Double-winding Wilson loops towards flux tube interaction in SU(N) lattice gauge theory

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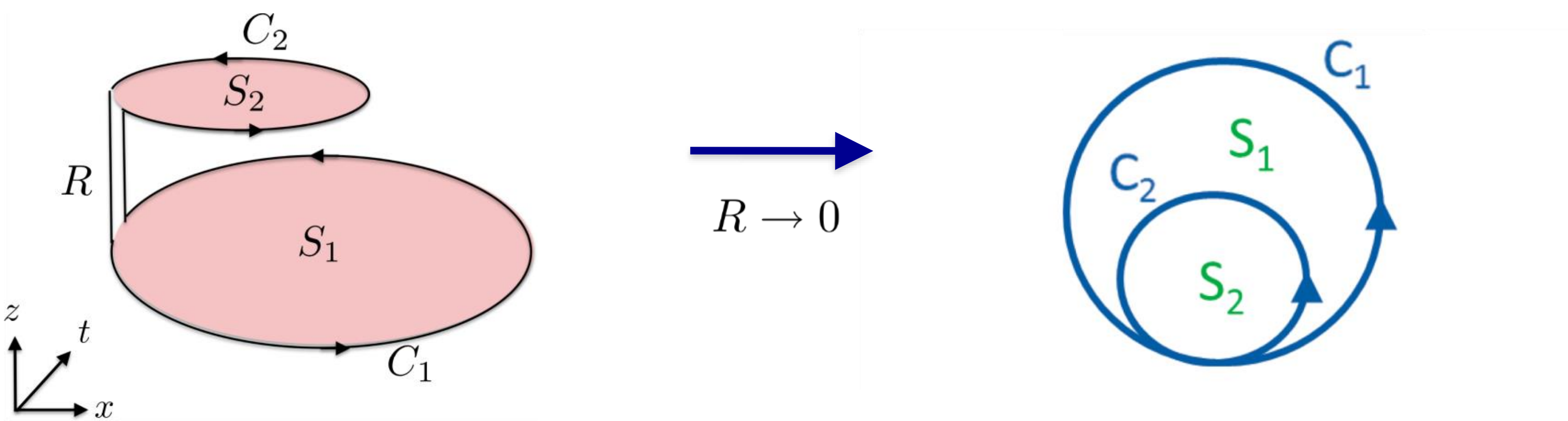
1. Introduction

What is double-winding Wilson loop operator?

In 2015, J. Greensite et al. introduced a following ``double-winding'' Wilson loop operator in lattice gauge theory: ([1] Phys.Rev.D91,054509(2015))

$$W(C) \equiv \mathrm{tr}[\prod_{\ell \in C} U_\ell], \quad (C = C_1 \times C_2)$$

This is a path-ordered product of (gauge) link variables along a closed contour C which is composed of two loops C_1 and C_2 .



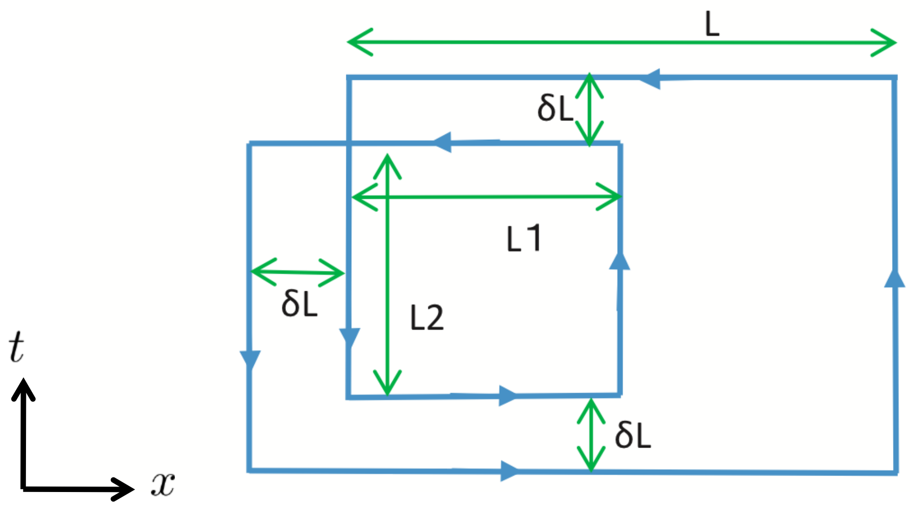
<shifted double-winding> W_S

<coplanar double-winding> W_C

■ Why do we consider such operator?

They considered such operator to examine possible mechanisms for quark confinement.

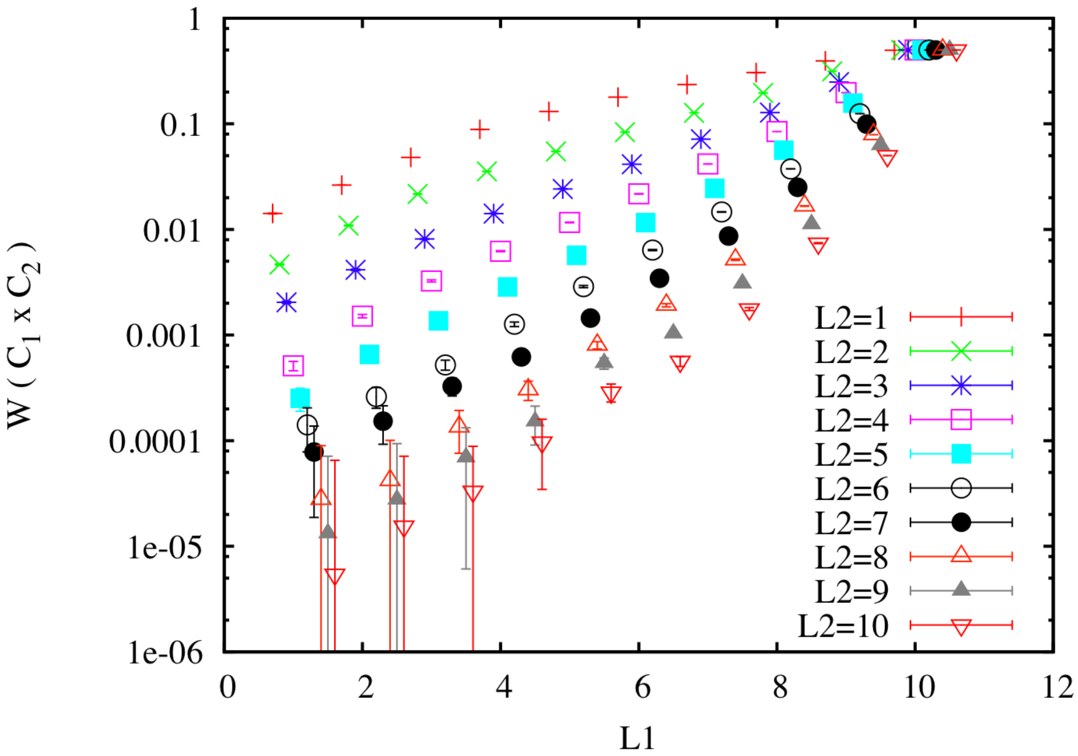
They studied the L_1-dependence of a coplanar double-winding Wilson loop average, $\langle W_C \rangle$, with the other lengths L, L_2, and δL(=0) being fixed:



Difference-of-areas law: $\langle W_C \rangle \simeq \exp[-\sigma|S_1 - S_2|] = \exp[-\sigma L_2(L - L_1)]$

Sum-of-areas law: $\langle W_C \rangle \simeq \exp[-\sigma'(S_1 + S_2)] = \exp[-\sigma' L_2(L + L_1)]$

They performed the numerical simulations for SU(2) LGT on 20^4-lattice at β=2.4, and studied L_1-dependence of ln < W_C > with the other lengths being fixed, L=10, L_2=1～10 and δL=0.



<reproduced from [1] Phys.Rev.D91,054509(2015)>

•They also studied L_1-dependence of ln < W_C > from center vortex d.o.f and abelian d.o.f .

Motivation

In this way, the study of double-winding Wilson loops itself is interesting because it can be used to test the confinement mechanism in QCD.
J. Greensite et al. showed difference-of-areas law for ``coplanar'' double-winding Wilson loop average in SU(2) LGT.

How about SU(3) LGT ? How about Large N ?

We study double-winding Wilson loops in SU(N) lattice gauge theory by using both strong coupling expansions and numerical simulations.

1) We examine how the area law falloff of a ``coplanar'' double-winding Wilson loop average, W_C, depends on the number of color N, which may contain information on the possible mechanism of quark confinement, e.g., magnetic monopole, center vortex, etc.

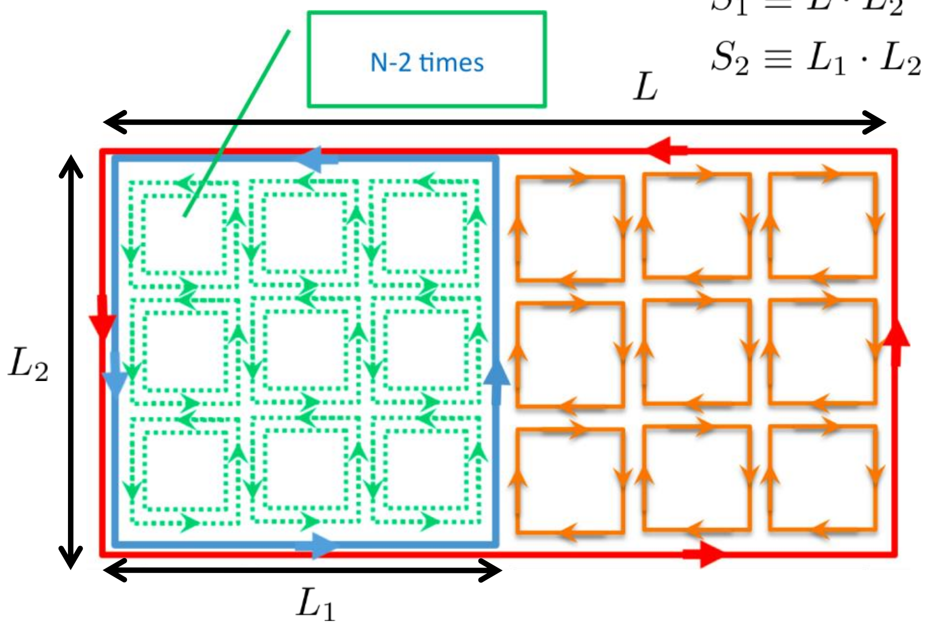
2) We evaluate ``shifted'' double-winding Wilson loop average, W_S, by changing the distance of a transverse direction, which may contain an information about interactions between two color flux tubes.

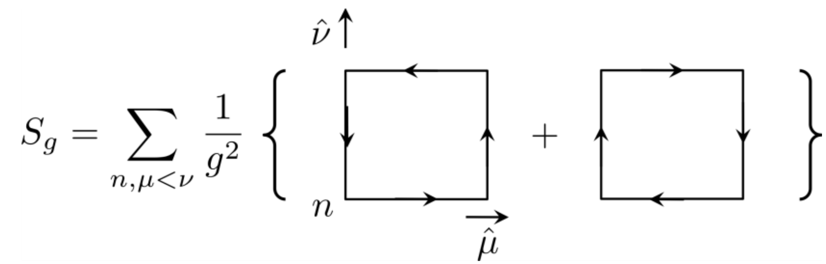
2. A “coplanar” double-winding Wilson loop in SU(N) LGT

strong coupling expansion

One of a set of plaquettes tiling the areas S_1 and S_2 which gives the non-trivial contribution to a coplanar double-winding Wilson loop average:

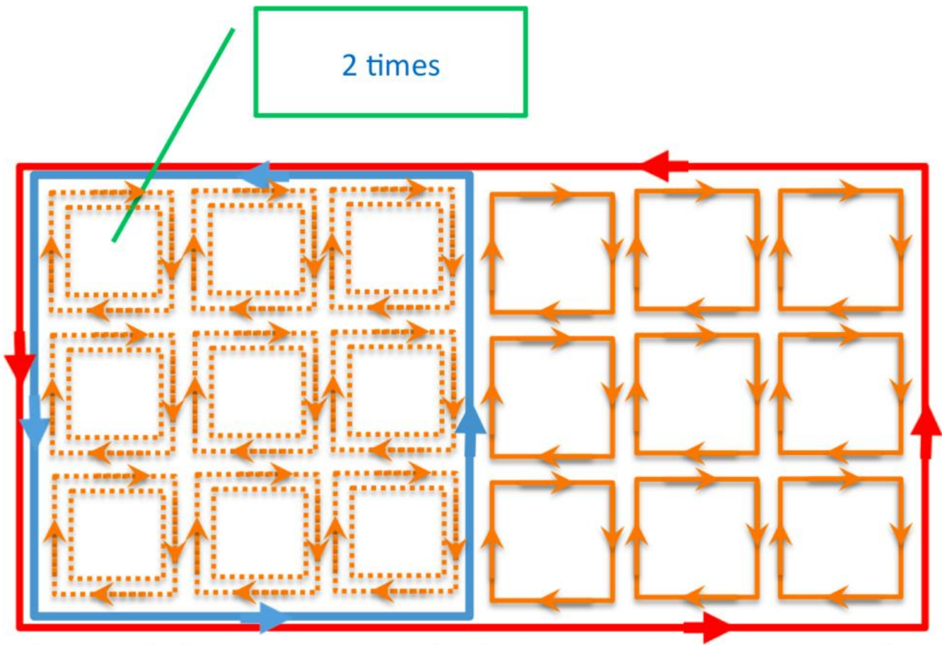
$$S_1 \equiv L \cdot L_2$$
$$S_2 \equiv L_1 \cdot L_2$$





$$\simeq p_N \left(\frac{1}{g^2 N} \right)^{(N-2)S_2 + (S_1 - S_2)}$$

Another set of plaquettes tiling the areas S_1 and S_2 which gives the non-trivial contribution to a coplanar double-winding Wilson loop average:



$$\simeq q_N \left(\frac{1}{g^2 N}\right)^{2S_2+(S_1-S_2)}$$
$$= q_N \left(\frac{1}{g^2 N}\right)^{S_1+S_2}$$

$$q_N = -\frac{N^{2S_2}}{2}\left\{\left[\frac{1}{N(N-1)}\right]^{S_2-1} - \left[\frac{1}{N(N+1)}\right]^{S_2-1}\right\} \quad (S_2 \geq 1)$$

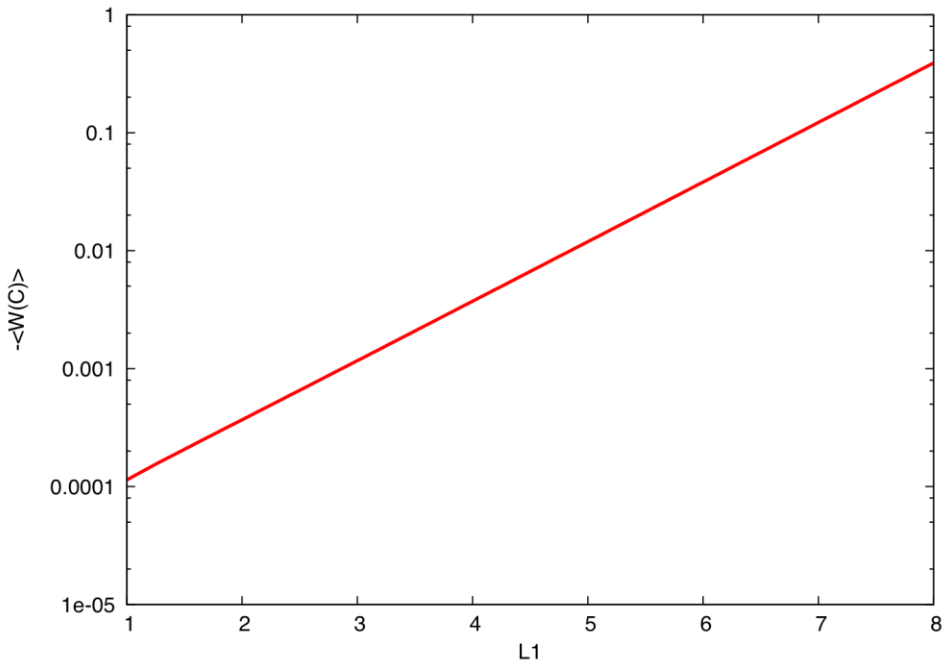
SU(2) : Difference-of-areas law [reconfirmed and improved]

$$\langle W_C \rangle = 2p_2 \left(\frac{1}{2g^2}\right)^{S_1 - S_2} + 2q_2 \left(\frac{1}{2g^2}\right)^{S_1+S_2} + \cdots$$

$$p_2 = -2, \quad q_2 = -\frac{4^{S_2}}{2}\left\{\left[\frac{1}{2}\right]^{S_2-1} - \left[\frac{1}{6}\right]^{S_2-1}\right\}$$

$$S_1 \equiv L \cdot L_2$$
$$S_2 \equiv L_1 \cdot L_2$$

L_1-dependence of $-\langle W_C \rangle$ from the S.C.E in SU(2) LGT:
($L=10$, $L_2=1$ and $1/g^2N=2.5/8$)



SU(3) : max-of-areas law [New]

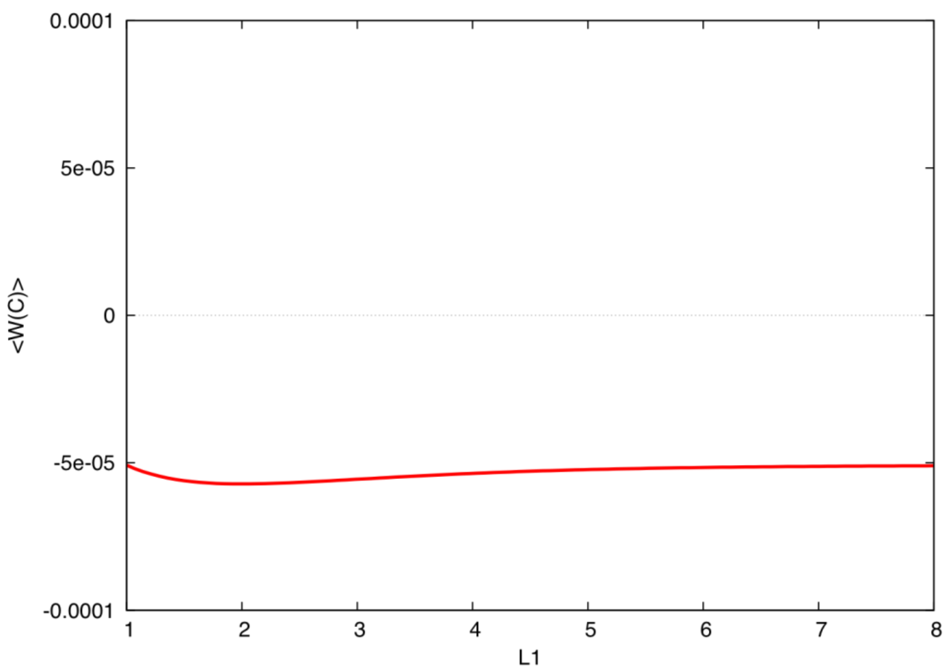
$Max(S_1, S_2)$

$$\langle W_C \rangle = p_3 \left(\frac{1}{3g^2}\right)^{S_1} + q_3 \left(\frac{1}{3g^2}\right)^{S_1+S_2} + \cdots$$

$$p_3 = -3, \quad q_3 = -\frac{9^{S_2}}{2}\left\{\left[\frac{1}{6}\right]^{S_2-1} - \left[\frac{1}{12}\right]^{S_2-1}\right\}$$

$$S_1 \equiv L \cdot L_2$$
$$S_2 \equiv L_1 \cdot L_2$$

L_1-dependence of $< W_C >$ from the S.C.E in SU(3) LGT:
($L=10$, $L_2=1$ and $1/g^2N=6.0/18$)



SU(N) (N≥4) : sum-of-areas law [New]

$$\langle W_C \rangle = p_N \left(\frac{1}{g^2 N}\right)^{(N-2)S_2+S_1-S_2} + q_N \left(\frac{1}{g^2 N}\right)^{S_1+S_2} + \cdots$$

For N≥4, we find that the second term in above equation gives the dominant contribution in the strong coupling expansion for $< W_C >$,
since the inequality holds,
$S_1+S_2 \le (N-2)S_2+S_1-S_2$, for N≥4.

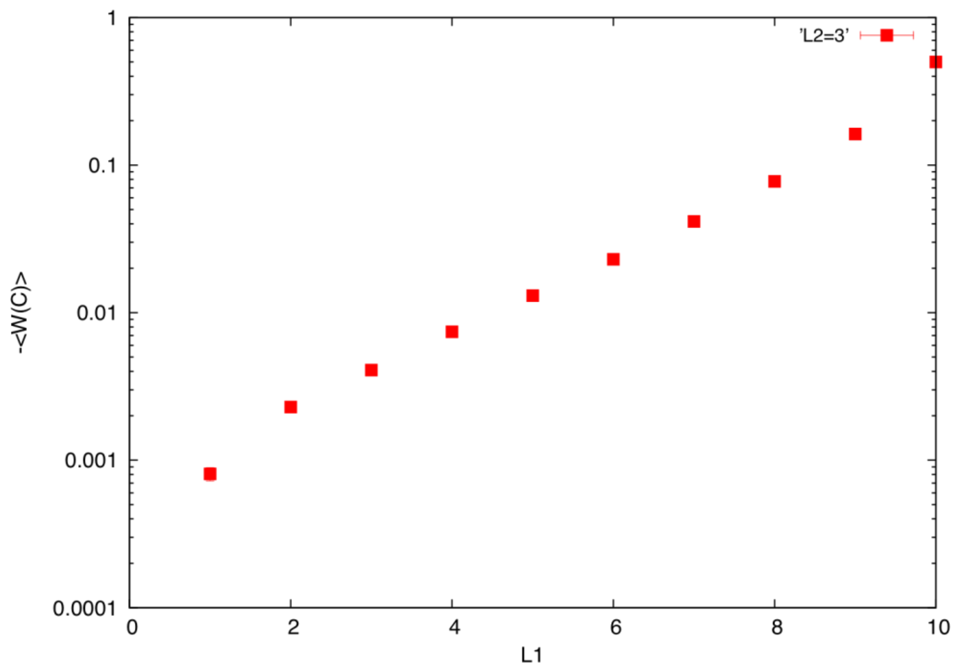
Thus we conclude that the sum-of-areas law of a coplanar double-winding Wilson loop is allowed for N≥4.

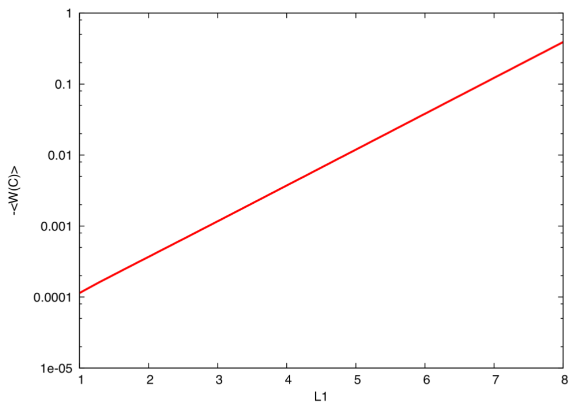
numerical simulation

SU(2) : Difference-of-areas law [reconfirmed]

Lattice set up:
- standard Wilson action
- 24^4-lattice, β=2.5
- 100 configurations

L_1-dependence of $-< W_C >$ from the numerical simulation in SU(2) LGT:
(L=10, L_2=3 and β=2.5)





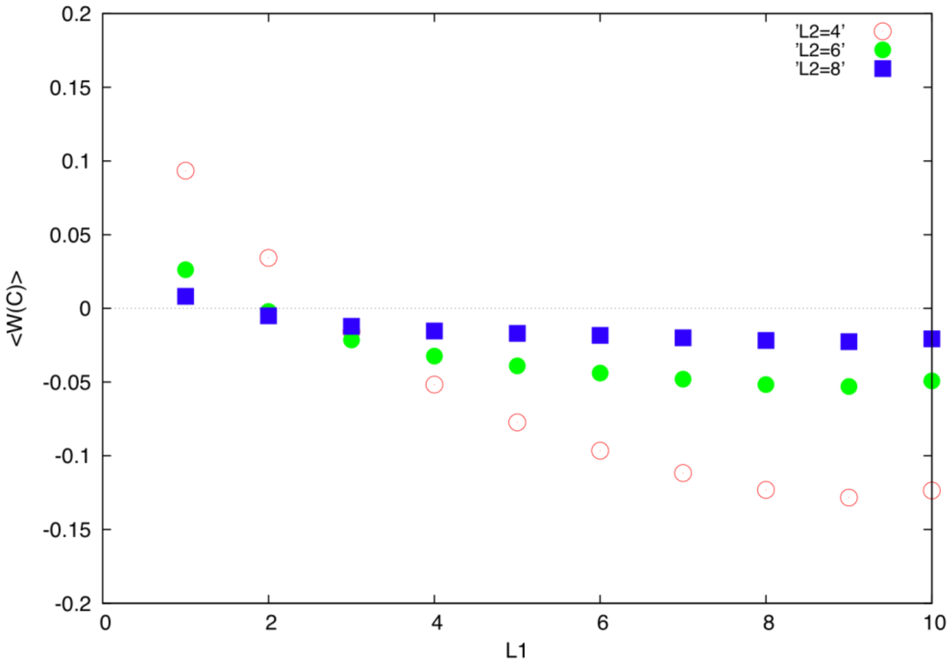
(S.C.E)

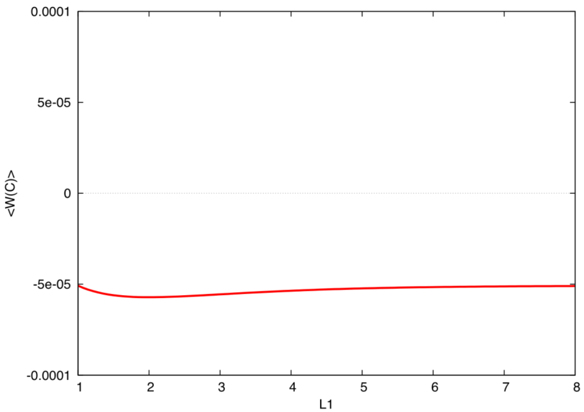
SU(3) : max-of-areas law [New]

Lattice set up:

- standard Wilson action
- 24^4-lattice, β=6.2
- 200 configurations
- APE smearing method (N'=12, α=0.1)

L_1-dependence of $\langle W_C \rangle$ from the numerical simulation in SU(3) LGT:
(L=10, L_2=4,6,8 and β=6.2)

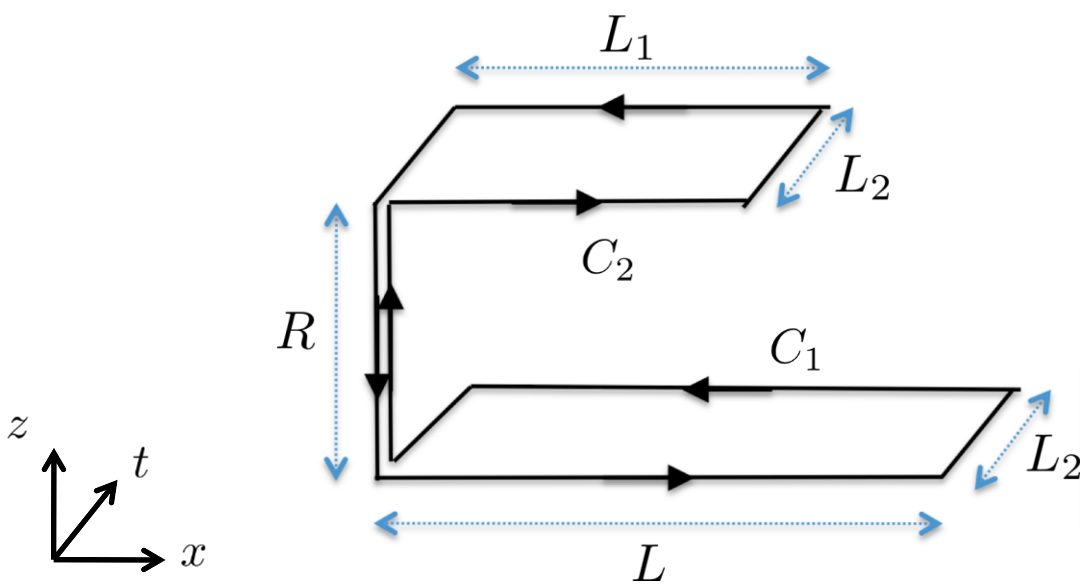




(S.C.E)

3. A "shifted" double-winding Wilson loop in SU(N) LGT

The setting up of a shifted double-winding Wilson loop operator (Ws):

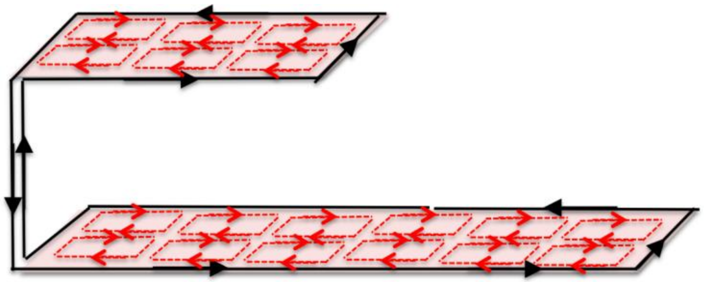


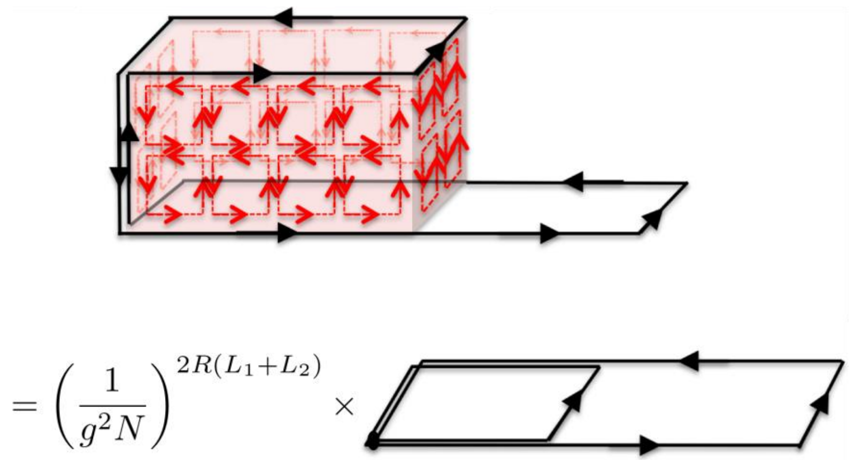
$$S_1 \equiv L \cdot L_2$$

$$S_2 \equiv L_1 \cdot L_2$$

strong coupling expansion

The diagrams which may gives a leading contribution in the S.C.E are given by a set of plaquettes tiling as follows:



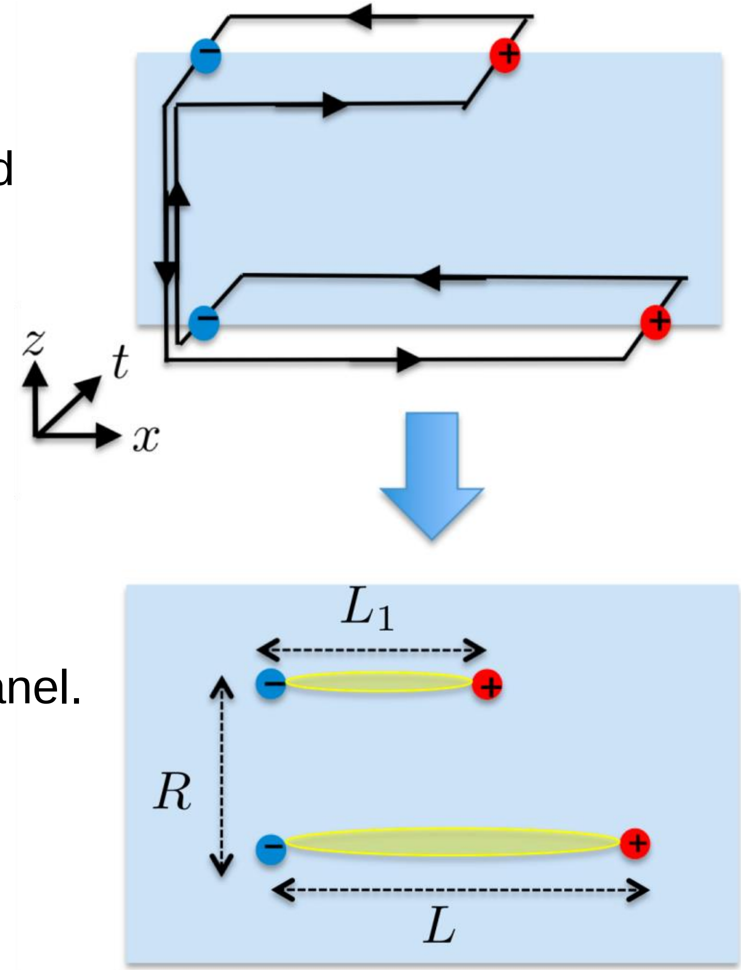


The result for SU(N):

$$\langle W_S \rangle \equiv \langle W(C_1 \times C_2) \rangle_{R \neq 0} = N \left(\frac{1}{g^2N}\right)^{S_1+S_2}$$
$$+ \left(\frac{1}{g^2N}\right)^{2R(L_1+L_2)} \times \left\{ p_N \left(\frac{1}{g^2N}\right)^{(N-2)S_2+S_1-S_2} + q_N \left(\frac{1}{g^2N}\right)^{S_1+S_2} \right\} + \cdots$$

flux tube interaction

As is explained in [1], the shifted double-winding Wilson loop (W_S) at a fixed time can be interpreted as a tetra-quark system consisting of two static quarks and two static antiquarks.



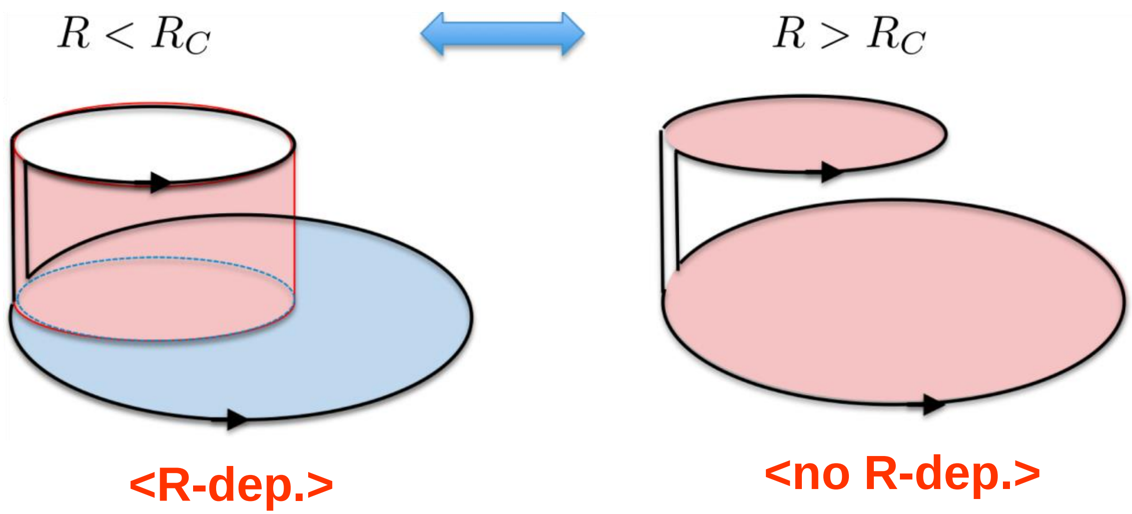
The pairs of quark-antiquarks are connected by a pair of color flux tubes, as seen in the bottom panel.

We study how interactions between the two color flux tubes change, when the distance R is varied.

$$\langle W_S \rangle = N \left(\frac{1}{g^2 N} \right)^{S_1+S_2}$$

$$+ \left(\frac{1}{g^2 N} \right)^{2R(L_1+L_2)} \times \left\{ p_N \left(\frac{1}{g^2 N} \right)^{(N-2)S_2+S_1-S_2} + q_N \left(\frac{1}{g^2 N} \right)^{S_1+S_2} \right\} + \cdots$$

We find that the second term dominates for $R<R_C$, and the first term dominates for $R>R_C$.



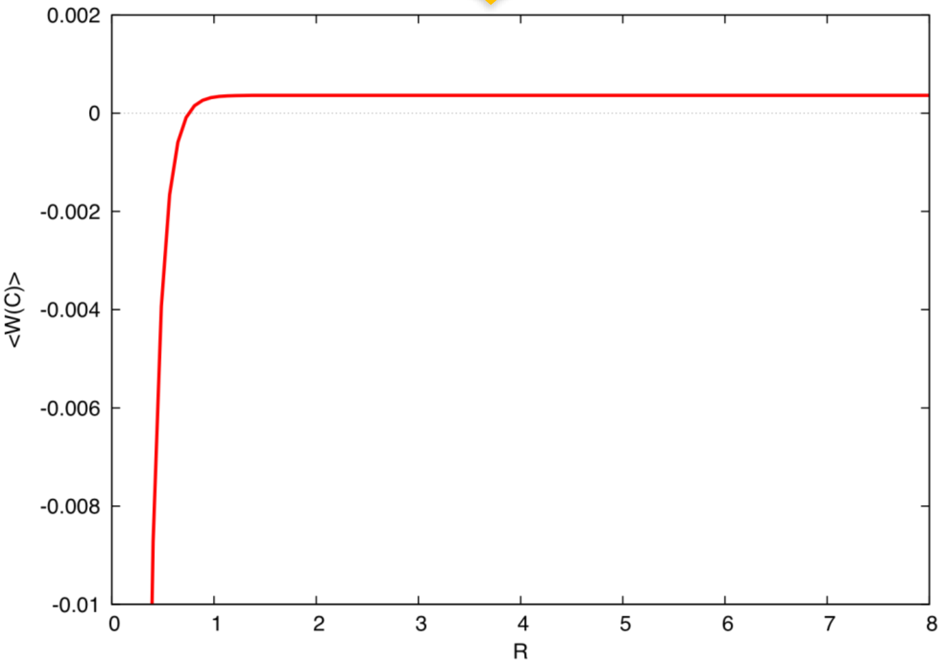
This means that the left diagram dominates for $R<R_C$, and the right diagram dominates for $R>R_C$.
Therefore, the dominant diagram switches from left to right at a certain value R_C as R increases, just like the minimal surface spanned by a soap film.

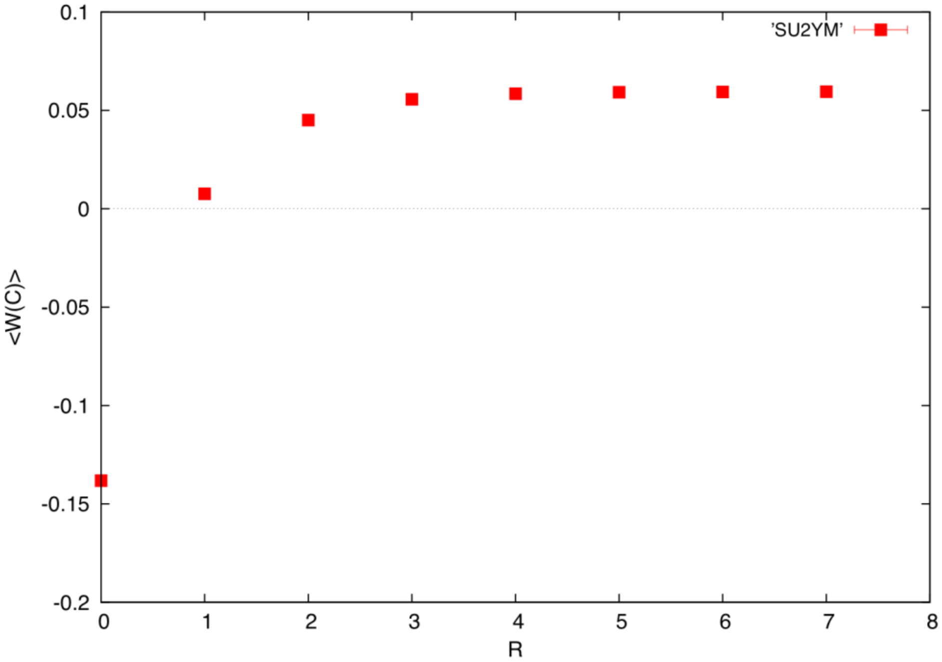
Results from S.C.E & numerical simulation [New]

SU(2) : R-dependence of a shifted double-winding Wilson loop average (Ws)

$$\langle W_S \rangle = 4\left(\frac{1}{2g^2}\right)^{S_1+S_2} + 2p_2\left(\frac{1}{2g^2}\right)^{S_1-S_2+2R(L_1+L_2)} + 2q_2\left(\frac{1}{2g^2}\right)^{S_1+S_2+2R(L_1+L_2)} + \cdots$$

($R_C \cong L_1/(1+L_1/T)$)



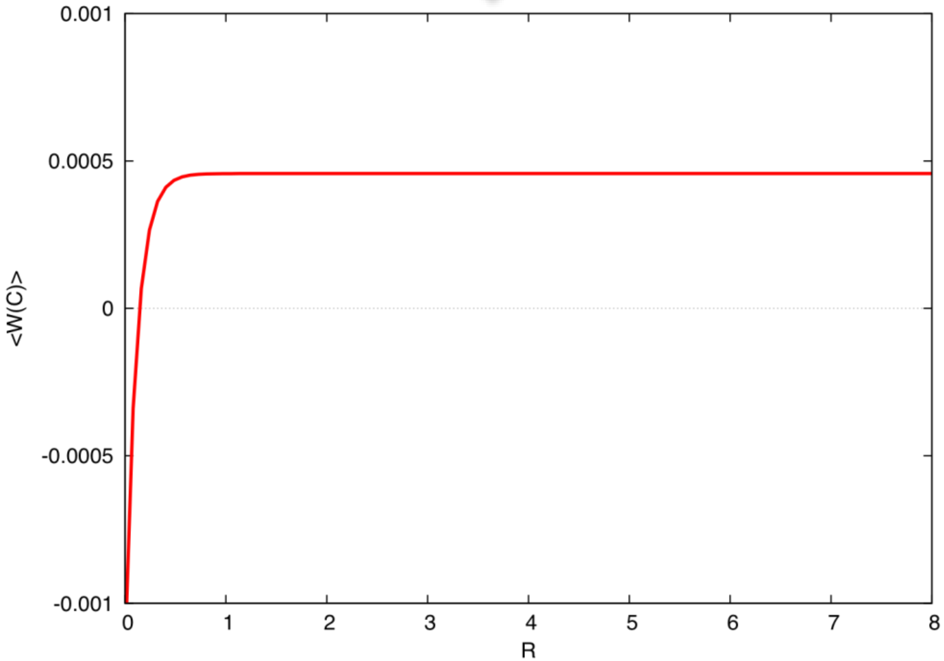


S.C.E
($1/2g^2=2.5/8$, $L=5$, $L_2=1$, $L_1=3$)

Numerical: 100 conf.
($\beta=2.5$, $L=5$, $L_2=2$, $L_1=3$)

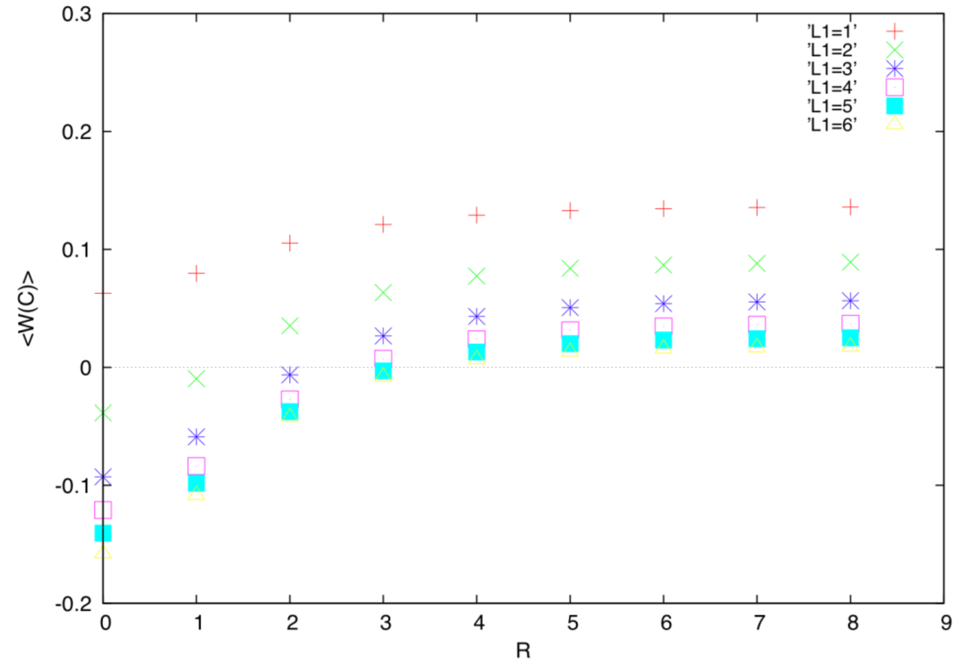
SU(3) : R-dependence of a shifted double-winding Wilson loop average (Ws)

$$\langle W_S \rangle = 3\left(\frac{1}{3g^2}\right)^{S_1+S_2} + p_3\left(\frac{1}{3g^2}\right)^{S_1+2R(L_1+L_2)} + q_3\left(\frac{1}{3g^2}\right)^{S_1+S_2+2R(L_1+L_2)} + \cdots$$



S.C.E

($1/3g^2$=6.0/18, L=5, L_2=1,L_1=3)



Numerical: 200 conf.

(β=6.2, L=8, L_2=8,L_1=1～6)

4. Conclusion and outlook

We have studied the double-winding Wilson loops in SU(N) lattice gauge theory by using both strong coupling expansion and numerical simulation.

(1)We have examined how the area law falloff of a ``coplanar'' double-winding Wilson loop average depends on the number of color N, by changing the size of minimal area S_2 of loop C_2.
We have reconfirmed the difference-of-areas law for N=2, and have found new results that ``max-of-areas law'' for N=3 and sum-of-areas law for $N \geq 4$.
These results are consistent with Matsudo-Kondo (Phys.Rev.D96,105011(2017)).

(2) We have evaluated a ``shifted'' double-winding Wilson loop average by changing the distance of a transverse direction,
and have found that their long distance behavior doesn't depend on the number of color N, but the short distance behavior depends on N.

outlook;

- Extract an information about interactions between two color flux tubes form ``shifted'' double-winding Wilson loop.
- Search an explicit expression for the Abelian operator which reproduce full double-winding Wilson loops.

Thank you !