QCD Equation of State

in external magnetic field and at finite baryon density

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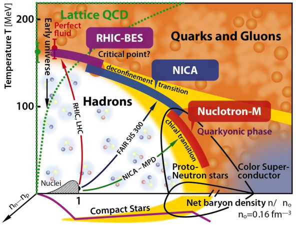
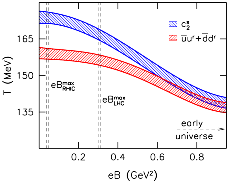
Motivation



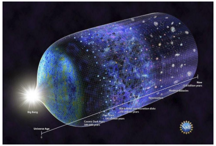
http://nica.jinr.ru/physics.php



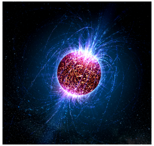
G. Endrodi, J.Phys.Conf.Ser. **503** (2014) 012009 [arXiv:1311.0648 [hep-lat]]

External magnetic field influences phase diagram!

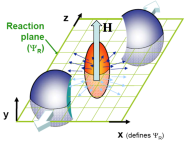
High density & Strong magnetic field



Early Universe



Neutron stars



Heavy Ion Collisions

Thermodynamics on the lattice

$$p = -\frac{\Omega}{V} = \frac{T}{V} \ln \mathcal{Z} \quad \longleftarrow \text{ cannot be measured directly}$$

$$/\star \quad \mathcal{Z} = \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}]\, \mathrm{e}^{-S(\psi,\bar{\psi},U)} \quad \star/$$

Derivatives of p can be measured!

$$n_q = \frac{N_q}{V} = \frac{\partial p}{\partial \mu_q} \quad \text{– quark number density}$$

$$n_q = \frac{T}{V} \cdot \frac{1}{\mathcal{Z}} \cdot \frac{\partial \mathcal{Z}}{\partial \mu_q} = -\frac{T}{V} \cdot \underbrace{\frac{1}{\mathcal{Z}} \int \mathcal{D}[U]\mathcal{D}[\psi,\bar{\psi}] \left(\frac{\partial S}{\partial \mu_q}\right) \mathrm{e}^{-S(\psi,\bar{\psi},U)}}_{\left\langle \frac{\partial S}{\partial \mu_q} \right\rangle} = -\frac{T}{V} \left\langle \frac{\partial S}{\partial \mu_q} \right\rangle$$

$$\mu_B = \mu_u + 2\mu_d$$
$$\mu_Q = \mu_u - \mu_d$$
$$\mu_S = \mu_d - \mu_s$$

$$n_B = (n_u + n_d + n_s)/3$$
$$n_Q = (2n_u - n_d - n_s)/3$$
$$n_S = -n_s$$

J. N. Günther et al., Nucl. Phys. A **967**, 720 (2017) [arXiv:1607.02493 [hep-lat]]

$$\frac{p}{T^4} = c_0(T) + c_2(T)\left(\frac{\mu_B}{T}\right)^2 + c_4(T)\left(\frac{\mu_B}{T}\right)^4 + c_6(T)\left(\frac{\mu_B}{T}\right)^6 + \mathcal{O}(\mu_B^8)$$

$$\frac{n}{\mu_B T^2} = \frac{T}{\mu_B} \cdot \frac{d(p/T^4)}{d(\mu_B/T)} = 2c_2 + 4c_4\left(\frac{\mu_B}{T}\right)^2 + 6c_6\left(\frac{\mu_B}{T}\right)^4 \quad \leftarrow \text{coefficients can be found from fit}$$

(μ_u, μ_d, μ_s are such that $\langle n_S \rangle = 0$, $\langle n_Q \rangle = 0.4 \langle n_B \rangle$)

Calculation of c_0:

S. Borsanyi et al., Phys. Lett. B **730**, 99 (2014) [arXiv:1309.5258 [hep-lat]].
G. S. Bali et al., JHEP **08**, 177 (2014) [arXiv:1406.0269 [hep-lat]].

Coefficients of the expansion

Our choice of chemical potentials:

$$\mu_u = \mu_d = \mu_q; \quad \mu_s = 0 \qquad \Rightarrow \qquad \mu_B = 3\mu_q; \quad \mu_Q = 0; \quad \mu_S = \mu_q.$$

Pressure expansion:

$$\frac{p}{T^4} = \sum_{n=0}^{n_{cut}} c_{2n}^B \theta_B^{2n} + \sum_{n=1}^{\overline{n}_{cut}} c_{2n}^S \theta_S^{2n} + c_{11}^{BS}\theta_B\theta_S + c_{22}^{BS}\theta_B^2\theta_S^2 + c_{13}^{BS}\theta_B\theta_S^3 + c_{31}^{BS}\theta_B^3\theta_S + \dots,$$

$\theta_B = \mu_B/T, \qquad \theta_S = \mu_S/T = \theta_B/3.$

Densities of the conserved charges:

$$\frac{n_B}{T^3} = \frac{\partial(p/T^4)}{\partial\theta_B} = \underbrace{\left(2c_2^B + \frac{c_{11}^{BS}}{3}\right)}_{2c_2}\theta_B + \underbrace{\left(4c_4^B + \frac{2c_{22}^{BS}}{9} + \frac{c_{13}^{BS}}{27} + c_{31}^{BS}\right)}_{4c_4}\theta_B^3 + \underbrace{\left(6c_6^B + \dots\right)}_{6c_6}\theta_B^5 + \dots$$

$$\frac{n_S}{T^3} = \frac{\partial(p/T^4)}{\partial\theta_S} = \left(\frac{2c_2^S}{3} + c_{11}^{BS}\right)\theta_B + \left(4\frac{c_4^S}{27} + \frac{2c_{22}^{BS}}{3} + \frac{c_{13}^{BS}}{3} + c_{31}^{BS}\right)\theta_B^3 + \left(6c_6^S + \dots\right)\theta_S^5 + \dots$$

Lattice setup

- Tree level improved Symanzik gauge action.
- Staggered $2+1$ fermionic action.
- Stout smearing improvement.
- Imaginary chemical potential: $\mu = i\mu_I$.
- External magnetic field:

$$\vec{B} = B\vec{e}_z; \qquad B = \text{const} \qquad\qquad A_y^{\text{ext}} = Bx/2, \quad A_x^{\text{ext}} = -By/2, \quad A_\mu^{\text{ext}} = 0, \ \mu = z, t$$

- Splitting of the rooted determinant:

$$\mathcal{Z} = \int \mathcal{D}U\, \mathrm{e}^{-S_G} [\det D(B, m_u, q_u)]^{\frac{1}{4}} [\det D(B, m_d, q_d)]^{\frac{1}{4}} [\det D(B, m_s, q_s)]^{\frac{1}{4}}$$

$$D(n|f) = \frac{1}{2a} \sum_\mu \eta_\mu(n) \left[u_\mu(B,q,n)\, \Xi_\mu\, U_\mu(n) \delta_{f,n+\hat{\mu}} - u^\star_\mu(B,q,f)\, \Xi^\star_\mu\, U^\dagger_\mu(f) \delta_{f,n-\hat{\mu}} \right] + m\, \delta_{f,n}$$

$$u_x(B,q,n_x,n_y,n_z,n_t) = \mathrm{e}^{-ia^2 qB\, n_y/2}, \quad n_x \neq N_x - 1, \qquad u_y(B,q,n_x,n_y,n_z,n_t) = \mathrm{e}^{ia^2 qB\, n_x/2}, \quad n_y \neq N_y - 1,$$

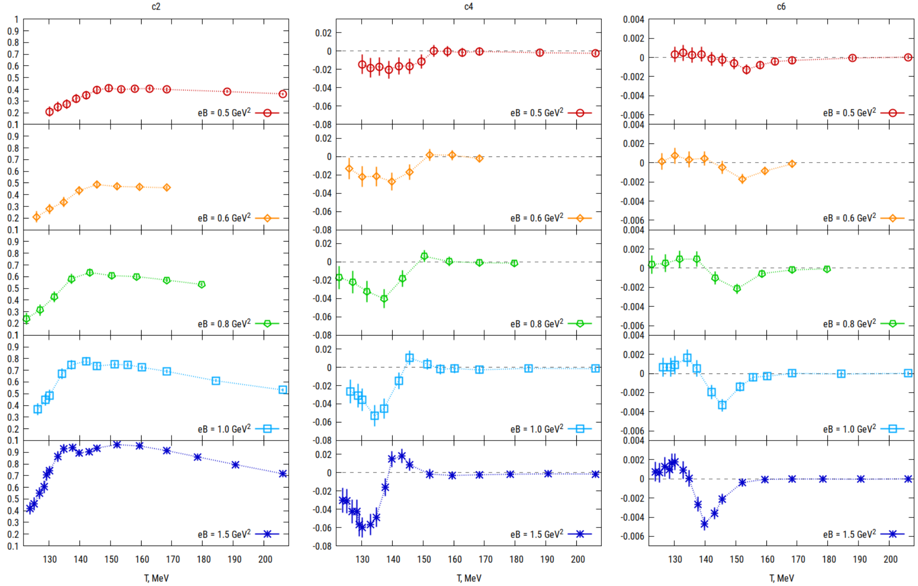
$$u_x(B,q,N_x-1,n_y,n_z,n_t) = \mathrm{e}^{-ia^2 qB(N_x+1)n_y/2}, \qquad u_y(B,q,n_x,N_y-1,n_z,n_t) = \mathrm{e}^{ia^2 qB(N_y+1)n_x/2}.$$

$$\Xi_\nu = \mathrm{e}^{ia\mu_I \times \delta_{\nu 4}}$$

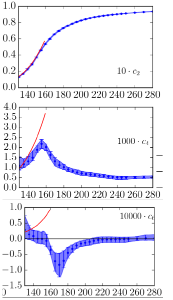
Periodic boundary conditions $\Rightarrow$ $eB = \dfrac{6\pi\, k}{N_x\, N_y\, a^2}, \quad k \in \mathbb{Z}$

Simulation parameters: 6×24^3 lattice;
$eB = 0.5,\ 0.6,\ 0.8,\ 1.0,\ 1.5$ GeV^2;
$T = 123 - 206$ MeV;
physical quark masses.

Results



J. N. Günther et al.



$eB = 0$ GeV^2

cont. extrapolated

Conclusions

- Simulations at non-zero chemical potential and with external magnetic field are carried out.
- First results on expansion coefficients c_2, c_4, c_6 in external magnetic field are obtained.
- Strong dependence of the EoS expansion coefficients on magnetic field is observed.

Plans for future:

- Increase statistics on 6×24^3 lattice.
- Perform simulations on larger lattices and take continuum limit.