QCD sum rule from lattice correlators

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Outline

- We propose a method to compute a spectral sum in the QCD sum rule from lattice correlators.
- The lattice results can replace OPE to extract the QCD parameters.

1. Borel transformation in QCD sum rule
2. Chebyshev expansion
3. Numerical result
4. Summary

Determination of the QCD parameters

the QCD parameters, such as α_s and m_q, can be determined by the matching:

$$\langle O \rangle_{\mathrm{OPE}} = \langle O \rangle_{\mathrm{lat}}$$

It requires

in OPE

- the typical energy scale is large enough to use perturbation theory

in LQCD

- discretization error under control

Determination of the QCD parameters

the QCD parameters, such as α_s and m_q, can be determined by the matching:

It re

we compute the Borel transform

$$\Pi(M^2) = \frac{1}{M^2}\int ds e^{-s/M^2}\rho(s) \quad \text{from} \quad C(t) = \sum_{\mathbf{x}} \langle J(t,\mathbf{x}) J(0,\mathbf{0})\rangle$$

following QCD sum rule (SVZ)

- Why $\Pi(M^2)$?
- How can we compute it?

- the t … der
  larg…
  pertu… theory

[Shifman, Vainshtein, Zakharov, 79]

Borel transform and OPE

$\Pi(M^2)$ is the Borel transform of HVP $\Pi(Q^2) = \int_0^\infty ds \frac{\rho(s)}{s+Q^2}$

def. of the Borel transformation ($Q^2 = -q^2$)

$$\mathscr{B}_M := \lim_{\substack{n,\, Q^2 \to \infty \\ Q^2/n = M^2}} \frac{Q^{2n}}{(n-1)!} \left(-\frac{\partial}{\partial Q^2}\right)^n$$

$$\mathscr{B}_M\left[\frac{1}{s+Q^2}\right] = \frac{1}{M^2} e^{-s/M^2}$$

$$\mathscr{B}_M[\Pi(Q^2)] = \frac{1}{M^2}\int_0^\infty ds e^{-s/M^2}\rho(s) = \Pi(M^2)$$

in large $Q^2 > 0$ region

$$\Pi(Q^2) = \Pi^{\text{pert}}(Q^2) + \frac{c_2}{Q^2} + \frac{c_4}{Q^4} + \frac{c_6}{Q^6} + \cdots$$

$$\mathscr{B}_M\left[\frac{1}{Q^{2n}}\right] = \frac{1}{(n-1)!}\frac{1}{M^{2n}}$$

OPE more convergent

$$\Pi(M^2) = \Pi^{\text{pert}}(M^2) + \frac{c_2}{M^2} + \frac{c_4}{M^4} + \frac{c_6}{2!M^6} + \cdots$$

Spectral rep. of correlator

current-current correlators

$$C(t) = \sum_{\mathbf{x}} \langle J_i(t,\mathbf{x}) J_i(0,\mathbf{0}) \rangle$$

spectral rep.

$$C(t) = \int d\omega e^{-\omega t} \omega^2 \rho(\omega^2) \qquad (\omega^2 = s)$$

How can we relate them?

$$\Pi(M^2) = \frac{2}{M^2} \int d\omega e^{-\omega^2/M^2} \omega \rho(\omega^2)$$

$e^{-\omega} \approx e^{-H}$: transfer matrix

expansion in $e^{-\omega}$

$$\int d\omega \rho(\omega) \left(\begin{array}{lcl} \frac{2\omega}{M^2} e^{-\omega^2/M^2} & = & a_0(M^2)\omega^2 + a_1(M^2)\omega^2 e^{-\omega} + a_2(M^2)\omega^2 e^{-2\omega} + \cdots \\ \Pi(M^2) & = & a_0(M^2)C(0) + a_1(M^2)C(1) + a_2(M^2)C(2) + \cdots \end{array} \right.$$

Chebyshev expansion

Chebyshev expansion:

$$\frac{2\omega}{M^2} e^{-\omega^2/M^2} \simeq \frac{c_0^*(M^2)}{2}\omega^2 + \sum_{j=1} c_j^*(M^2) T_j^*(e^{-\omega})\omega^2$$

$$\Pi(M^2) \simeq \frac{c_0^*(M^2)}{2} C(0) + \sum_{j=1} c_j^*(M^2)\langle T_j^* \rangle$$

$c_j^*(M^2)$ determined by the form $\frac{2}{M^2\omega} e^{-\omega^2/M^2}$

(shifted) Chebyshev polynomial

$$T_1^*(x) = 2x - 1,\ T_2^*(x) = 8x^2 - 8x + 1, \cdots$$

$$\langle T_1^* \rangle = 2C(1) - C(0),\ \langle T_2^* \rangle = 8C(2) - 8C(1) + C(0), \cdots$$

correlators from lattice simulations

Setup

- JLQCD ensemble

  Nf = 2+1 Möbius domain-wall fermion

β	a^{-1}[GeV]	$L^3 \times T(\times L_5)$	#meas	am_{ud}	am_s
4.17	2.453(4)	$32^3 \times 64\ (\times 12)$	800	0.007	0.04
4.35	3.610(9)	$48^3 \times 96\ (\times 8)$	600	0.0042	0.025
4.47	4.496(9)	$64^3 \times 96\ (\times 8)$	400	0.0030	0.015

am_s is also valence quark mass.

- $J_i = \bar{s}\gamma_i s$

ground state: ϕ meson

- $m_\phi \sim 1\ \mathrm{GeV}$

$$\Pi^{\mathrm{lat}}(M^2) = \frac{c_0^*(M^2)}{2} + \sum_{j=1}^{N} c_j^*(M^2)\langle T_j^* \rangle$$

Chebyshev expansion

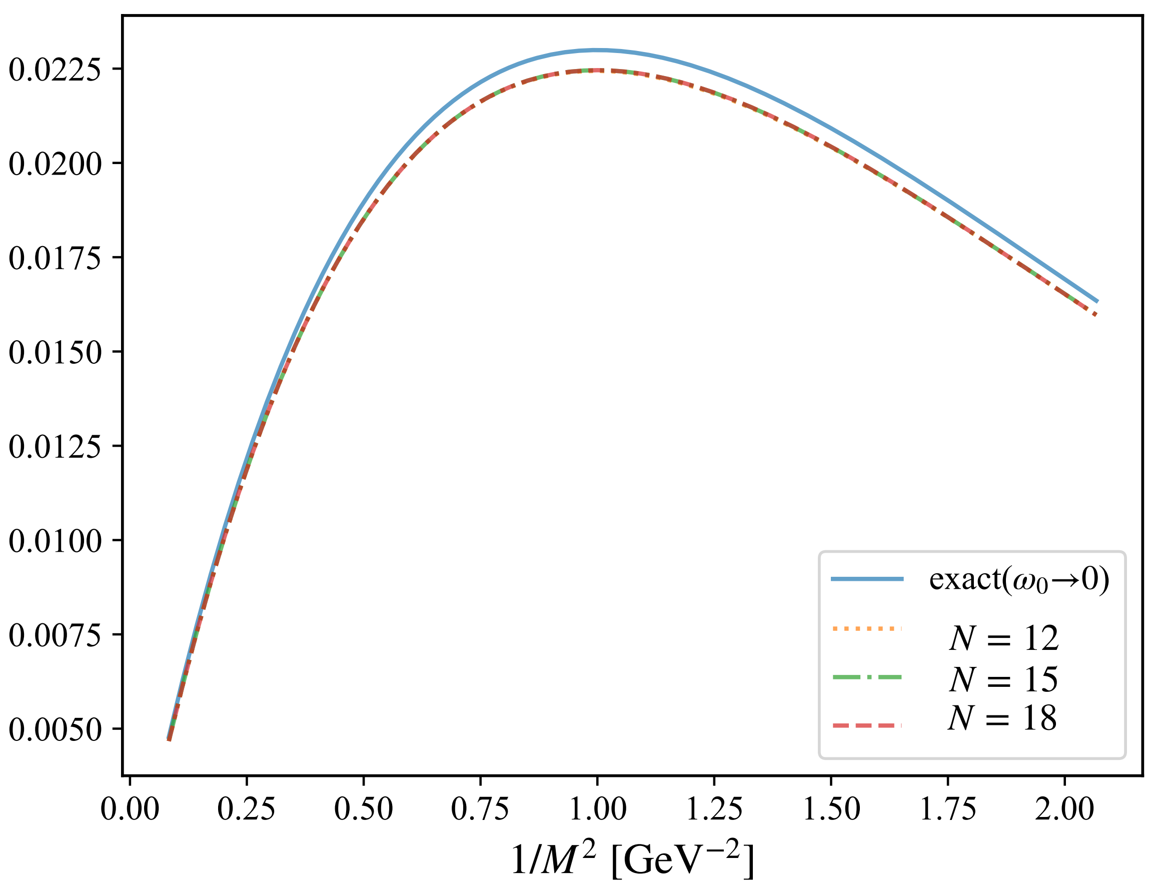
lattice simulation

Convergence of expansion

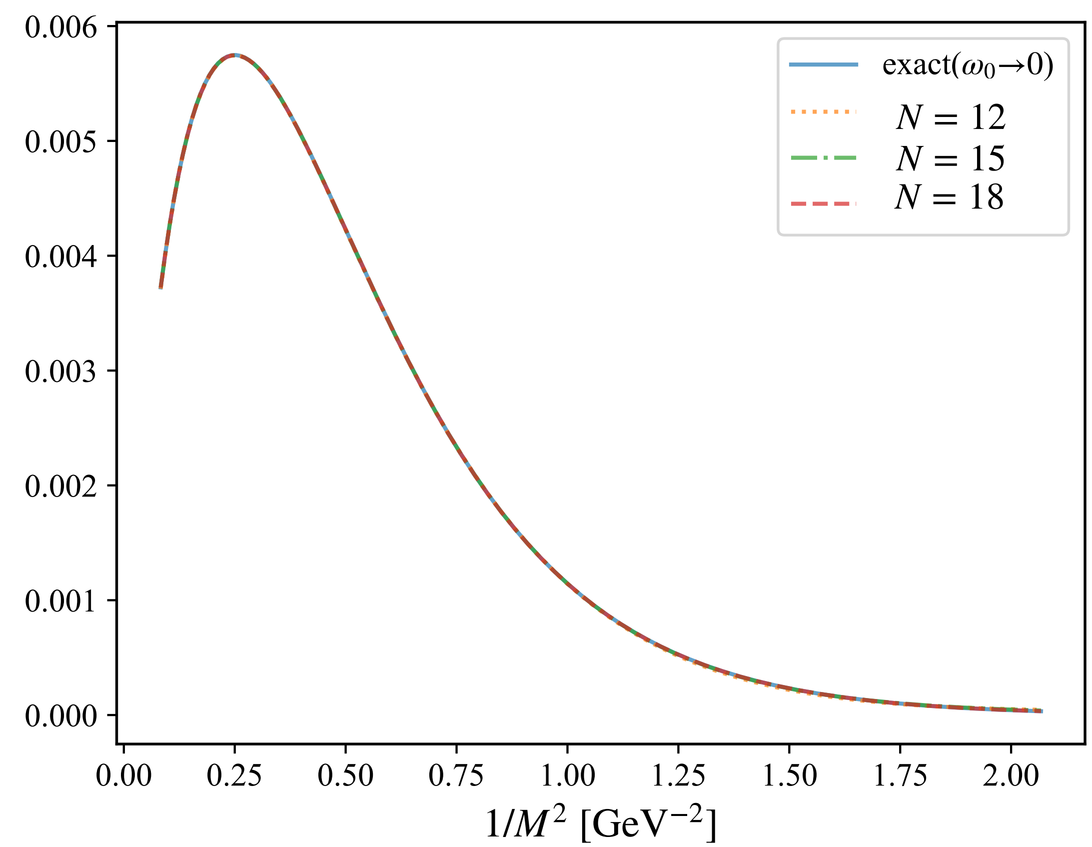
$$\frac{2}{M^2\omega}e^{-\omega^2/M^2}\tanh(\omega/\omega_0) \simeq \frac{c_0^*(M^2)}{2} + \sum_{j=1}^{N} c_j^*(M^2)T_j^*(e^{-\omega})$$

introduced to regularize infrared div.

$\omega = 1\ \mathrm{GeV}, \omega_0 = 0.45\ \mathrm{GeV}$



$\omega = 2\ \mathrm{GeV}, \omega_0 = 0.45\ \mathrm{GeV}$



Nearly perfect approximation with $N = 12$.

Convergence of expansion

$$\frac{2}{M^2\omega}e^{-\omega^2/M^2}\underline{\tanh(\omega/\omega_0)} \simeq \frac{c_0^*(M^2)}{2} + \sum_{j=1}^{N} c_j^*(M^2)T_j^*(e^{-\omega})$$

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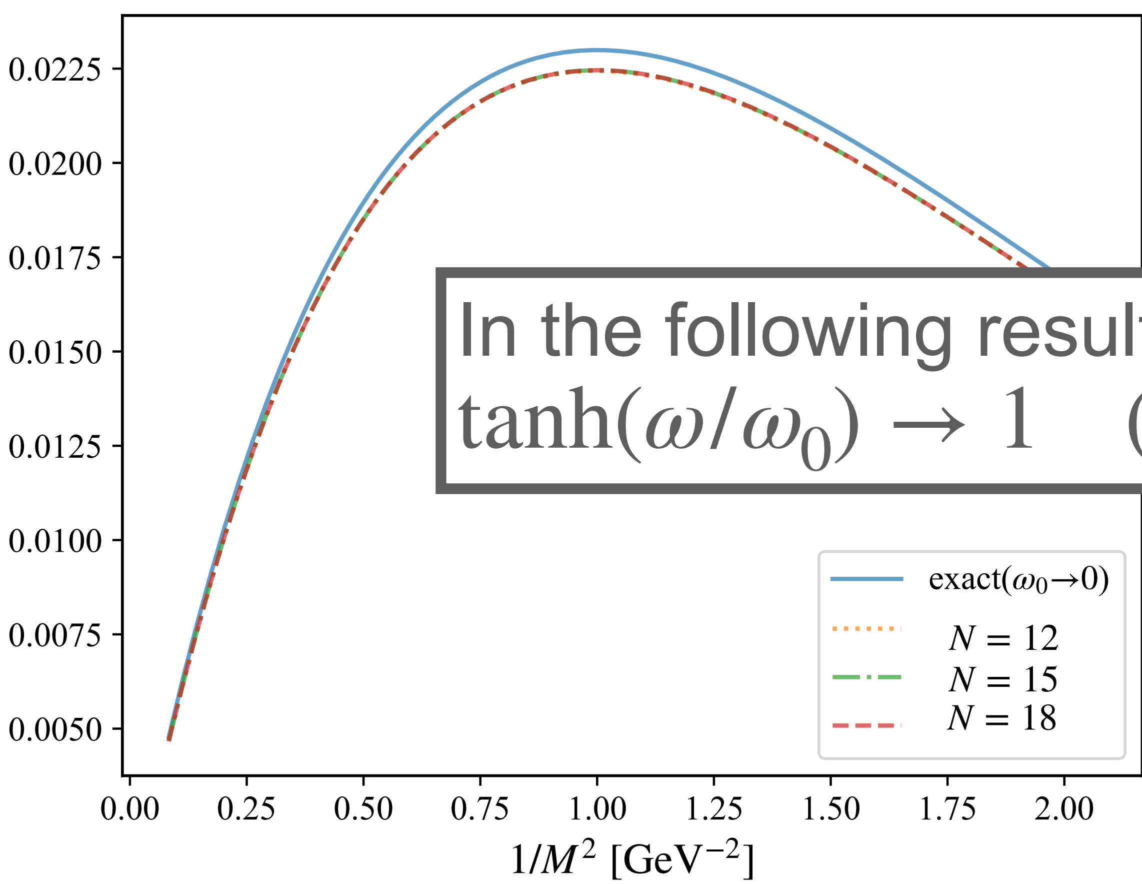
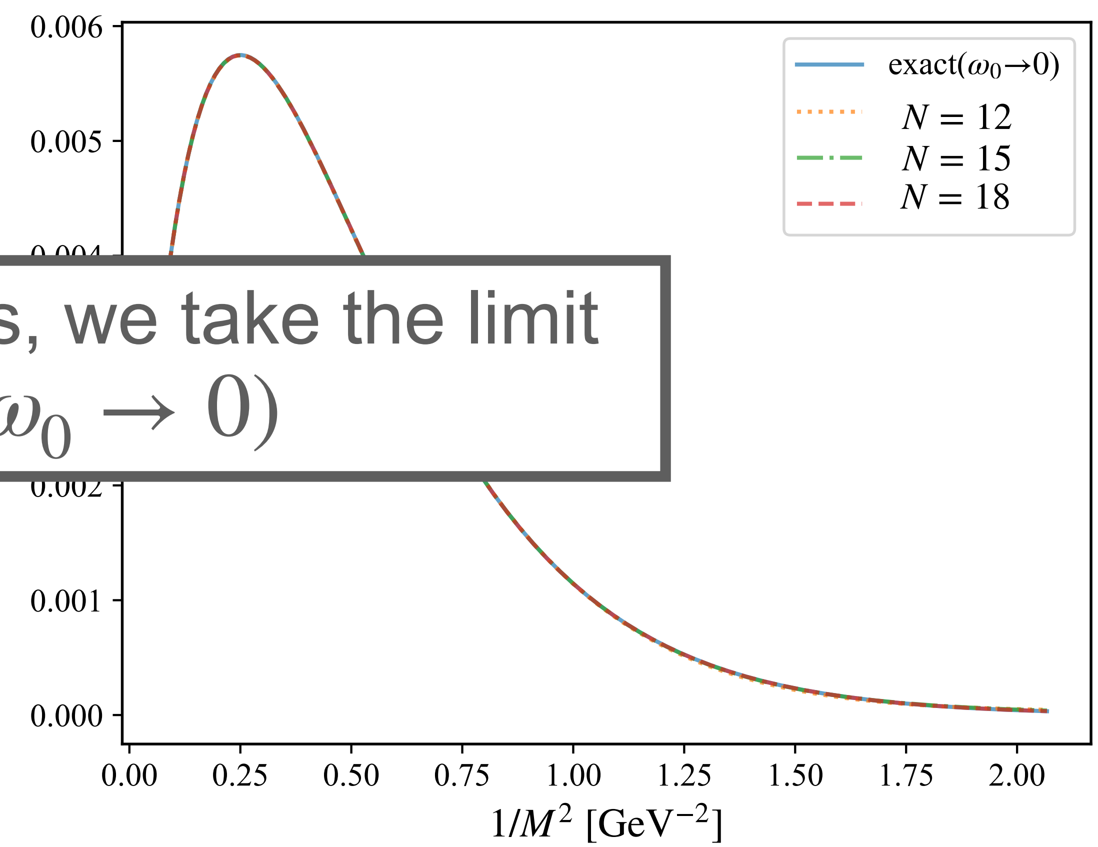


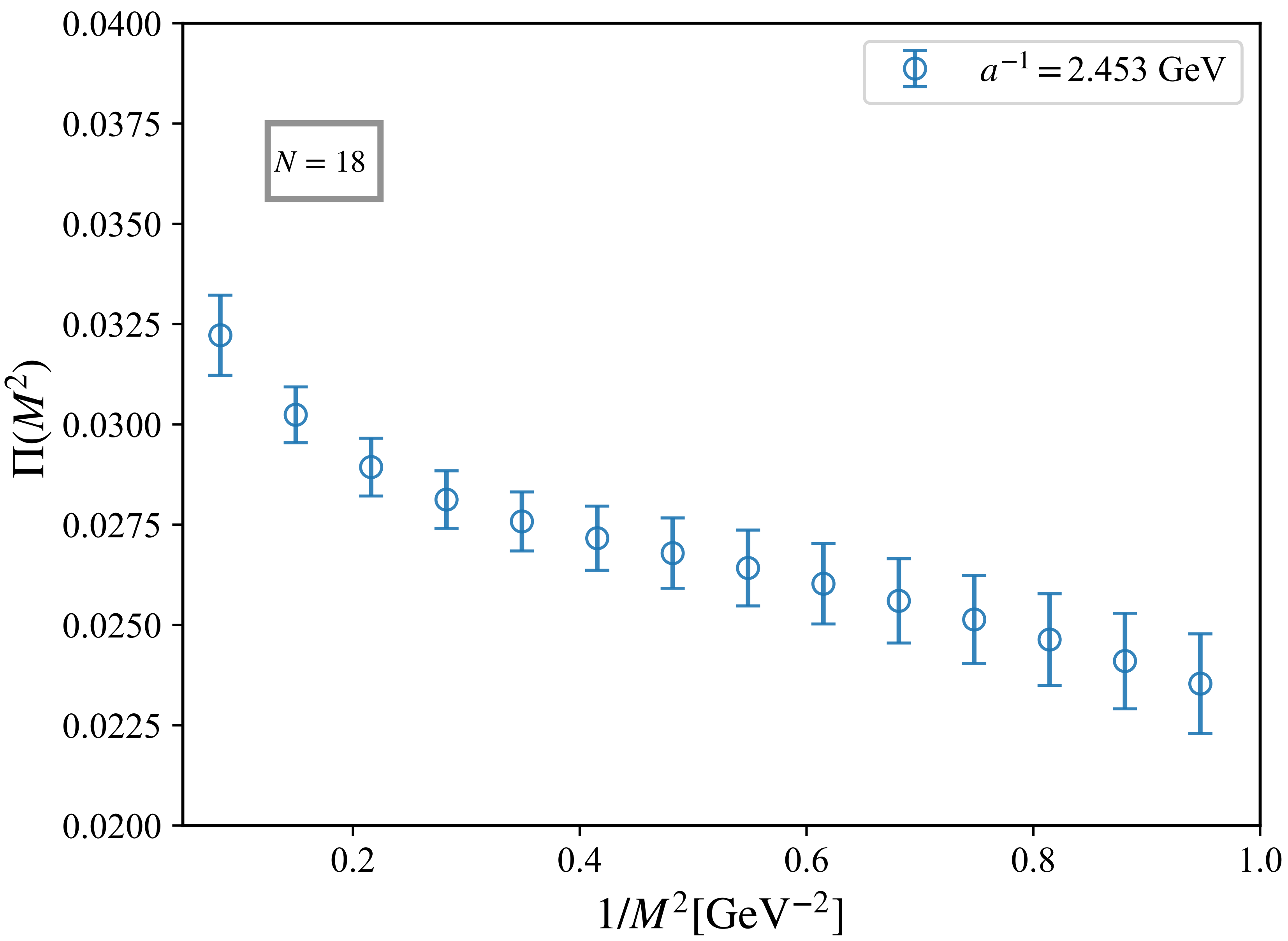
$\omega = 2$ GeV, $\omega_0 = 0.45$ GeV



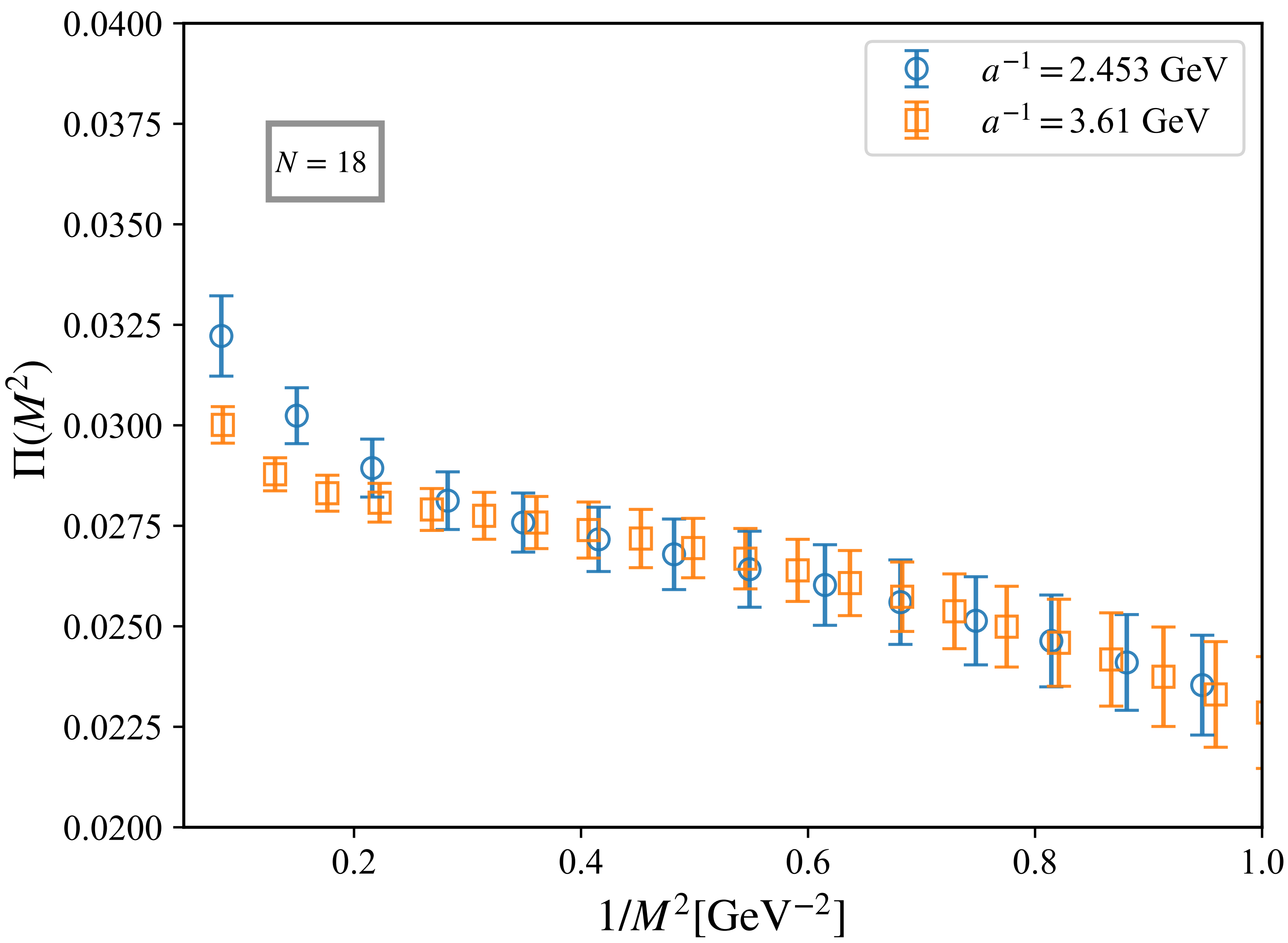
In the following results, we take the limit
$\tanh(\omega/\omega_0) \to 1 \quad (\omega_0 \to 0)$

Nearly perfect approximation with $N = 12$.

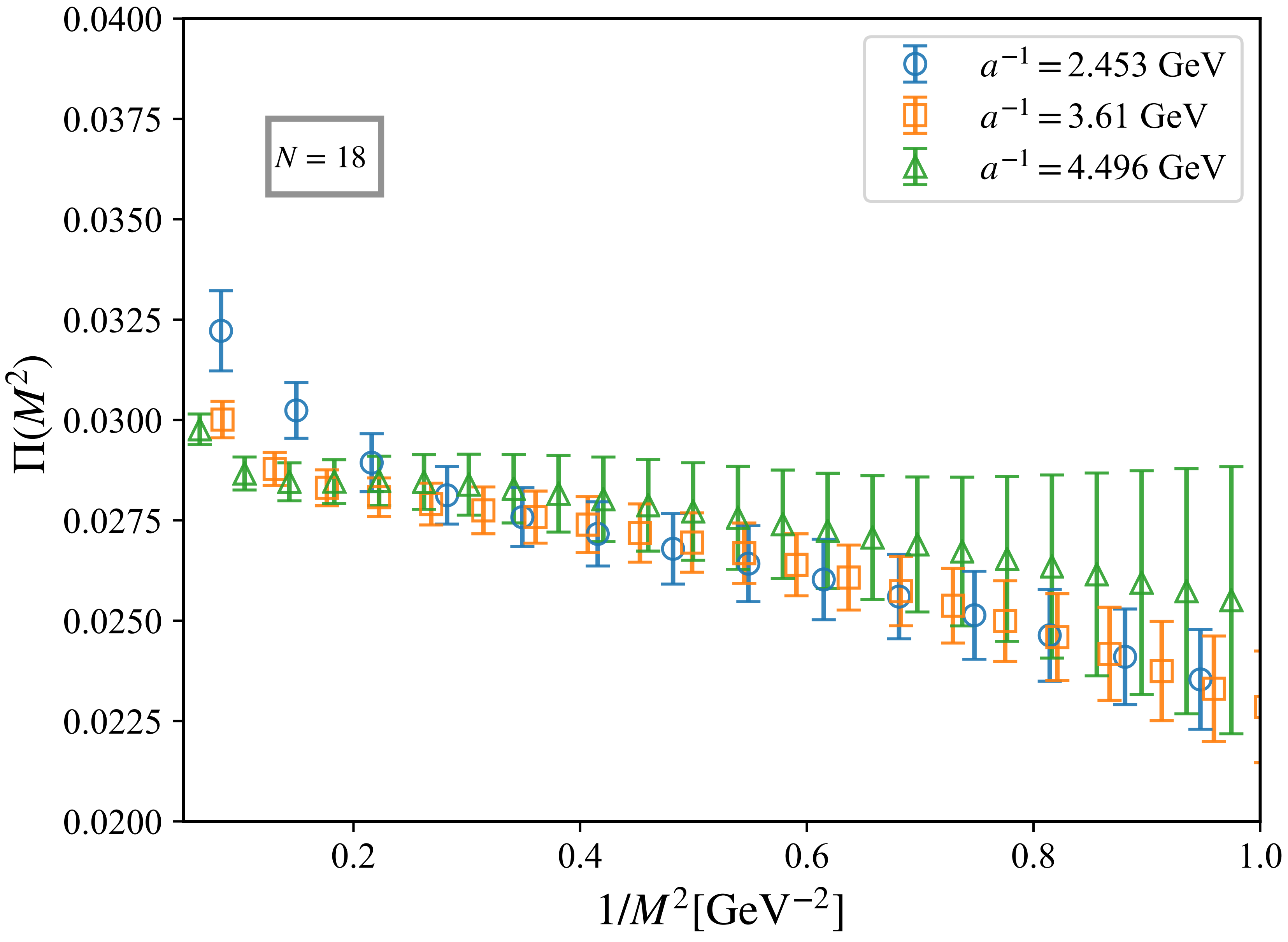
Lattice results



Lattice results



Lattice results

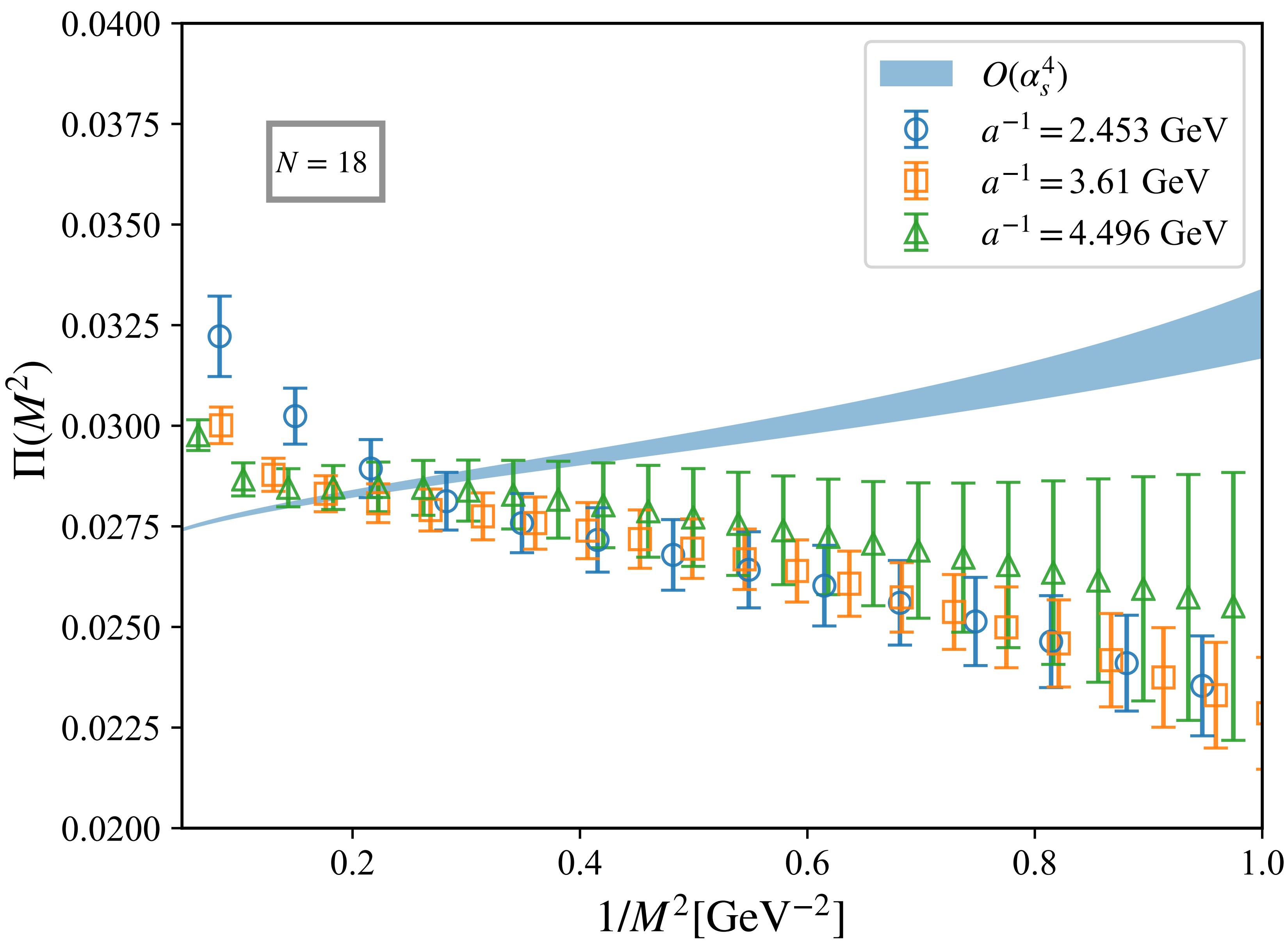


- $\Pi^{\mathrm{lat}}(M^2)$ are renormalized and matched to $\overline{\mathrm{MS}}$

  X-space method [Tomii et al., 2016]
- discretization effects are small

Comparison with OPE

input
$\mu^2 = M^2 e^{-\gamma_E}$
$m_s(2\text{ GeV}) = 0.0920(11)$ GeV
$\Lambda_{\overline{\rm MS}}^{n_f=3} = 0.0332(17)$ GeV
$\langle \bar{s}s \rangle = (0.296(11)\text{ GeV})^3$
$\langle \frac{\alpha_s}{\pi} G^2 \rangle = 0.0120(36)\text{ GeV}^4$



massless perturbation
at 5-loop level

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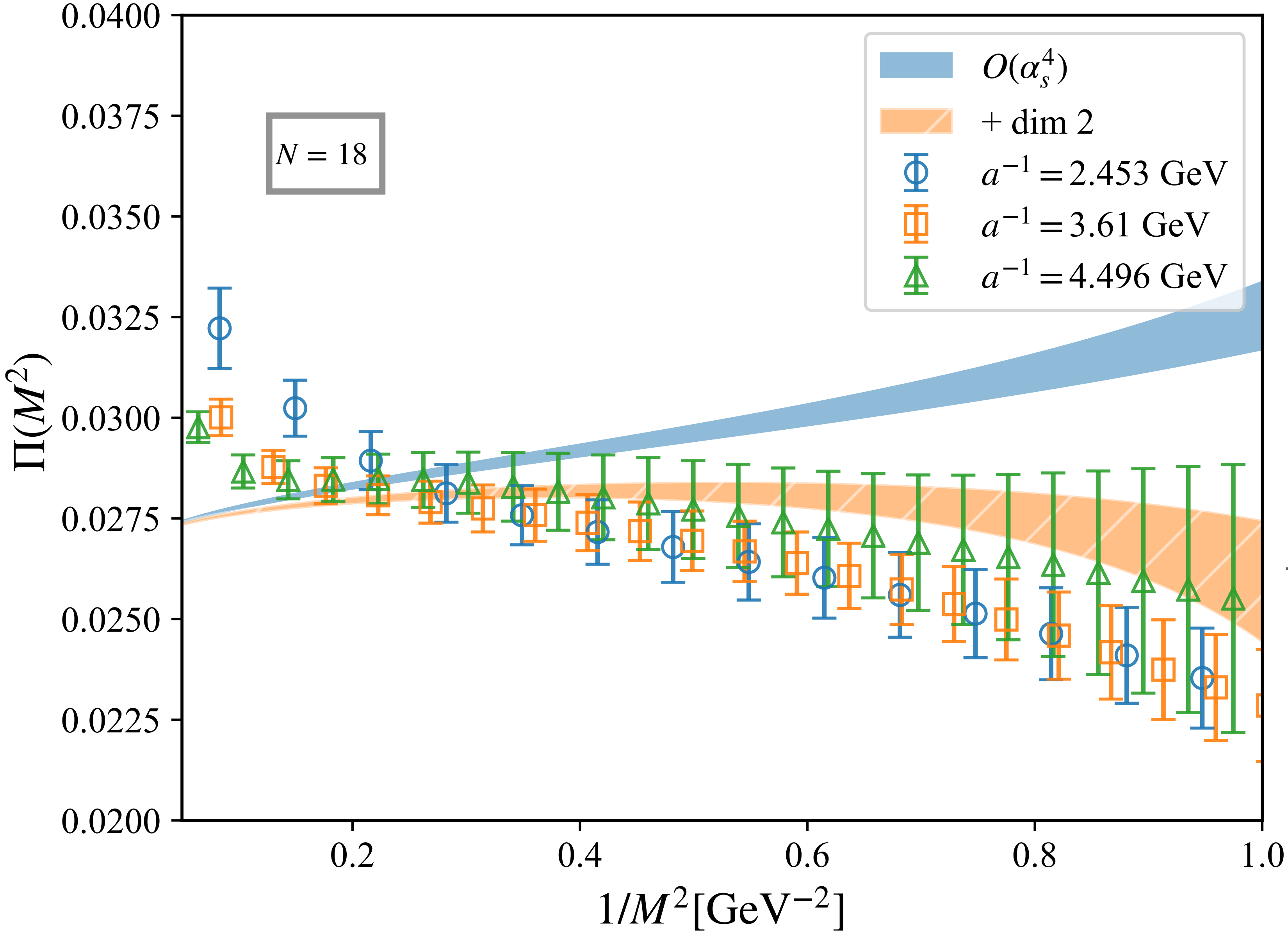

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$+\frac{m_s^2}{M^2}$

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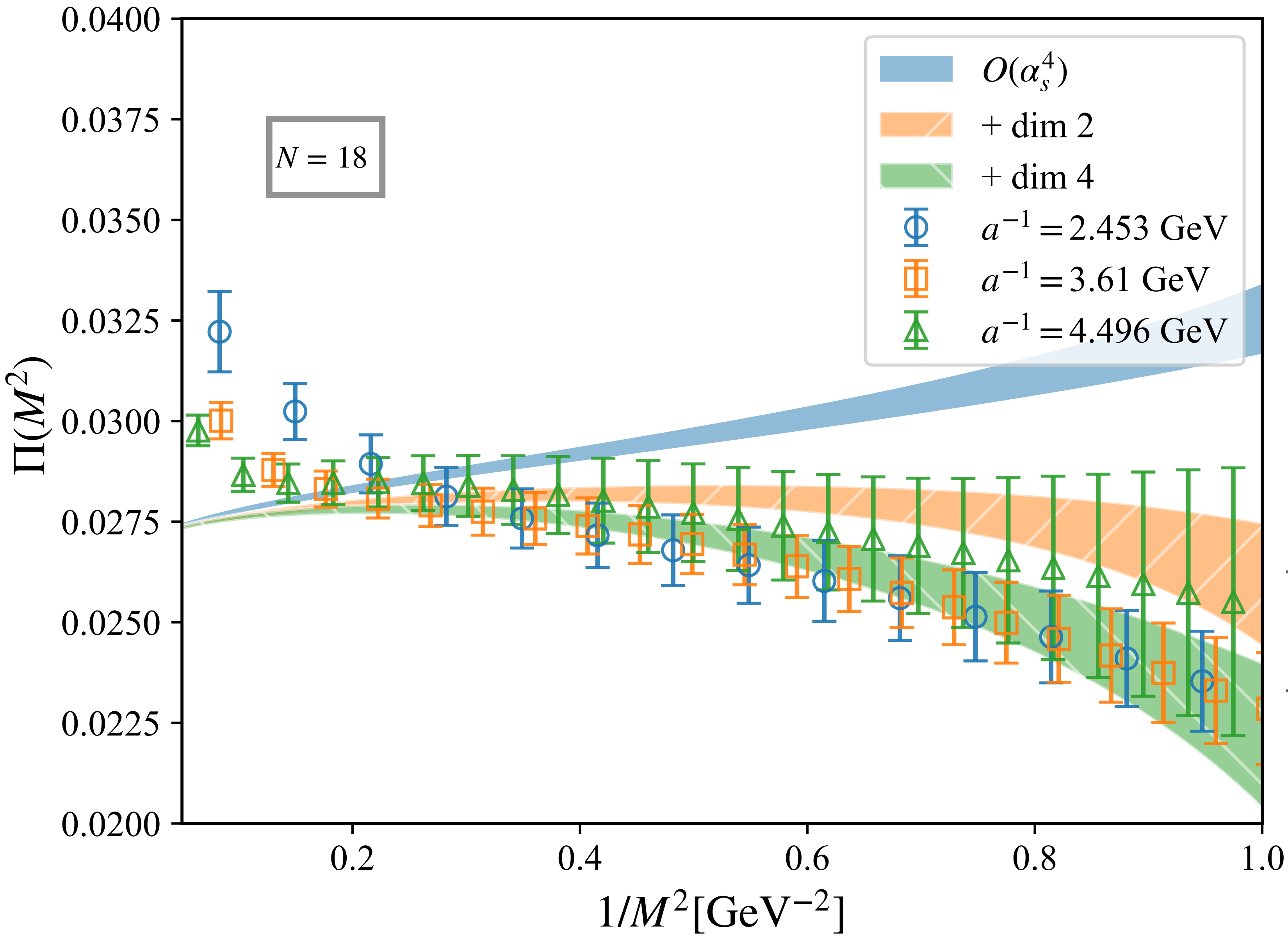

massless perturbation
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$+\frac{m_s^2}{M^2}$

$+\frac{m_s^4}{M^4}, \frac{m_s \langle \bar{s}s \rangle}{M^4}, \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{M^4}$

Comparison with OPE

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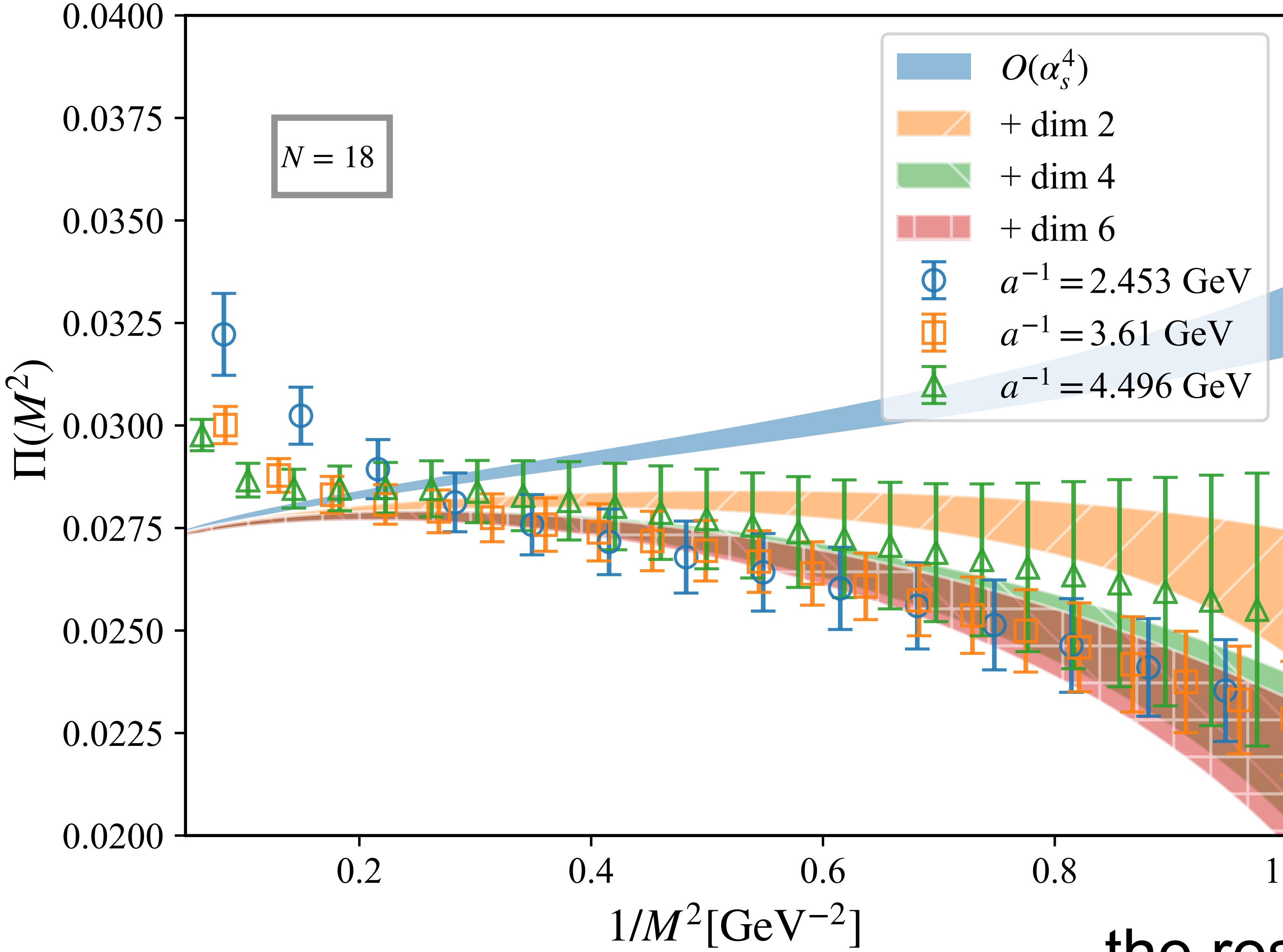


massless perturbation at 5-loop level

$+\frac{m_s^2}{M^2}$

$+\frac{m_s^4}{M^4}, \frac{m_s\langle \bar{s}s \rangle}{M^4}, \frac{\langle \frac{\alpha_s}{\pi} G^2 \rangle}{M^4}$

$+\frac{m_s^6}{M^6}, \frac{\langle \bar{s}s\bar{s}s \rangle}{M^6}, \frac{m_s^2\langle \frac{\alpha_s}{\pi} G^2 \rangle}{M^6}$

the results are consistent with “resonable” condensates

Uncertainties of condensates

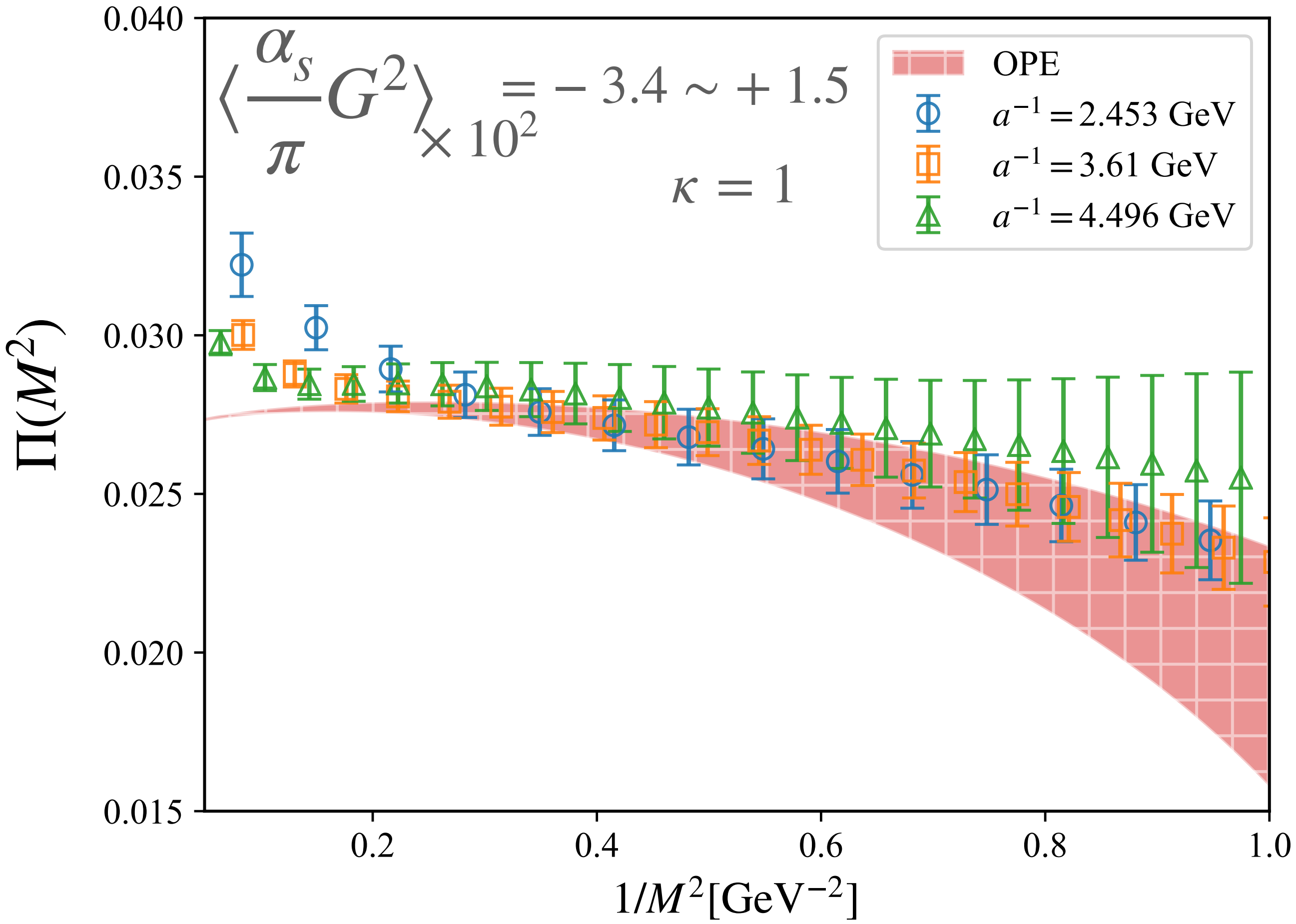
- uncertainty in $\langle \frac{\alpha_s}{\pi} G^2 \rangle \times 10^2$

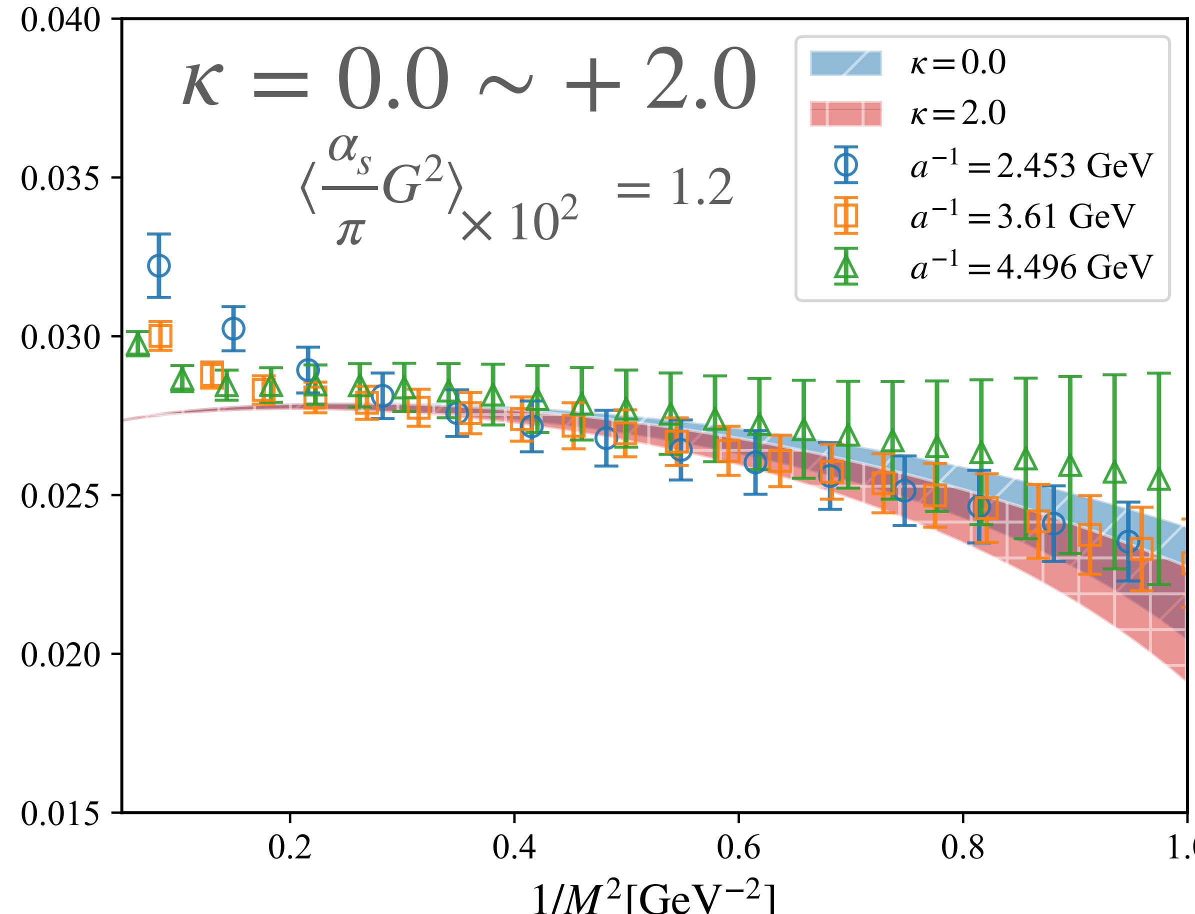
charmonium	$1.20(36)$	[SVZ]
tau decay	$0.6(12)$	[Geshkenbein et al, 01]
	$-3.4 \sim -0.5$	[Davier et al, 14]

- factorization $\langle \bar{s}s\bar{s}s \rangle \propto \kappa \langle \bar{s}s \rangle^2$

κ introduced to take the violation into account

κ is not well-known: 1 ± 1?





Lattice results can constrain these condensates.

Summary

OPE

$$\mathscr{B}_M[\Pi^{\rm OPE}(Q^2)] = \Pi^{\rm OPE}(M^2)$$

↔

lattice

$$\Pi^{\rm lat}(M^2) = \frac{c_0^*(M^2)}{2} + \sum_{j=1}^{N} c_j^*(M^2)\langle T_j^* \rangle$$

- We propose a method to compute the Borel transform of HVP in the QCD sum rule from lattice correlator.
- The scale parameter M^2 is continuous and easily adjustable in our method.
- can be used to extract physical parameters including the condensates and to renormalize lattice operators.