Abelian and monopole dominance without gauge fixing in pure SU(3) gauge theory

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Introduction

Color confinement in QCD is still an important unsolved problem.

- Color confinement (dual Meissner picture)

  [G. ‘t Hooft (1976), S. Mandelstam (1976)]

  - The QCD vacuum is a kind of a dual superconducting state caused by condensation of magnetic monopoles.
  - The color charges are then confined inside hadrons due to formation of the color-electric flux tube through the dual Meissner effect.
- Abelian projection [G. ‘t Hooft (1981)]
  - SU(3) (SU(2)) QCD can be reduced to an Abelian U(1)xU(1) (U(1)) theory by adopting a partial gauge fixing, and the color-magnetic monopoles appear.
- Violation of non-Abelian Bianchi identities (VNABI) [T. Suzuki (2014)]
  - The VNABI can be regarded as Abelian-like monopole currents in the continuum QCD. (N^2-1 Abelian monopoles appear in SU(N))

Introduction

- Previous works
  - The confining properties are dominated by the Abelian fields in Maximally Abelian (MA) gauge in SU(2) and SU(3) gauge theories. (Abelian dominance) [T. Suzuki and I. Yotsuyanagi (1990), etc.]
  - The confining properties are also dominated by monopoles in MA gauge (monopole dominance) [H. Shiba et al. (1994), J. D. Stack et al. (1994), etc.]
  - Abelian dominance and monopole dominance without gauge fixing are observed in SU(2) gauge theory. [T. Suzuki et al. (2007)]
    - SU(2) -> three Abelian and monopole components
  - Scaling behaviors of monopole densities based on the VNABI are observed in various smooth gauge fixing conditions like MA gauge and maximal center gauge in SU(2) gauge theory. [T. Suzuki et al. (2018)]

- Based on the idea of the VNABI, it is important to study the case of SU(3) gauge theory without gauge fixing.

VNABI and Abelian monopoles

- Using a covariant derivative $D_\mu = \partial_\mu - igA_\mu$, we get the following commutation relation,

$$[D_\mu, D_\nu] = -igG_{\mu\nu} + [\partial_\mu, \partial_\nu]$$

where $G_{\mu\nu}$ is a non-Abelian field strength.

- The second term $[\partial_\mu, \partial_\nu]$ can not be neglected when a line singularity exists.
- The Jacobi identities $\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0$

→ $D_\nu G^*_{\mu\nu} = \partial_\nu f^*_{\mu\nu} = k_\mu$

$$f_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\lambda^a/2$$

→ VNABI is equivalent to eight Abelian-like magnetic monopole currents in SU(3) gauge theory.

Abelian projection

- Definition of Abelian link variables
  - Non-Abelian SU(3) link variable $U_\mu(s)$
  - Abelian link variables $u^a_\mu(s) = \exp(i\theta^a_\mu(s)\lambda^a)$ using Gell-Mann matrices (a=1-8) are extracted from SU(3) link variable to maximize the overlap R for each index a

$$R = \sum_{s,\mu} \mathrm{ReTr}\{u^a_\mu(s) U^\dagger_\mu(s)\}$$

  - Example for a=1

$$\theta^1_\mu(s) = \tan^{-1}\left\{\frac{\mathrm{Im}(U^{12}_\mu(s) + U^{12}_\mu(s))}{\mathrm{Re}(U^{11}_\mu(s) + U^{22}_\mu(s))}\right\}$$

Polyakov loop correlation function (PCLF)

- Polyakov loop and PLCF

$$P(s) = \prod_{k=0}^{N_t-1} U_4(s+k\hat{4}) \Longrightarrow -\frac{1}{T}\ln\langle \mathrm{Tr}P(\vec{s}_1)\mathrm{Tr}P(\vec{s}_2)^*\rangle = V(r) \quad (r = |\vec{s}_2 - \vec{s}_1|)$$

$V(r)$: static potential

- Abelian, monopole and photon parts of Polyakov loop

$$P_{\mathrm{A}}^a(s) = \exp\{i\sum_{k=0}^{N_t-1}\theta_4^a(s+k\hat{4})\} \Longrightarrow -\frac{1}{T}\ln\langle P_A^a(\vec{s}_1)P_A^a(\vec{s}_2)^*\rangle = V_A^a(r)$$

$V_A^a(r)$: Abelian static potential

$$\theta_4^a(s) = -\sum_{s'} D(s-s')[\partial'_\nu\Theta_{\nu4}^a(s') + \partial_4(\partial'_\nu\theta_\nu^a(s\prime))]$$

$D(s-s')$: lattice Coulomb propagator

$\Theta_{\mu\nu}^a(s) = \partial_\mu\theta_\nu^a(s) - \partial_\nu\theta_\mu^a(s) = \bar{\Theta}_{\mu\nu}^a(s) + 2\pi n_{\mu\nu}^a(s)$: Abelian field strength

$\bar{\Theta}_{\mu\nu}^a(s) \in [-\pi,\pi]$, $n_{\mu\nu}^a(s)$: integer

$$P_{\mathrm{A}}^a = P_{\mathrm{ph}}^a \cdot P_{\mathrm{mon}}^a$$

$$P_{\mathrm{ph}}^a(s) = \exp\{-i\sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_\nu\bar{\Theta}_{\nu4}^a(s')\} \Longrightarrow -\frac{1}{T}\ln\langle P_{ph}^a(\vec{s}_1)P_{ph}^a(\vec{s}_2)^*\rangle = V_{ph}^a(r)$$

$V_{Ph}^a(r)$: photon static potential

$$P_{\mathrm{mon}}^a(s) = \exp\{-2\pi i\sum_{k=0}^{N_t-1}\sum_{s'} D(s+k\hat{4}-s')\partial'_\nu n_{\nu4}^a(s')\} \Rightarrow -\frac{1}{T}\ln\langle P_{mon}^a(\vec{s}_1)P_{mon}^a(\vec{s}_2)^*\rangle = V_{mon}^a(r)$$

$V_{mon}^a(r)$: monopole static potential

Numerical simulations

- T=0 system
  - PLCF at zero-temperature is very noisy.
  - Calculate PLCF and APLCF by applying multilevel algorithm to reduce the errors.

M. Lüscher and P. Weisz, JHEP 0109 (2001) 010, [hep-lat/0108014].
M. Lüscher and P. Weisz, JHEP 0207 (2002) 049, [hep-lat/0207003].
Y. Koma and M. Koma, Phys. Rev. D 95, 094513 (2017), [hep-lat/170306247]

- T≠0 system
  - Monopole and photon Polyakov loops are non-local operators.
  - Evaluate APLCF, MPLCF and PPLCF by employing random gauge transformations at finite temperature in the confinement phase.

T. Suzuki et al., Phys.Rev.D77:034502,2008, [hep-lat/07064366].
T. Suzuki et al., Phys.Rev.D80:054504,2009, [hep-lat/09070583].
T. Sekiguchi and K. I., Int. J. Mod. Phys. A31, 1650149 (2016), [hep-lat/161201670]

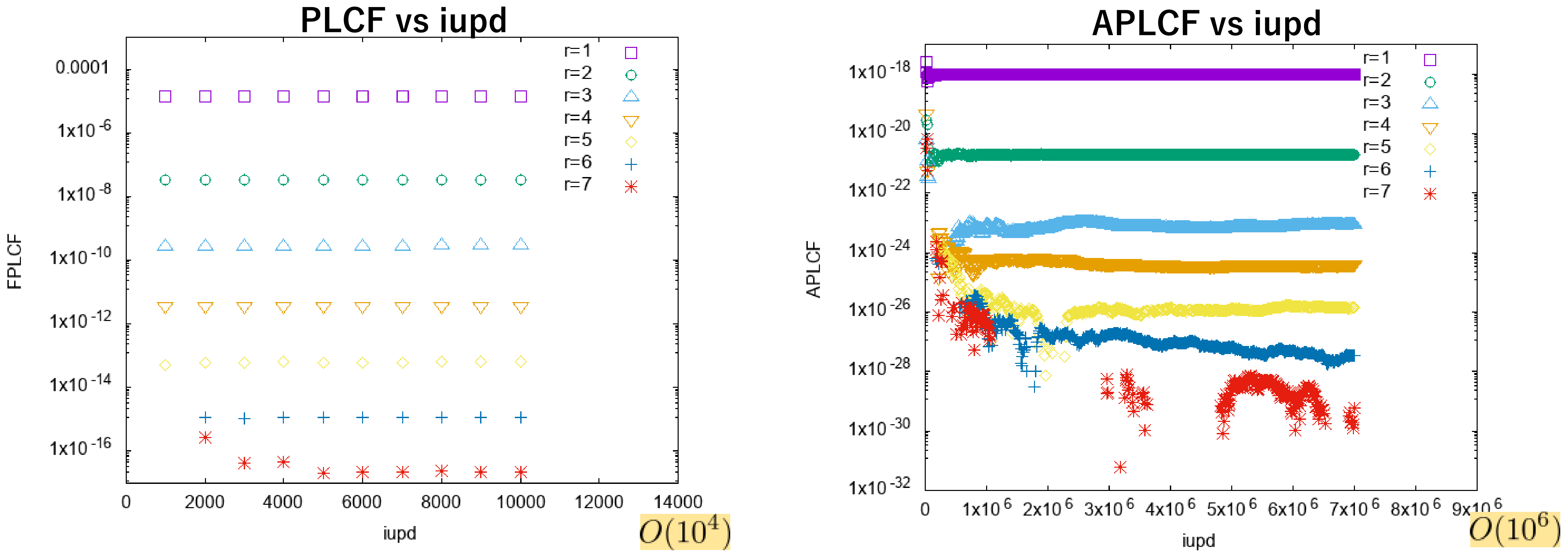
In both cases, it does not impose any gauge fixing conditions.

Numerical results (T=0)

SU(3) Wilson action, 16^4, $\beta = 5.60$

Multilevel algorithm

1. Divide the lattice volume into several sublattices along the time direction.
2. Take the average of a parts of PLCF over internal update (iupd) on each sublattice.
3. Compute the PLCF from the product of sublattice average of its components



A huge number of internal updates are needed for the calculation of APLCF.

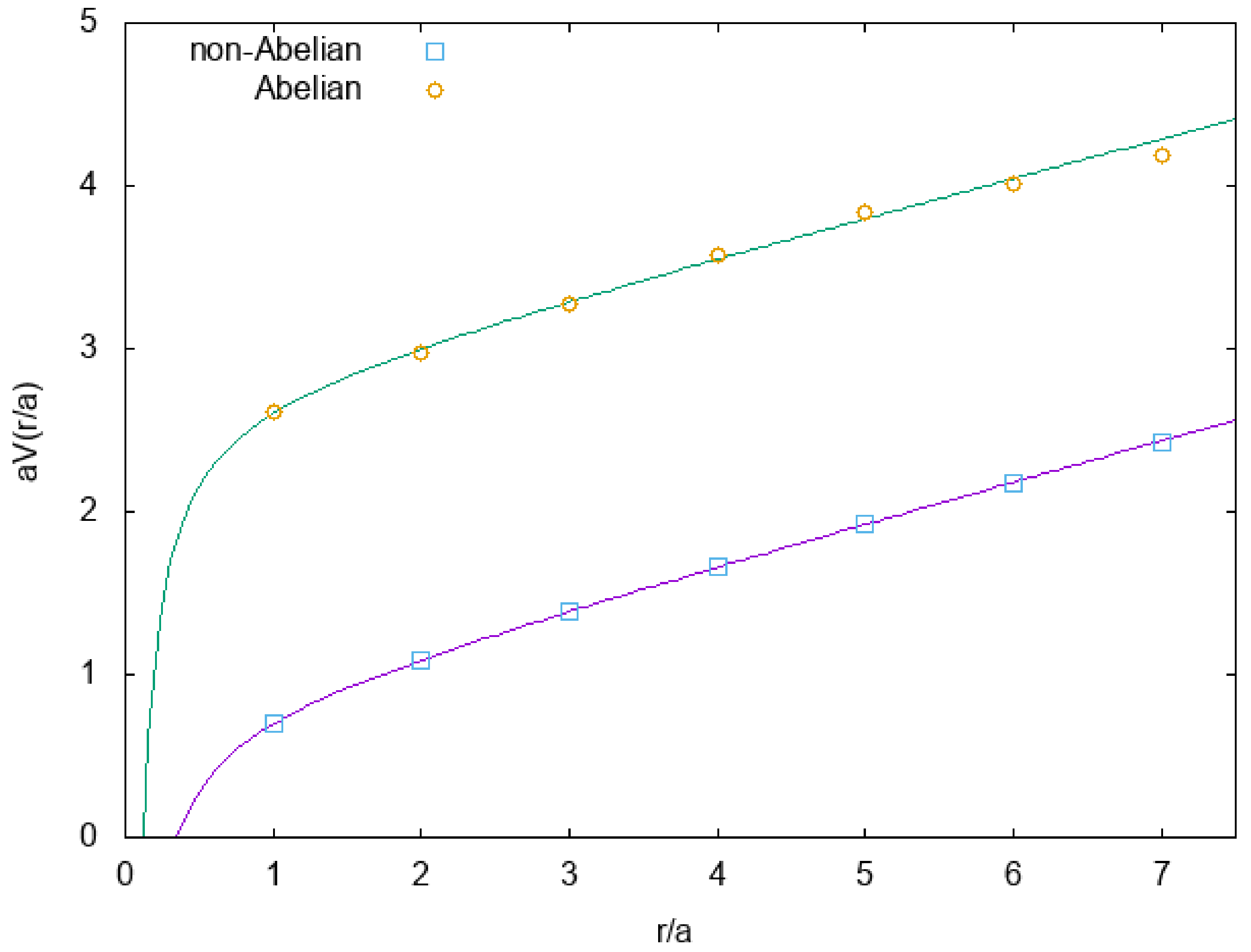
Numerical results (static potential)

T=0 (multilevel algorithm)

SU(3) Wilson action
16^4、 β =5.60
Abelian : color-1 component
iupd : 10,000 for PLCF
7,000,000 for APLCF
fitting function

$$V(r) = \sigma r - \frac{\alpha}{r} + C$$

	σ	α	C	fit-range
non-Abelian	0.249(2)	0.29(1)	0.73(1)	1-6
Abelian	0.23(2)	0.3(1)	2.7(1)	1-6



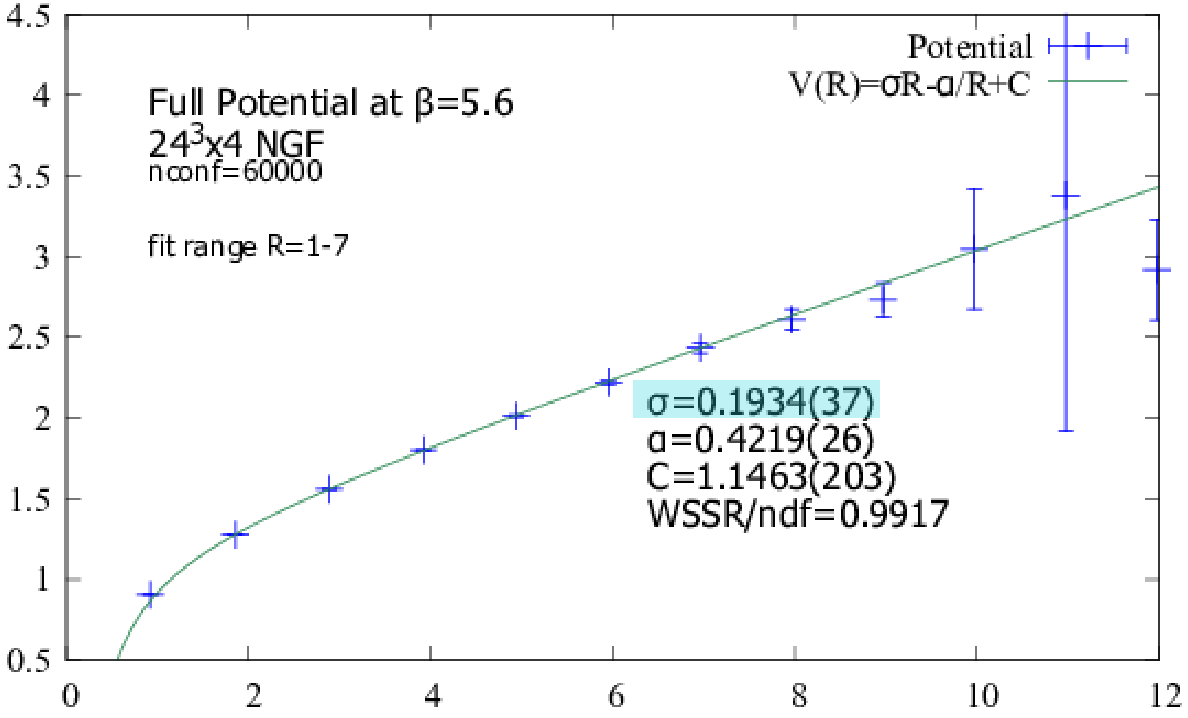
This result show Abelian dominance only for one-color component.

$T \neq 0$ (random gauge transformation)

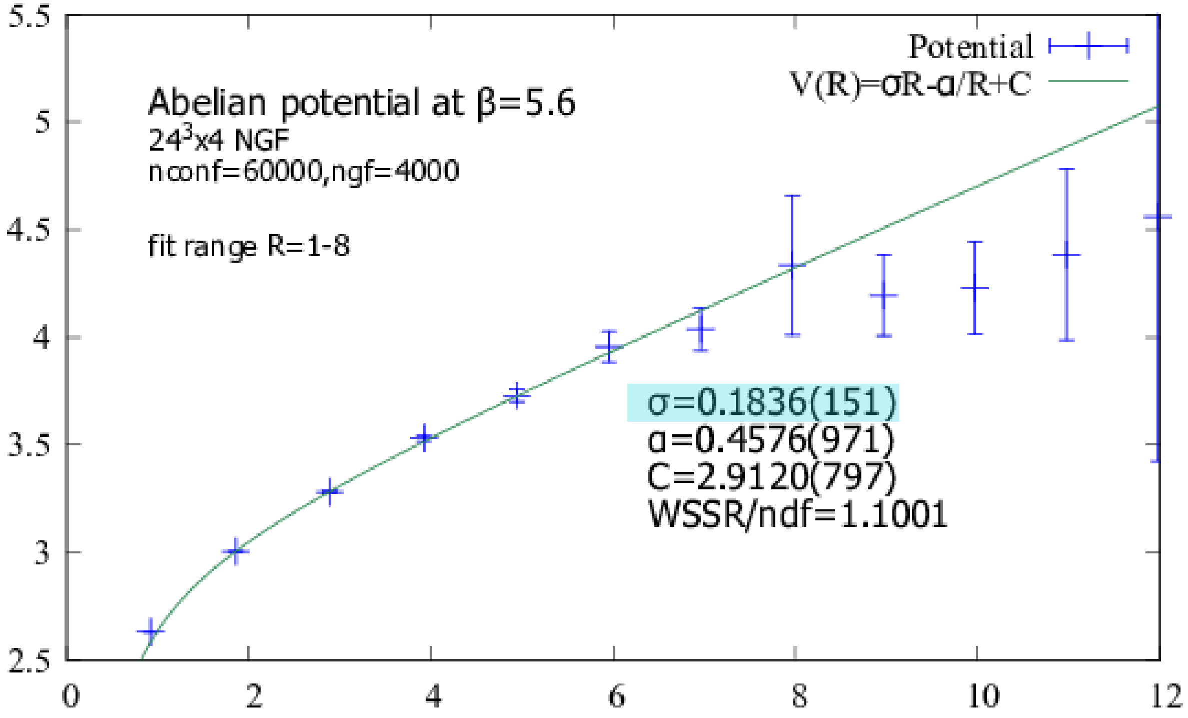
$$V(r) = \sigma r - \frac{\alpha}{r} + C$$

- non-Abelian and Abelian static potential

SU(3) Wilson action
24^3x4, β =5.60, T～0.8Tc,
# of conf.=60,000, # of random gauge transformation = 4000



non-Abelian potential

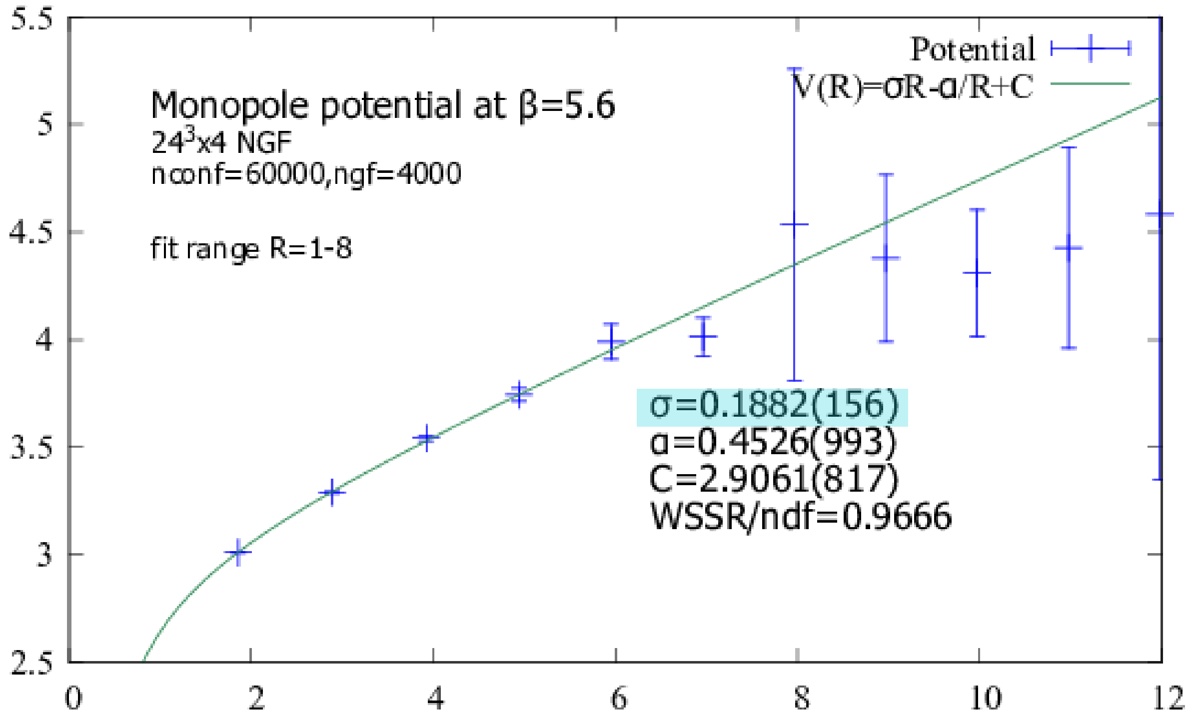


Abelian potential

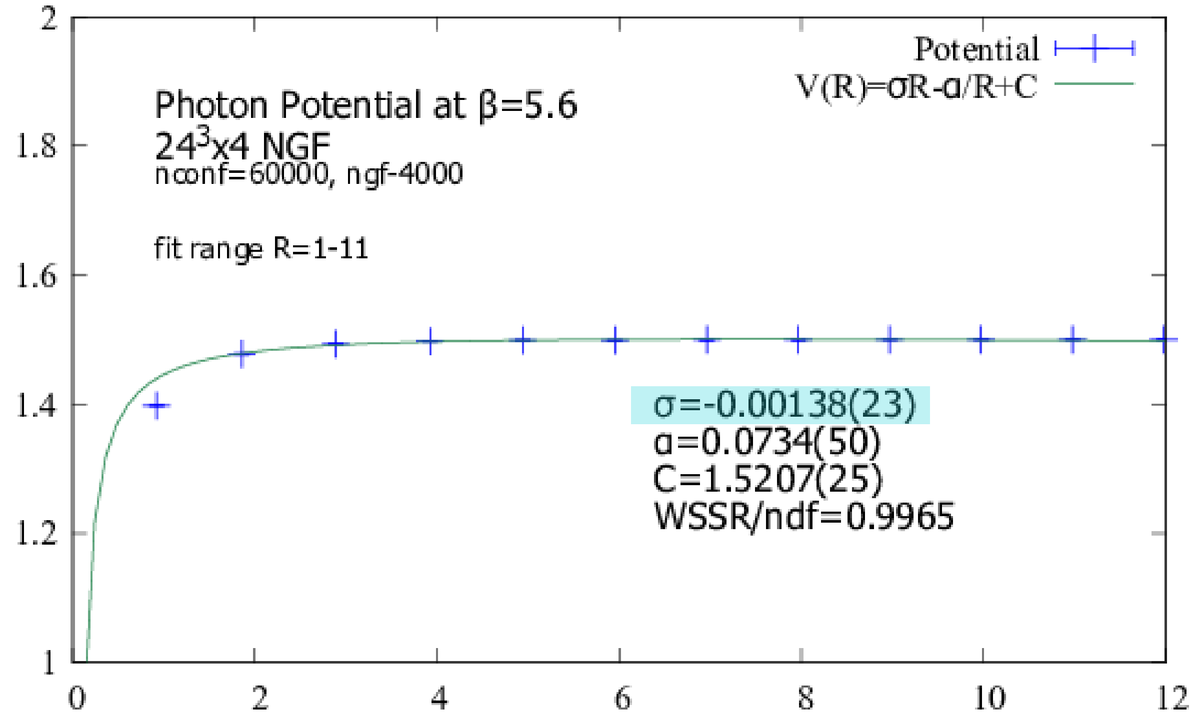
T≠0 (random gauge transformation)

- Monopole and photon static potential

$$V(r) = \sigma r - \frac{\alpha}{r} + C$$



monopole potential



photon potential

Results(T ≠ 0)

- Fitting function for static potentials $V(r) = \sigma r - \frac{\alpha}{r} + C$

FPLCF	nconf=60000			
fit range	σ	α	C	$WSSR/ndf$
[1:7]	0.1934(37)	0.4219(26)	1.1463(203)	0.9917

APLCF	nconf=60000	ngf=4000		
fit range	σ	α	C	$WSSR/ndf$
[1:8]	0.1836(151)	0.4576(971)	2.9120(797)	1.1001

MPLCF	nconf=60000	ngf=4000		
fit range	σ	α	C	$WSSR/ndf$
[1:8]	0.1882(156)	0.4526(993)	2.9061(817)	0.9666

PPLCF	nconf=60000	ngf=4000		
fit range	σ	α	C	$\sqrt{WSSR/ndf}$
[1:11]	-0.0014(2)	0.073(5)	1.521(3)	0.9965

- These results show perfect Abelian dominance and monopole dominance for string tension

Summary

- Calculate static potentials in SU(3) gauge theory at zero(16^4, beta=5.6, multilevel) and finite temperature(24^3x4, beta=5.6, random gauge transformation)
- Perfect Abelian dominance for string tension without gauge fixing are observed only for one-color component.
- Perfect monopole dominance are also obtained in finite temperature system.

- It is necessary to check scaling behaviors and finite volume effects.
- Study the contribution of monopole for various physical quantities (hadron mass, chiral symmetry breaking, etc.)