Semileptonic $B \to \pi \ell \nu$ decays

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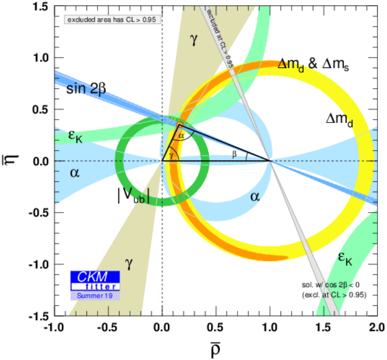
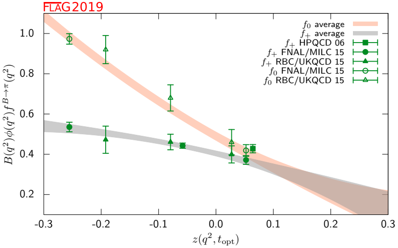
Motivation

- Test unitarity of CKM matrix
- 2-3σ discrepancy between exclusive ($B \to \pi\ell\nu$) and inclusive ($B \to X_u\ell\nu$)
- Lepton universality ratio predictions.



CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41, 1-131 (2005) [hep-ph/0406184], updated results and plots available at: http://ckmfitter.in2p3.fr

Motivation



- Relatively little data for heavy-light decays in 2019 FLAG averages

[Flavour Lattice Averaging Group: http://flag.unibe.ch/2019/MainPage]

Goal

- Differential $B \to \pi \ell \nu$ decay rate:

$$\underbrace{\frac{d\Gamma(B \to \pi\ell\nu)}{dq^2}}_{\text{Experiment}} = \eta_{EW} \frac{G_F^2}{24\pi^3} \times \underbrace{|V_{ub}|^2}_{CKM} \times \left(\frac{(q^2 - m_\ell^2)^2 \sqrt{E_\pi^2 - M_\pi^2}}{q^4 M_B^2} \right.$$

$$\left. \times \left[\left(1 + \frac{m_\ell^2}{2q^2}\right) M_B^2 (E_\pi^2 - M_\pi^2) \underbrace{|f_+(q^2)|^2}_{non-pert.} + \frac{3m_\ell^2}{8q^2} (M_B^2 - M_\pi^2)^2 \underbrace{|f_0(q^2)|^2}_{non-pert.} \right] \right)$$

q^2 — momentum transfer to $\ell\nu$

- Requires a theoretical computation of the form factors.

Heavy Quark Action

- RHQ Action for b quarks, Columbia interpretation
  [Christ et al. PRD 76 (2007) 074505] [Lin and Christ PRD 76 (2007) 074506]
  - Builds on original Fermilab action [El-Khadra et al. PRD 55 (1997) 3933]
  - Related to Tsukuba interpretation [S. Aoki et al. PTP 109 (2003) 383]
  - Clover action with anisotropic clover term
  - Uses 3 parameters $(m_0 a, c_p, \zeta)$ that can be non-pertubatively tuned to remove $\mathcal{O}((m_0 a)^n)$, $\mathcal{O}(\vec{p}a)$, $\mathcal{O}((\vec{p}a)(m_0 a)^n)$ errors [PRD 86 (2012) 116003]
  - We can use current improvement terms to get $\mathcal{O}(a)$ improved discretisation errors

Strategy

1. Simulate form factors at various lattice spacings, masses, daughter energies
2. Extrapolate the results to the continuum
3. Assess sources of systematic errors and factor these into the error budget
4. Extrapolate continuum result over full q^2 range.

Strategy

- In order to find f_+, f_0, seek to compute the hadronic matrix element for the flavour-changing vector currents $\langle\pi|\mathcal{V}^\mu|B\rangle$
- Standard parameterisation in terms of the scalar and vector form factors f_+ and f_0:

$$\langle\pi|\mathcal{V}^\mu|B\rangle = f_+(q^2)\left(p_B^\mu + p_\pi^\mu - \frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right) + f_0(q^2)\left(\frac{M_B^2 - M_\pi^2}{q^2}q^\mu\right)$$

Strategy

- Parallel and perpendicular form factors $f_{\|}$ and $f_{\perp}$ are simpler to relate to lattice data in the rest frame of the B-meson:

$$\langle \pi | \mathcal{V}^\mu | B \rangle = \sqrt{2M_B}\left[v^\mu f_{\|}(E_\pi) + p_\perp^\mu f_\perp(E_\pi) \right]$$

with
v^μ — B-meson 4-velocity
$p_\perp^\mu$ — $p_\pi^\mu - (p_\pi \cdot v) v^\mu$
p_π^μ — momentum of pseudoscalar particle

$$f_{\|} = \frac{\langle \pi | \mathcal{V}^0 | B \rangle}{\sqrt{2M_B}} \qquad f_\perp = \frac{\langle \pi | \mathcal{V}^i | B \rangle}{\sqrt{2M_B}} \frac{1}{p^i}$$

- Neatly separates into spatial and temporal components.

Strategy

- The matrix elements can be extracted from three-point correlation functions by cancelling off the other contributions.
- These other contributions involve two-point correlation functions of the involved mesons and their energies.
- Energies can be extracted from the two-point correlation functions.

Matrix Element Ratios

- Three-point functions:

$$C_{3,\mu} = \sum_{n,m} \langle 0|\mathcal{O}_\pi^\dagger|\pi_n\rangle\langle\pi_n|\mathcal{V}^\mu|B_m\rangle\langle B_m|\mathcal{O}_B|0\rangle \frac{e^{-tE_\pi^{(n)}}e^{-(t-t_{\text{sink}})E_B^{(m)}}}{4E_\pi^{(n)}E_B^{(m)}}$$

- Two-point functions:

$$C_2^X = \sum_n \langle 0|\mathcal{O}_X^\dagger|X_n\rangle\langle X_n|\mathcal{O}_X|0\rangle \frac{e^{-E_X^{(n)}}}{2E_X^{(n)}}$$

Matrix Element Ratio

- Make plateau fits to the ratio

$$\frac{C_{3,\mu}}{\sqrt{C_2^{\pi} C_2^{B}}} \sqrt{\frac{4 E_P^0 E_B^0}{e^{-tE_\pi^0} e^{-(t-t_{\text{sink}})E_B^0}}} = \langle \pi_0 | \mathcal{V}^\mu | B_0 \rangle$$

- Alternatively, include excited states by making the substitution

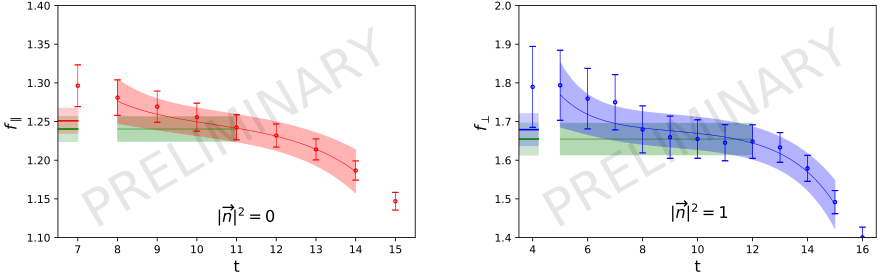
$$C_2^X \longrightarrow C'^X_2 = \left(C_2^X - Z_X^1 e^{-tE_X^1} \right)$$

and include additional terms in the fit

$$\langle \pi_1 | \mathcal{V}^\mu | B_0 \rangle \sqrt{\frac{Z_\pi^1}{Z_\pi^0}} \sqrt{\frac{E_\pi^0}{E_\pi^1}} \sqrt{e^{-t\Delta E_\pi^1}} \; + \; \langle \pi_0 | \mathcal{V}^\mu | B_1 \rangle \sqrt{\frac{Z_B^1}{Z_B^0}} \sqrt{\frac{E_B^0}{E_B^1}} \sqrt{e^{-(t-t_{\text{sink}})\Delta E_B^1}}$$

Ratio Fits

- For this analysis, excited state fits preferred



Ground state + excited state fits on the C1 ensemble

Ensembles

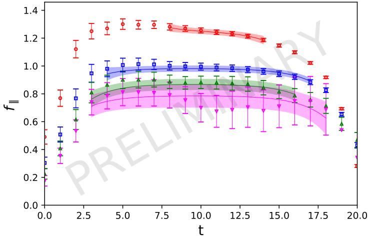
	$L^3 \times T \ / \ a^4$	a^{-1} / GeV	m_π / MeV
C1	$24^3 \times 64$	1.78	340
C2	$24^3 \times 64$	1.78	430
M1	$32^3 \times 64$	2.38	300
M2	$32^3 \times 64$	2.38	360
M3	$32^3 \times 64$	2.38	410
F1S	$48^3 \times 96$	2.77	270
C0*	$48^3 \times 96$	1.73	139

- 2+1f ensembles: Degenerate light quark
- Sea quarks: Domain-wall fermions
- **F1S** ensemble: New for this update of RBC-UKQCD 2015 analyses.
- *Physical pion ensemble C0 planned for future inclusion.

$B \to \pi$ analysis

$B \to \pi$ Form Factors

- Extract energies from two-point functions.
- Calculate $f_{\parallel}$ and $f_{\perp}$ from lattice data.
- Use this to find f_0 and f_+:



$f_{\parallel}$ on C1 ensemble

$$f_0(q^2) = \frac{\sqrt{2M_B}}{M_B^2 + E_\pi^2}\left[(M_B - E_\pi)f_{\parallel}(q^2) + (E_\pi^2 - M_\pi^2)f_{\perp}(q^2)\right]$$

$$f_+(q^2) = \frac{1}{\sqrt{2M_B}}\left[f_{\parallel}(q^2) + (M_B - E_\pi)f_{\perp}(q^2)\right]$$

$B \to \pi$ Chiral Continuum Fits

- Extrapolate to physical pion mass and zero lattice spacing simultaneously
- Use NLO hard-pion SU(2) HMχPT [PRD 67 (2003) 054010]

$$f(M_\pi, E_\pi, a) = \frac{c_1 \Lambda}{E_\pi + \Delta} \left(1 + \frac{\delta f}{(4\pi f_\pi)^2} + c_2 \frac{M_\pi^2}{\Lambda^2} + c_3 \frac{E_\pi}{\Lambda} + c_4 \left(\frac{E_\pi}{\Lambda} \right)^2 + c_5 (a\Lambda)^2 \right)$$

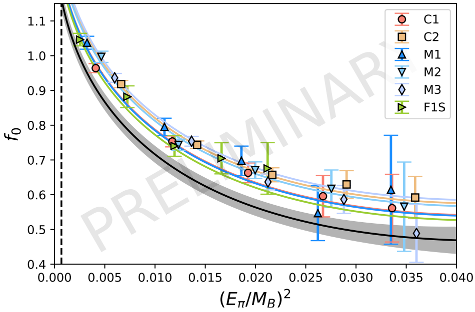
where $\delta f = -\frac{3}{4}(3g_b^2 + 1) M_\pi^2 \log\left(\frac{M_\pi^2}{\Lambda}\right)$

$$\Delta_+ = M_B - M_{B*} \approx 45.2 \text{ MeV}$$

- Don't include pole for f_0: $M_{B^*}(0^+)$ predicted well above $M_B + M_\pi$

$B \to \pi$ Chiral Continuum Fits

- Four (for f_+) or five (for f_0) values of E_π per ensemble
- Six ensembles/pion masses over three lattice spacings
- Simultaneously fit coefficients c_{1-5} to all data
- Continuum form factor given by $f(M_\pi^{\text{phys}}, E_\pi, a = 0)$



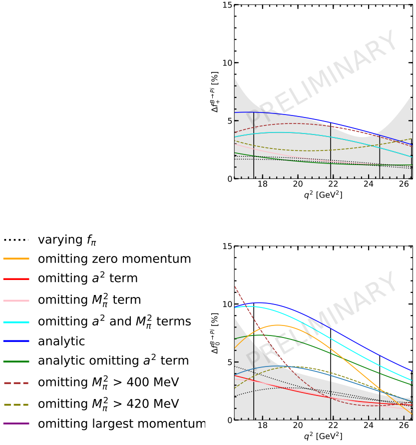
Systematic Error Analysis

- In preparation for an extrapolation of the continuum results to the full q^2 range, we construct **synthetic data points**.

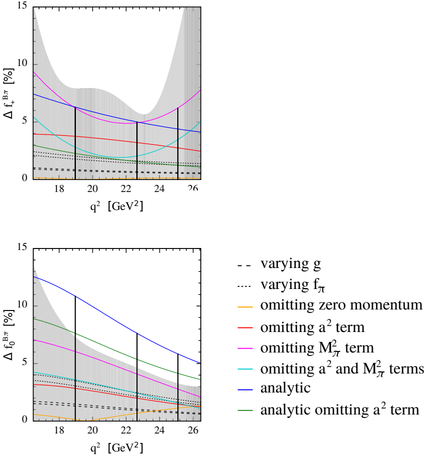
$$q^2 = (M_B^2 + M_\pi^2 - 2E_\pi M_B)$$

- The synthetic data points are constructed at **reference** q^2 **values** and account for both systematic and statistical errors.
- The extrapolation coefficients are lattice-independent as a result.
- Systematic error analysis required for this construction: discretisation error, lattice scale uncertainty, variations in chiral continuum fit ansatz...

This analysis



2015



Full q^2 range extrapolation

z-expansion

- Model-independent q^2 extrapolation scheme
- Change variables from q^2 to z with

$$z(q^2, t_0) = \frac{\sqrt{1 - q^2/t_+} - \sqrt{1 - t_0/t_+}}{\sqrt{1 - q^2/t_+} + \sqrt{1 - t_0/t_+}}$$

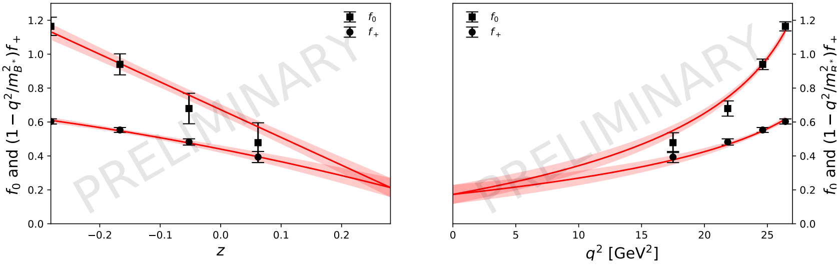
$$t_+ = (M_B + M_\pi)^2$$

$$t_0 = (M_B + M_\pi)(\sqrt{M_B} - \sqrt{M_\pi})^2$$

- Allows the form factors to be expanded as a power series in z
- Two common forms: Boyd-Grinstein-Lebed (BGL) and Bourrely-Caprini-Lellouch (BCL)

z-expansions

- z-expansions usually given the kinematic constraint that $f_0 = f_+$ at $q^2 = 0$.
- Can be combined with experimental data to determine $|V_{ub}|$.
- By integrating over q^2, predictions of the lepton-universality ratios can be obtained.
- z-expansions shown are BCL fits



Summary

- Updates to RBC-UKQCD 2015 results in the pipeline
- More precisely determined lattice spacing and inclusion of third lattice spacing via F1S ensemble reduces errors
- More details available in conference proceedings: arXiv:1912.09946
- Currently **still preliminary**