The Pion Vector Form Factor from Lattice QCD at the physical point

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Flavor Singlet Project for ETMC
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Project Overview

ETMC Flavor Singlet Project

- List of ensembles covering three different lattice spacing
  - study discretization effects
  - study finite size effects
- Two ensemble at physical point
- Lattice Volume $L \sim 1.92 - 5.44$ fm
- Pion masses $\sim 135 - 340$ MeV
- Four dynamical flavors $(N_f = 2 + 1 + 1)$ [C. Alexandrou et al., Phys. Rev. D98 (2018) 054518]
- Twisted mass lattice QCD at maximal twist $\rightarrow$ automatic $\mathcal{O}(a)$-improvement [R. Frezzotti, S. Sint, Nucl. Phys. B Proc. Suppl. 106, 814-816(2002)], [R. Frezzotti, G. C. Rossi, JHEP 08, 007(2004)]

Presentation Outline

Pion Vector Form factor calculation

- Analysis of excited state contaminations in $F_\pi^{V,\,\text{bare}}(Q^2)$
- Quark mass dependence of Z_V
- Check validity of Vector Meson Dominance
- Compare different analysis approaches for $F_\pi^V(Q^2)$
- Comparison with experimental results
  - $\pi - e$ scattering : CERN [Amendolia et al., Phys. Lett. B146 (1984) 116-120]

Lattice Simulation Setup

[C. Alexandrou et al., Phys. Rev. D93 (2018) 054518]

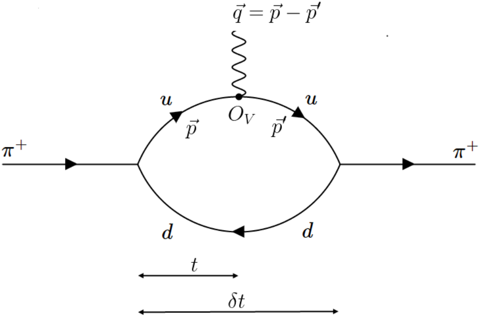
name	β	$(w_0/a)^2$	L (fm)	T/a	a (fm)[1]	$a\mu_\ell$	M_π (MeV)	$M_\pi L$	c_{sw}
cA211.53.24	1.726	3.096(12)	2.23	48	0.09273(75)	0.0053	$\approx$ 340	3.83	1.74
cA211.40.24	1.726	3.156(11)	2.23	48	0.09273(75)	0.004	$\approx$ 300	3.38	1.74
cA211.30.32	1.726	3.222(9)	2.97	64	0.09273(75)	0.003	$\approx$ 250	3.76	1.74
cB211.25.24	1.778	4.402(8)	1.92	48	0.08000(62)	0.0025	$\approx$ 250	2.43	1.69
cB211.25.32	1.778	4.402(8)	2.56	64	0.08000(62)	0.0025	$\approx$ 250	3.24	1.69
cB211.25.48	1.778	4.402(8)	3.84	96	0.08000(62)	0.0025	$\approx$ 250	4.87	1.69
cB211.072.64	1.778	4.517(10)	5.12	128	0.08000(62)	0.00072	$\approx$ 135	3.50	1.69
cC211.06.80	1.836	6.266(9)	5.44	160	0.06802(52)	0.0006	$\approx$ 135	3.72	1.65

Table Summary

- $M_\pi = 135, 250, 340$ MeV
- $L = 1.92 \sim 5.44$ (fm), $M_\pi L = 2.43 \sim 4.87$
- Wilson gradient flow, w_0 is used as intermediate lattice scale, and the pion decay constant f_π is used for setting the scale

[1] Lattice spacing preliminary results, thanks to *S. Simula*

Pion Vector Form Factor Target Diagram



Vector current operator, O_V

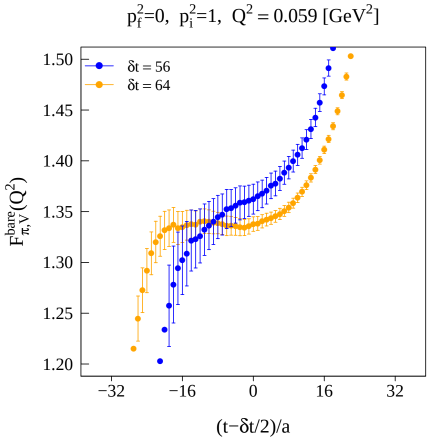
- Insert vector current operator between pion creation and annihilation operator

$$O_V(x) = \frac{2}{3}\bar{u}(x)\gamma_\mu u(x) - \frac{1}{3}\bar{d}(x)\gamma_\mu d(x) \approx \bar{u}(x)\gamma_\mu u(x)$$

$$O_{\pi^+}(t, x) = \bar{d}(t, x)\gamma_5 u(t, x)$$

- In Vector Form Factor calculation, there is no disconnected contribution

Excited states contamination on $F_\pi^{V,\,\text{bare}}(Q^2)$

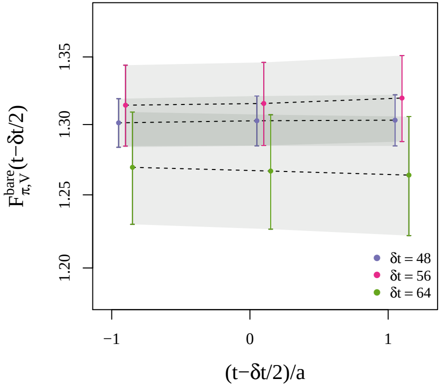


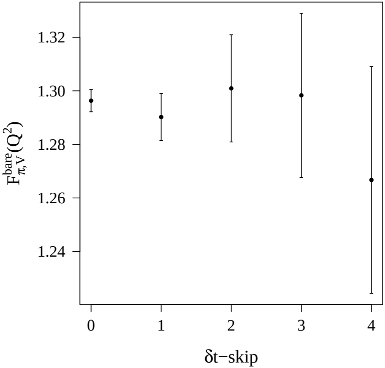
- M_π^{phy}, $L = 5.12$ fm, $a \sim 0.08$ fm
- $\delta t = 56$, $\delta t = 64$
- δt is source-sink separation
- Supposed one pion state is dominated
- Overshooting at the boundary $\rightarrow$ Excited state contamination, eg. two/three pion states.
- Extract bare form factor in Plateau region near $t - \delta t/2 = 0$

$$C_{3pt}^{\pi\pi} \xrightarrow[\delta t - t \to \infty]{t \to \infty} G(\vec{p_f}) \frac{e^{-E(p_f)(\delta t - t)}}{2E(p_f)} \langle \pi_+(\vec{p}_f)|O_V|\pi_+(\vec{p}_i)\rangle \frac{e^{-E(p_i)(t)}}{2E(p_i)} G^\dagger(\vec{p_i})$$

$$Z_V \frac{\langle \pi_+(\vec{p}_f)|O_V|\pi_+(\vec{p}_i)\rangle}{E(p_f) + E(p_i)} = F_\pi^{V,bare}(Q^2), \qquad G(\vec{p}) = \langle 0|O_{\pi^+}(\vec{p})|\pi_+(\vec{p})\rangle$$

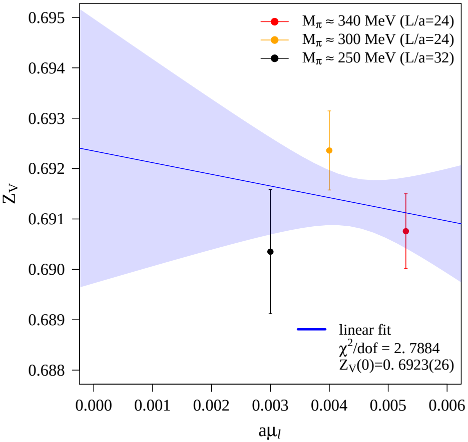
Excited states contamination on $F_\pi^{V,\,\text{bare}}(Q^2)$





- M_π^{phy}, $L = 5.12$ fm, $a \sim 0.08$ fm
- $p_f^2 = 1$, $p_i^2 = 2$, $Q^2 = 0.059$ GeV^2, $\chi^2/dof = 2.92/8$
- $\delta t - skip$ is number of skipped source-sink separation on $F_\pi^{V,\,\text{bare}}(Q^2)$ fitting
- $\delta t \in \{24, 36, 48, 56, 64\}, \delta t - skip = 2 \Leftrightarrow \delta t \in \{48, 56, 64\}$

Quark mass dependence of Z_V



$$Z_V(g_0^2, a\mu_\ell) = Z_V(g_0^2, 0)\,(1 + A(g)\cdot a\mu_\ell)$$

[A. Gérardin et al., Phys. Rev. D99 (2019)]

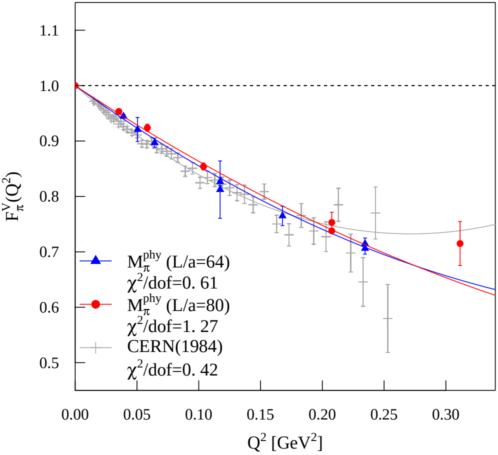
- No statistically significant mass dependence $\rightarrow$ one can use Z_V at individual ensembles

$$Z_V \xrightarrow[\delta t - t\rightarrow\infty]{t\rightarrow\infty} \frac{C^{\pi}_{2pt}(t,\vec{0})}{C^{\pi}_{3pt}(t,t',\vec{0},\vec{0})}$$

M_π (MeV)	L (fm)	$a\mu_\ell$	Z_V
$\approx$ 340	2.23	0.0053	0.69076(74)
$\approx$ 300	2.23	0.004	0.69236(78)
$\approx$ 250	2.97	0.003	0.6904(12)

- $\beta = 1.726, a = 0.09273(75)$ fm
- $a\mu_\ell$ is twisted quark mass.

$F_\pi^V(Q^2)$ - Results at the Physical Pion Mass, VMD check



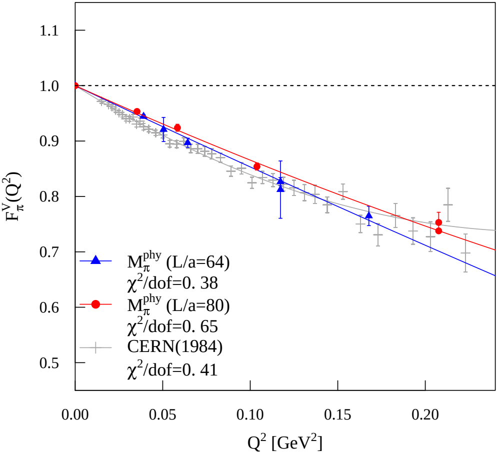
M_π (MeV)	L (fm)	a (fm)	$\langle r^2\rangle_V$ (fm^2)	c_π (fm^4)
$\approx$ 135	5.12	~0.080	0.374(23)	0.0023(6)
$\approx$ 135	5.44	~0.068	0.344(23)	0.0016(7)
139.57(exp)	∞	0	0.461(30)	0.0055(5)

- L = 5.12 fm, 5.44 fm, Exp. data, CERN [Amendolia et al., Phys. Lett. B146 (1984) 116-120]
- $Q^2 \in [0\ \mathrm{GeV}^2, 0.45\ \mathrm{GeV}^2]$
- All fits have χ^2/dof ~1

$$F_\pi^V(Q^2) = 1 - \frac{\langle r^2\rangle_V}{6}Q^2 + c_\pi Q^4 + \mathcal{O}(Q^6)$$

- NLO coef. $s_\pi = \langle r^2\rangle_V/6$
- $c_\pi/s_\pi^2 \sim 0.68(12) \to$ VMD is weakly valid ($s_\pi^2 \sim c_\pi$)

$F_\pi^V(Q^2)$ - Small Q^2 region, VMD check



M_π (MeV)	L (fm)	a (fm)	$\langle r^2\rangle_V$ (fm^2)	c_π (fm^4)
$\approx$ 135	5.12	~0.080	0.349(24)	0.0004(10)
$\approx$ 135	5.44	~0.068	0.332(25)	0.0012(7)
139.57(exp)	∞	0	0.462(32)	0.0056(4)

- $Q^2 \in \left[0\,\mathrm{GeV}^2, 0.225\,\mathrm{GeV}^2\right]$
- All fits have χ^2/dof ~0.5

$$F_\pi^V(Q^2) = 1 - \frac{\langle r^2\rangle_V}{6}Q^2 + c_\pi Q^4 + \mathcal{O}(Q^6)$$

- NLO coef. $s_\pi = \langle r^2\rangle_V/6$
- $c_\pi/s_\pi^2 \sim 0.48(20) \rightarrow$ VMD is weakly valid ($s_\pi^2 \sim c_\pi$)
- In smaller Q^2 region, linear fits are more dominant
- Large difference between experimental/simulation results $\rightarrow$ Need improved analysis method to reduce NNLO residuals

Reduced NNLO effect on $\langle r^2 \rangle_V$ by using $1/F_\pi^V(Q^2)$

NNLO contribution of $1/F_\pi^V(Q^2)$ [C. Alexandrou, Phys. Rev. D47(2018) 014508]

- Using NLO and NNLO expansions of $F_\pi(Q^2)$
- $1/F_\pi(Q^2)$ has small NNLO effects than $F_\pi(Q^2)$

$$F_\pi(Q^2) = 1 + \Delta F_\pi^{\text{NLO}}(Q^2) + \Delta F_\pi^{\text{NNLO}}(Q^2) + \dots$$

$$\frac{1}{F_\pi(Q^2)} = 1 - \Delta F_\pi^{\text{NLO}}(Q^2) + \Delta K^{\text{NNLO}}(Q^2) + \dots$$

$$F_\pi(Q^2) \times \frac{1}{F_\pi(Q^2)} = 1 - \left(\Delta F_\pi^{\text{NLO}}(Q^2)\right)^2 + \Delta F_\pi^{\text{NNLO}}(Q^2) + \Delta K^{NNLO}(Q^2) + \dots$$

$$\Longrightarrow \Delta K^{NNLO}(Q^2) = \left(\Delta F_\pi^{\text{NLO}}(Q^2)\right)^2 - \Delta F_\pi^{\text{NNLO}}(Q^2)$$

$$1/F_\pi(Q^2) = 1 - \Delta F_\pi^{\text{NLO}}(Q^2) + \left[\left(\Delta F_\pi^{\text{NLO}}(Q^2)\right)^2 - \Delta F_\pi^{\text{NNLO}}(Q^2)\right] + \dots$$

Reduced NNLO effect on $\langle r^2 \rangle_V$ by using $1/F_\pi^V(Q^2)$

Vector Form factor Q^2 extrapolation, NLO and NNLO effect

$$\Delta F_\pi^{\mathrm{NLO}}(Q^2) = s_\pi Q^2, \qquad \Delta F_\pi^{\mathrm{NNLO}}(Q^2) = c_\pi Q^4$$

$$F_\pi(Q^2) = 1 + s_\pi Q^2 + c_\pi Q^4 + \ldots$$

Strong VMD case, $s_\pi^2 \gg c_\pi$

- Use $F_\pi(Q^2)$ for Q^2-fitting

$$\frac{1}{F_\pi(Q^2)} = 1 - s_\pi Q^2 + (s_\pi^2 - c_\pi)Q^4 + \ldots$$

Weakly VMD case, $s_\pi^2 \sim c_\pi$

- Use $1/F_\pi(Q^2)$ for Q^2-fitting
- Especially, $c_\pi / s_\pi^2 \approx 1$, then $1/F_\pi^V(Q^2)$ NNLO effect goes negligible

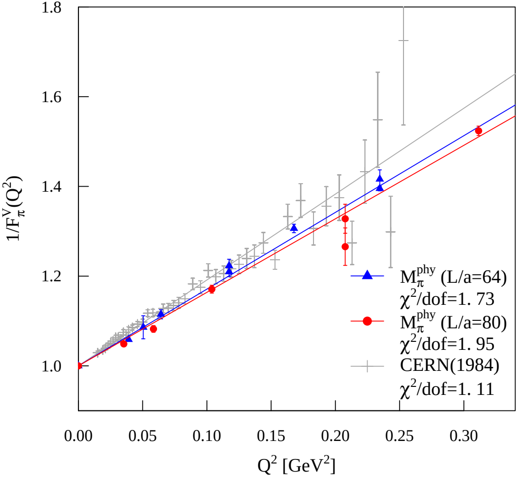
$F_\pi^V(Q^2)$ and $1/F_\pi^V(Q^2)$ fitting result comparison

M_π (MeV)	L (fm)	a (fm)	$Q^2 \in \{0,\ 0.225\,\mathrm{GeV}^2\}$			$Q^2 \in \{0,\ 0.45\,\mathrm{GeV}^2\}$		
			$1/F_\pi^V(Q^2)$	$F_\pi^V(Q^2)$		$1/F_\pi^V(Q^2)$	$F_\pi^V(Q^2)$	
			$\langle r^2\rangle_V$ (fm^2)		c_π/s_π^2	$\langle r^2\rangle_V$ (fm^2)		c_π/s_π^2
$\approx$ 135	5.12	0.08000(62)	0.414(14)	0.349(24)	0.12(30)	0.400(6)	0.374(2)	0.60(9)
$\approx$ 135	5.44	0.06802(52)	0.335(14)	0.332(25)	0.38(20)	0.383(9)	0.344(2)	0.49(14)
139.57(exp)	∞	0	0.448(36)	0.462(32)	0.95(11)	0.450(6)	0.463(3)	0.93(13)

*PDG('14,'16,'18), Pion squared vector radius : 0.452(11) fm^2

- In most cases, $\langle r^2\rangle_V$ and c_π/s_π^2 decrease in small Q^2 fitting window
- In weak VMD cases, generally $1/F_\pi^V(Q^2)$ fitting gives more close $\langle r^2\rangle_V$ to PDG.
- Yellow box: CERN result with $F_\pi^V(Q^2)$ has $\mathcal{O}(\sim 2\%)$ and $1/F_\pi^V(Q^2)$ has $\mathcal{O}(\sim 0.5\%)$ to PDG.

$1/F_\pi^V(Q^2)$ results comparison with experiment



M_π (MeV)	L (fm)	a (fm)	$M_\pi L$	$\langle r^2\rangle_V$ (fm^2)
$\approx$ 135	5.12	~0.080	3.50	0.400(6)
$\approx$ 135	5.44	~0.068	3.72	0.383(9)
139.57(exp)	∞	0	-	0.450(6)

- $Q^2 \in [0\,\mathrm{GeV}^2, 0.45\,\mathrm{GeV}^2]$
- All fits have χ^2/dof ~1.5
- Observation and remarks
  - Two ensemble difference $\mathcal{O}(\sim 4\%)$
  - Difference with exp. $\mathcal{O}(11 \sim 15\%)$
  - Need more investigation → FSE, Discretization correction

$$1/F_\pi^V(Q^2) = 1 + \frac{\langle r^2\rangle_V}{6} Q^2 + \mathcal{O}(Q^4)$$

Conclusions and Outlook

Conclusion

- Excited states gives systematical error to bare form factor
- Vector current Renormalization constant is independent of light quark mass
- VMD is weakly valid$\rightarrow$ Use of $1/F_\pi^V(Q^2)$ gives smaller NNLO effect

Outlook

- In order to manage remaining systematics, we should perform combined analysis of all ensemble, including three different lattice spacing
- Consider discretization effect, FSE effect

Thanks for your attention!