Chiral phase transition temperature in (2+1)-flavor QCD

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for
HotQCD collaboration

BMBF project

ALICE Germany

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CRC-TR 211
Strong-interaction matter under extreme conditions

Overview

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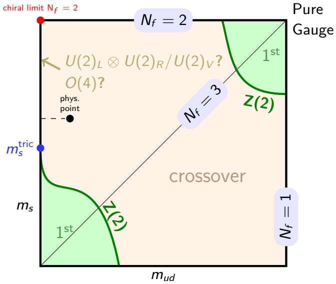
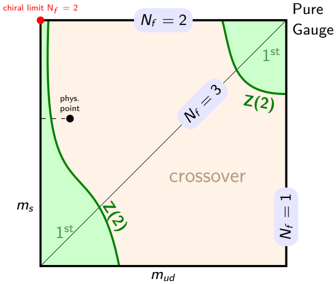
Introduction?

- Key question : What is the chiral transition temperature, T_c^0?
- Related question : What is the nature of the thermal phase transition in the chiral limit?
- Two possible scenarios[1]





[1] O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.

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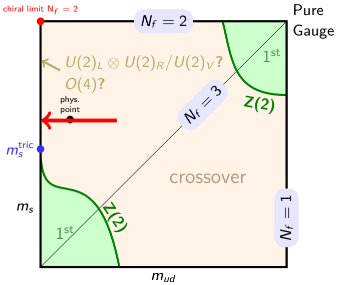

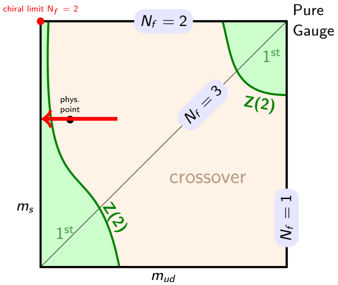

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Basic quantities

In terms of temperature T and symmetry breaking field $H = m_l/m_s$ the scaling variables are defined as :

$$t = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0} \qquad \text{and} \qquad h = \frac{1}{h_0}\frac{m_l}{m_s} = \frac{1}{h_0}H$$

Scaling variable :

$$z = \frac{t}{h^{\frac{1}{\beta\delta}}} = z_0\left(\frac{T - T_c^0}{T_c^0}\right)\left(\frac{1}{H^{1/\beta\delta}}\right); \quad z_0 = \frac{h_0^{\frac{1}{\beta\delta}}}{t_0}$$

Chiral condensate : $\langle\bar{\psi}\psi\rangle_f = \dfrac{T}{V}\dfrac{\partial \ln Z}{\partial m_f}$

Chiral susceptibility : $\chi_m^{fg} = \dfrac{\partial}{\partial m_g}\langle\bar{\psi}\psi\rangle_f$

Scaling relations

Renormalization group invariant (RGI) definition of order parameter :

$$M = \frac{m_s}{f_K^4}\left((\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d) - \frac{m_u + m_d}{m_s}\langle\bar{\psi}\psi\rangle_s \right) \equiv \frac{\Sigma_{\mathsf{sub}}}{f_K^4}$$

RGI definition of order parameter susceptibility :

$$\chi_M = \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d} \right) M$$

Close to chiral limit, singular part behaves as :

$$\begin{aligned} M &= h^{1/\delta} f_G(z) \\ \chi_M &= \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \end{aligned}$$

$f_G(z)$ and $f_\chi(z)$ are universal scaling functions which have been precisely determined from various spin models.

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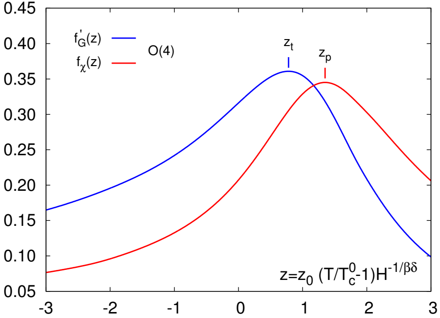
Mass scaling of conventional estimators

Mass scaling of different estimators of T_{pc} :

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right) \qquad X = t, p$$



In chiral limit :

$$\begin{aligned} M &= h^{1/\delta} f_G(z) \\ \chi_M &= \frac{1}{h_0} h^{1/\delta-1} f_\chi(z) \\ \frac{\partial M}{\partial T} &= \frac{1}{t_0 T_c^0} h^{1/\delta - 1/\beta\delta} f'_G(z) \end{aligned}$$

$f_G(z)$ and $f_\chi(z)$ are universal scaling functions which have been precisely determined from various spin models.

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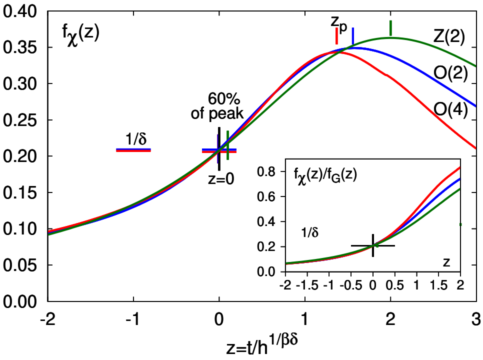
Scaling functions : Some intriguing facts

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

- Our approach : Use z_X at or close to 0.
- We choose to work with $X = \delta$ and 60 :

$$\frac{H\chi_M(T_\delta, H)}{M(T_\delta, H)} = \frac{1}{\delta}$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max}$$



Dependence on quark mass (H) reduced by two orders of magnitude $\Rightarrow$

	z_p	$\bar{z}_{60\%}$
$O(2)$	1.56	-0.005
$O(4)$	1.37	-0.013
$Z(2)$	2.00	0.10

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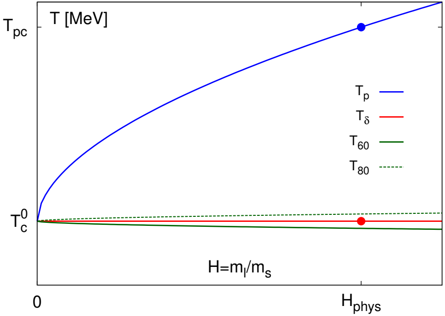
Improved estimators : basic philosophy

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

- Our approach : Use z_X at or close to 0.
- We choose to work with $X = \delta$ and 60.

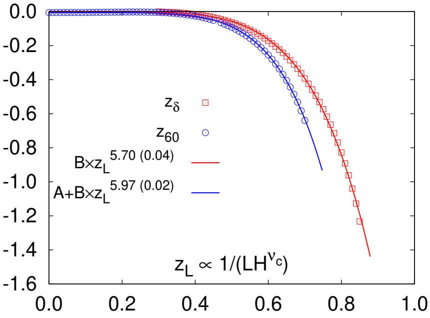


Because of the the reduced variation w.r.t. H, up to the regular contributions, the pseudo-critical temperatures defined by the improved estimators at any finite value of H, *e.g.* H_{phys}, already gives a close estimate of T_c^0.

Improved estimators on finite volumes

- System size (L) is also a scaling field resulting into additional scaling variable $z_L \propto 1/\left(LH^{\nu_c}\right)$.
- We have used O(4) finite size scaling function for our calculations.[1]

$$T_X(H,L) = T_c^0\left(1 + \frac{z_X(z_L)}{z_0}H^{1/\beta\delta}\right)$$



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Improved estimators on finite volumes

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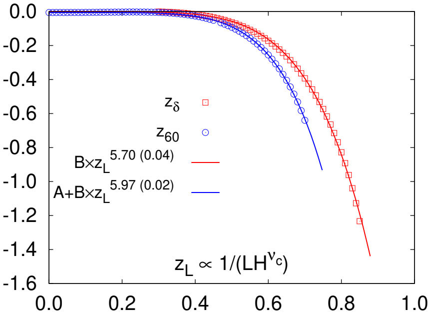
- We have used O(4) finite size scaling function for our calculations.[1]
- Both the estimators seems to approach thermodynamic limit faster than $1/V$. Determine temperature $T_\delta(H,L)$ which satisfies :

$$T_X(H,L) = T_c^0\left(1 + \frac{z_X(z_L)}{z_0} H^{1/\beta\delta}\right)$$



$$\frac{H\chi_M\left(T_\delta,H,L\right)}{M\left(T_\delta,H,L\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0}\lim_{a\to 0}\lim_{L\to\infty} T_\delta\left(H,L\right).$$

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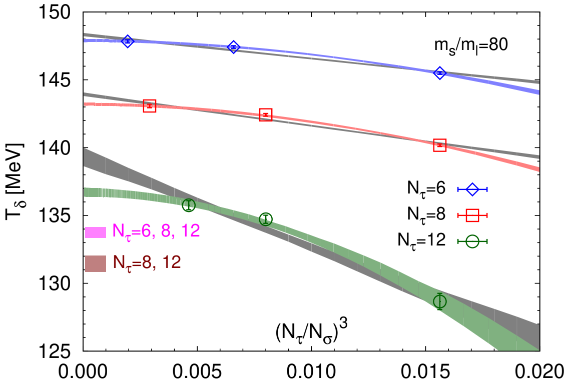
T_c^0 in continuum : 'Proper' limits



- Results for fixed H have been extrapolated to thermodynamic limit.
- Systematic uncertainty comes in form of difference between $O(4)$ and $1/V$ extrapolations.
- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

Chiral extrapolation : $T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$

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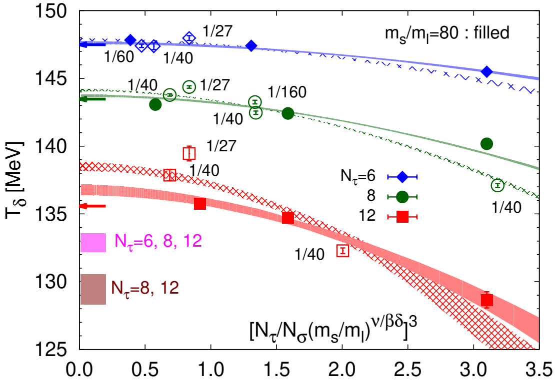
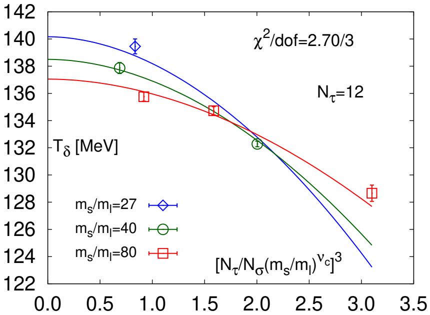


- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

- Results for fixed N_τ have been extrapolated to thermodynamic limit and chiral limit simultaneously using $O(4)$ scaling functions.



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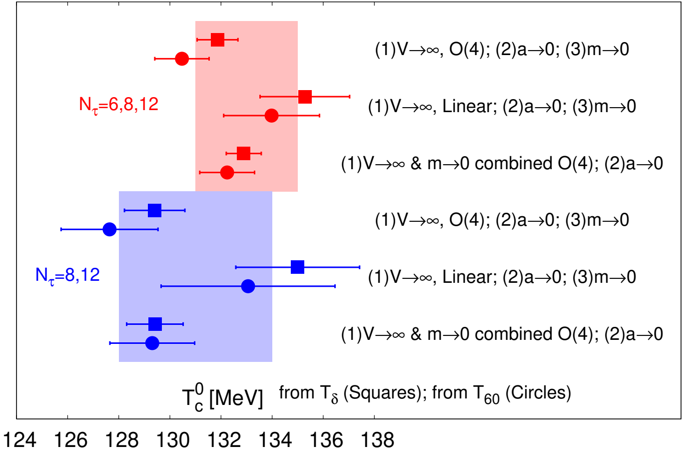
T_c^0 : A single number



Final number we have quoted : $T_c^0 = 132^{+3}_{-6}$ MeV.

HotQCD; PRL 123, 062002 (2019).

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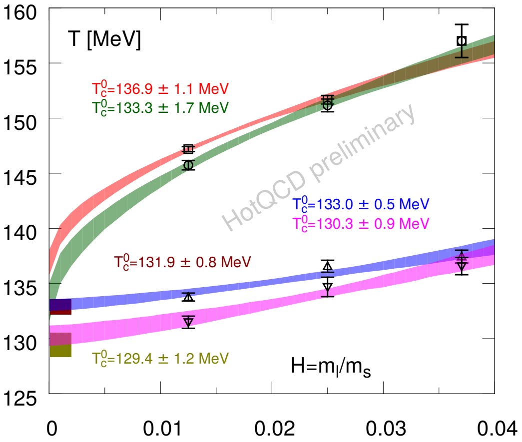
Preliminary comparison with conventional estimator

- Disclaimer : All T_{pc} numbers and T_δ for $H = 1/27$ are not infinite volume extrapolated.
- A little tension can be seen for T_{pc} calculation for $H = 1/40$.
- Still compares well.
- In thermodynamic limit, as we have seen earlier, T_{pc} will presumably increase which may pull down T_c^0, more closer to the current estimate.



- Stability of new estimators are vivid.

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Summary

- Scaling fits with $O(4)$ exponents worked reasonably.
- Within error same results obtained taking chiral and continuum extrapolations in different order.
- Current estimate of T_c^0, in continuum, is 132^{+3}_{-6} MeV.
- Comparison with conventional estimators works reasonably well.

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