Digital Quantum Simulation of the Schwinger Model with Theta Term via Adiabatic State Preparation

Masazumi Honda

(本多正純)

YUKAWA INSTITUTE FOR THEORETICAL PHYSICS

CGP
Center for Gravitational Physics
Yukawa Institute for Theoretical Physics

KYOTO UNIVERSITY FOUNDED 1897

Ref.: arXiv:2001.00485 [hep-lat]
(submitting to PRL)

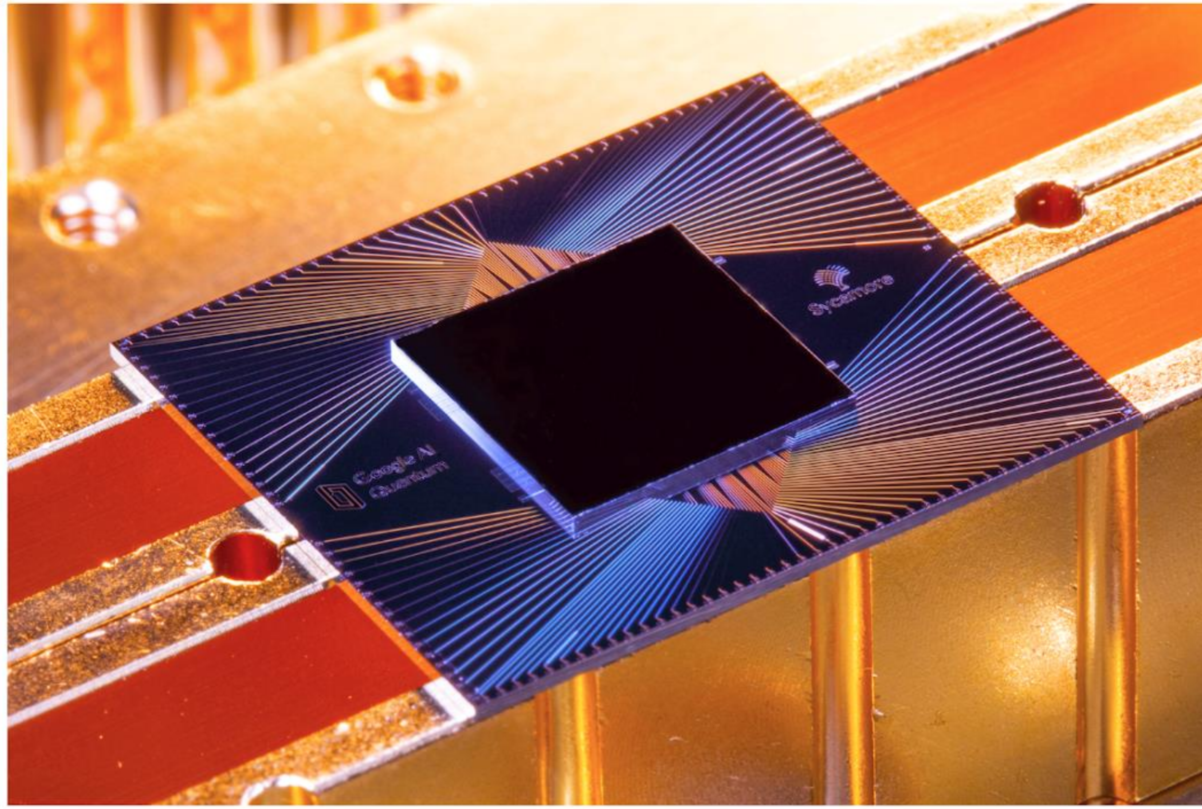
based on a collaboration with

Bipasha Chakraborty (Cambridge U.), Taku Izubuchi (BNL & RIKEN BNL),

Yuta Kikuchi (RIKEN BNL) & Akio Tomiya (RIKEN BNL)

6th, August, 2020 Asia-Pacific Symposium for Lattice Field Theory (APLAT2020)

Quantum computer sounds growing well…



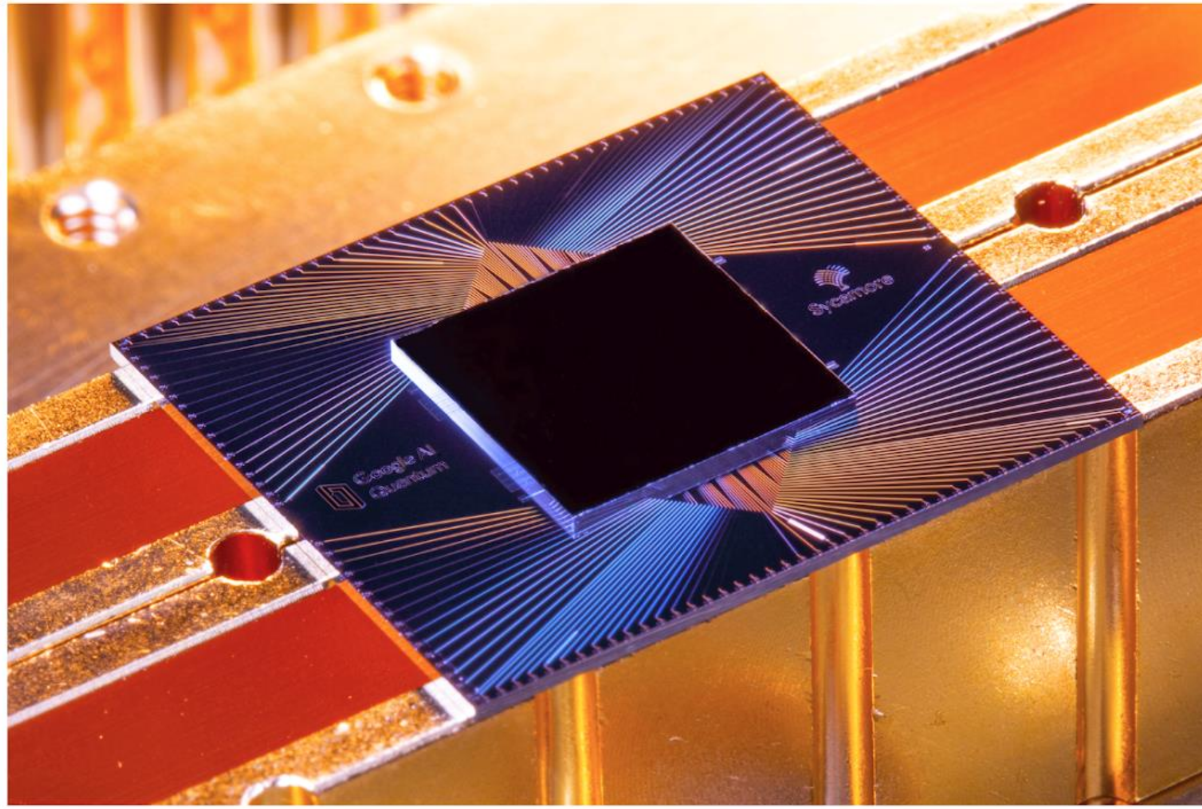
Article

Quantum supremacy using a programmable superconducting processor

https://doi.org/10.1038/s41586-019-1666-5

Frank Arute[1], Kunal Arya[1], Ryan Babbush[1], Dave Bacon[1], Joseph C. Bardin[1,2], Rami Barends[1],

Quantum computer sounds growing well…



Article

Quantum supremacy using a programmable superconducting processor

This talk = How can we use it for particle physics?

This talk is on

[See also talks by Hanada, Kawai, Lamm, Meurice, Sturzu, Wang]

Application of Quantum Computation
to
Quantum Field Theory (QFT)

- <u>Generic motivation:</u>

  simply would like to use powerful computers?

- <u>Specific motivation:</u>

This talk is on [See also talks by Hanada, Kawai, Lamm, Meurice, Sturzu, Wang]

Application of Quantum Computation to Quantum Field Theory (QFT)

- <u>Generic motivation:</u>

simply would like to use powerful computers?

- <u>Specific motivation:</u>

Quantum computation is suitable for Hamiltonian formalism

⟶ We don't perform (path) integral

⟶ Liberation from sign problem in Monte Carlo?

(∃ various approaches & talks within the framework of path integral but I skip it)

Cost of Hamiltonian formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Cost of Hamiltonian formalism

We have to play with huge vector space

since QFT typically has ∞-dim. Hilbert space

regularization needed!

Technically, computers have to

memorize huge vector & multiply huge matrices

Quantum computers do this job?

In this talk, we focus on

Schwinger model with topological term in Minkowski space

1+1d QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu + igA_\mu)\psi - m\bar{\psi}\psi$$

topological "theta term"

supposed to be difficult in the conventional approach:

- real time
- ∃ sign problem even in Euclidean case when θ isn't small

In this talk, we focus on

Schwinger model with topological term in Minkowski space

1+1d QED

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^\mu(\partial_\mu + igA_\mu)\psi - m\bar{\psi}\psi$$

topological "theta term"

supposed to be difficult in the conventional approach:

- real time
- $\exists$ sign problem even in Euclidean case when θ isn't small

Results:

[Tensor Network approach: Banuls-Cichy-Jansen-Saito '16 , Funcke-Jansen-Kuhn '19, etc.]

- Construction of true vacuum
- Computation of $\langle\bar{\psi}\psi\rangle$ & consistency check/prediction
- Estimation of computational resource

Contents

1. Introduction
2. Schwinger model as qubits
3. Algorithm to prepare vacuum
4. Results
5. Summary & Outlook

QFT as Quantum Bit (=Qubit) ?

Qubit = Quantum system w/ 2-dim. Hilbert space

(ex. up/down spin system)

Quantum computer = a combination of qubits

QFT as Quantum Bit (=Qubit) ?

Qubit = Quantum system w/ 2-dim. Hilbert space

(ex. up/down spin system)

Quantum computer = a combination of qubits

To put QFT on quantum computer,

1. "Regularize" Hilbert space (make it finite-dim.!)
2. Rewrite the regularized theory in terms of qubits

QFT as Quantum Bit (=Qubit) ?

Qubit = Quantum system w/ 2-dim. Hilbert space

(ex. up/down spin system)

Quantum computer = a combination of qubits

To put QFT on quantum computer,

1. "Regularize" Hilbert space (make it finite-dim.!)
2. Rewrite the regularized theory in terms of qubits

Schwinger model = the simplest nontrivial example w/ gauge interaction in this context

— 1+1d gauge field has only 1-dim. physical Hilbert sp.

— Lattice fermion has finite-dim. Hilbert sp.

Schwinger model w/ topological term

Continuum:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

Using “chiral anomaly”, the same physics can be studied by [Fujikawa’79]

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi}$$

Schwinger model w/ topological term

Continuum:

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{g\theta}{4\pi}\epsilon_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}\psi$$

Using "chiral anomaly", the same physics can be studied by [Fujikawa'79]

$$\boxed{\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\gamma^{\mu}(\partial_{\mu} + igA_{\mu})\psi - m\bar{\psi}e^{i\theta\gamma^5}\psi}$$

Taking temporal gauge $A_0 = 0$, $(\Pi = \dot{A}^1)$

$$\hat{H} = \int dx\left[-i\bar{\psi}\gamma^1(\partial_1 + igA_1)\psi + m\bar{\psi}e^{i\theta\gamma^5}\psi + \frac{1}{2}\Pi^2\right]$$

Physical states are constrained by Gauss law:

$$0 = -\partial_1\Pi - g\bar{\psi}\gamma^0\psi$$

Lattice theory w/ staggered fermion

Hamiltonian:

[Susskind, Kogut-Susskind '75]

$$\hat{H} = -i \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[\chi_n^\dagger e^{i\phi_n} \chi_n - \text{h.c.} \right]$$

$$+ m \cos\theta \sum_{n=1}^{N} (-1)^n \chi_n^\dagger \chi_n + J \sum_{n=1}^{N-1} L_n^2 \qquad \left(w = \frac{1}{2a}, J = \frac{g^2 a}{2} \right)$$

Commutation relation:

$$\{\chi_n^\dagger, \chi_m\} = \delta_{mn}, \ \ \{\chi_n, \chi_m\} = 0, \ \ [\phi_n, L_m] = i\delta_{mn}$$

Gauss law:

$$L_n - L_{n-1} = \chi_n^\dagger \chi_n - \frac{1 - (-1)^n}{2}$$

Schwinger model as qubits

1. Take open b.c. & solve Gauss law: [cf. Martinez-Muschik-Schindler-Nigg-Erhard '16]

$$L_n = \sum_{\ell=1}^{n-1} \left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right] \qquad \text{(took } L_0 = 0\text{)}$$

2.

3.

Schwinger model as qubits

1. Take open b.c. & solve Gauss law: [cf. Martinez-Muschik-Schindler-Nigg-Erhard '16]

$$L_n = \sum_{\ell=1}^{n-1} \left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right]$$ (took $L_0 = 0$)

2. Redefine fermion to absorb ϕ_n : $\chi_n \to \prod_{\ell<n} \left[e^{-i\phi_\ell} \right] \chi_n$

3.

Schwinger model as qubits

1. Take open b.c. & solve Gauss law: [cf. Martinez-Muschik-Schindler-Nigg-Erhard '16]

$$L_n = \sum_{\ell=1}^{n-1} \left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right]$$

(took $L_0 = 0$)

2. Redefine fermion to absorb ϕ_n : $\chi_n \to \prod_{\ell<n} \left[e^{-i\phi_\ell} \right] \chi_n$

3. Map to spin system: $\chi_n = \left(\prod_{\ell<n} iZ_\ell \right) \frac{X_n - iY_n}{2}$ (X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

Schwinger model as qubits

1. Take open b.c. & solve Gauss law: [cf. Martinez-Muschik-Schindler-Nigg-Erhard '16]

$$L_n = \sum_{\ell=1}^{n-1} \left[\chi_\ell^\dagger \chi_\ell - \frac{1-(-1)^\ell}{2} \right] \qquad \text{(took } L_0 = 0\text{)}$$

2. Redefine fermion to absorb ϕ_n : $\chi_n \to \prod_{\ell<n} \left[e^{-i\phi_\ell} \right] \chi_n$

3. Map to spin system: $\chi_n = \left(\prod_{\ell<n} iZ_\ell \right) \frac{X_n - iY_n}{2}$ (X_n, Y_n, Z_n: $\sigma_{1,2,3}$ at site n)

"Jordan-Wigner transformation"

[Jordan-Wigner'28]

Finally,

$$\hat{H} = H_{ZZ} + H_{\pm} + H_Z$$

$$\begin{cases} H_{ZZ} = \frac{J}{2} \sum_{n=2}^{N-1} \sum_{1 \le k < \ell \le n} Z_k Z_\ell, \\ H_{\pm} = \frac{1}{2} \sum_{n=1}^{N-1} \left(w - (-1)^n \frac{m}{2} \sin\theta \right) \left[X_n X_{n+1} + Y_n Y_{n+1} \right], \\ H_Z = \frac{m \cos\theta}{2} \sum_{n=1}^{N} (-1)^n Z_n - \frac{J}{2} \sum_{n=1}^{N-1} (n \bmod 2) \sum_{\ell=1}^{n} Z_\ell \end{cases}$$

Qubit description of the Schwinger model !!

Contents

1. Introduction

2. Schwinger model as qubits

3. Algorithm to prepare vacuum

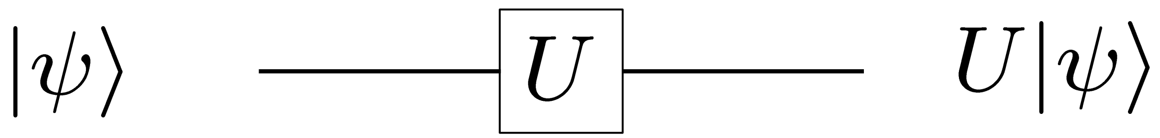
4. Results

5. Summary & Outlook

“Rule” of Quantum Computation

Use only the following 2 operations:

- Action of unitary operator: $|\psi\rangle \;\rightarrow\; U|\psi\rangle$

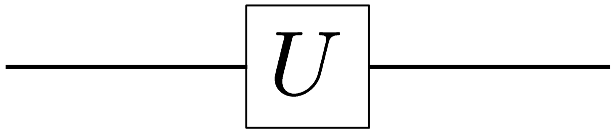


“Rule” of Quantum Computation

Use only the following 2 operations:

- Action of unitary operator: $|\psi\rangle \;\to\; U|\psi\rangle$

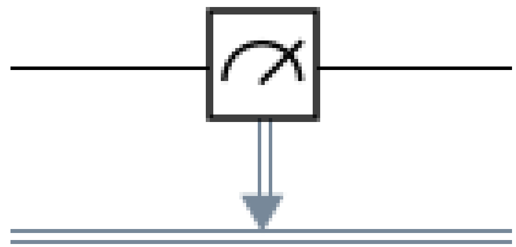
$|\psi\rangle$



$U|\psi\rangle$

- Measurement:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



c (classical number)

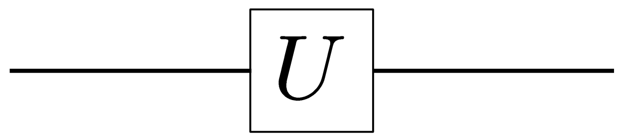
$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

“Rule” of Quantum Computation

Use only the following 2 operations:

- Action of unitary operator: $|\psi\rangle \rightarrow U|\psi\rangle$

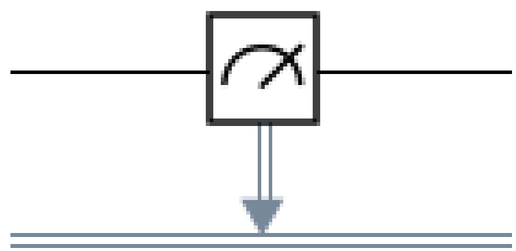
$|\psi\rangle$



$U|\psi\rangle$

- Measurement:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



c (classical number)

$$\begin{cases} c = 0 \text{ w/ probability } |\alpha|^2 \\ c = 1 \text{ w/ probability } |\beta|^2 \end{cases}$$

How can we obtain the vacuum?

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|vac_0\rangle$ is known and unique

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|vac_0\rangle$ is known and unique

Step 2: Consider the time evolution

$$\mathcal{T}\exp\left(-i\int_0^T dt\ H_A(t)\right)|\mathrm{vac}_0> \quad \text{w/} \quad H_A(0) = H_0,\ H_A(T) = \hat{H}$$

Adiabatic state preparation of vacuum

Step 1: Choose an initial Hamiltonian H_0 of a simple system whose ground state $|vac_0\rangle$ is known and unique

Step 2: Consider the time evolution

$$\mathcal{T} \exp\left(-i \int_0^T dt \ H_A(t)\right)|\text{vac}_0> \quad \text{w/} \quad H_A(0) = H_0, \ H_A(T) = \hat{H}$$

Step 3: Use the adiabatic theorem

If the system w/ the Hamiltonian $H_A(t)$ has a unique gapped vacuum, then the desired ground state is obtained by

$$|\text{vac}> = \lim_{T\to\infty} \mathcal{T} \exp\left(-i \int_0^T dt \ H_A(t)\right)|\text{vac}_0>$$

Adiabatic state preparation of vacuum (Cont'd)

$$|\text{vac}> = \lim_{T\to\infty} \mathcal{T} \exp\left(-i\int_0^T dt\ H_A(t)\right)|\text{vac}_0>$$

$$\simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\text{vac}_0>$$

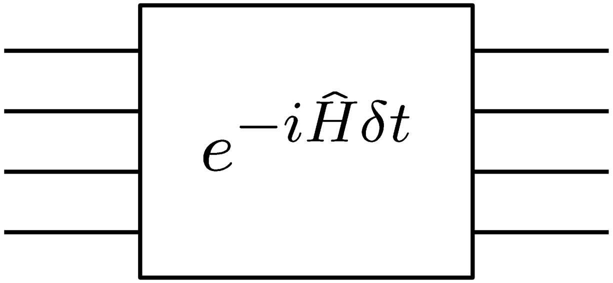
$$\left(U(t) = e^{-iH_A(t)\delta t}\right)$$

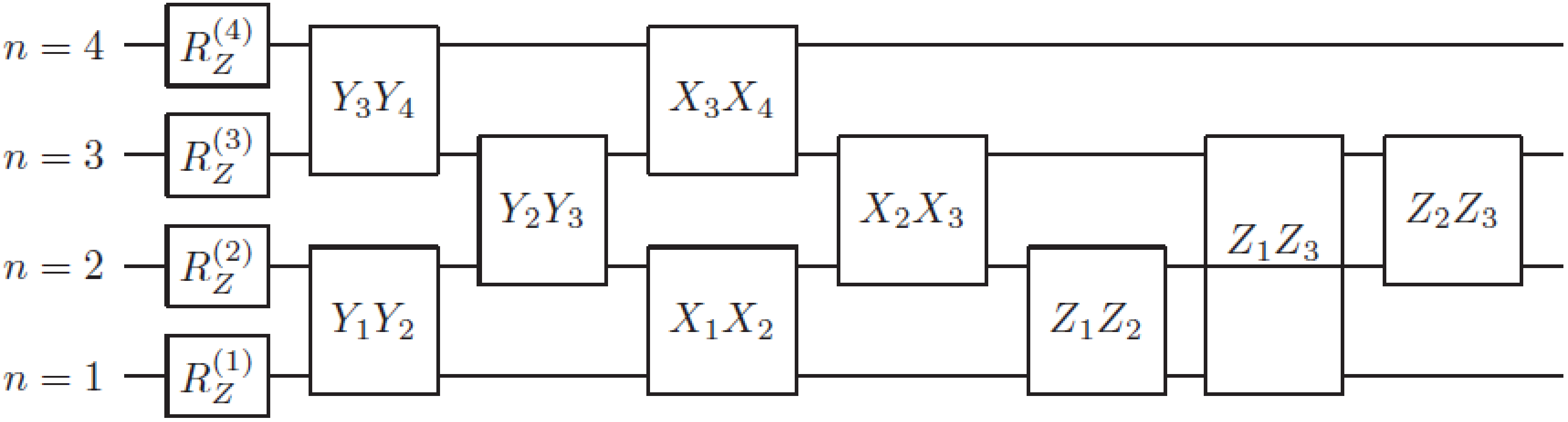
Here we choose

$$\begin{cases} H_0 = H_{ZZ} + H_Z|_{m\to m_0,\theta\to 0} \quad \Rightarrow \quad |\text{vac}_0\rangle = |0101\cdots 01\rangle \\ H_A(t) = \hat{H}\Big|_{w\to w(t),\theta\to\theta(t),m\to m(t)} \\ w(t) = \frac{t}{T}w,\ \theta(t) = \frac{t}{T}\theta,\ m(t) = \left(1-\frac{t}{T}\right)m_0 + \frac{t}{T}m \end{cases}$$

m_0 can be any positive number in principle
but it is practically chosen to have small systematic error

Quantum circuit for time evolution op. (N=4)





Results

<u>Skipped contents:</u>

- processes of taking ∞ volume & continuum limits
- how to estimate systematic errors, etc...

(Classical) simulator for Quantum computer

In real quantum computer,

Qubits in quantum circuit ≠ isolated system

➡ Interactions w/ environment cause errors

(Classical) simulator for Quantum computer

In real quantum computer,

Qubits in quantum circuit ≠ isolated system

→ Interactions w/ environment cause errors

Here we use

Simulator = tool to simulate quantum computer by classical computer

- Doesn't have errors → ideal answers

  (More precisely, classical computer also has errors but its error correction is established)

- The same code can be run in quantum computer w/ speed-up

Useful to test algorithm & estimate computational resources

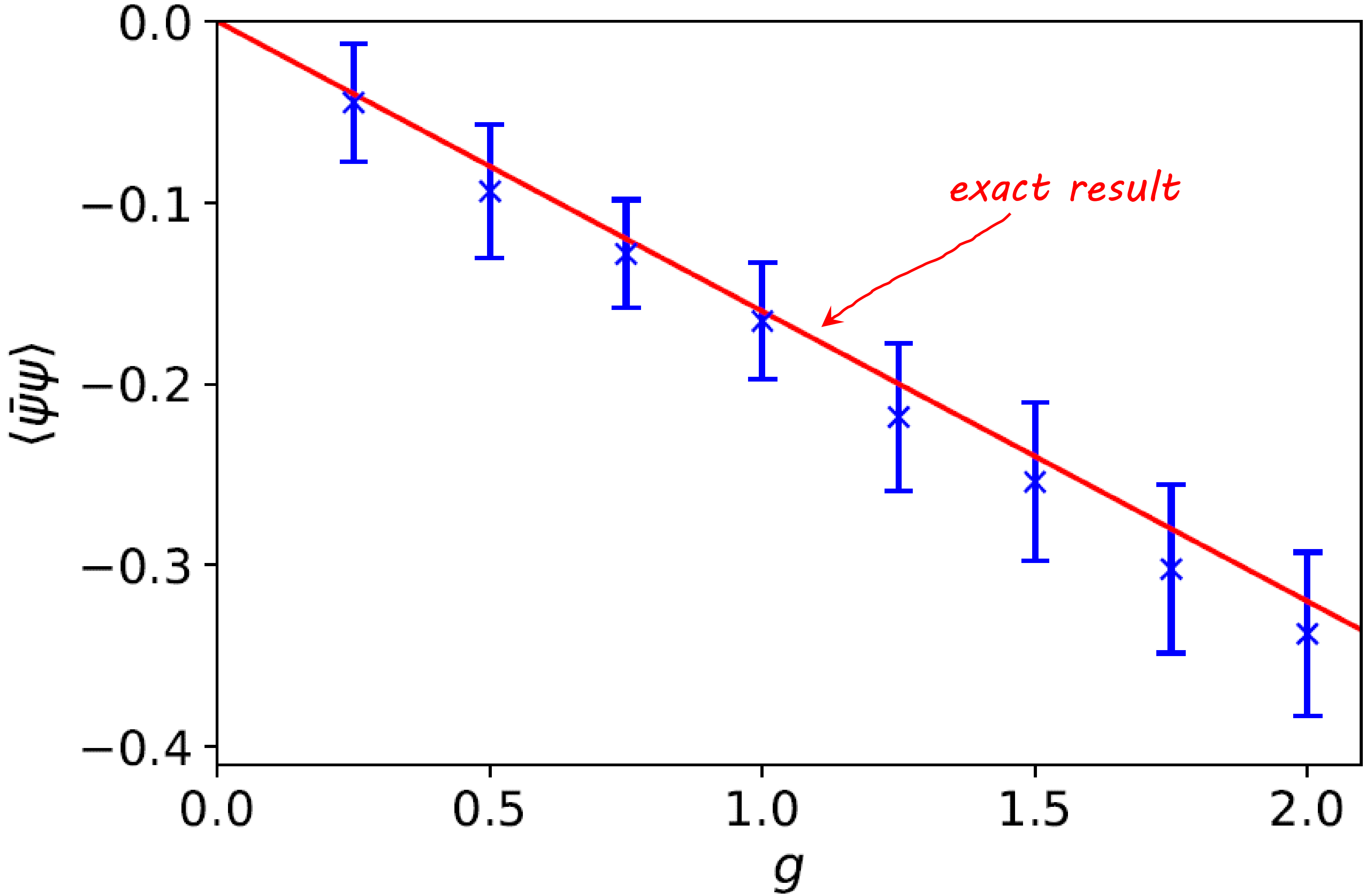
(~# of qubits, gates)

Massless case (after continuum & ∞ volume limit)

$T = 100, \delta t = 0.1, N_{\max} = 16$, 1M shots

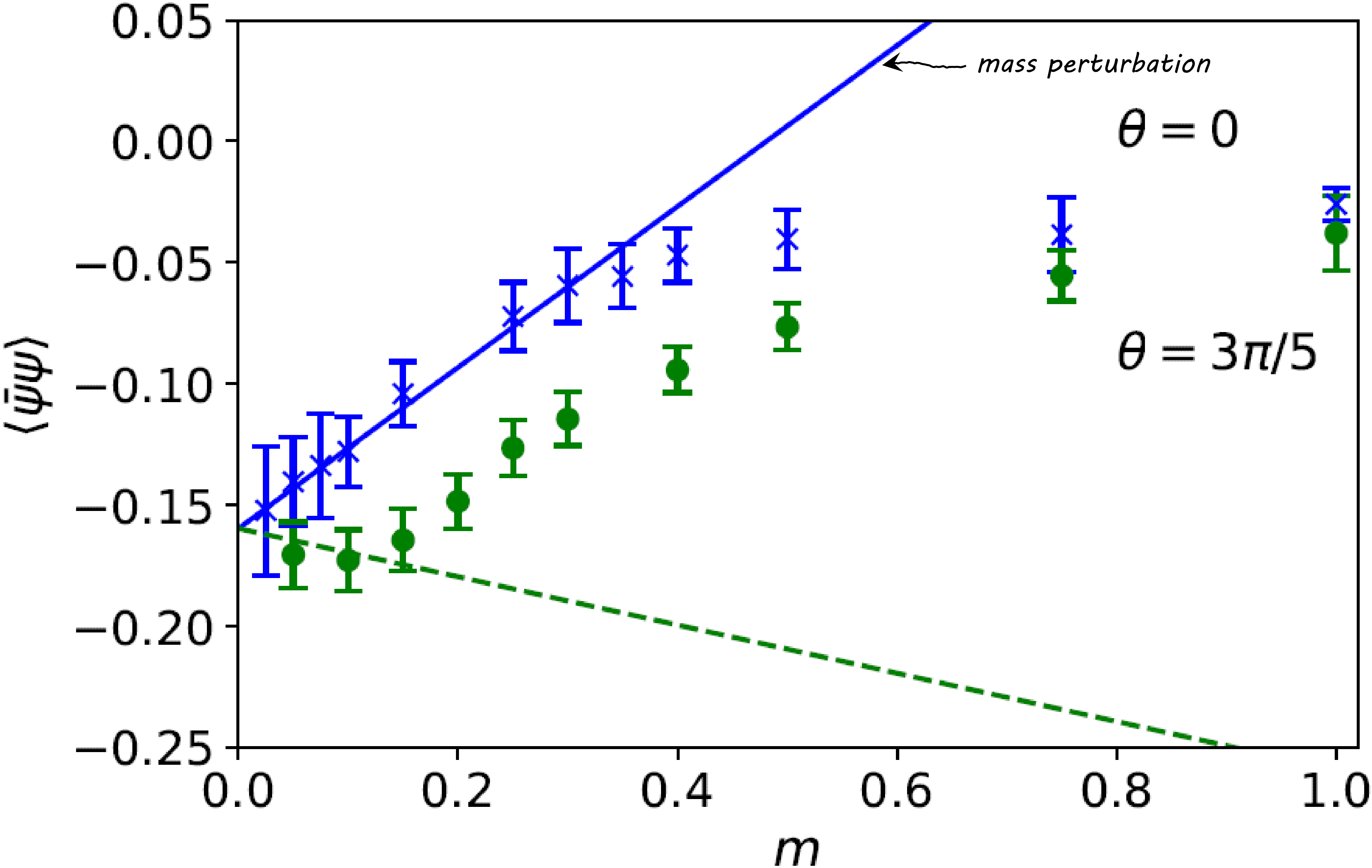
#(measurements)

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

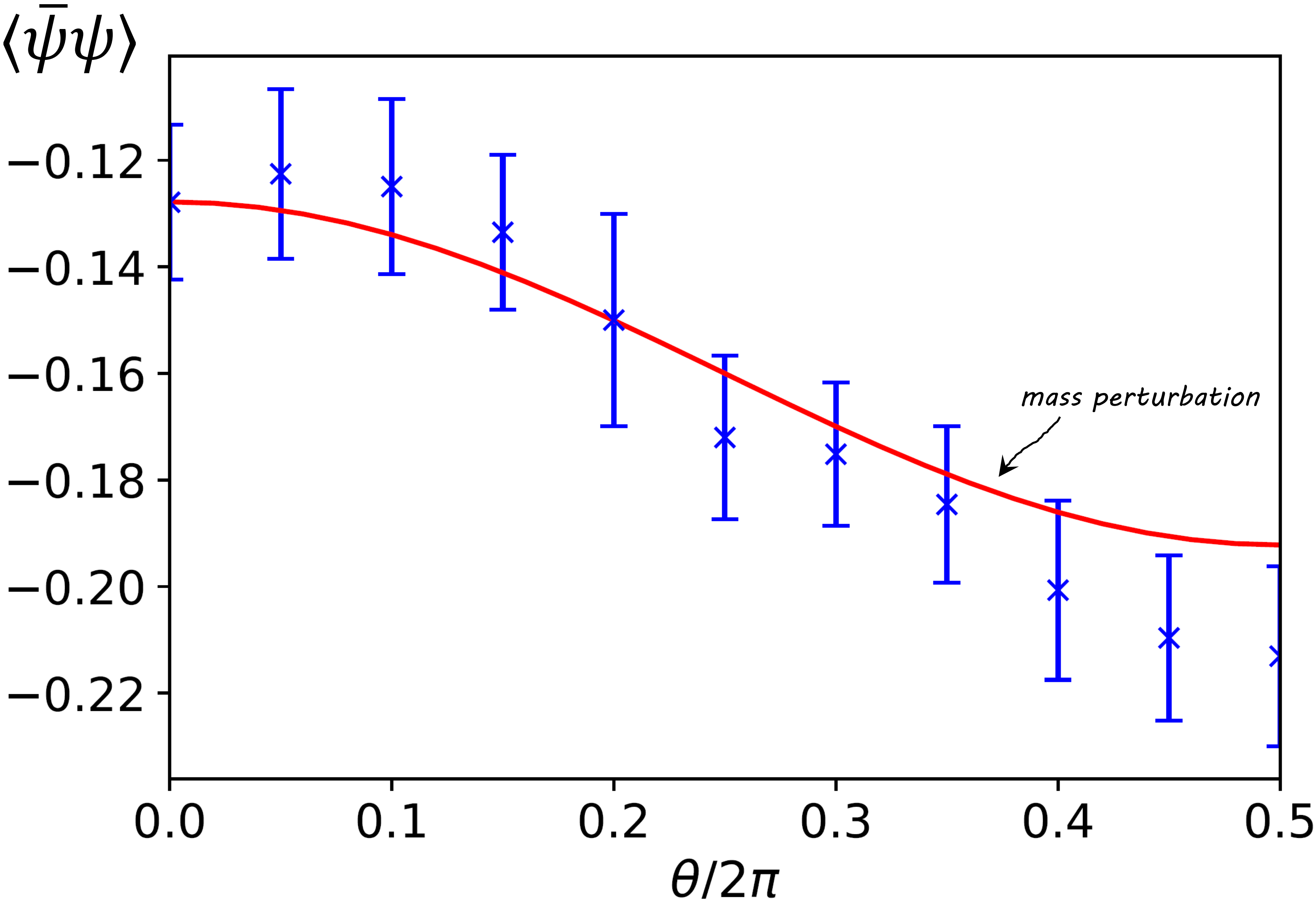


Result for massive case at $g = 1$

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]



θ dependence at $m = 0.1$ & $g = 1$



Summary & Outlook

Summary: [See also talks by Hanada, Kawai, Lamm, Meurice, Sturzu, Wang]

- Quantum computation is suitable for Hamiltonian formalism which is free from sign problem

- Instead we have to deal with huge vector space
  (Quantum computer may do this job?)

- constructed the vacuum of Schwinger model w/ θ term by adiabatic state preparation

- found agreement with the exact result for $m = 0$ & mass perturb.

(A part of) Works in progress:

- Searching critical point at $\theta = \pi$ [Chakraborty-MH-Kikuchi-Izubuchi-Tomiya]

- Confinement/screening [MH-Itou-Kikuchi-Nagano-Okuda]

- Matrix QM & (non-)SUSY QFTs [Hanada's talk, Buser-Gharibyan-Hanada-MH-Liu]

Thanks!

Appendix

Estimation of systematic errors

[Chakraborty-MH-Kikuchi-Izubuchi-Tomiya '20]

Approximation of vacuum:

$$|\text{vac}> \simeq U(T)U(T-\delta t)\cdots U(2\delta t)U(\delta t)|\text{vac}_0> \equiv |\text{vac}_A\rangle$$

Approximation of VEV:

$$\langle \mathcal{O} \rangle \equiv \langle \text{vac}|\mathcal{O}|\text{vac}\rangle \simeq \langle \text{vac}_A|\mathcal{O}|\text{vac}_A\rangle$$

Introduce the quantity

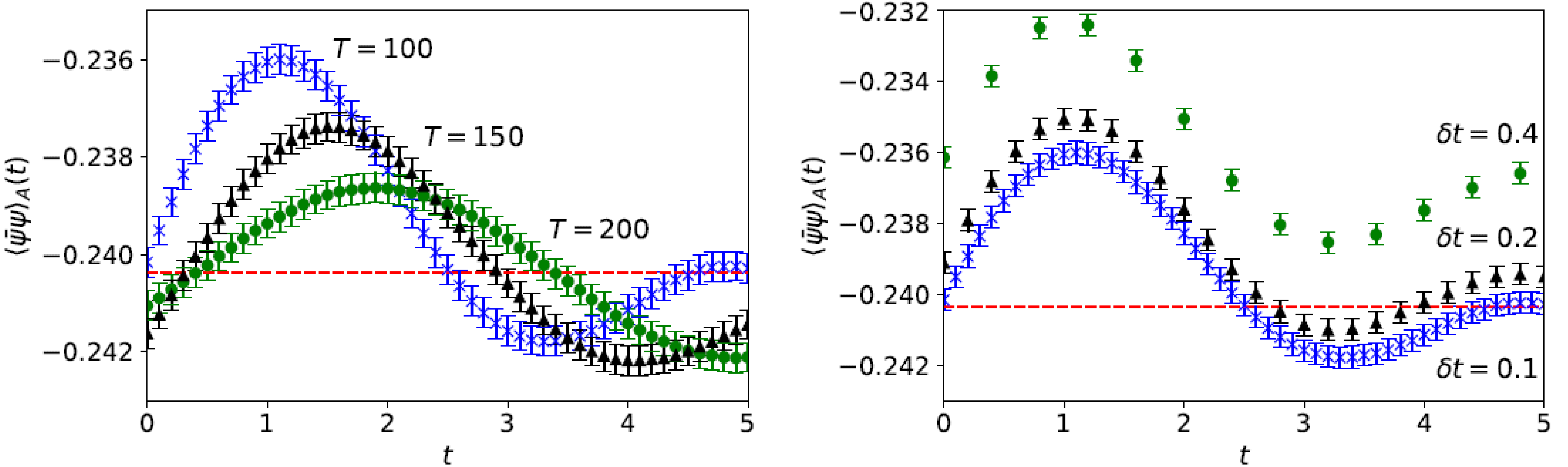
$$\langle \mathcal{O} \rangle_A(t) \equiv \langle \text{vac}_A|e^{i\hat{H}t}\mathcal{O}e^{-i\hat{H}t}|\text{vac}_A\rangle$$

- independent of t if $|\text{vac}_A\rangle = |\text{vac}\rangle$
- dependent on t if $|\text{vac}_A\rangle \neq |\text{vac}\rangle$

This quantity describes intrinsic ambiguities in prediction

⇨ Useful to estimate systematic errors

Estimation of systematic errors (Cont'd)



Oscillating around the correct value

Define central value & error as

$$\frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)+\min\langle\mathcal{O}\rangle_A(t)\right) \quad \& \quad \frac{1}{2}\left(\max\langle\mathcal{O}\rangle_A(t)-\min\langle\mathcal{O}\rangle_A(t)\right)$$

Thermodynamic & Continuum limit

$g = 1, m = 0, N_{\text{max}} = 16, T = 100, \delta t = 0.1, 1M$ shots

#(measurements)