi}}{M_{J/\psi}}$$

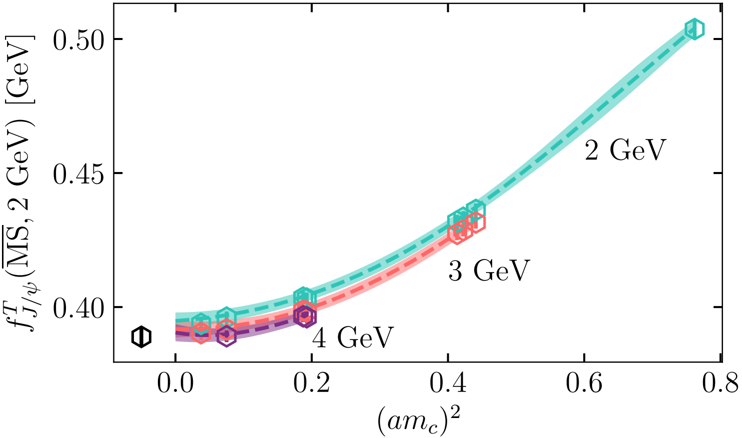


$f_{J/\psi} = 410.4(1.7)$ MeV, $f^{\mathrm{expt}}_{J/\psi} = 406.5(3.7)$ MeV

$f^T_{J/\psi}$

- Insert $\overline{\psi}\sigma_{\mu\nu}\psi$ instead of $\overline{\psi}\gamma_\mu\psi$
- This doesn't correspond to an experimental observable like $f_{J/\psi}$
- Does appear in EFT analyses of meson leptonic decay rates
  - Could look for lepton flavour violating interactions in ratio of J/ψ leptonic decays [1607.00815]
- Need to renormalise the tensor current (also needed for e.g. $B \to K\ell^+\ell^-$, W. Parrott Wed. 17:20)
- Use the RI-SMOM scheme: account for nonperturbative condensate contributions that decrease with scale μ
  - Use a fit that captures this [1805.06225]

$f^T_{J/\psi}$



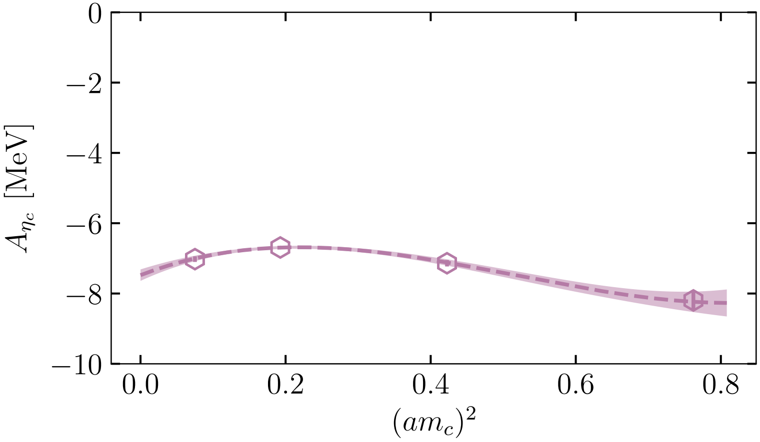
$$f^T_{J/\psi}(\overline{\text{MS}}, 2\ \text{GeV}) = 0.3889(33)\ \text{GeV}$$

$$\frac{f^T_{J/\psi}}{f_{J/\psi}} = 0.9569(52)$$

Coulomb interaction effect on charmonium masses

- As the Coulomb interaction is attractive it must yield a negative contribution to the mass
- The full quenched QED shift is positive
- Quenched QED includes both Coulomb effect and self energy
- $M(q_1, q_2) = M(0,0) + Aq_1q_2 + Bq_1^2 + Cq_2^2$
- Would be interesting to separate the contributions
  - Can compare Coulomb contribution to expectation from potential calculation (-3 MeV)
- Calculate with both q_1, q_2 and $q_1, -q_2$ to extract A
- For $q_1, -q_2$ the meson has the unphysical charge of $4/3$
  - Subtract known finite volume effects starting at $1/L$

η_c Coulomb effect



Summary

- Properties of the low-lying charmonium spectrum can be determined with very high precision using the HISQ action
- We use this as a testing ground to address various sytematics including leading QED effects

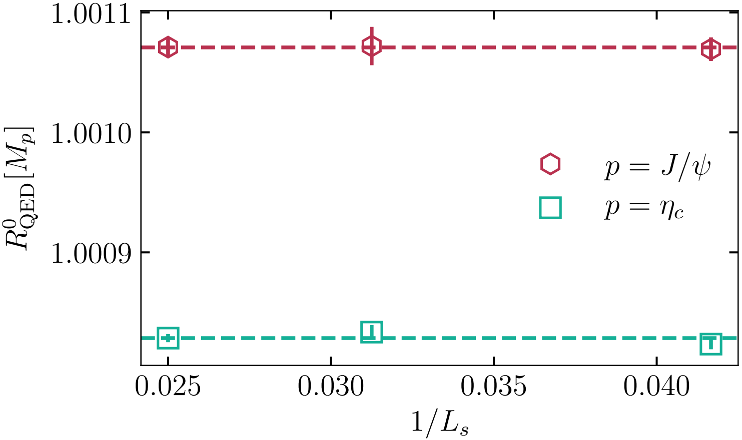
$$
\begin{aligned}
M_{J/\psi} - M_{\eta_c} &= 0.1203(11)\ \text{GeV} \\
\Delta M_{\eta_c}^{\text{annihln}} &= +7.3(1.2)\ \text{MeV} \\
f_{J/\psi} &= 0.4104(17)\ \text{GeV} \\
f^T_{J/\psi}(2\ \text{GeV}) &= 0.3927(27)\ \text{GeV} \\
a_\mu^c &= 14.638(47) \times 10^{-10}
\end{aligned}
$$

See [2005.01845] & [2008.02024]
(and upcoming works) for further details

Backup Slides

QED mass shifts volume dependence

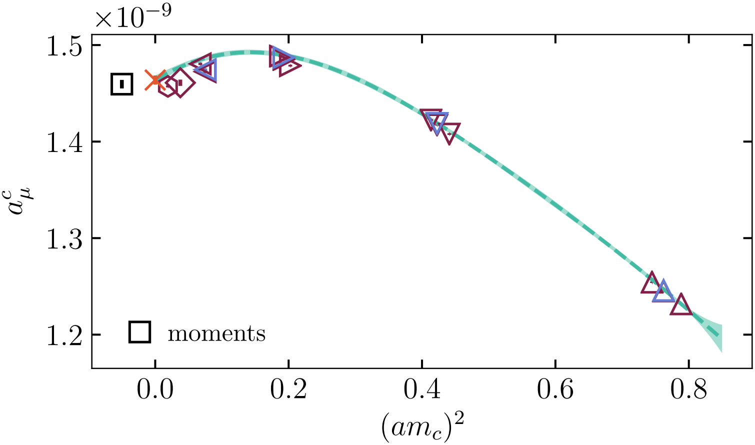
$$R^0_{\mathrm{QED}}[M] = \frac{M[\mathrm{QCD+QED}]}{M[\mathrm{QCD}]}$$



Finite-volume dependence not visible (electrically neutral mesons)

a_μ^c

$$G_n = Z_V^2 \sum_t t^n C_V(t),\; G_n = (-1)^{n/2} \frac{\partial^n}{\partial q^n} q^2 \hat{\Pi}(q^2) \Big|_{q^2=0}$$



0.3% precision

J/ψ Coulomb effect