

The dual Meissner effect due to the violation of non-Abelian Bianchi identity

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Purpose

Do the violation of non-Abelian Bianchi identities(VNABI) explain the dual Meissner effect ?

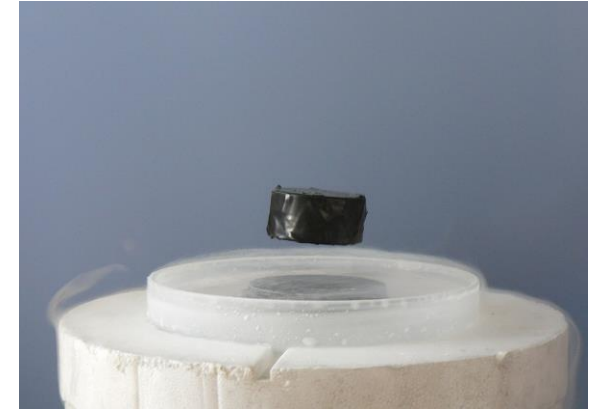
Introduction

The dual Meissner effect

G. 't Hooft, in Proceedings of the EPS International, edited by A. Zichichi (Editrice Compositori, Bologna, 1976), p. 1225.

S. Mandelstam, Phys. Rep. 23, 245 (1976).

Fig 1. Magnetic flux pinning



Wikipedia

QCD

Superconductivity

Color electric flux-tube



Magnetic flux-tube

Condensation of
color magnetic monopoles



Condensation of pair
of electrons (Cooper pairs)

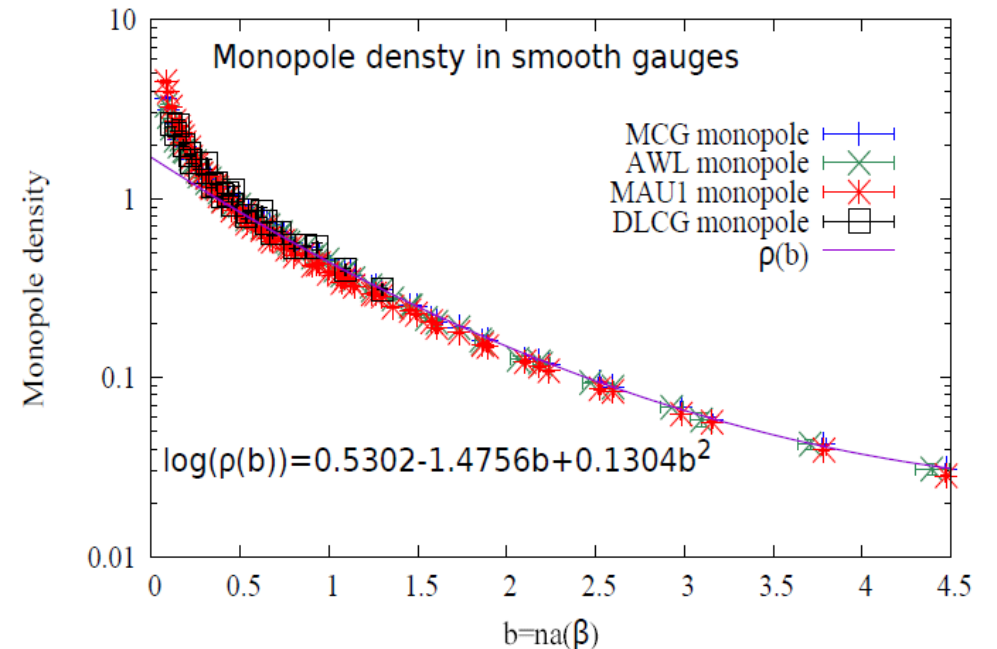
The violation of non-Abelian Bianchi identities(VNABI)

T.Suzuki,K.Ishiguro,V.Bornyakov,Phys.Rev.D97:034501 (2018)

T.Suzuki,Phys.Rev.D97,034509 (2018)

$$D_\nu G_{\mu\nu}^* = \partial_\nu f_{\mu\nu}^* = k_\mu$$

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A_\nu^a - \partial_\nu A_\mu^a) \frac{\lambda^a}{2}$$



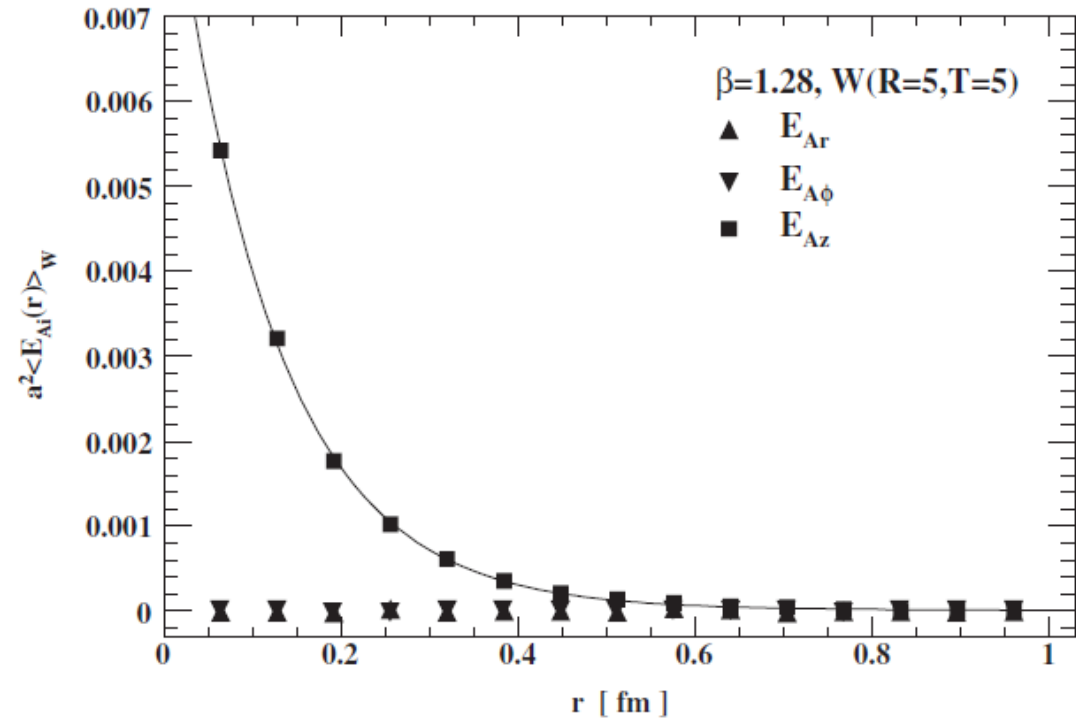
In SU(2) gauge theory, Abelian color electric fields and monopole currents can be evaluated without gauge fixing.

T.Suzuki,M.Hasegawa,K.Ishiguro,Y.Koma,T.Sekido,Phys.Rev.D80:054504(2009)

TABLE III. Best fitted values of the string tension σa^2 , the Coulombic coefficient c , and the constant μa for the potentials V_{NA} , V_A , V_{mon} , and V_{ph} .

$24^3 \times 4$	σa^2	c	μa	FR (R/a)	χ^2/N_{df}
V_{NA}	0.181(8)	0.25(15)	0.54(7)	3.9–8.5	1.00
V_A	0.183(8)	0.20(15)	0.98(7)	3.9–8.2	1.00
V_{mon}	0.183(6)	0.25(11)	1.31(5)	3.9–6.7	0.98
V_{ph}	$-2(1) \times 10^{-4}$	0.010(1)	0.48(1)	4.9–9.4	1.02
$24^3 \times 6$					
V_{NA}	0.072(3)	0.49(6)	0.53(3)	4.0–9.0	0.99
V_A	0.073(4)	0.41(7)	1.09(3)	3.7–10.9	1.00
V_{mon}	0.073(4)	0.44(10)	1.41(4)	3.9–9.3	1.00
V_{ph}	$-1.7(3) \times 10^{-4}$	0.0131(1)	0.4717(3)	5.1–9.4	0.99
$36^3 \times 6$					
V_{NA}	0.072(3)	0.48(9)	0.53(3)	4.6–12.1	1.03
V_A	0.073(2)	0.47(6)	1.10(2)	4.3–11.2	1.03
V_{mon}	0.073(3)	0.46(7)	1.43(3)	4.0–11.8	1.01
V_{ph}	$-1.0(1) \times 10^{-4}$	0.0132(1)	0.4770(2)	6.4–11.5	1.03
$24^3 \times 8$					
V_{NA}	0.0415(9)	0.47(2)	0.46(8)	4.1–7.8	0.99
V_A	0.041(2)	0.47(6)	1.10(3)	4.5–8.5	1.00
V_{mon}	0.043(3)	0.37(4)	1.39(2)	2.1–7.5	0.99
V_{ph}	$-6.0(3) \times 10^{-5}$	0.0059(3)	0.46649(6)	7.7–11.5	1.02

Abelian color electric field



Abelian projection in SU(3) gauge theory

Previous research

G. 't Hooft, Nucl. Phys. B 190 (1981) 455.

F. Brandstaeter et al., Phys. Lett. B 272 (1991) 631.

Gauge fixing : Maximal Abelian

$$R = \sum_{s,\mu,a} \text{Tr}(U_\mu(s)\lambda_a U_\mu^\dagger(s)\lambda_a)$$

$$\lambda_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \lambda_2 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \lambda_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Projection : SU(3) \rightarrow U(1) \times U(1)

$$u(s, \mu) = \text{diag}[u_1, u_2, u_3]$$

$$u_i(s, \mu) = \exp\{i\arg[U_{ii}^{MA}(s, \mu)] - \frac{1}{3}i\phi(s, \mu)\}$$

$$\phi(s, \mu) = \sum_i \arg[U_{ii}^{MA}(s, \mu)] \bmod{2\pi}$$

Our research

Gauge fixing : None

Projection : SU(3) \rightarrow U(1)

$$R_1 = \sum_{s,\mu} \text{Re Tr}[e^{i\theta_\mu^1(s)\lambda^1} U_\mu^\dagger(s)]$$

$$\rightarrow \theta_\mu^1(s) = \tan^{-1} \frac{\{\text{Im}(U_{12}) + \text{Im}(U_{21})\}}{\{\text{Re}(U_{11}) + \text{Re}(U_{22})\}}$$

Method

- Wilson action $V = 24^3 \times 8, \beta = 5.9$

- Monopole on the lattice

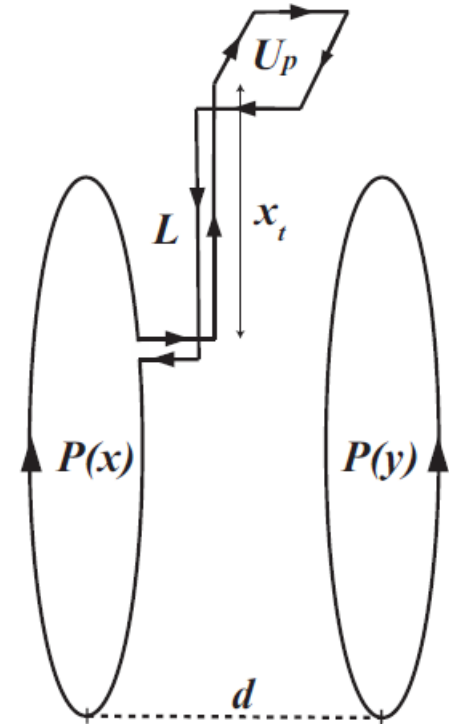
$$k_{\mu}^a(s) = \frac{1}{4\pi} \epsilon_{\mu\nu\rho\sigma} \partial_{\nu} \bar{\theta}_{\rho\sigma}^a(s),$$

$$\bar{\theta}_{\mu\nu}^a(s) = \theta_{\mu\nu}^a(s) - 2\pi n_{\mu\nu}^a(s)$$

T. A. DeGrand and D. Toussaint, Phys. Rev. D22, 2478 (1980)

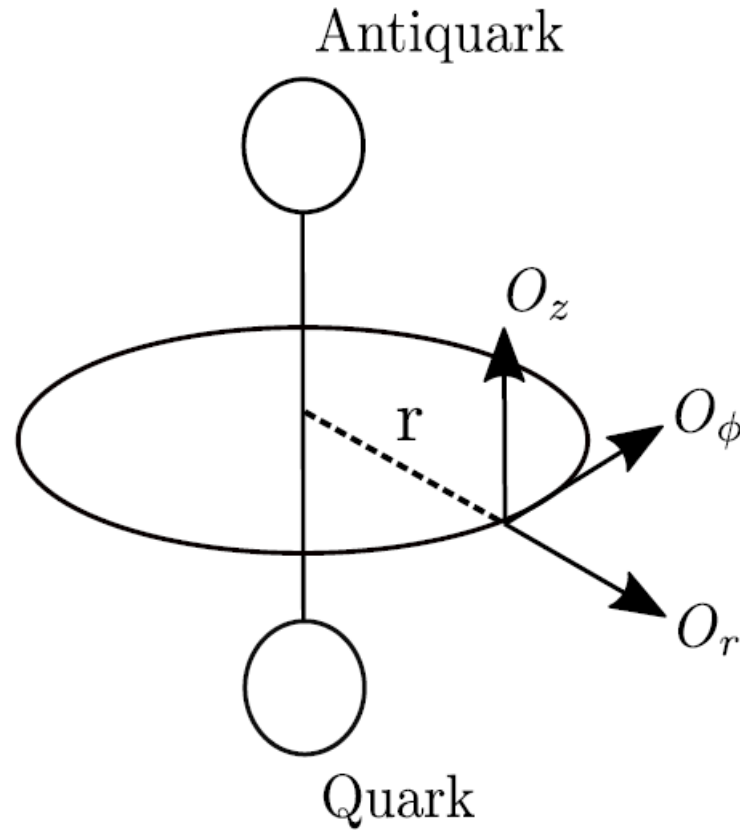
- APE smearing
- HYP smearing
- Random gauge transformation

- Flux-tube profile



P.Cea, L.Cosmai, F.Cuteri, A.Papa, EPJ Web Conf. 175 (2018) 12006

The definition of the cylindrical coordinate



Results

Fig.2 SU(3) color electric field

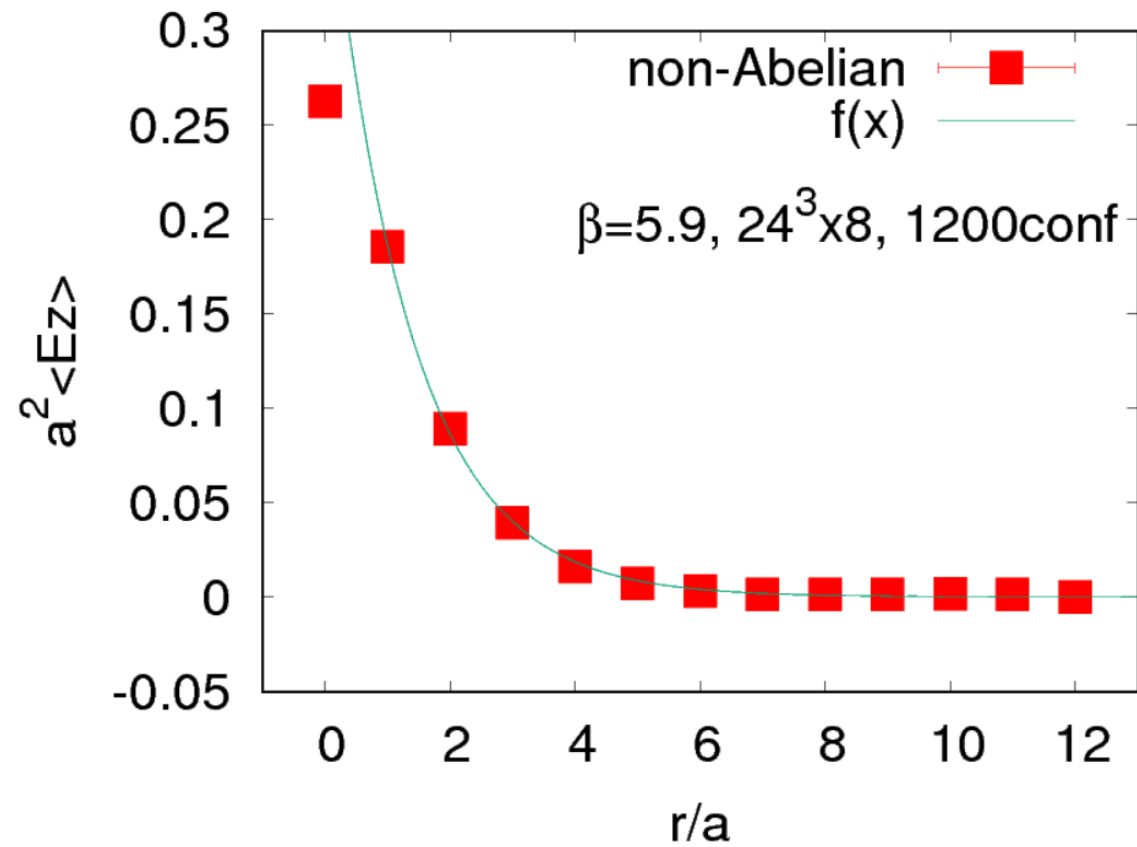
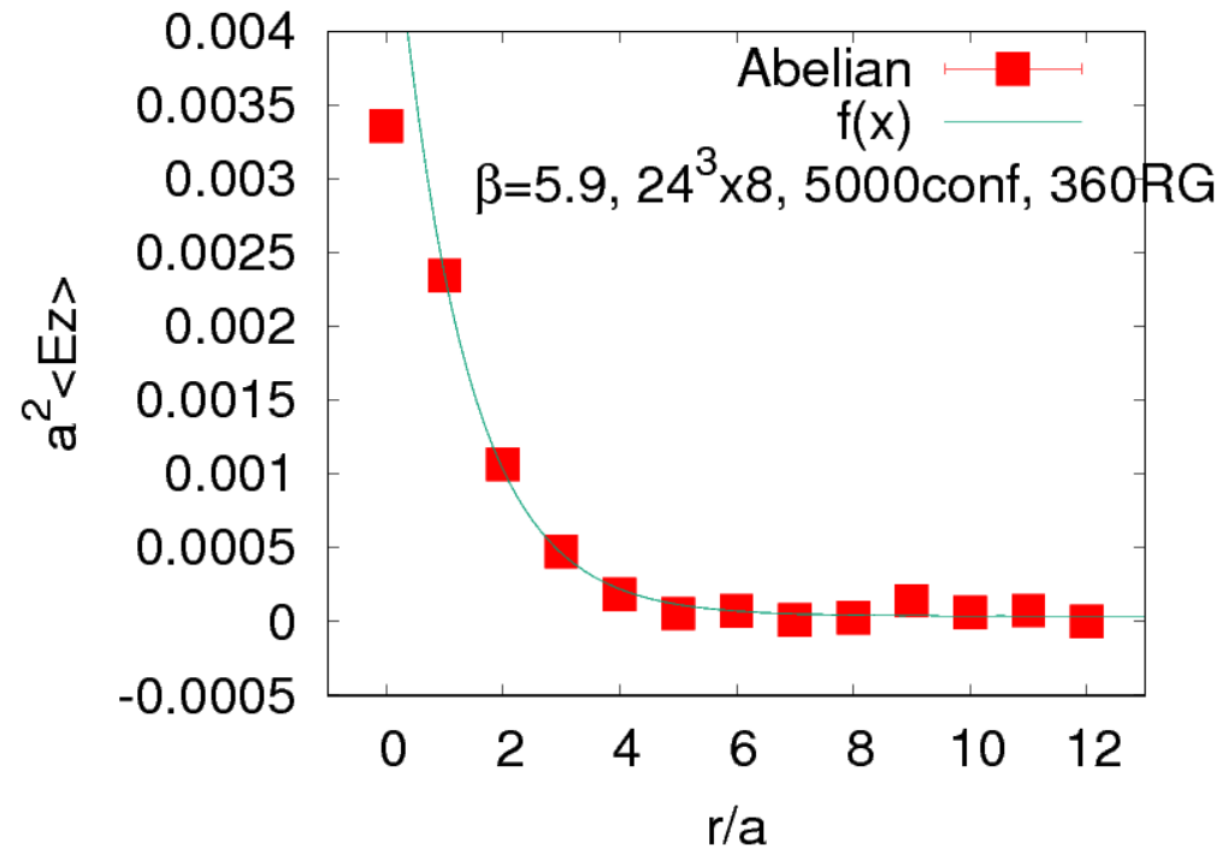


Fig.3 U(1) color electric field



The comparison between non-Abelian and Abelian.

	λ	A	c	FR(r/a)	$\chi^2/N_{d.o.f}$
non-Abelian	1.29(2)	0.401(8)	0.0003(6)	1-12	0.606036
Abelian	1.18(5)	0.0054(2)	0.00003(1)	1-12	0.636806

Fitting function :

$$f(r) = A \exp\left(-\frac{r}{\lambda}\right) + c$$

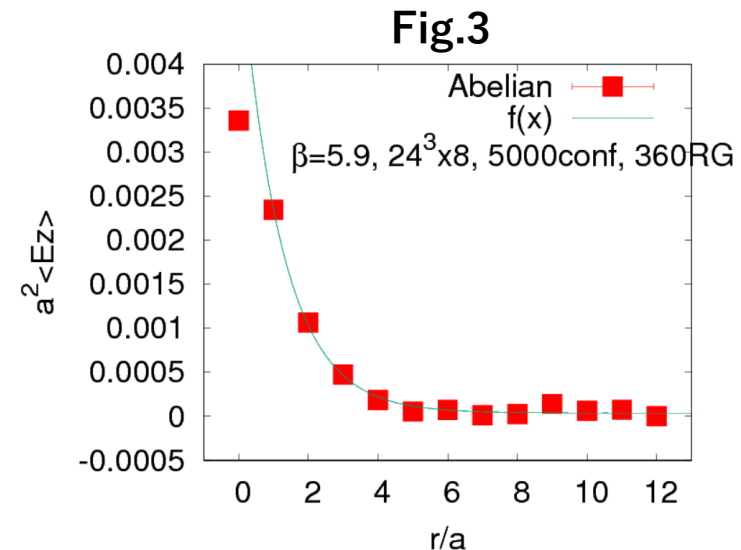
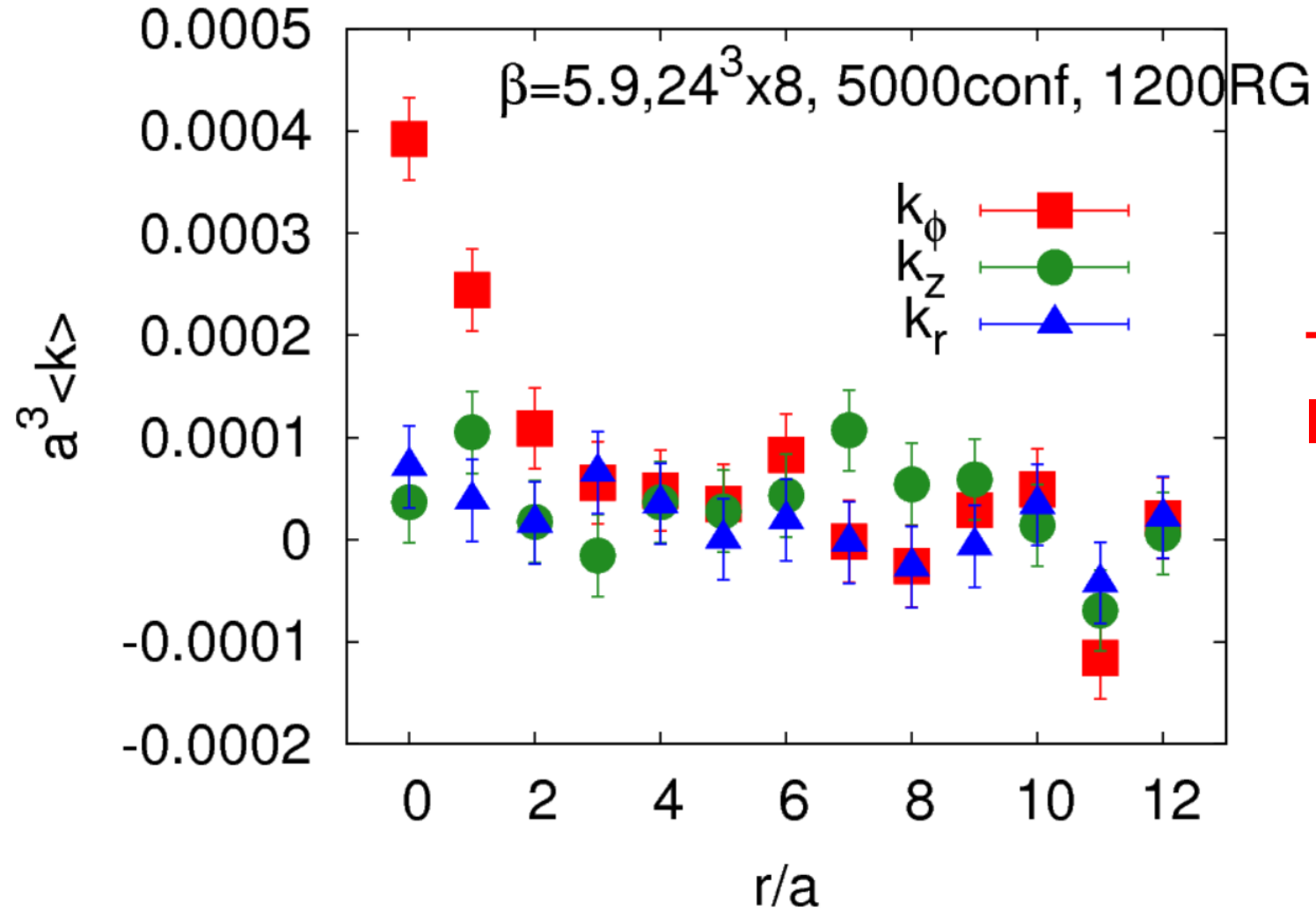


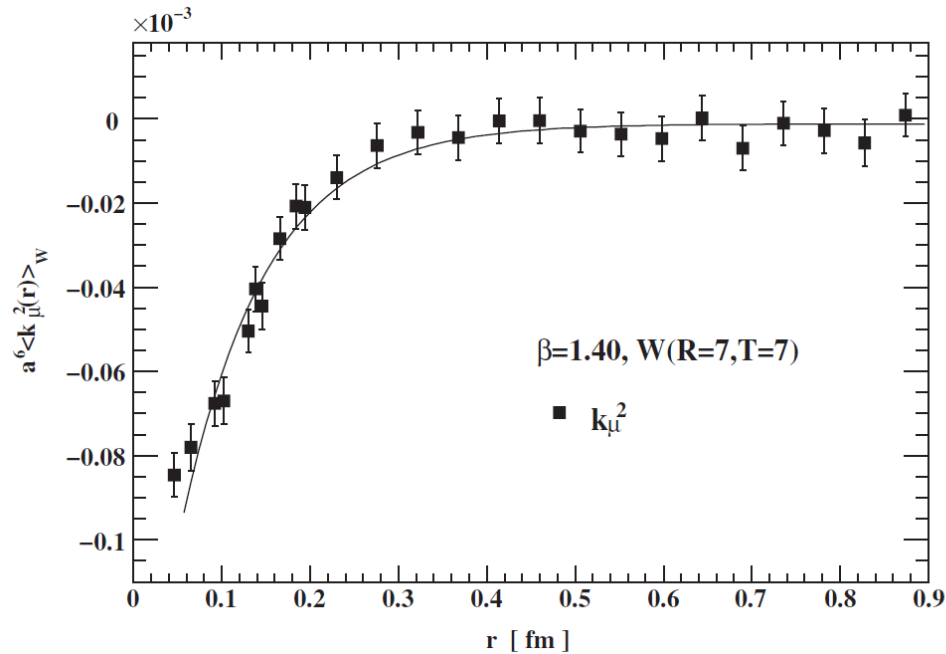
Fig.4 Monopole current (VNABI)



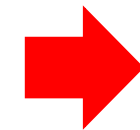
The signal is very noisy.
It is in progress.

The vacuum type in SU(3) without gauge fixing is in progress.

The squared monopole density in SU(2)



$$\langle k^2(r) \rangle = B \exp\left(-\frac{\sqrt{2}r}{\xi}\right)$$



Important!

SU(3)

Summary

1. In SU(3) gauge theory, the Abelian projection becomes SU(3) to U(1) projection due to the violation of non-Abelian Bianchi identity(VNABI).
2. These U(1) color electric fields are squeezed by the monopole currents.
3. To get the Abelian flux-tube profile without gauge fixing needs a huge number of statistics. It is necessary to use the smoothing procedure like random gauge transformation.
4. It is important to decide the vacuum type in SU(3) gauge theory without gauge fixing. To decide the vacuum type, we need to get the coherence length by the squared monopole density. It is in progress.