First Principle Calculation of γW-box diagram corrections

Base on Phys. Rev. Lett. 124, 192002 (2020)

Reporter: Pengxiang Ma
Peking University
2020.08.05

APLAT 2020

CKM matrix

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- In the Standard Model, the Cabibbo–Kobayashi–Maskawa(CKM) matrix is a 3×3 unitary matrix.

- We focus on the first-row CKM unitarity: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1.$

$$\begin{cases} |V_{ud}| = 0.97370 \pm 0.00014, \\ |V_{us}| = 0.2245 \pm 0.0008, \\ |V_{ub}| = (3.82 \pm 0.24) \times 10^{-3}. \end{cases} \qquad [*]$$

- V_{ud} is the main contributor and the most accurately-determined element.

- [*]: P.A. Zyla et al. (Particle Data Group), to be published in Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

Precious calculations of $|V_{ud}|$

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superallowed β decay :(most accurate) $$|V_{ud}|^2 = \frac{2984.432(3)\mathrm{s}}{\mathcal{F}t\left(1+\Delta_R^V\right)}, \quad |V_{ud}| = 0.97370(14); \quad [1]$$

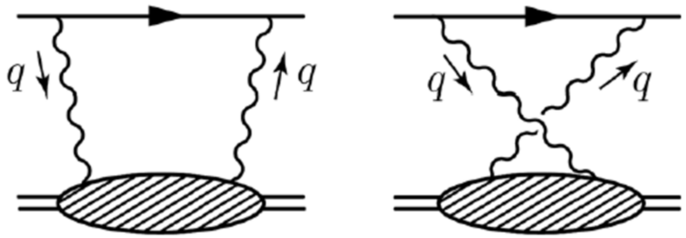
neutron β decay: $$|V_{ud}|^2 = \frac{5024.7\mathrm{s}}{\tau_n\left(1+3g_A^2\right)\left(1+\Delta_R^V\right)}, \quad |V_{ud}| = 0.9733(4); \quad [2]$$

pion β decay: $$\Gamma_{\pi\ell 3} = \frac{G_F^2|V_{ud}|^2 m_\pi^5 \left|f_+^\pi(0)\right|^2}{64\pi^3}(1+\delta)I_\pi, \quad |V_{ud}| = 0.9739(29). \quad [3]$$

- [1]: C. Y. Seng, M. Gorchtein, H. H. Patel, and M. J. Ramsey-Musolf, Phys. Rev. Lett. 121, 241804 (2018).
- [2]: P.A. Zyla et al. (Particle Data Group), to be published in Prog. Theor. Exp. Phys. 2020, 083C01 (2020).
- [3]: A. Czarnecki, W. J. Marciano, and A. Sirlin, (2019),arXiv:1911.04685 [hep-ph].

axial γW-box correction

- beyond tree level, the electroweak radiative corrections involving the axial-vector current become important and ultimately dominate the theoretical uncertainties.



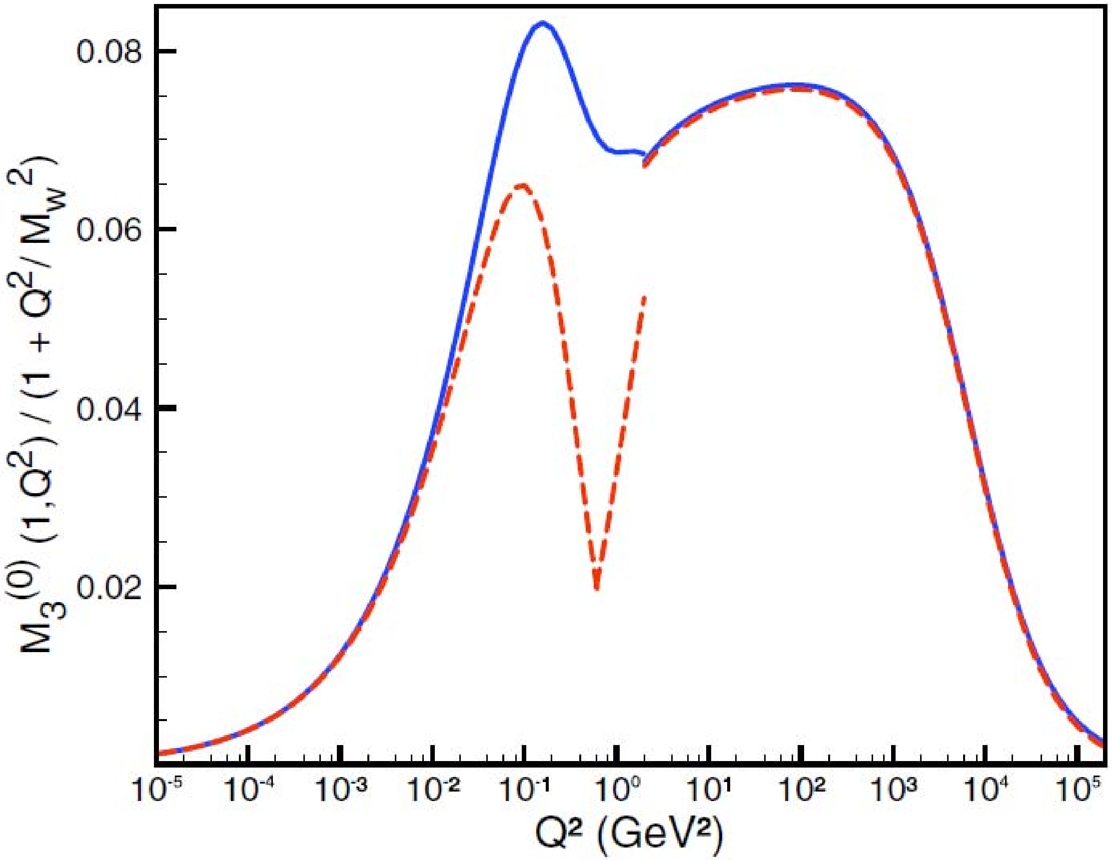
- According to current algebra*, only the axial γW-box contribution is sensitive to hadronic scales.
- The relevant hadronic tensor is

$$T_{\mu\nu}^{VA} = \frac{1}{2}\int d^4x e^{iqx} \left\langle H_f(p) \left| T\left[J_\mu^{em}(x) J_\nu^{W,A}(0)\right] \right| H_i(p) \right\rangle$$

$$J_\mu^{em} = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d \quad J_\nu^{W,A} = \bar{u}\gamma_\nu\gamma_5 d$$

*: A. Sirlin. Rev. Mod. Phys. 1978-07: 573–605.

axial γW-box correction



Marciano and Sirlin,
Phys. Rev. Lett. 2006-01: 032002.
vector dominance model(VDM).
$|V_{ud}^{\text{old}}| = 0.97420(18)_{\text{RC}}\,(10)_{\mathcal{F}t}$

↓

C.-Y. Seng, M. Gorchtein and M. J. Ramsey-Musolf.
Phys. Rev. D, 2019-07: 013001.
dispersion relation & data-driven analysis.
$|V_{ud}^{\text{new}}| = 0.97370(10)_{\text{RC}}\,(10)_{\mathcal{F}_t}$

↓

first principle calculation.

$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9984 \pm 0.0004,$ a violation of CKM unitarity by 4σ!

Methodology

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- Lattice QCD can give a first principle calculation of γW-box contribution.

- We take the pion semileptonic decay as an example.

- Perturbation theory and lattice QCD can be used in different regimes:
  - operator product expansion(OPE) can be used in large Q^2 regime,
  - Lattice QCD can be used in small Q^2 regime.

- Introduce a momentum scale Q_{cut},
  - we use the lattice data for $Q<Q_{cut}$;
  - we use perturbation theory for $Q>Q_{cut}$.

axial γW-box correction

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- axial γW-box correction can be calculated as

$$\square_{\gamma W}^{VA}\Big|_H = \underbrace{\frac{1}{F_+^H}\frac{\alpha_e}{\pi}\int_0^\infty dQ^2 \frac{m_W^2}{m_W^2+Q^2} \times \int_{-\sqrt{Q^2}}^{\sqrt{Q^2}} \frac{dQ_0}{\pi} \frac{\left(Q^2-Q_0^2\right)^{\frac{3}{2}}}{\left(Q^2\right)^2}}_{\text{Loop integral}} \underbrace{T_3\left(Q_0, Q^2\right)}_{\text{hadronic function}}$$

Loop integral
Analytically known

hadronic function
Need lattice calculation

- Simplified version:

$$\square_{\gamma W}^{VA}\Big|_H = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2} M_H\left(Q^2\right)$$

Calculation with lattice QCD

axial γW-box correction $\square_{\gamma W}^{VA}$

↑

hadronic function $M_H\left(Q^2\right)$

↑

matrix element $\langle H_f(P)|T[J_\mu^{em}(t,\vec{x})J_\nu^{W,A}(0)]|H_i(P)\rangle$

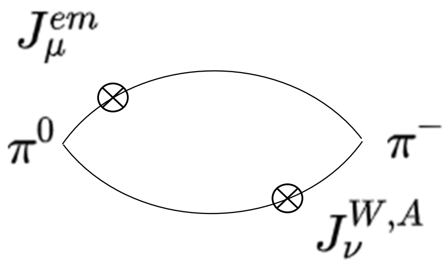
↑

Correlation function $\langle \phi_{\pi^0}(t_f)J_\mu^{em}(x)J_\nu^{W,A}(y)\phi_{\pi^-}^\dagger(t_i)\rangle$

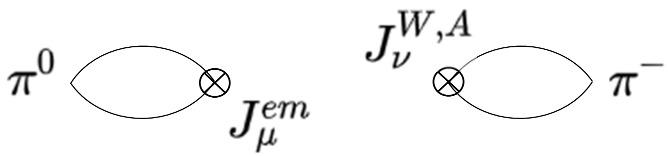
The correlation function can be written as:

$$C(t_f,x,y,t_i)=\frac{\sqrt{2}}{6}H_2(x-y)[\nu+4,\mu]+\frac{\sqrt{2}}{6}P_1^{\text{backward}}(x-y)[\nu+4,\mu] + \frac{\sqrt{2}}{6}P_2^{\text{backward}}(x-y)[\nu+4,\mu]+\cdots$$

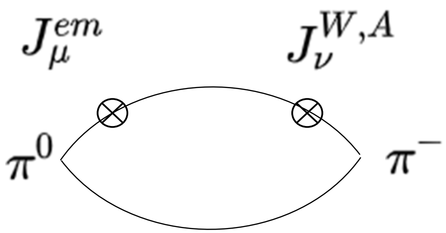
Four-point function



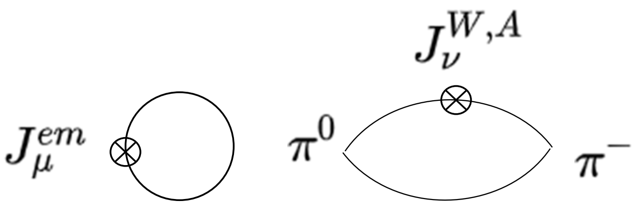
Type (A)
Sub-dominant.



Type (B)
does not contribute under the γ_5-Hermitian.



Type (C)
Dominant.



Type (D)
vanishes in the flavor SU(3) limit.

Calculation with perturbation theory

- For large Q^2, we utilize the operator product expansion (OPE) at leading twist:

$$\frac{1}{2}\int d^4x e^{-iQx} T\left[J_\mu^{em}(x) J_\nu^{W,A}(0)\right] = \frac{i}{2Q^2}\left\{C_a\left(Q^2\right)\delta_{\mu\nu}Q_\alpha - C_b\left(Q^2\right)\delta_{\mu\alpha}Q_\nu - C_c\left(Q^2\right)\delta_{\nu\alpha}Q_\mu\right\} J_\alpha^{W,A}(0) + \frac{1}{6Q^2} C_d\left(Q^2\right)\epsilon_{\mu\nu\alpha\beta}Q_\alpha J_\beta^{W,V}(0) + \cdots$$

- For the pion decay, the first three operators vanish.
- The Wilson coefficient can be calculated with perturbation theory, up to four-loop accuracy*:

$$C_d\left(Q^2\right) = \sum_n c_n a_s^n, \quad a_s = \frac{\alpha_s\left(Q^2\right)}{\pi}$$

$$\begin{aligned} c_0 &= 1, \quad c_1 = -1 \\ c_2 &= -4.583 + 0.3333 n_f \\ c_3 &= -41.44 + 7.607 n_f - 0.1775 n_f^2 \\ c_4 &= -479.4 + 123.4 n_f - 7.697 n_f^2 + 0.1037 n_f^3 \end{aligned}$$

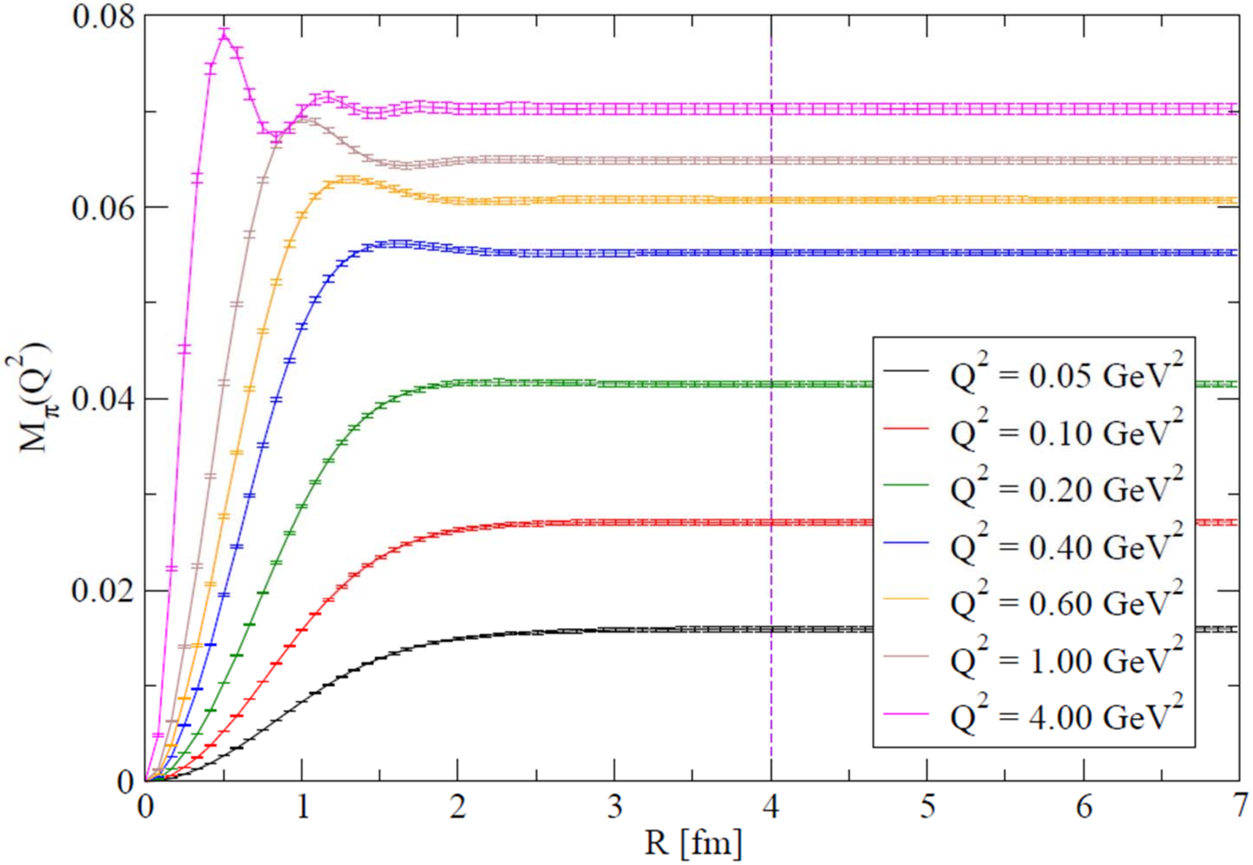
*: P. A. Baikov, K. G. Chetyrkin and J. H. Kühn. Phys. Rev. Lett. 2010-03.

Lattice Setup

ensemble	M_π/MeV	L	T	a^{-1}/GeV	N_{conf}	N_r	Δt/a
24D	141.2(4)	24	64	1.015	46	1024	8
32D	141.4(3)	32	64	1.015	32	2048	8
32D-fine	143.0(3)	32	64	1.378	71	1024	10
48I	135.5(4)	48	96	1.730	28	1024	12
64I	135.3(2)	64	128	2.359	62	1024	18

- We use five lattice QCD gauge ensembles at the physical pion mass.

result



- The integral is saturated at large R, indicating that the finite-volume effects are well under control.

- R: Integral range

$$M_H\left(Q^2\right)=-\frac{1}{6}\frac{1}{F_+^H}\frac{\sqrt{Q^2}}{m_H}\int d^4x\omega(t,\vec{x})\epsilon_{\mu\nu\alpha0}x_\alpha\mathcal{H}_{\mu\nu}^{VA}(t,\vec{x})$$

result

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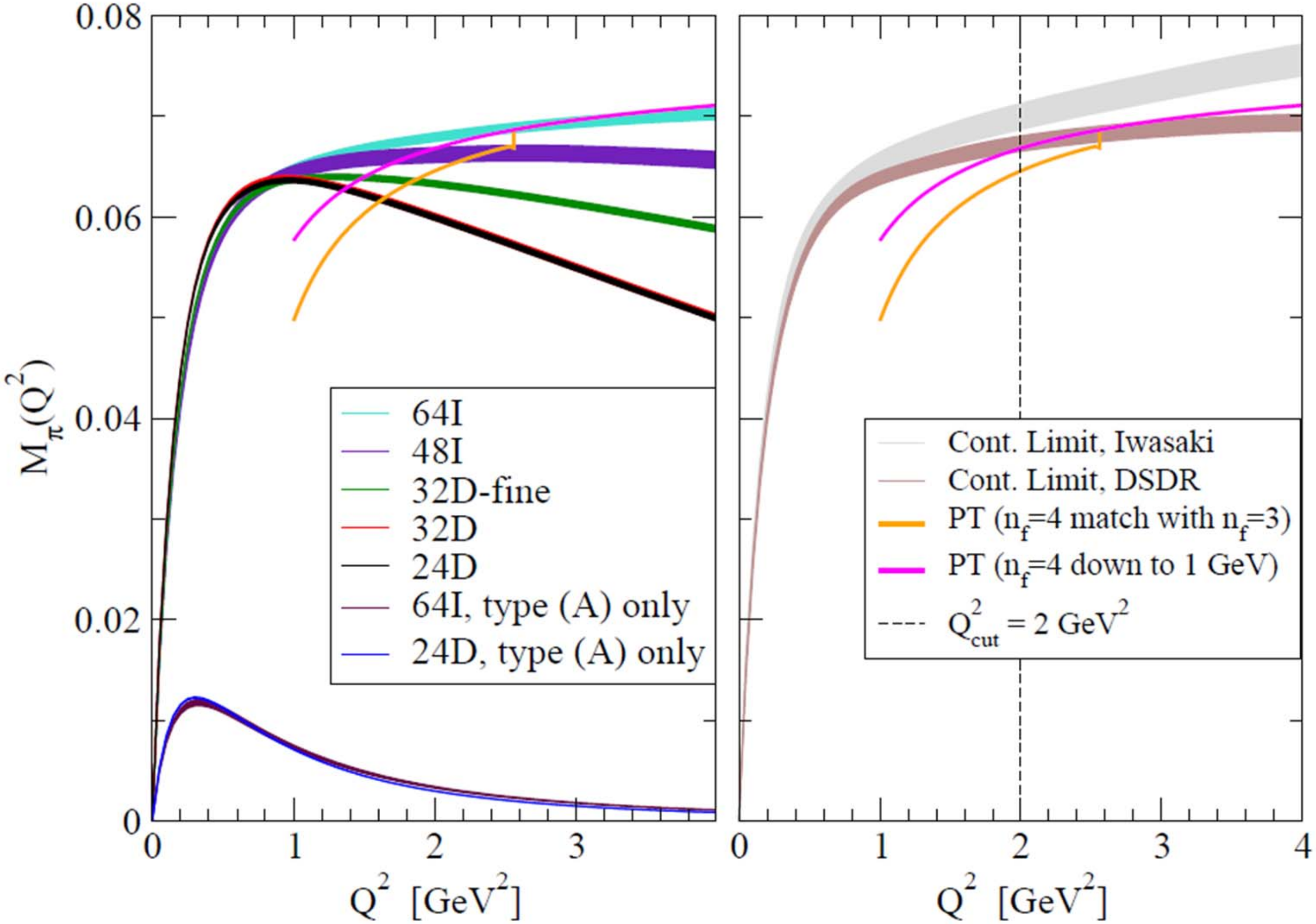

- For Q^2<1GeV^2, lattice data are consistent with each other.
- For Q^2>1GeV^2, the lattice discretization effects dominate the uncertainties.

$$\Box^{VA}_{\gamma W}\Big|_H = \frac{3\alpha_e}{2\pi}\int \frac{dQ^2}{Q^2}\frac{m_W^2}{m_W^2+Q^2}M_H\left(Q^2\right)$$

Error analysis

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- In the calculation of lattice QCD:
  - statistical.
  - lattice discretization effects.
  - lattice finite-volume effects
- In the calculation of perturbation theory:
  - Higher loop contribution.
  - Higher twist contribution.
- Errors mentioned above will change as Q_{cut} changes.

Error analysis

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- the perturbative determination of M_H with 3-flavor theory and 4-flavor theory has discrepancy of 14% at Q^2=1GeV^2, which can be used to calculate the systematic uncertainties of perturbation theory.

- At different Q_{cut},

$$\square^{VA}_{\gamma W}\Big|_{\pi} = 2.816(9)_{\rm stat}\,(24)_{\rm PT}\,(18)_{\rm a}\,(3)_{\rm FV}\times 10^{-3} \quad \text{at} \quad Q^2_{\rm cut} = 1\text{GeV}^2$$

$$\square^{VA}_{\gamma W}\Big|_{\pi} = 2.830(11)_{\rm stat}\,(9)_{\rm PT}\,(24)_{\rm a}\,(3)_{\rm FV}\times 10^{-3} \quad \text{at} \quad Q^2_{\rm cut} = 2\text{GeV}^2$$

$$\square^{VA}_{\gamma W}\Big|_{\pi} = 2.835(12)_{\rm stat}\,(5)_{\rm PT}\,(30)_{\rm a}\,(3)_{\rm FV}\times 10^{-3} \quad \text{at} \quad Q^2_{\rm cut} = 3\text{GeV}^2$$

stat: statistical. PT: perturbative truncation.

a: lattice discretization effects. FV: lattice finite-volume effects

Calculation of δ and $|V_{ud}|$

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$$\begin{cases} \Gamma_{\pi\ell 3} = \dfrac{G_F^2 |V_{ud}|^2 m_\pi^5 |f_+^\pi(0)|^2}{64\pi^3}(1+\delta)I_\pi \\ \delta = \dfrac{\alpha_e}{2\pi}\left[\bar{g} + 3\ln\dfrac{m_Z}{m_p} + \ln\dfrac{m_Z}{m_W} + \tilde{a}_g\right] + \delta_{\mathrm{HO}}^{\mathrm{QED}} + 2\Box_{\gamma W}^{VA} \end{cases}$$

- $\bar{g}$: the infrared contributions involving the vector γW-box and the bremsstrahlung corrections.
- $\tilde{a}_g$: the O(α_s) QCD correction to all oneloop diagrams except the axial γW box.
- $\delta_{\mathrm{HO}}^{\mathrm{QED}}$: the leading-log higher-order QED effects.
- δ=0.0334(10)→0.0332(3).
- $V_{ud} = 0.9739(28)_{\mathrm{exp}}(5)_{\mathrm{th}} \rightarrow V_{ud} = 0.9740(28)_{\mathrm{exp}}(1)_{\mathrm{th}}$

Conclusion

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- we perform the first realistic lattice QCD calculation of the γW-box correction to the pion semileptonic decay.
- $\Box_{\gamma W}^{VA}\big|_{\pi} = 2.830(11)_{\rm stat}\ (26)_{\rm sys}\ \times 10^{-3}$
- $V_{ud} = 0.9739(28)_{\rm exp}(5)_{\rm th} \rightarrow V_{ud} = 0.9740(28)_{\rm exp}(1)_{\rm th}$
- The uncertainty of the theoretical prediction for the pion semileptonic decay rates is reduced by a factor of 3.
- To further improve the determination of |Vud| with pion semileptonic decay, it requires better-quality experimental input.
- This study further can be used in the first principles computation of the γW-box correction to the neutron decay.

Thank you for listening!