Shear viscosity of classical Yang-Mills field with use of scaling invariance

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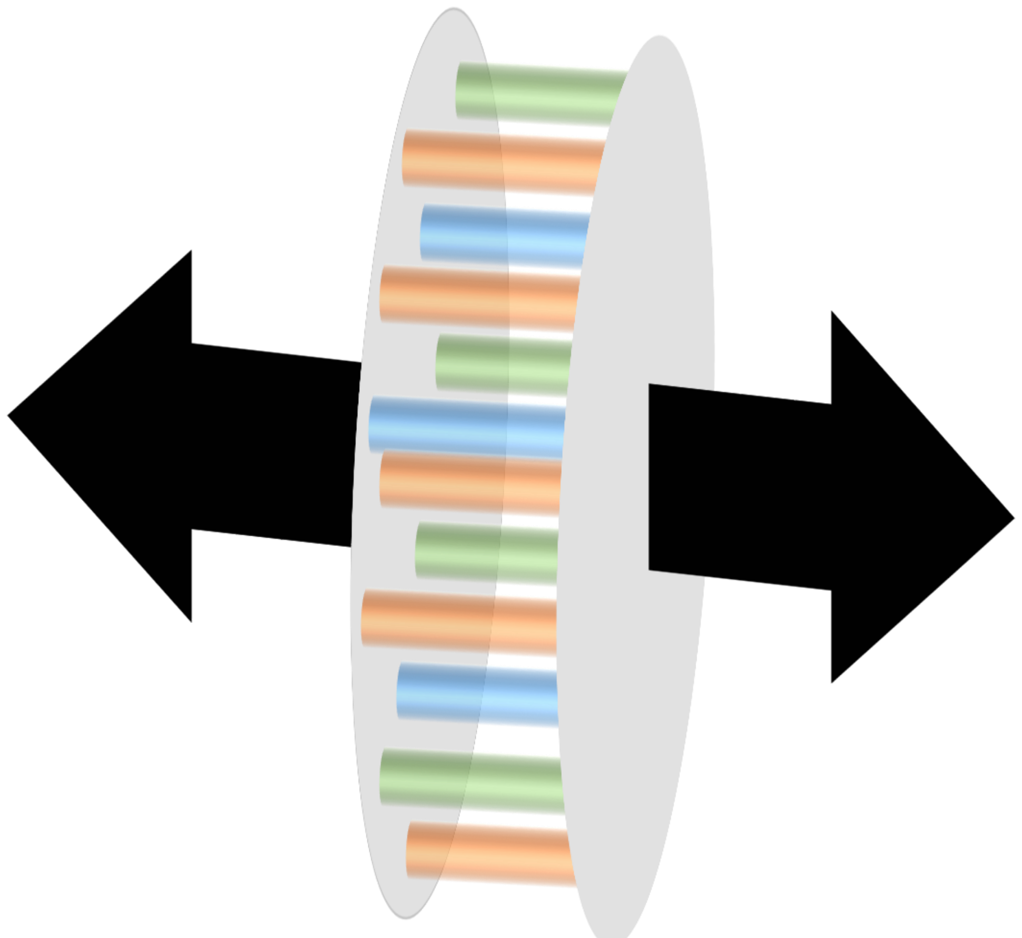
- *Introduction*
- Theory
- Numerical Results
- Summary

Initial stage of relativistic heavy-ion collisions

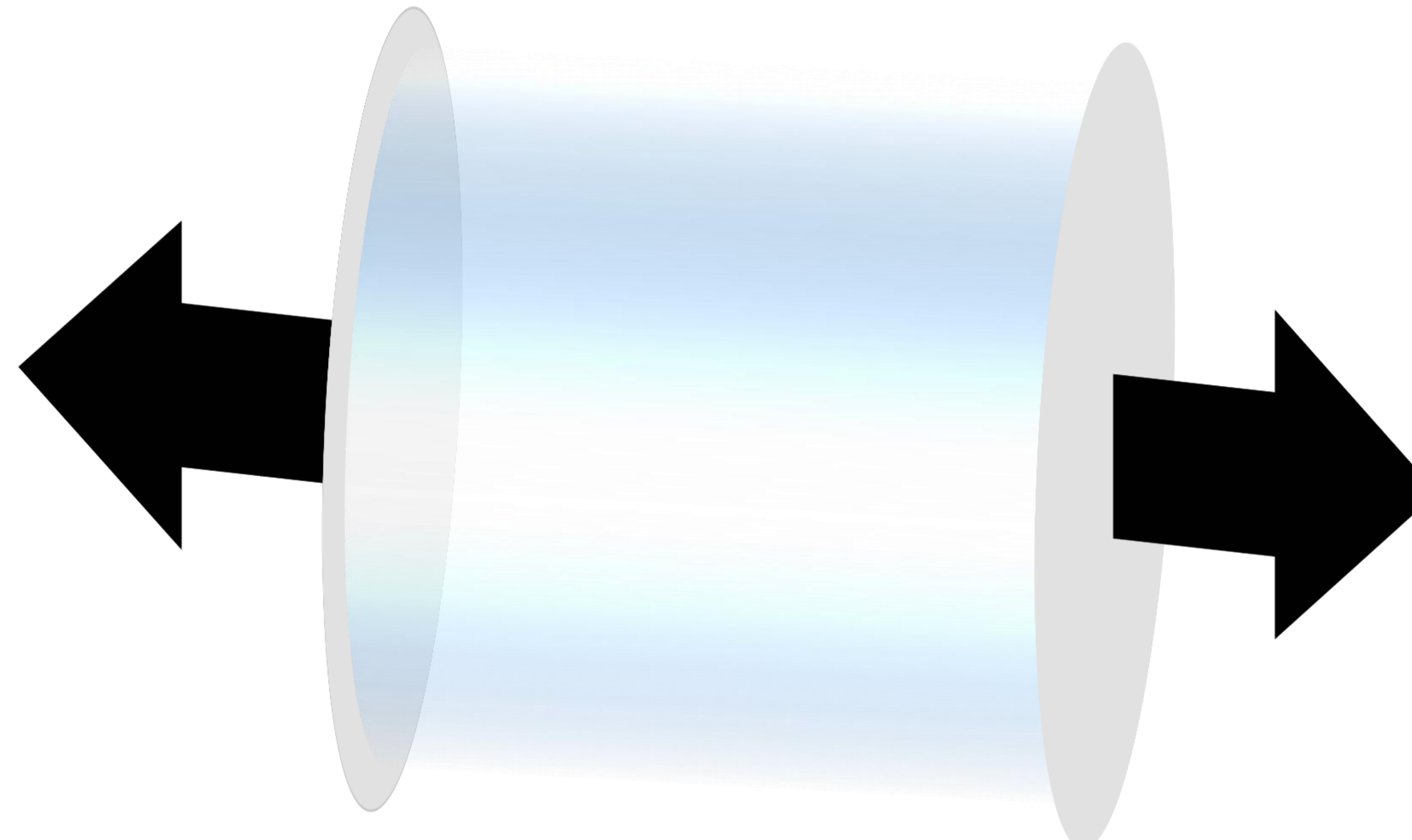


Flux tube($\tau \sim 0$ fm/c)

- Strong gluon fields

(based on the Color Glass Condensate)

Classical Yang-Mills(CYM) theory



Quark Gluon Plasma($\tau \gtrsim 0.6$ fm/c)

- Quark and gluon matter
- Viscous fluid

Relativistic viscous hydrodynamics

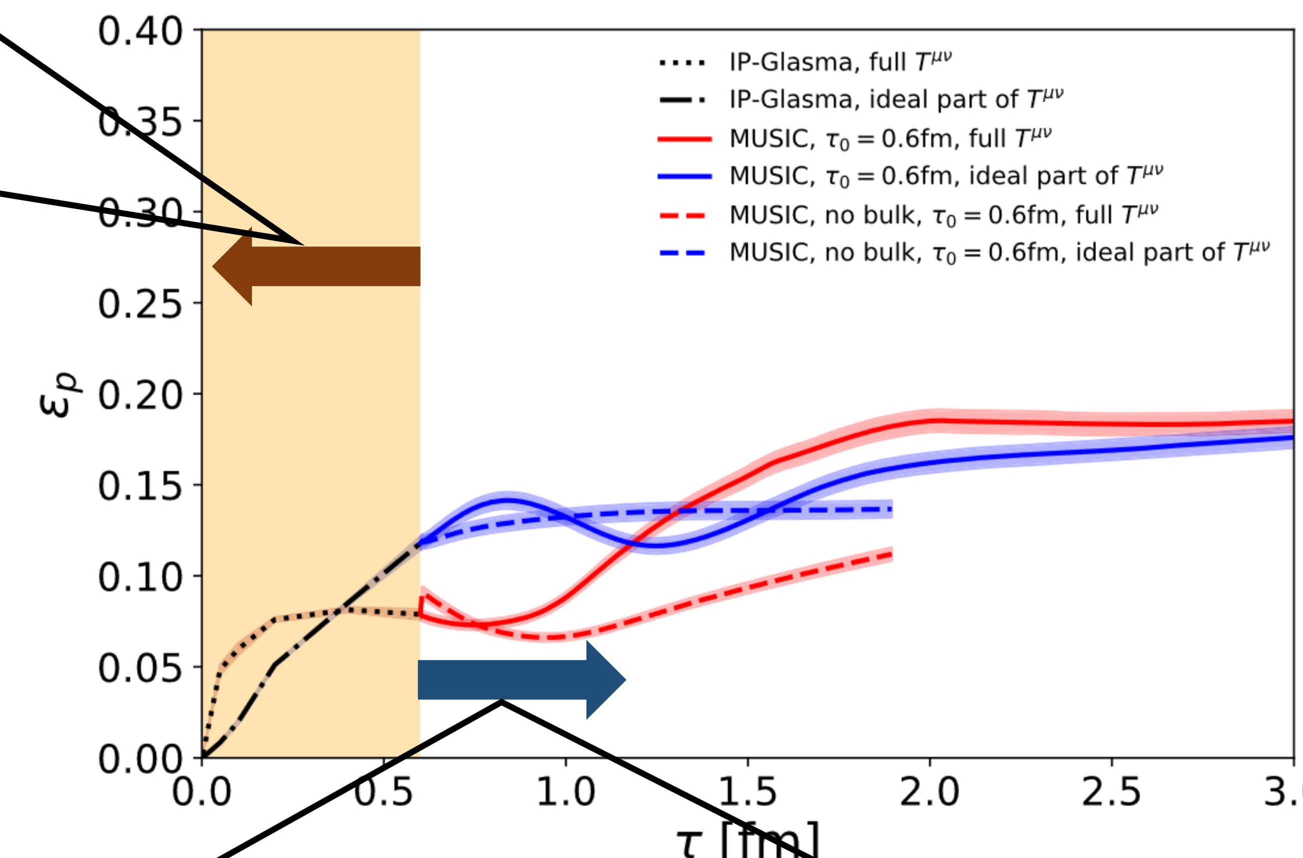
IP-glasma model + Relativistic viscous hydrodynamics

The relativistic viscous hydrodynamics in combination with the initial condition given by the IP-glasma model successfully reproduces the experimental data at mid-rapidity

IP-glasma model :

- The initial condition is given as the CYM fields, taking account for the event-by-event fluctuations of the nucleon positions in the transverse plane
- The time evolution is obtained by solving the classical equation of motion in the Yang-Mills theory

B. Schenke, C. Shen and P. Tribedy (2018)



Relativistic viscous hydrodynamics :

- The initial condition is obtained from the energy-momentum tensor of the CYM fields at a switching time
- The shear viscosity is set as the small value, which is close to the lower bound predicted by superstring theories with an Einsteinian classical limit, $1/4\pi$

$$\eta/s \sim 0.1 - 0.2$$

This talk

The CYM fields is used as the initial condition of the relativistic viscous hydrodynamics in order to reproduce the experimental data. However, the transport property of the CYM fields hasn't been clarified.

⬇

We investigate the shear viscosity of the CYM fields by using the Green-Kubo formula

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Green-Kubo formula

In the Green- Kubo approach, we can extract the shear viscosity η from the thermal expectation value of the time-correlation function of energy-momentum(EM) tensor,

$$C(t) = \frac{V}{3} \sum_{i \neq j} \left\langle \tau_{ij}(t) \tau_{ij}(0) \right\rangle ,$$

$$\left(\begin{array}{ll} \tau_{ij}(t) = \frac{1}{V} \int d^3x \, T_{ij}(x) & \\ T_{ij}(x) & : \text{EM tensor} \\ V & : \text{volume} \end{array} \right)$$

by using the Green-Kubo formula,

$$\eta = \frac{1}{2T} \lim_{\omega \to 0} \int_{-\infty}^{\infty} dt \, e^{i\omega t} C(t).$$

How to examine thermal expectation value in classical fields

Step1

prepare N classical field configurations independently($N \gg 1$)

- same energy as each other
- non-equilibrium

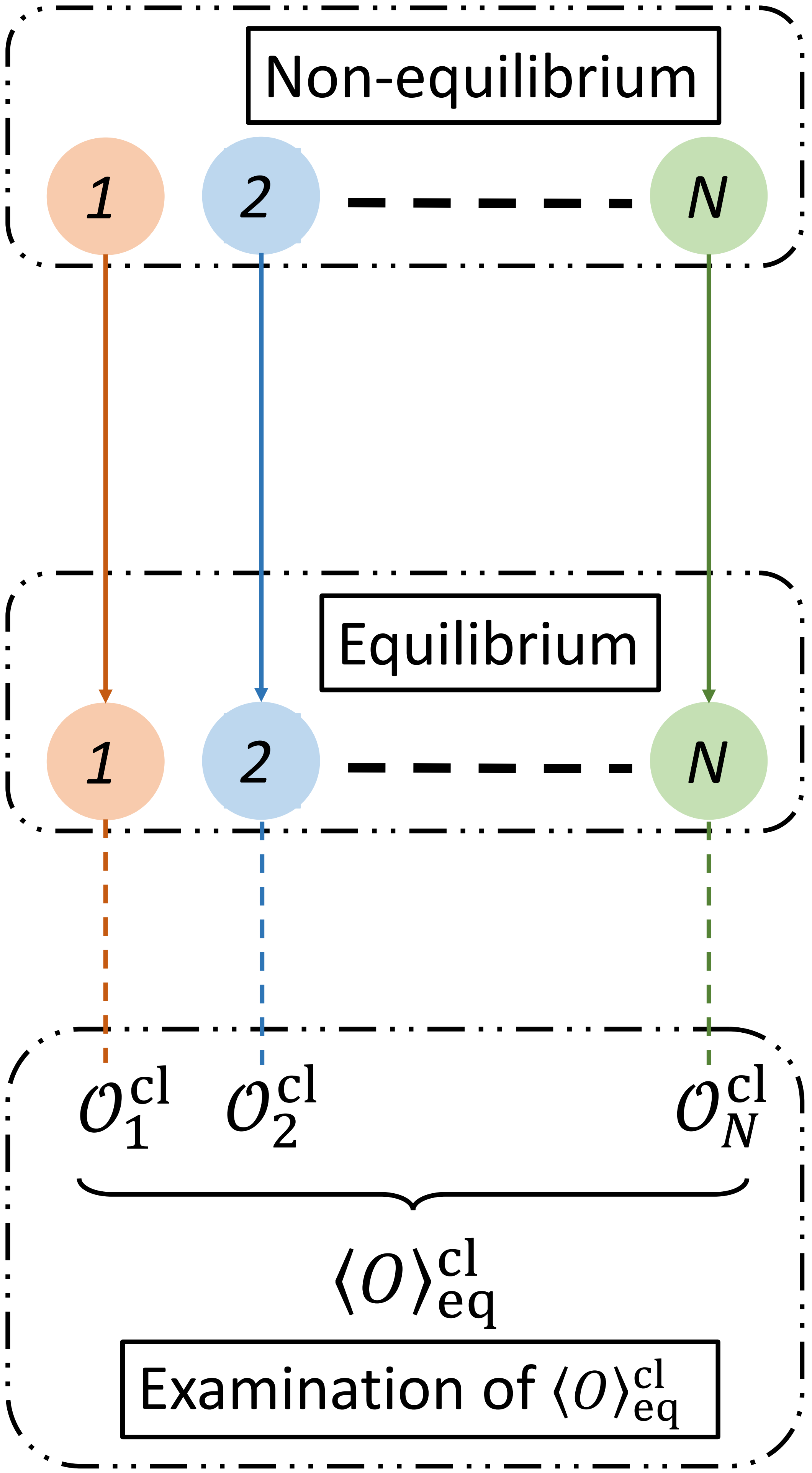
Step2

evolve each configuration with the classical equation of motion until it equilibrates

Step3

calculate a observable in each configuration, $\mathcal{O}_i^{\mathrm{cl}}(i = 1,2,\cdots N)$, and regard the average of them as the expectation value,

$$\langle O \rangle_{\mathrm{eq}}^{\mathrm{cl}} \sim \frac{1}{N} \sum_{i=1}^{N} O_i^{\mathrm{cl}}$$

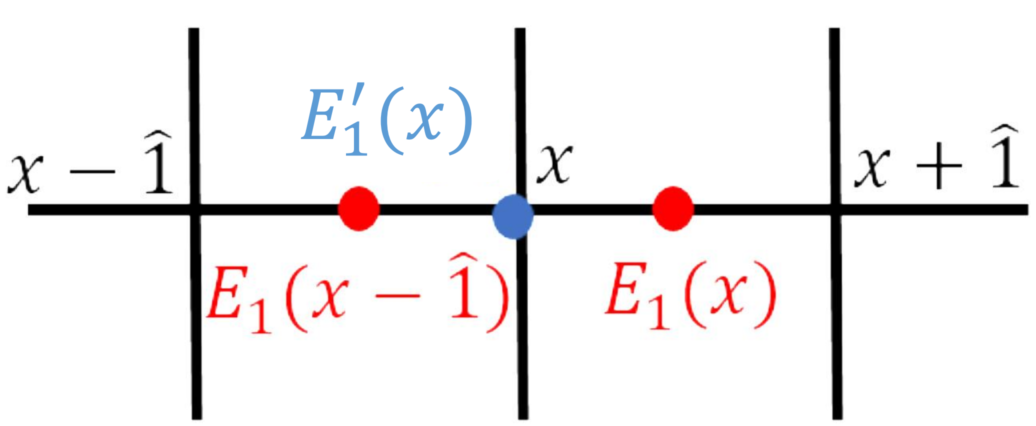


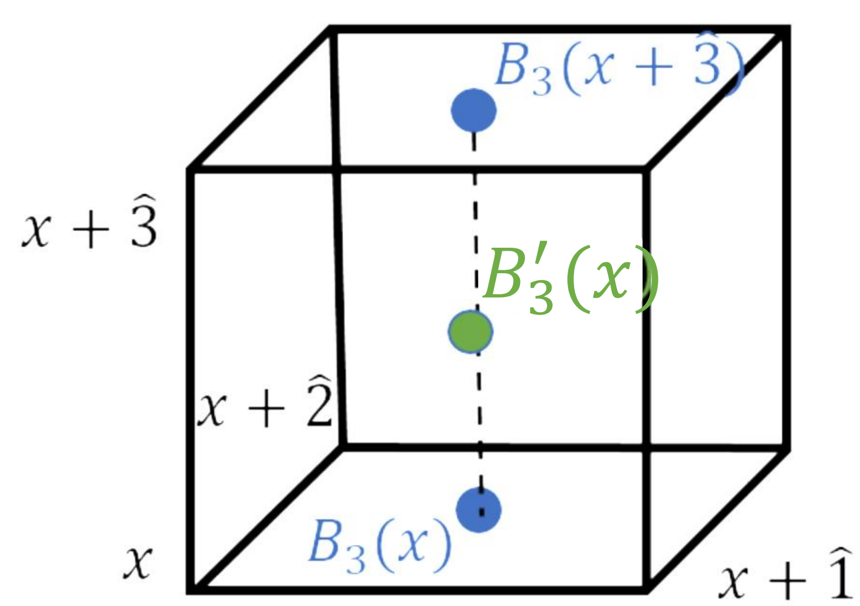
CYM theory on lattice

We employ the Hamiltonian formalism to describe the time evolution of the CYM fields(A. Krasnitz *et al*.(2003), T. Lappi(2003), P. Romatschke *et al*.(2006))

- Temporal gauge +Coulomb gauge $A_0^a(x) = 0,\ \sum_i \partial_i A_i^a(x) = 0$
- Hamiltonian $H = \frac{1}{2}\sum_{a,i,\vec{x}}\left[E_i^a(x)^2 + B_i^a(x)^2\right] \quad \left(E_i^a(x) = \partial_0 A_i^a(x), B_i^a(x) = \frac{1}{2}\epsilon_{ijk}F_{jk}^a(x)\right)$
- Equation of motion $\partial_0 A_i^a(x) = -\frac{\partial H}{\partial E_i^a(x)}, \partial_0 E_i^a(x) = \frac{\partial H}{\partial A_i^a(x)}$
- Space-averaged energy-momentum tensor (off-diagonal matrix elements, $i \neq j$)

$$\tau_{ij}(x) = \frac{1}{V}\sum_{a,\vec{x}}\left[E'^a_i(x)E'^a_j(x) + B'^a_i(x)B'^a_j(x)\right], \quad \left(E'^a_i(x) = \frac{\left[E_i^a(x)+E_i^a(x-\hat{\imath})\right]}{2}, B'^a_i(x) = \frac{\left[B_i^a(x)+B_i^a(x+\hat{\imath})\right]}{2}\right)$$





*The CYM theory is the infrared effective model with a finite ultraviolet cut-off.
*We don’t take a continuum limit and don’t renormalize.

Scaling property of CYM theory

The dependence of the shear viscosity η on the coupling g and lattice temperature T is represented by a scaling function $f_\eta(g^2T)$ as $\eta = Tf_\eta(g^2T)$ due to the scaling property of the CYM theory

<Scaling property of CYM theory>

Gibbs ensemble of CYM field $\quad e^{\frac{H(A,E,g)}{T}} = e^{\frac{H(A/\gamma,E/\gamma,\gamma g)}{T/\gamma^2}}$

$$\Rightarrow \langle A^m E^n \rangle_{(g,T)} = \gamma^{m+n} \langle A^m E^n \rangle_{(\gamma g, T/\gamma^2)}$$

$\Rightarrow T^{-\frac{m+n}{2}} \langle A^m E^n \rangle_{(g,T)}$ is a function of g^2T

$$\Rightarrow \begin{aligned} &C(t;g,T) = T^2 f_C(t;g^2T) \\ &\eta(g,T) = Tf_\eta(g^2T) \end{aligned}$$

We extract scaling function from numerical calculations over a wide range of g^2T

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Set up

- Number of sites : $16^3, 24^3, 32^3$
- Coupling constant and lattice temperature : $\mathrm{g} = 0.15 \sim 20,\ T \sim 1$

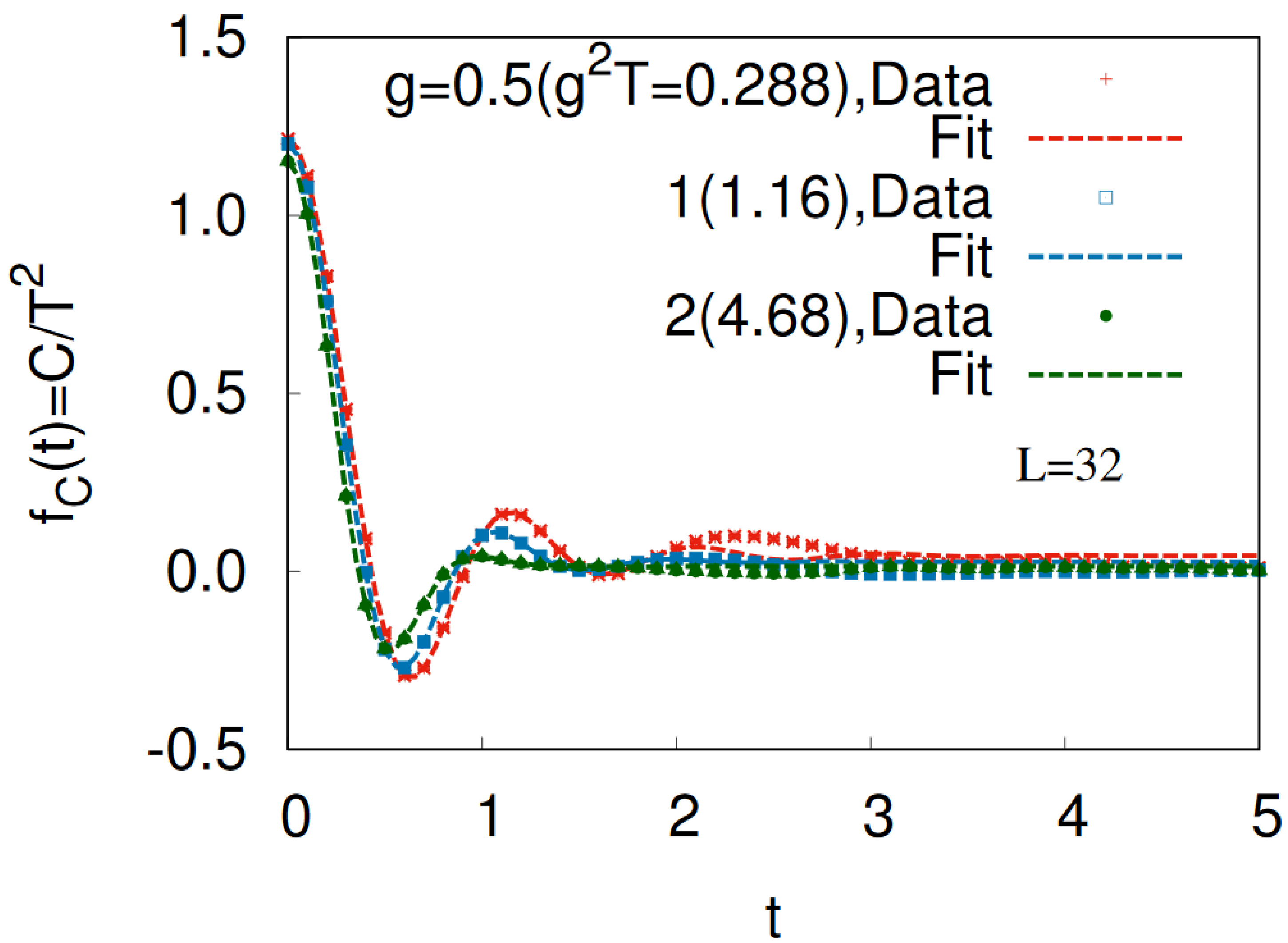
  $\Rightarrow$ $\mathrm{g}^2\mathrm{T} = 0.0256 \sim 499$

- SU(2) Yang-Mills theory

Time-correlation function of EM tensor

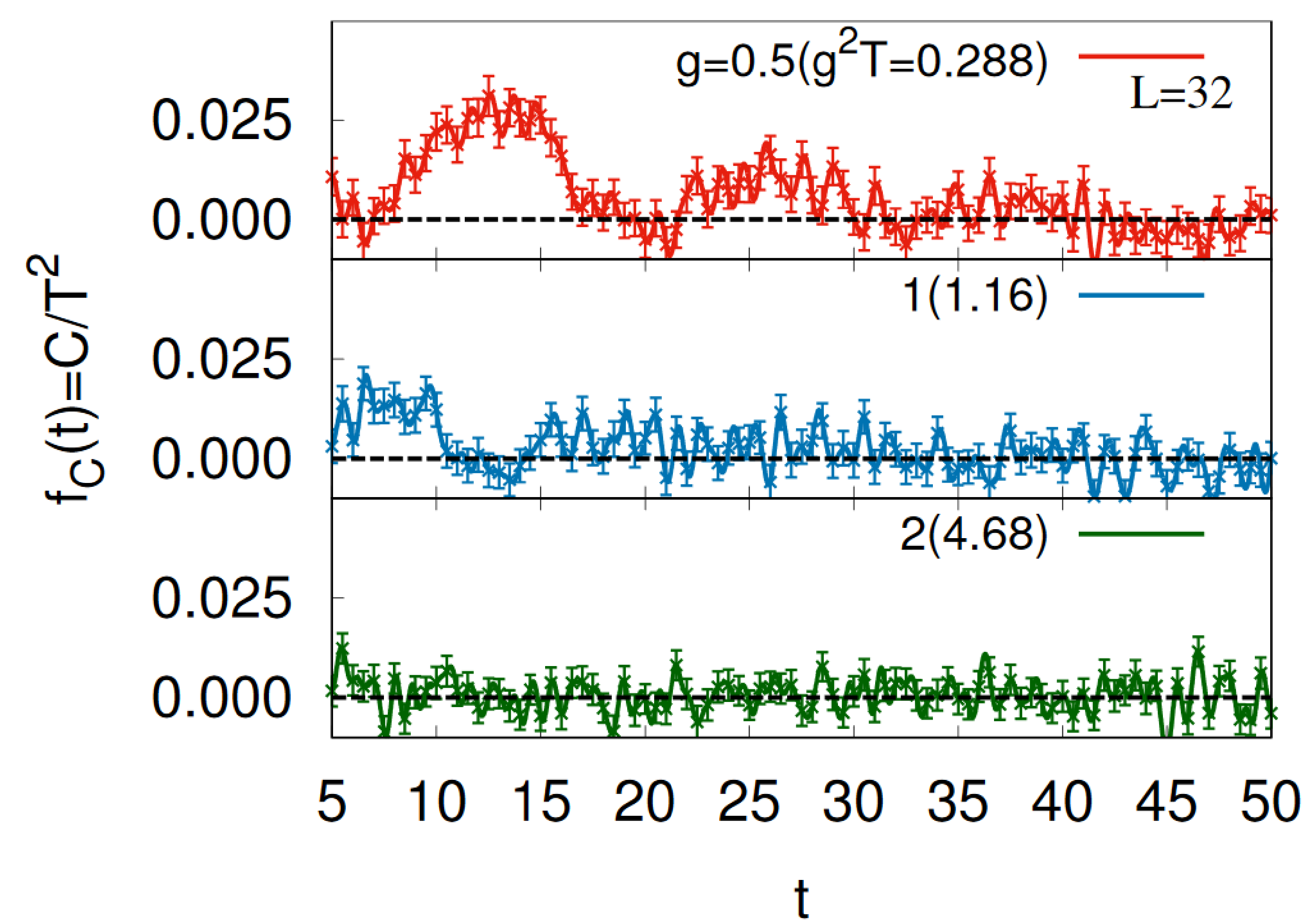
$t < 1.5$

- ✔ Damped oscillatory behavior
- ✔ Amplitude decreases by around two orders of magnitude



$t > 5$

- ✔ Small remaining correlation decay
- ✔ g^2T dependence is not weak

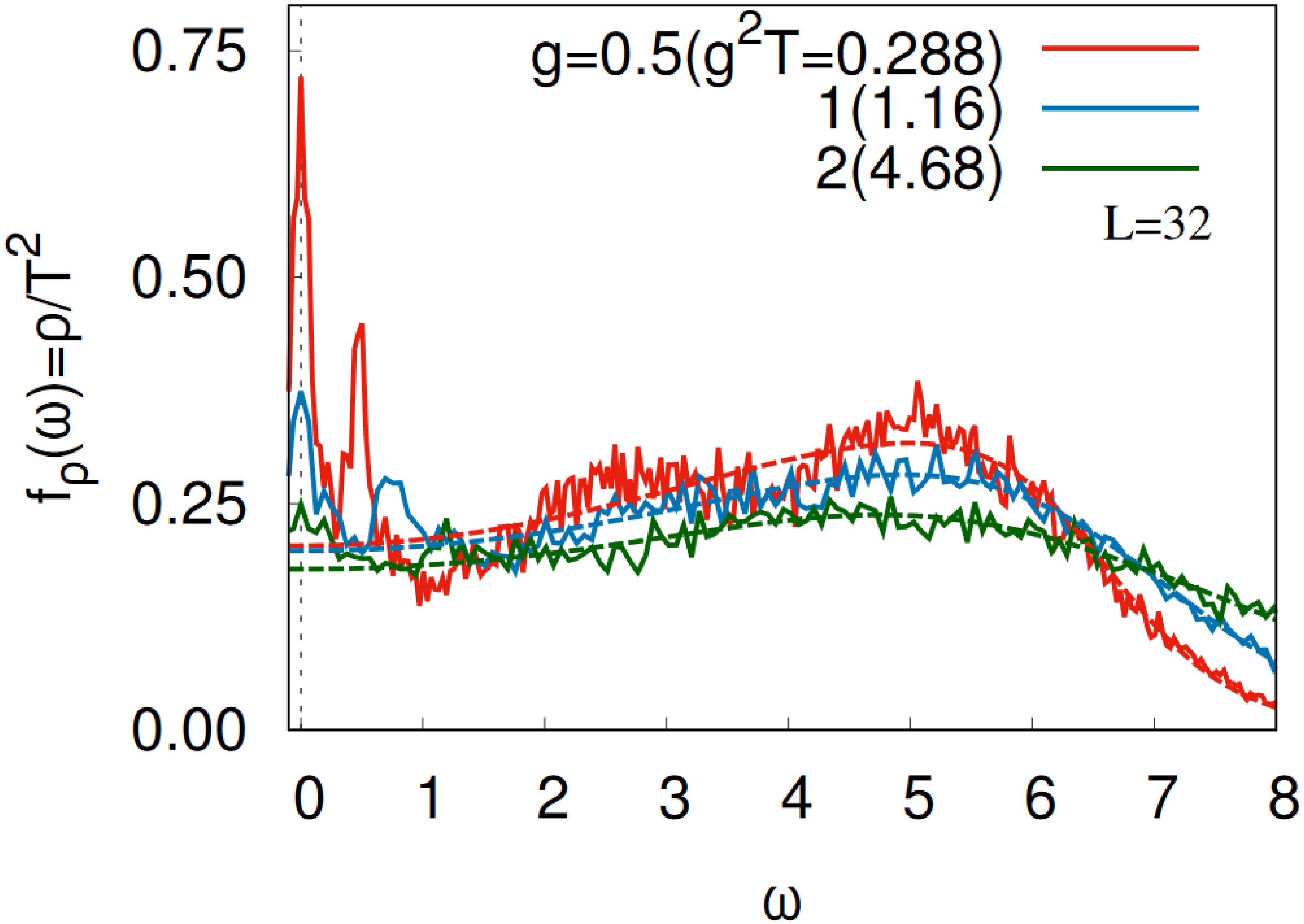


Fourier transformation of $C(t)$, $\rho(t)$

$$\rho(\omega) = \int_{-\infty}^{\infty} dt e^{i\omega t} C(t) \sim \int_{-t_{\rm cut}}^{t_{\rm cut}} dt e^{i\omega t} C(t)$$

Solid line : $f_\rho(\omega; g^2T) = \rho(\omega; g, T)/T^2$

Dashed line : Fourier transformation of damped oscillator



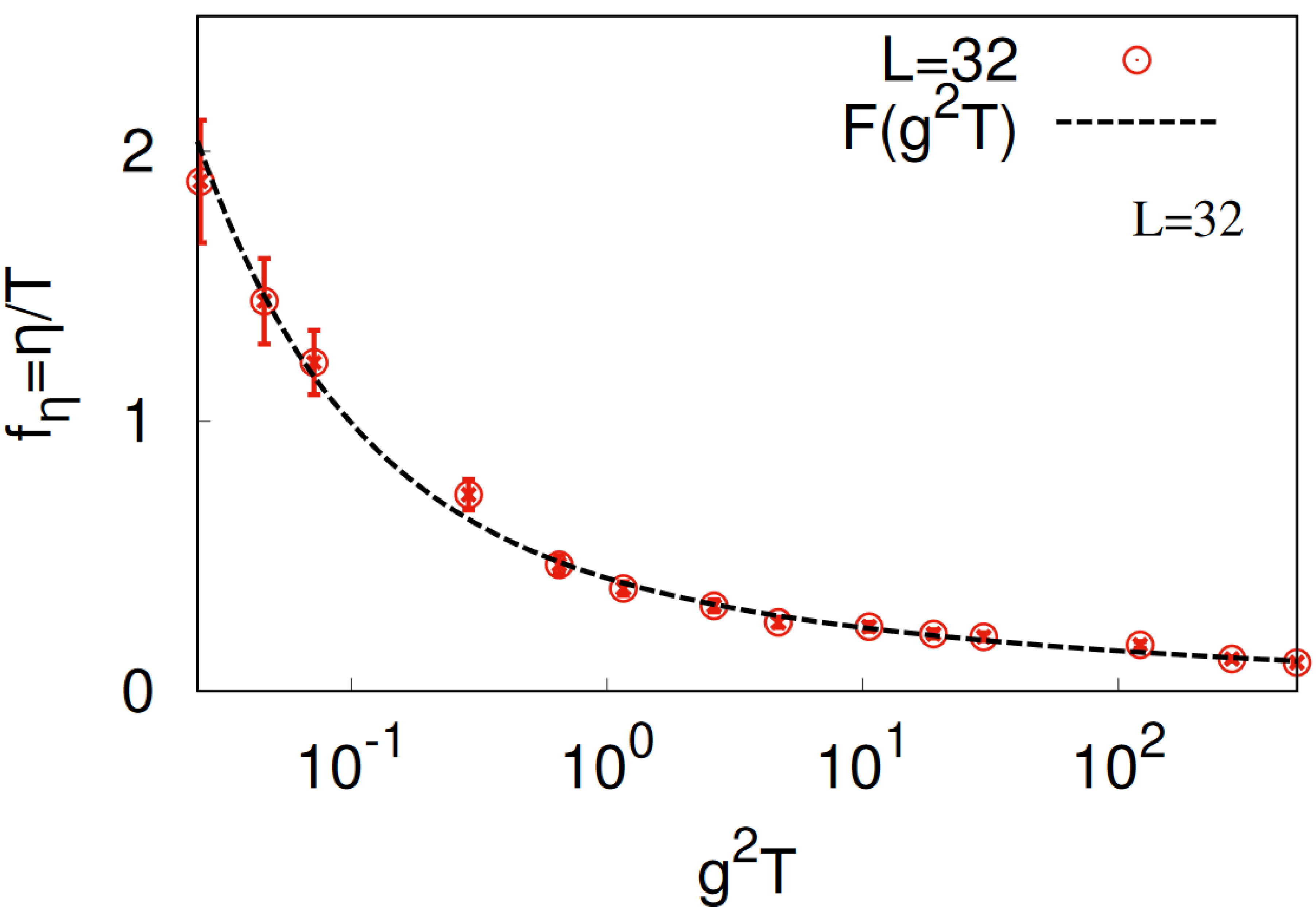
- ✔ The broad bump around $\omega \sim 5$ reflects the damped oscillatory behavior of $C(t)$
- ✔ With decreasing g^2T, the height of the sharp peak at $\omega \sim 0$ increases and its width narrows. Thus the $\omega \sim 0$ peak may be due to the slow decay part of $C(t)$

Shear viscosity : g^2T dependence

g^2T dependence of $f_\eta(g^2T)$ is well described by a polynomial function with parameters $\alpha, \beta, \gamma, \delta$,

$$F(x) = \alpha x^{-\beta/2} + \gamma x^{-\delta/2} \quad (\beta > \delta)$$

$$\alpha = 0.09 \pm 0.07, \ \beta = 1.49 \pm 0.39,$$
$$\gamma = 0.33 \pm 0.06, \ \delta = 0.35 \pm 0.07.$$



It turns out that η of the CYM field is proportional to $1/g^{1.10-1.88}$ at weak coupling, which has a weaker dependence on g than that in the leading-order perturbation theory $\eta \propto 1/g^4 \ln g^{-1}$[1] but consistent with that of the "anomalous viscosity" $\eta \propto 1/\mathrm{g}^{1-1.5}$[2] under the strong disordered field in a turbulent plasma.

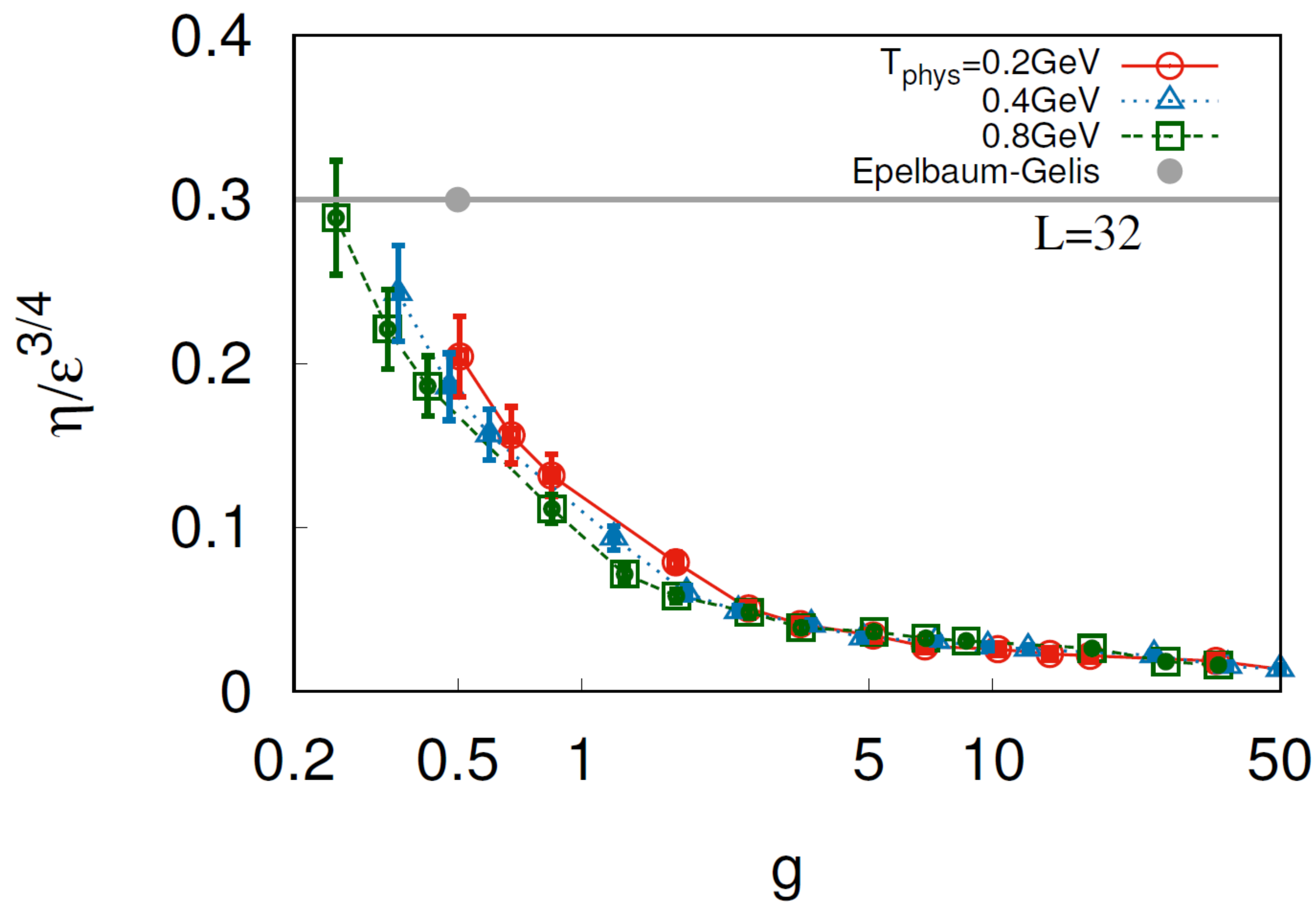
[1] P. B. Arnold, G. D. Moore and L. G. Yaffe (2000), (2003)
[2] M. Asakawa, S. A. Bass and B. Müller (2006), V. Chandra (2012), J. Hong *et al.* (2014)

Shear viscosity : physical unit

$a^{-1} = 2\ \mathrm{GeV}, \mathrm{g} = 0.5$
: same as value in previous work
(T. Epelbaum and F. Gelis (2013))

$T_{\mathrm{phys}} = 0.2\ \mathrm{GeV}, 0.4\ \mathrm{GeV}$ and $0.8\ \mathrm{GeV}$
: temperature at the start of the hydrodynamical simulation(RHIC and LHC)

$\varepsilon_{\mathrm{lat}} \sim 3\left(N_c^2 - 1\right)T_{\mathrm{lat}} = 9T_{\mathrm{lat}}$
: harmonic oscillator approximation within the acuracy of 9%-15%



The obtained shear viscosity is also found to be roughly consistent with that estimated through the analysis of the near-equilibrium dynamics of the CYM fields in the boost invariant expanding geometry with recourse to a hydrodynamic equation[3].

[1] T. Epelbaum and F. Gelis (2013))

Summary and Future work

- We calculated the shear viscosity of the classical Yang-Mills (CYM) fields on a lattice by applying the Green-Kubo formula to the time-correlation function $C(t)$ of the energy-momentum(EM) tensor in equilibrium.
- The correlation function C(t) was found to exhibit damped oscillatory behavior at early times followed by a slow decay which produces a sharp peak at $\omega \sim 0$ in the Fourier transformation of C(t).
- It turns out that η of the CYM field is proportional to $1/g^{1.10-1.88}$ at weak coupling, which has a weaker dependence on g than that in the leading-order perturbation theory $\eta \propto 1/g^4 \ln g^{-1}$(P. B. Arnold *et al.*(2000),(2003)) but consistent with that of the "anomalous viscosity" $\eta \propto 1/\mathrm{g}^{1-1.5}$ (M. Asakawa *et al.*(2006),V. Chandra(2012),Hong *et al.*(2014)) under the strong disordered field in a turbulent plasma.
- The obtained shear viscosity is also found to be roughly consistent with that estimated through the analysis of the near-equilibrium dynamics of the CYM fields in the expanding geometry with recourse to a hydrodynamic equation(T. Epelbaum *et al.*(2013)).

We found two interesting behaviors of η of the CYM field. However it is beyond the scope of this work to clarify the physical origins of those. It is a feature work.