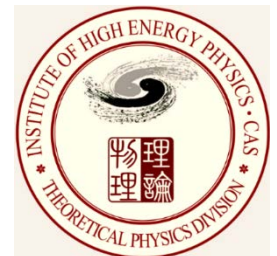


# Charmed meson decay constants from 2+1-flavor lattice QCD

**Zhaofeng Liu**  
**Institute of High Energy Physics, Beijing**

**2020.08.06**  
**APLAT 2020**



# Outline

- **Motivation**
- **Lattice setup and data analyses**
- **Results and summary**

## Collaborators:

**Ying Chen, Wei-Feng Chiu, Ming Gong, Yunheng Ma**

# What we calculate

- $f_{D_{(s)}^{(*)}}$  and their ratios ( $f_\phi$  is also computed)

$$\langle \mathbf{0} | \bar{q}(\mathbf{0}) \gamma_\mu \gamma_5 c(\mathbf{0}) | P(p) \rangle = i f_P p_\mu \quad q = d, s$$

$$(m_q + m_c) \langle \mathbf{0} | \bar{q}(\mathbf{0}) \gamma_5 c(\mathbf{0}) | P(p) \rangle = f_P m_{PS}^2$$

$$\langle 0 | \bar{q}(0) \gamma^\mu q'(0) | V(p, \lambda) \rangle = f_V m_V e_\lambda^\mu$$

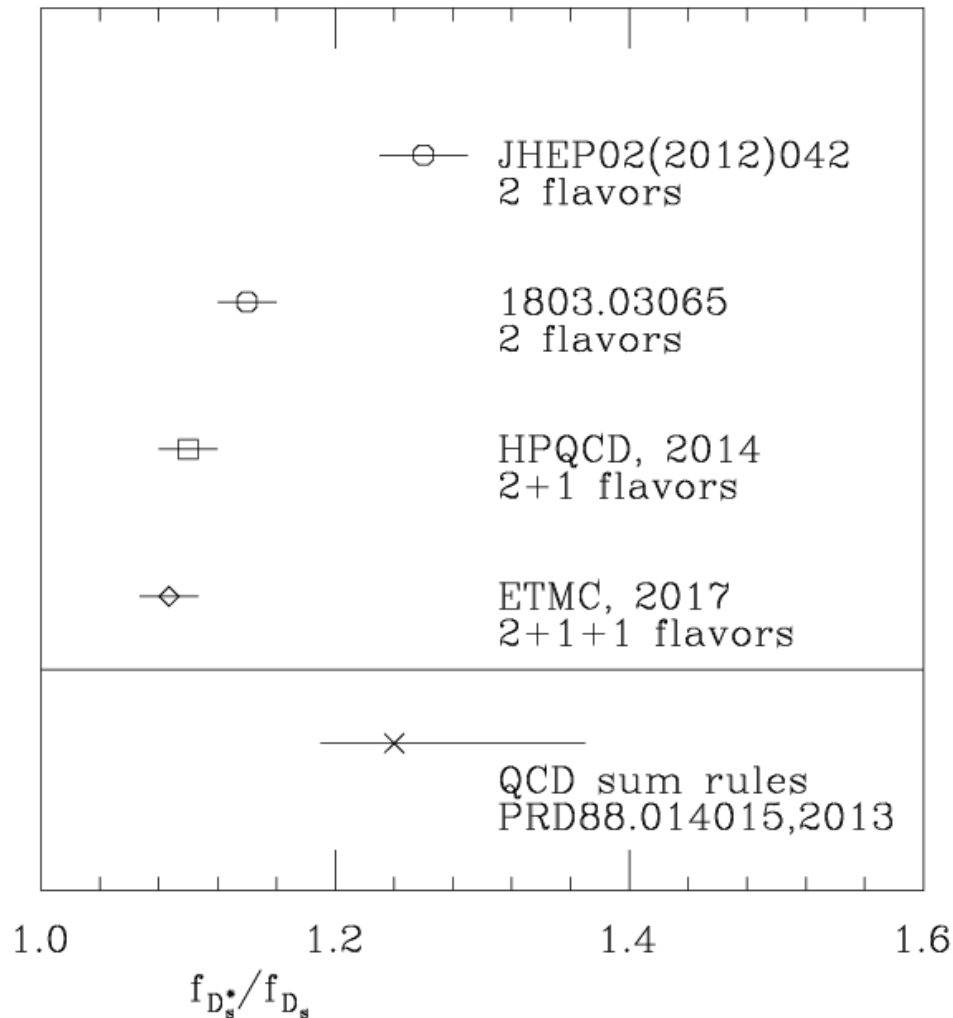
- $f_V^T / f_V$

$$\langle 0 | \left( \bar{q}(0) \sigma^{\mu\nu} q'(0) \right) (\mu) | V(p, \lambda) \rangle = i f_V^T(\mu) (e_\lambda^\mu p^\nu - e_\lambda^\nu p^\mu)$$

# Motivation

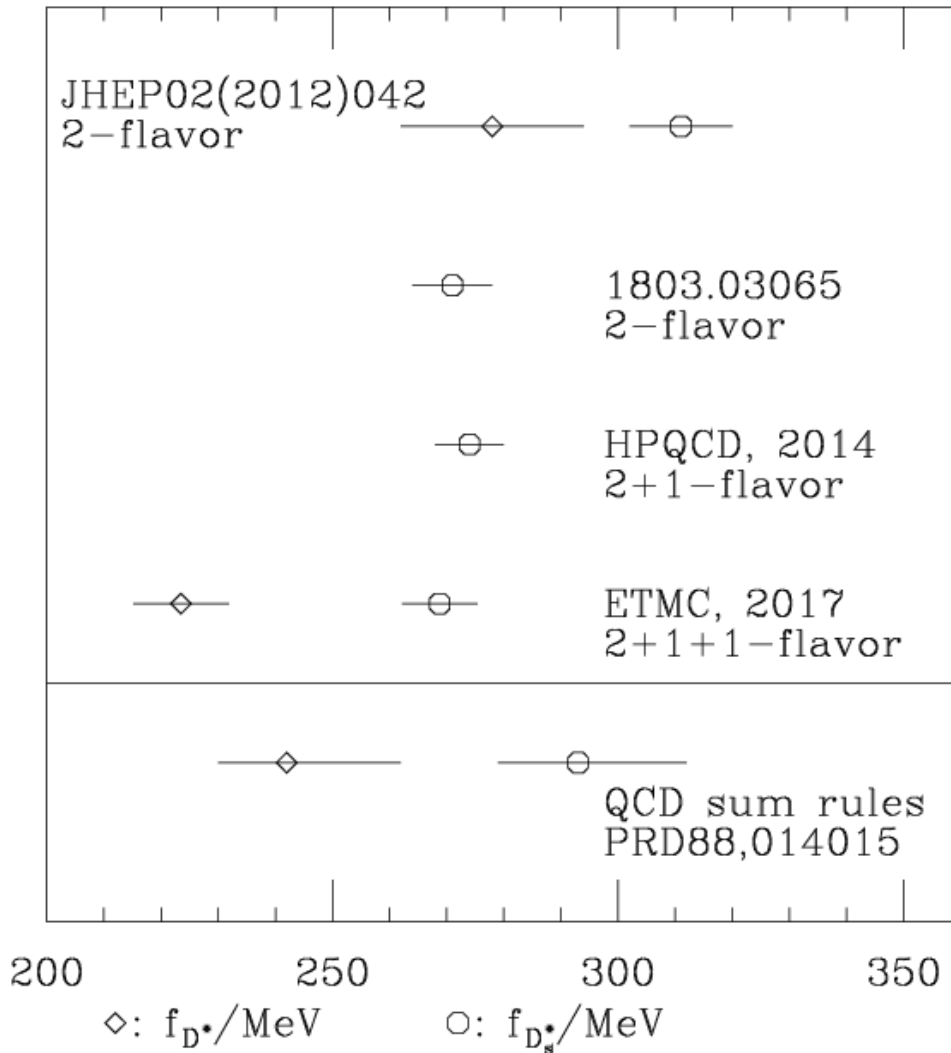
- **CKM elements**  $\Gamma(P \rightarrow \ell\nu) = \frac{G_F^2 |V_{q_1 q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$
- $f_V$  is not easy to measure
  - Leptonic decay BRs are much smaller than those of strong decays
- Test the accuracy of HQET,  
$$f_V/f_{PS} = 1 + \mathcal{O}(1/m_Q)$$
- $f_V^T/f_V$  for  $D^*$  &  $D_s^*$  can be used as inputs for LCSR in calculations of  $B \rightarrow V$  form factors at low  $q^2$
- Input parameters for QCD factorization in studies of nonleptonic B decays, e.g.,  $B \rightarrow D^{(*)} M$

$$f_{D_s^*}/f_{D_s}$$



- **Becirevic et al., JHEP02 (2012) 042**
  - $4\alpha$ , 2-flavor, tmQCD
- **Blossier, Heitger, Post, PRD98.054506 (1803.03065)**
  - $2\alpha$ , 2-flavor, Clover fermions
- **HPQCD, PRL112.212002 (2014)**
  - $2\alpha$ , 2+1-flavor, HISQ+asqtad
- **ETMC, PRD96.034524 (2017)**
  - $3\alpha$ , 2+1+1-flavor, tmQCD
- **Sea quark effects from the strange quark?**

# $f_{D^*}$ and $f_{D_s^*}$



- **Becirevic et al., JHEP02 (2012) 042**
  - $4a$ , 2-flavor, tmQCD
- **Blossier, Heitger, Post, PRD98.054506 (1803.03065)**
  - $2a$ , 2-flavor, Clover fermions
- **HPQCD, PRL112.212002 (2014)**
  - $2a$ , 2+1-flavor, HISQ+asqtad
- **ETMC, PRD96.034524 (2017)**
  - $3a$ , 2+1+1-flavor, tmQCD
- **Sea quark effects from the strange quark?**

# Lattice setup

- **2+1-flavor ensemble (RBC/UKQCD Collab.)**
- **Physical sea quark mass:**  
 $m_{\pi}^{\text{sea}} = 139.2(4) \text{ MeV}$
- **45 configurations**

$L^3 \times T$	$48^3 \times 96$
$a^{-1}(\text{GeV})$	$1.730(4)$
$N_{\text{conf}}$	$45$
$am_l^{(\text{val})}$	$0.0017, 0.0024, 0.0030, 0.0060$
$m_{\pi}/\text{MeV}$	$114(2), 135(2), 149(2), 208(2)$
$am_s^{(\text{val})}$	$0.0580, 0.0650$
$am_c^{(\text{val})}$	$0.6800, 0.7000, 0.7200, 0.7400$

- **Overlap valence and domain wall fermion sea**
- **Partial quenching effects are small:  $\Delta_{\text{mix}} = 0.030(6)(5) \text{ GeV}^4$**   
[\[PRD86.014501, 2012\]](#)
- **4 light val. quark masses:  $m_{\pi} \sim 114 - 208 \text{ MeV}$**
- **$Lm_{\pi} = 3.2/3.7/4.1/5.8$**
- **2 strange val. quark masses, slightly  $< m_s^{\text{phy.}}$**

# $m_\pi, f_\pi, m_K$

- $m_\pi$  &  $m_K$  are extracted from pseudoscalar density 2-point functions
- Use to fix the physical light and strange valence quark masses

$am_l^{\text{val}}$	0.0017	0.0024	0.0030	0.0060
$m_\pi/\text{MeV}$	114(2)	135(2)	149(2)	208(2)
$f_\pi/\text{MeV}$	130.3(9)	131.0(9)	131.6(8)	---

- A linear interp. in  $m_\pi^2$  gives  $f_\pi = 131.3(6)$  MeV
- Consistent with the RBC/UKQCD result on the same ensemble [[arXiv:1411.7017\(hep-lat\)](https://arxiv.org/abs/1411.7017)]



# D-meson 2-point functions

- Coulomb gauge wall source propagators are used to improve overlapping with the ground state
- Sink operators are with spacial displacement

$$O_{\Gamma}(\vec{x}, t; \vec{r}) = \bar{\psi}_1(\vec{x}, t) \Gamma \psi_2(\vec{x} + \vec{r}, t)$$

$$\Gamma = \gamma_5 \text{ or } \gamma_i$$

$$\vec{r} = \mathbf{0}: \text{local operator}$$

- Same  $r = |\vec{r}|$  averaged to get the correct  $J^P$

$$C_P(r, t) = \frac{1}{N_r} \sum_{\vec{x}, |\vec{r}|=r} \langle 0 | O_{\gamma_5}(\vec{x}, t; \vec{r}) O_{\gamma_5}^{(W)\dagger}(0) | 0 \rangle,$$

$$C_V(r, t) = \frac{1}{3N_r} \sum_{\vec{x}, i, |\vec{r}|=r} \langle 0 | O_{\gamma_i}(\vec{x}, t; \vec{r}) O_{\gamma_i}^{(W)\dagger}(0) | 0 \rangle$$

$$C^W(t) = \langle 0 | O^{(W)}(t) O^{(W)\dagger}(0) | 0 \rangle$$

# Data analysis

1. Simultaneous correlated fittings to several correlators

Common parameter:  $m_H$

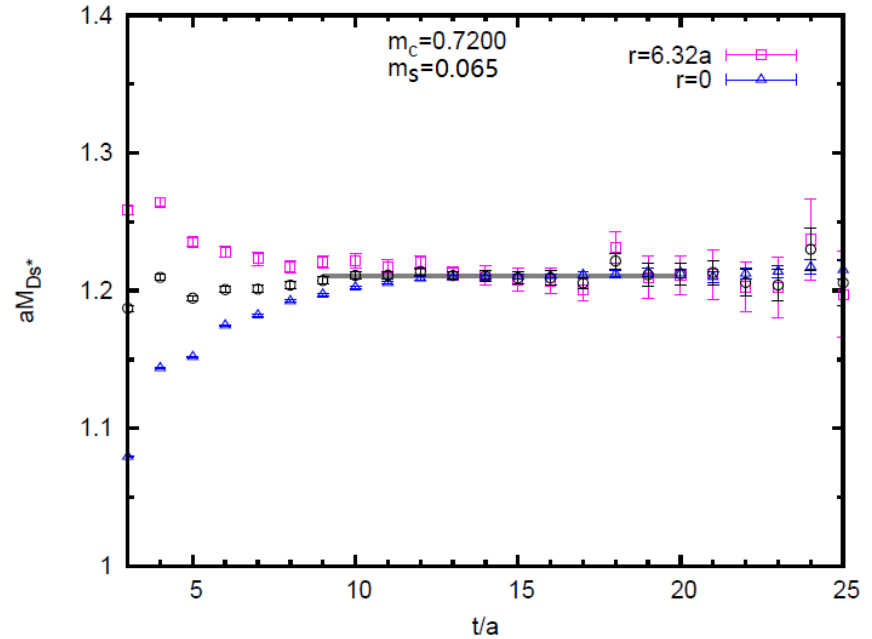
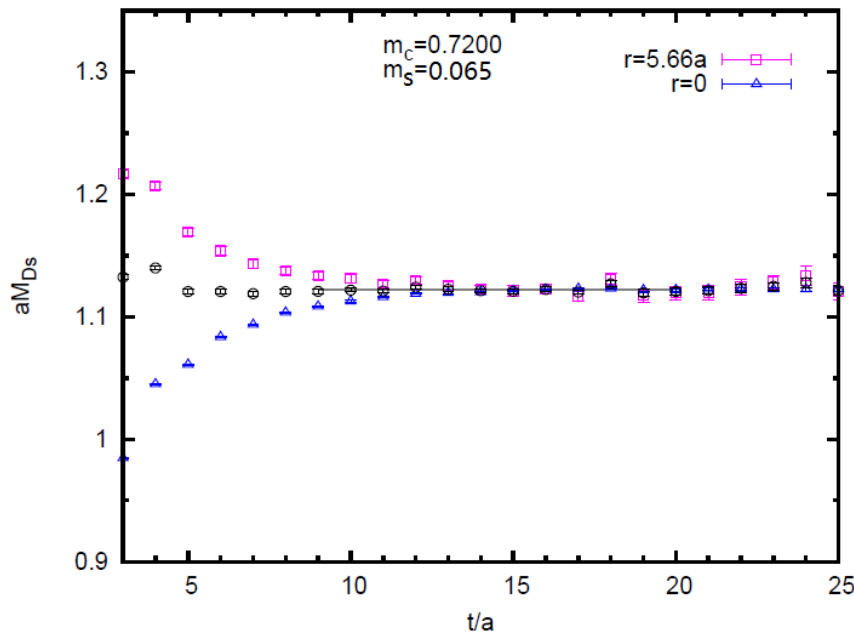
2. Fit combined correlators

$$C(\omega, t) = C(r = 1, t) + \omega C(r, t)$$

Adjust  $r$  and  $\omega$  to get the best mass plateau

- ✓ The two methods give consistent  $m_H$
- ✓ The result of  $m_H$  is insensitive to  $\omega$
- Combine the spectral weights from  $C(r = 0, t)$  and  $C^W(t)$  to get the decay constants

# Mass plateau

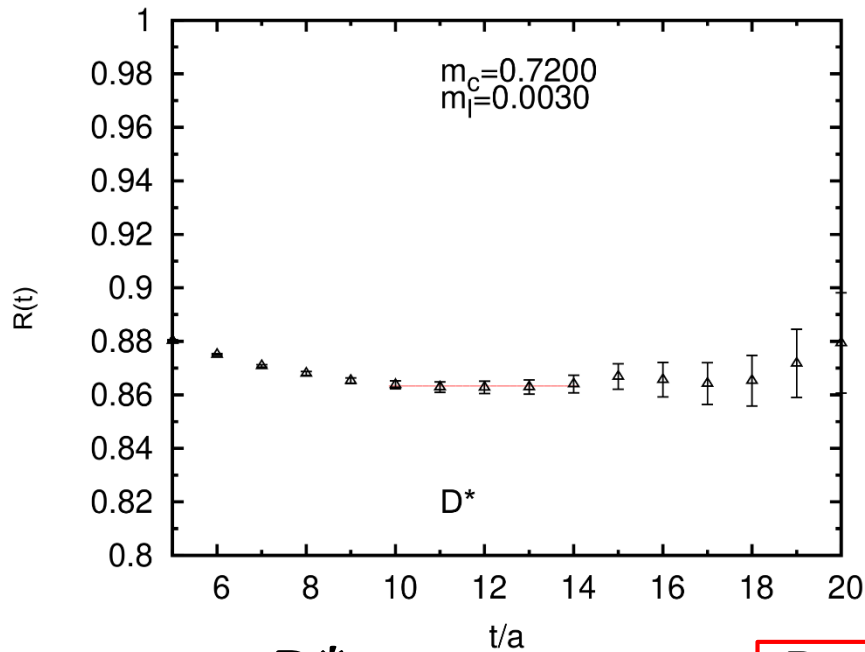


- **Black circles:**  $C(\omega, t) = C(r = 1, t) + \omega C(r, t)$
- $t_{\max}$ :  $\frac{\delta C(t)}{C(t)} < 10\%(5\%)$  for V(PS) mesons
- $t_{\min}$ : varied to get stable results,  $\chi^2/\text{dof} \lesssim 1.0$

# $f_V^T / f_V$

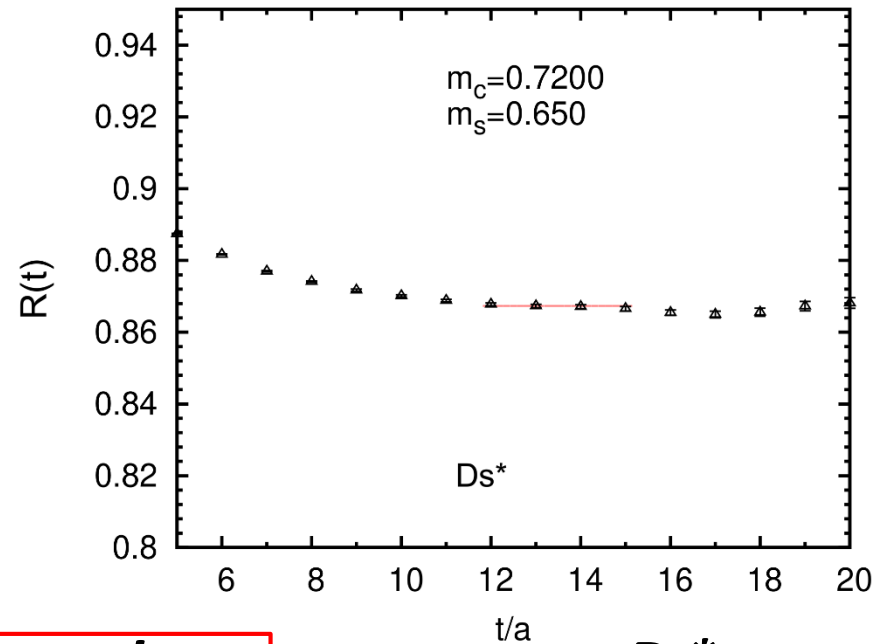
$$R(t) = C_T(t) / C_V(r=0, t) = \frac{\sum_i \langle T_{0i} O_V^{(W)\dagger} \rangle}{\sum_i \langle V_i O_V^{(W)\dagger} \rangle} \xrightarrow{t \rightarrow \infty} \frac{f_T}{f_V}$$

$$C_T(r=0, t) = \frac{1}{3} \sum_{\vec{x}, i} \langle 0 | O_{\sigma_{0i}}(\vec{x}, t) O_{\gamma_i}^{(W)\dagger}(0) | 0 \rangle$$



$D^*$

**Bare values**



$D_s^*$

# Interp./extrap. to physical point

- $m_{\pi}^2, m_{SS}^2 \equiv 2m_K^2 - m_{\pi}^2$  and  $m_{D_s}$  are used to set the physical quark masses
- Our quark masses are close to their physical values
- Linear Interp./extrap. in  $m_{\pi}^2, m_{SS}^2$  and  $m_{D_s}$
- For a meson mass or decay constant:

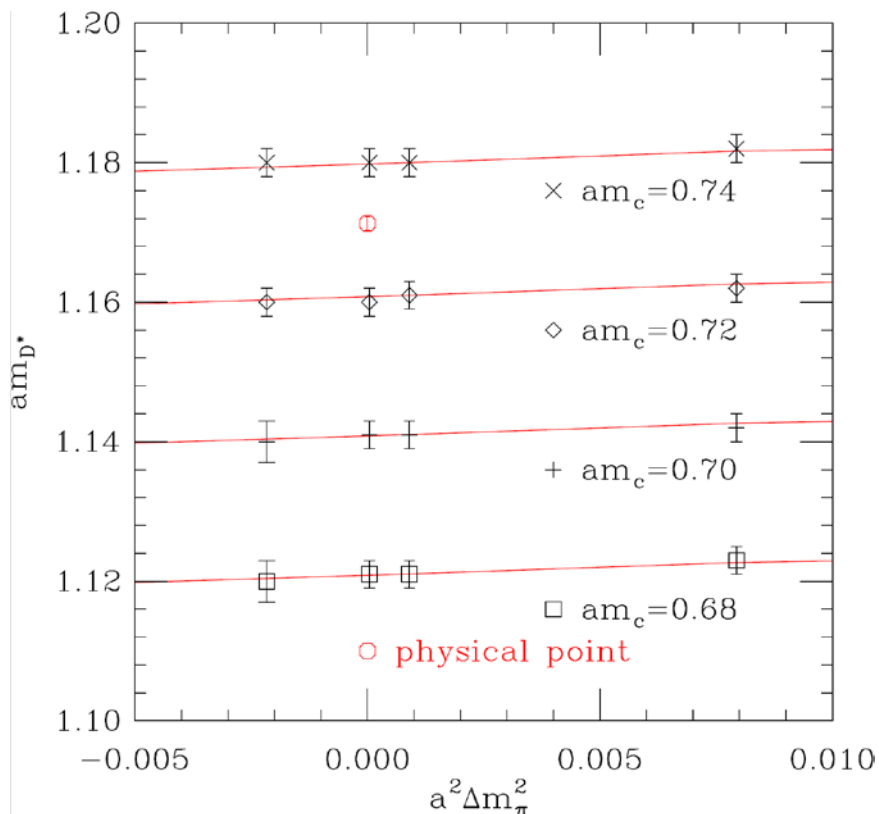
$$A(m_{u/d}, m_s, m_c) = A^{(phy)} + b_1 \Delta m_{\pi}^2(m_{u/d}) + b_2 \Delta m_{SS}^2(m_s) + b_3 \Delta m_{D_s}(m_c)$$

$$\Delta m_{\pi}^2 = m_{\pi}^2 - m_{\pi}^2(\text{phy}), \quad \Delta m_{SS}^2 = m_{SS}^2 - m_{SS}^2(\text{phy}), \quad \Delta m_{D_s} = m_{D_s} - m_{D_s}^{\text{phy}}$$

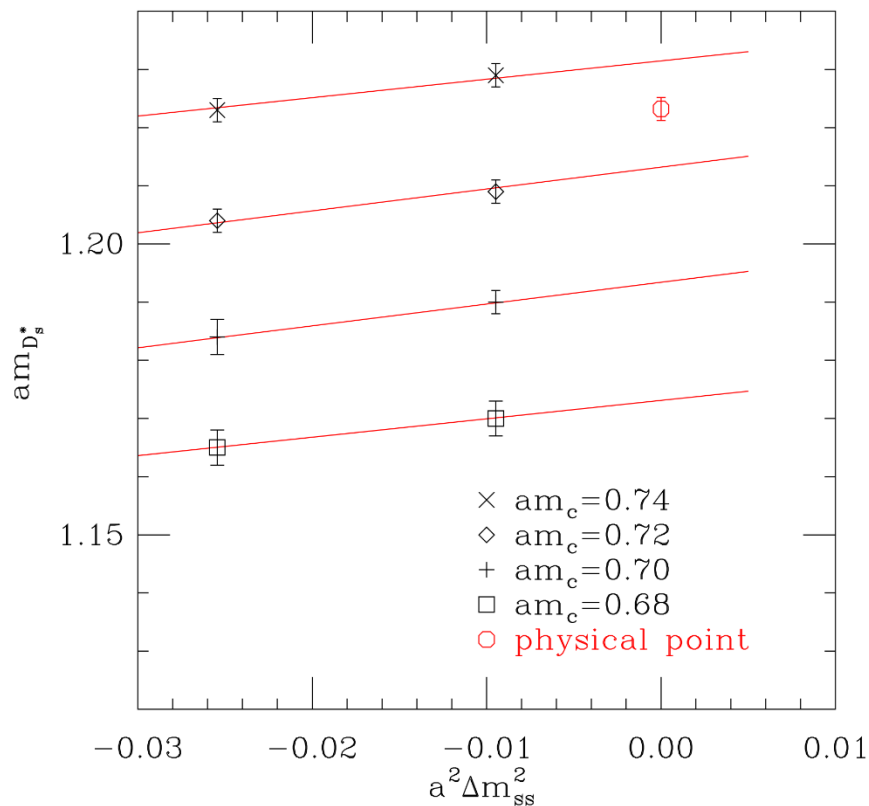
- Supported by the data with good  $\chi^2/\text{dof}$

# Interp./extrap. to physical point

$m_{D^*}$

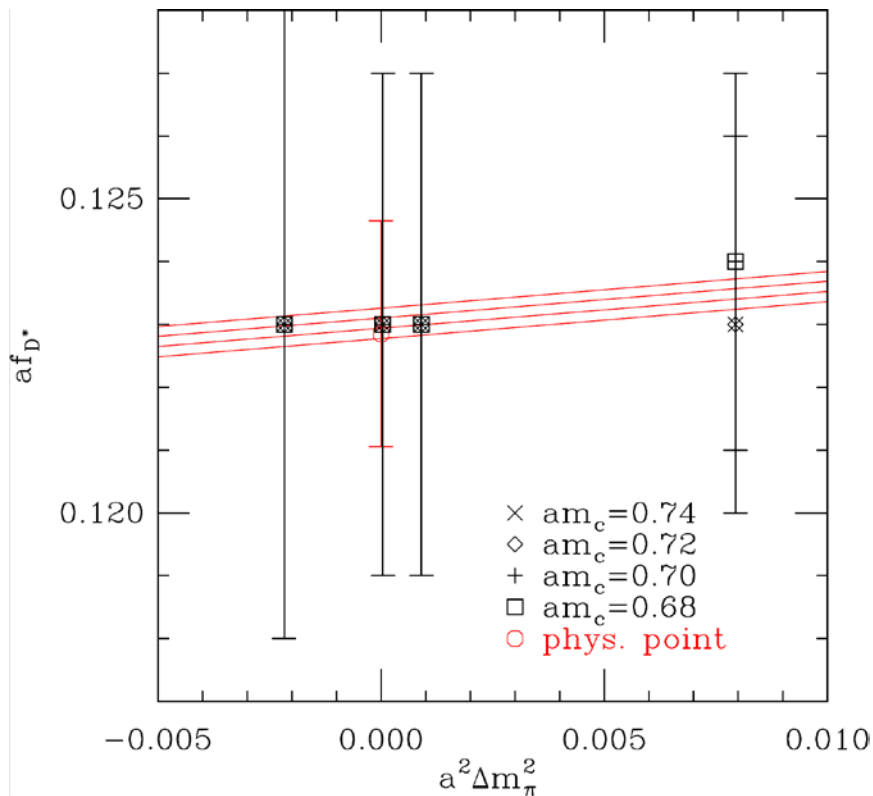


$m_{D_s^*}$

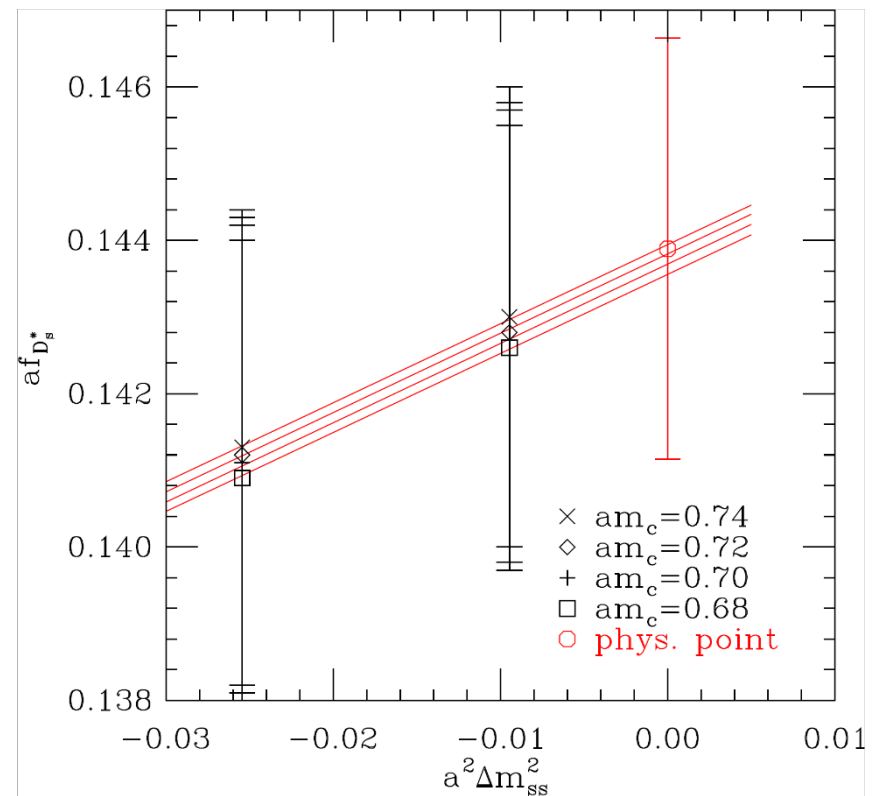


# Interp./extrap. to physical point

$f_{D^*}$



$f_{D_s^*}$



- Quark mass dependences of decay constants can barely be seen with the statistical uncertainties

# RI/(S)MOM renormalization

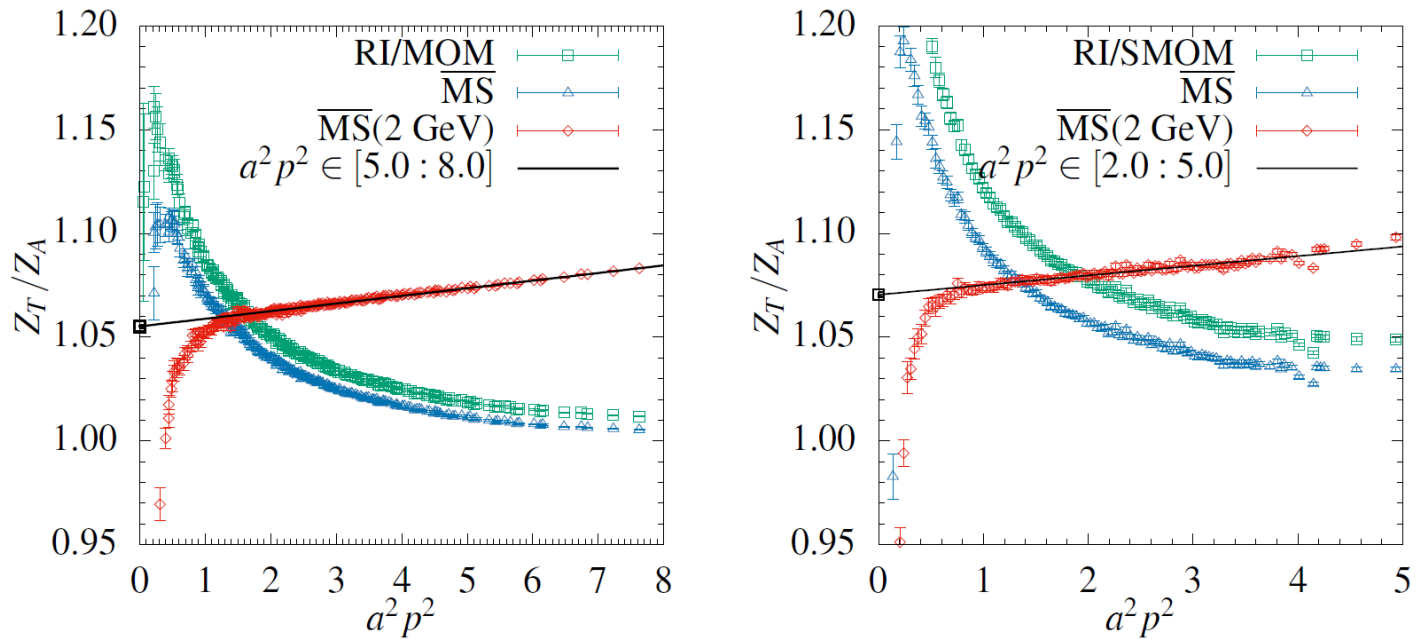
- Renormalization constants are needed to get
  - $f_P$  using the local axial vector current
  - $f_V$  using the local vector current
  - $f_V^T$  from the tensor operator
- RI/MOM and RI/SMOM schemes are used and matched to the  $\overline{\text{MS}}$  scheme
- Matching hadronic matrix elements calculated on the lattice to the continuum

$$\bar{\psi}\Gamma\psi, \quad \Gamma = I, \gamma_5, \gamma_\mu, \gamma_\mu\gamma_5, \sigma_{\mu\nu}$$

Bi et al., arXiv:1710.08678(PRD97.094501)



# RI/(S)MOM renormalization



## $Z_T$ in the RI/MOM, RI/SMOM and $\overline{\text{MS}}$ schemes

- Final results for all renormalization constants

TABLE IX. Matching factors to the  $\overline{\text{MS}}$  scheme for the quark field and bilinear quark operators.

$Z_A$	$Z_q(2 \text{ GeV})$	$Z_T(2 \text{ GeV})$	$Z_S(2 \text{ GeV})$	$Z_P(2 \text{ GeV})$
1.1025(16)	1.216(23)	1.163(34)	1.118(29)	1.123(56)

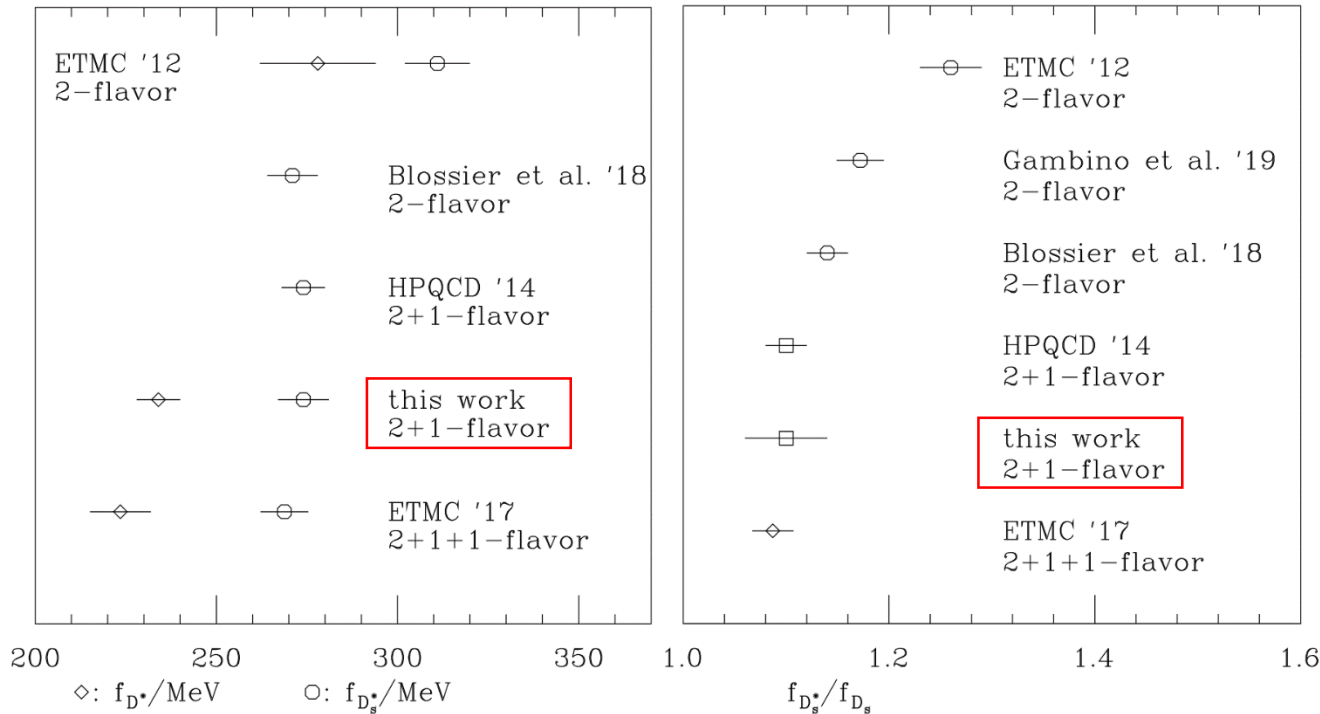
# Results

	$D$	$D^*$	$D_s$	$D_s^*$	$\phi$
Mass/MeV	1873(5)	2026(5)	-	2116(6)	1018(17)
$M^{\text{exp}}/\text{MeV}$	1869.6	2010.3	1968.3	2112.2	1019.5
$f_M/\text{MeV}$	213(2)(4)	234(3)(5)	249(5)(5)	274(5)(5)	241(9)(2)
$f_V^T/f_V$		0.91(3)(2)		0.92(3)(2)	

- The mass of  $D^*$  is  $\sim 1\%$  higher than experiments
- $f_{D_s} = 249(5)$  MeV **vs.**  $254(2)(4)$  MeV [[PRD92.034517](#)]
- The 1<sup>st</sup> error from stat. and interp./extrap.
- The 2<sup>nd</sup> error from Z-factors and finite  $a$  ( $\sim 2\%$ )
- $f_D$  agrees with FLAG2019 (2+1-flavor):  $209.0(2.4)$  MeV
- $f_{D_{(s)}^*}^T/f_{D_{(s)}^*}$  are the first lattice QCD results

# Results

- Heavy quark symmetry breaking ( $\sim 10\%$ )
  - $f_V/f_{PS} = 1 + \mathcal{O}(1/m_Q)$
  - $f_{D^*}/f_D = 1.10(2)(2)$ ,  $f_{D_s^*}/f_{D_s} = 1.10(3)(2)$
- SU(3) flavor symmetry breaking ( $\sim 17\%$ )
  - $f_{D_s}/f_D = 1.163(14)(23)$ ,  $f_{D_s^*}/f_{D^*} = 1.17(2)(2)$



- [Becirevic et al., JHEP02 \(2012\) 042](#)
- [Blossier, Heitger, Post PRD98.054506 \(2018\)](#)
- [Gambino et al., J. Phys. Conf. Ser. 1137, 012005 \(2019\)](#)
- [HPQCD, PRL112.212002 \(2014\)](#)
- [ETMC, PRD96.034524 \(2017\)](#)

# Summary

- $f_{D_{(s)}^{(*)}}$ ,  $f_{D_{(s)}^{*}}^T / f_{D_{(s)}^{*}}$  and  $f_{\phi}$  are calculated with overlap fermion on 2+1-flavor domain wall fermion configurations
- RI/(S)MOM are used for renormalization
- More lattice spacings are needed to better control discretization effects

Thanks for your attention!