Three-Photon Decay of J/ψ from Lattice QCD

Yu Meng, Chuan Liu, Ke-Long Zhang

Peking University

APLAT-2020, 6 August 2020

Introduction

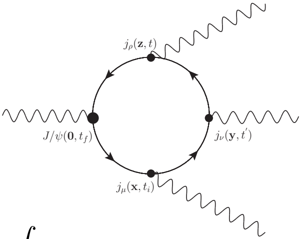
- The rare decay $J/\psi \to 3\gamma$ acts as a probe of higher-order QCD effects.
- Experimental difficulties: poor knowledge of matrix element
  - Crystal Ball, $\mathcal{B} < 5.5 \times 10^{-5}$, PRL 44,712(1980)
  - CLEOc, $\mathcal{B} = (1.2 \pm 0.3 \pm 0.2) \times 10^{-5}$, PRL 101,101801(2008)
  - BESIII, $\mathcal{B} = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}$, PRD 87,032003(2013)
- Theoretical difficulties: perturbation fails
  - For $\eta_c \to 2\gamma$, both photons are hard with half energies of the charmonium, perturbation is expected to work better;
  - For $J/\psi \to 3\gamma$, there exits a soft photon, hindering the perturbation calculation.

We present the first lattice calculation for rare decay $J/\psi \to 3\gamma$.

Y.Meng,C.Liu and K-L.Zhang.(2019).

Decay amplitude on lattice

$$M(t_f, t; t', t_i) = \epsilon_\mu \epsilon_\nu \epsilon_\rho \epsilon_\alpha \mathcal{M}_{\mu\nu\rho\alpha}$$



$$\mathcal{M}_{\mu\nu\rho\alpha} = \frac{e^3}{\frac{Z(\boldsymbol{p})}{2E(\boldsymbol{p})} e^{-E(\boldsymbol{p})(t_f - t)}} \times \int dt' e^{-\omega_2 |t' - t|} \int dt_i e^{-\omega_1 |t_i - t|}$$

$$\times \sum_{\boldsymbol{x},\boldsymbol{y},\boldsymbol{z}} e^{i(\boldsymbol{q}_3 \cdot \boldsymbol{z} + \boldsymbol{q}_2 \cdot \boldsymbol{y} + \boldsymbol{q}_1 \cdot \boldsymbol{x})} \left\langle 0 \left| \hat{T} \left\{ \mathcal{O}^\alpha_{J/\psi}(\mathbf{0}, t_f) j_\rho(\boldsymbol{z}, t) j_\nu(\boldsymbol{y}, t') j_\mu(\boldsymbol{x}, t_i) \right\} \right| 0 \right\rangle$$

- Local current $j_\mu(x) = Z_V Q_c \bar{c} \gamma_\mu c(x)$;
- Using 'sequential' method to calculate the four-point function;
- The three photons can't be on-shell simultaneously, with virtualities Q_i^2.

Decay amplitude $\rightarrow$ decay width

- Conventional: amplitude parameterization. For $\eta_c \rightarrow 2\gamma$:

$$\mathcal{M}_{\mu\nu} = 2(\frac{2}{3}e)^2 m_{\eta_c}^{-1} F(Q_1^2, Q_2^2)\epsilon_{\mu\nu\rho\alpha} q_1^\rho q_2^\sigma, \quad \Gamma(\eta_c \rightarrow 2\gamma) = \pi\alpha_{em}^2(\frac{16}{81}) m_{\eta_c} |F(0,0)|^2$$

J.J.Dudek and E.G.Edwards.(2006)

- For three-photon case: $\mathcal{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3) = \sum_{perm} \mathscr{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3)$

$$\begin{aligned}
&\mathscr{M}_{\mu\nu\rho\alpha}(q_1, q_2, q_3) \\
&= \mathcal{F}_{123} \frac{1}{q_1 \cdot q_3} \left(\frac{q_3^\mu q_1^\rho}{q_1 \cdot q_3} - g^{\mu\rho} \right) q_1^\alpha \left(\frac{q_3^\nu}{q_2 \cdot q_3} - \frac{q_1^\nu}{q_1 \cdot q_2} \right) \\
&+ \mathcal{G}_{123} \left[\frac{1}{q_2 \cdot q_3} \left(\frac{q_1^\alpha q_3^\mu}{q_1 \cdot q_3} - g^{\alpha\mu} \right) \left(\frac{q_1^\nu q_2^\rho}{q_1 \cdot q_2} - g^{\nu\rho} \right) + \frac{1}{q_1 \cdot q_3} \left(\frac{q_1^\nu}{q_1 \cdot q_2} - \frac{q_3^\nu}{q_2 \cdot q_3} \right) (q_1^\rho g^{\alpha\mu} - q_1^\alpha g^{\mu\rho}) \right] \\
&+ \mathcal{H}_{123} \frac{1}{q_1 \cdot q_3} \left(\frac{q_1^\alpha q_3^\mu}{q_1 \cdot q_3} - g^{\alpha\mu} \right) \left(\frac{q_3^\nu q_2^\rho}{q_2 \cdot q_3} - g^{\nu\rho} \right)
\end{aligned}$$

G.S.Adkins.(1996).

- It is a redundant process for lattice simulation.

Decay amplitude → decay width

- The three-body decay width:

$$\Gamma_3 = \frac{1}{3!}\frac{1}{2m}\int \frac{d^3q_1}{(2\pi)^3 2\omega_1}\frac{d^3q_2}{(2\pi)^3 2\omega_2}\frac{d^3q_3}{(2\pi)^3 2\omega_3}(2\pi)^4\delta(p-q_1-q_2-q_3)\overline{|\mathcal{M}|^2}$$
$$= \frac{m}{1536\pi^3}\int_0^1 dx\int_{1-x}^1 dy\overline{|\mathcal{M}|^2}, \quad x\equiv 1-2q_2\cdot q_3/m^2, y\equiv 1-2q_1\cdot q_2/m^2$$

- New approach: amplitude summation, define $\mathcal{T}-$function

$$\mathcal{T}\equiv\overline{|\mathcal{M}|^2}=\frac{1}{3}\sum_{\mu\nu\rho\alpha}\sum_{\lambda_1\lambda_2\lambda_3\lambda_0}|\epsilon_\mu^{\lambda_1}\epsilon_\nu^{\lambda_2}\epsilon_\rho^{\lambda_3}\epsilon_\alpha^{\lambda_0}\mathcal{M}_{\mu\nu\rho\alpha}|^2=\frac{1}{3}\sum_{\mu\nu\rho\alpha}|\mathcal{M}_{\mu\nu\rho\alpha}|^2$$

- The decay width of $J/\psi\to 3\gamma$:

$$\Gamma(J/\psi\to 3\gamma)=\frac{m_{J/\psi}}{1536\pi^3}\int_0^1 dx\int_{1-x}^1 dy\mathcal{T}(x,y)$$

Input parameters

- **Photon momenta:**
  - On-shell as possible: fix photon 1 and 3 on-shell exactly, minimize Q_2;
  - The (x, y) cover the physical region as possible, i.e. $x \in [0, 1], y \in [1 - x, 1]$;
  - Fewer momenta to meet above requirements.

Ensemble	Q_1^2	Q_3^2	n_1	n_3	n_2	ω_1	ω_3	ω_2	x	y	$Q_2^2(GeV^2)$
I	0	0	111	-1-1-2	001	0.4680	0.6525	0.2134	0.7017	0.9783	-0.1541
	0	0	111	-20-1	1-10	0.4680	0.5967	0.2692	0.7017	0.8946	-0.4096
	0	0	002	11-1	-1-1-1	0.5343	0.4680	0.3316	0.8011	0.7017	-0.6077
	0	0	002	11-2	-1-10	0.5343	0.6525	0.1471	0.8011	0.9783	-0.5690
II	0	0	210	-1-11	-10-1	0.4257	0.3320	0.2905	0.8123	0.6335	0.0932
	0	0	002	10-2	-100	0.3810	0.4257	0.2415	0.7269	0.8123	0.1857
	0	0	002	11-1	-1-1-1	0.3810	0.3320	0.3352	0.7269	0.6335	0.0187

- **On-shell fitting:**

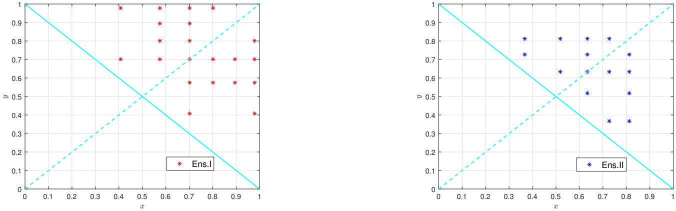
$$\mathcal{T}(x, y, Q_1^2, Q_2^2, Q_3^2) = \mathcal{T}(x, y) + \text{const} \times \sum_i Q_i^2$$

- **Twisted Mass Ensembles:**

Ens	β	a(fm)	V/a^4	$a\mu_{\text{sea}}$	m_π(MeV)	N_{conf}
I	3.9	0.085	$24^3 \times 48$	0.004	315	40
II	4.05	0.067	$32^3 \times 64$	0.003	300	20

Input parameters

- $xy-$distribution



- Exchange symmetry: $\mathcal{T}(x,y,z) = \mathcal{T}(y,x,z) = ...$

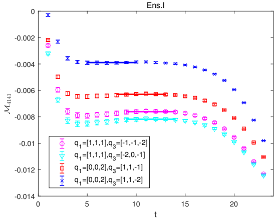
- Current renormalization constant: $Z_V^{I,II} = 0.6347(26), 0.6640(27)$.

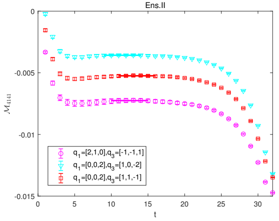
$$Z_V^{(\mu)} = \frac{p^\mu}{E(\mathbf{p})} \frac{1/2 \sum_k \Gamma^{(2)}_{\psi_k \psi_k}(\mathbf{p}, t_{\text{source}} = T/2, t_{\text{sink}} = 0)}{\sum_k \Gamma^{(3)}_{\psi_k \gamma^\mu \psi_k}(\mathbf{p}, t_{\text{source}} = T/2, t_{\text{sink}} = 0, t)}$$

J.J.Dudek, E.G.Edwards and D.G.Richards.(2006)

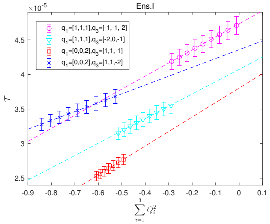
Matrix elements

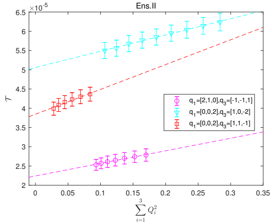
- Four-point function $\mathcal{M}_{\mu\nu\rho\alpha}$:





- On-shell fitting: $\mathcal{T}(x, y, Q_1^2, Q_2^2, Q_3^2)$

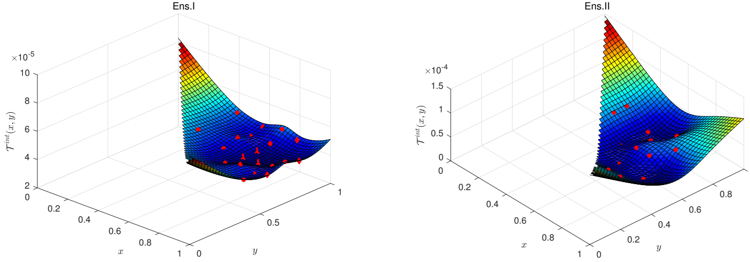




Cubic spline interpolation

- Decay width:

$$\Gamma(J/\psi \to 3\gamma) = 1.530(15)\text{eV}, 1.715(47)\text{eV}$$



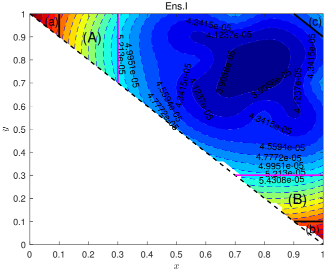
- Existing problems:
  - The intermediate contribution $J/\psi \to \gamma\eta_c \to 3\gamma$ to be removed;
  - Estimate the systematical error caused by cubic spline interpplation, for the region without data covered.

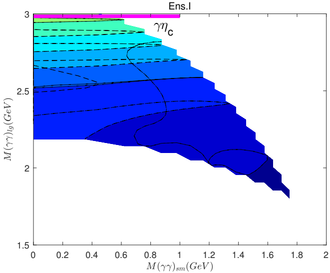
- Dalitz variables:

$$\frac{M(\gamma\gamma)_{lg/sm}}{m_{J/\psi}} = \begin{matrix}\max\\ \min\end{matrix} \left\{\sqrt{1-x}, \sqrt{1-y}, \sqrt{x+y-1}\right\}$$

  - Dalitz plot is the direct observable for the experiments.
  - Bands in Dalitz plot indicate the intermediate two-body states.

- Dalitz plot





Dalitz analysis

- Removing the $\gamma\eta_c$ contribution by setting the cut $M_{\rm cut} = m_{\eta_c}$

  (a) : $\sqrt{1-x} > M_{\rm cut}/m_{J/\psi}$
  (b) : $\sqrt{1-y} > M_{\rm cut}/m_{J/\psi}$
  (c) : $\sqrt{x+y-1} > M_{\rm cut}/m_{J/\psi}$

$$\Rightarrow 0.031\text{eV}(\text{Ens.I}), \qquad 0.034\text{eV}(\text{Ens.II})$$

- Regarding the region without (x, y) covered as systematic error, i.e.

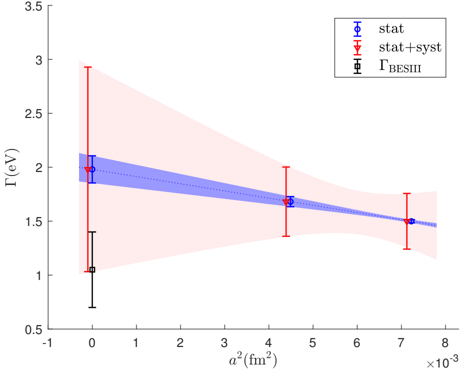
  (A) $x \in [0.1, 0.3], y \in [1-x, 1]$
  (B) $x \in [1-y, 1], x \in [0.1, 0.3]$

$$\Rightarrow 0.243\text{eV}(\text{Ens.I}), \qquad 0.274\text{eV}(\text{Ens.II})$$

- The pure decay width:

$$\Gamma(J/\psi \to 3\gamma) = 1.499(15)(243)\ \text{eV};\ \ 1.681(47)(274)\ \text{eV}$$

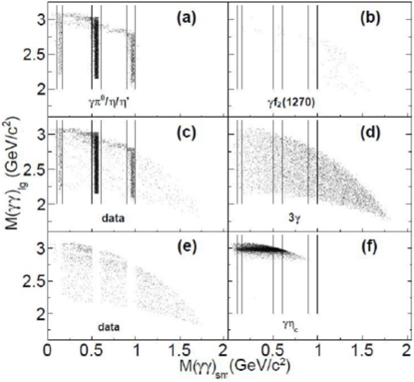
Naive continuum extrapolation



$$\mathcal{B}(J/\psi \to 3\gamma) = 2.13(14)(89) \times 10^{-5}$$

Dalitz plot in experiments

- BESIII

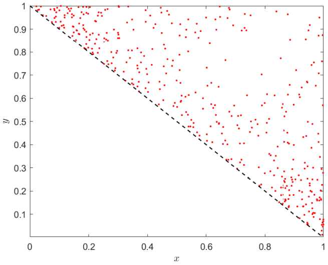


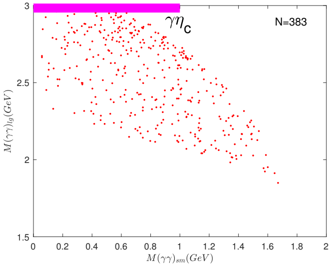
$$\mathcal{B} = (1.13 \pm 0.18 \pm 0.2) \times 10^{-5}, \quad N_{J/\psi} = 389.$$

Dalitz plot on lattice

- Normalized $\mathcal{T}$-function distribution:

$$\tilde{\mathcal{T}}(x,y) = \frac{\mathcal{T}^{int}(x,y)}{\int_0^1 dx \int_{1-x}^1 dy \mathcal{T}^{int}(x,y)}$$





- No obvious bands on vertical region for the range $M(\gamma\gamma)_{sm} \in [0.1, 0.16]$,$[0.5, 0.6]$,$[0.9, 1]$, which correspond to the dominant sources $\gamma\pi_0/\eta/\eta^{'}$ in experiments.

Importance of Dalitz analysis

- Providing a direct comparison with the experiments.
- The parametric analytical expression for the $\mathcal{T}_{a\to 0}(x,y)$ could be used as the theoretical input for the matrix element of $J/\psi \to 3\gamma$ for the experiments.
- The J/ψ events in BESIII are 100 times greater than ever before, a higher precision result of $J/\psi \to 3\gamma$ could be expected with $\mathcal{T}_{a\to 0}(x,y)$ utlized.

TABLE III. Summary of the relative systematic uncertainties. $\mathcal{B}_{3\gamma}$ and $\mathcal{B}_{\gamma\eta_c}$ stand for the measurements of branching fractions $\mathcal{B}(J/\psi \to 3\gamma)$ and $\mathcal{B}(J/\psi \to \gamma\eta_c, \eta_c \to \gamma\gamma)$, respectively. A dash (–) means the uncertainty is negligible.

Source	Uncertainties (%)	
	$\mathcal{B}_{3\gamma}$	$\mathcal{B}_{\gamma\eta_c}$
Signal model	15	–
η_c width	–	5
η_c line shape	1	1
Resolution	3	9
$M(\pi^+\pi^-)_{\rm recoil}$ window	4	4
π^0, η, η' rejection	0.5	5
PWA model	2	2
Photon detection	3	3
Tracking	2	2
Number of good photons	0.5	0.5
Kinematic fit and χ^2_{4C} requirement	2	2
Fitting	5	5
Number of $\psi(3686)$	0.8	0.8
$\mathcal{B}(\psi(3686) \to \pi^+\pi^- J/\psi)$	1.2	1.2
Total	18	14

Taken from BESIII.

New result for $\eta_c \to 2\gamma$

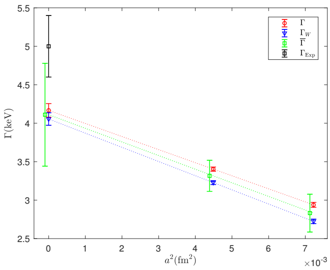
- Previous results:

Methods	$\mathcal{B} \times 10^{-4}$	$\delta\mathcal{B} \times 10^{-4}$	Refs
Quenched Wilson	0.83	0.50	J.J.Dudek *et at.*(2006)
$N_f = 2$ twisted mass	0.351	0.004	CLQCD(2016)
NRQCD	3.1 ∼3.2	-	F.Feng(2017)
Exp	1.57	0.12	PDG(2018)

- Amplitude summation:
  - $\Gamma : \epsilon_\mu \epsilon_\nu \to -g_{\mu\nu}$
  - $\Gamma_W : \epsilon_\mu \epsilon_\nu \to -g_{\mu\nu} + (q^i_\mu \bar{q}^i_\nu + \bar{q}^i_\mu q^i_\nu)/2\omega_i^2$

$$\mathcal{B}(\eta_c \to 2\gamma) = 1.29(3)(18) \times 10^{-4}$$

PRD 102,034502(2020).



Conclusion and Outlook

- **Conclusion**
  - We present the first lattice calculation for $J/\psi \to 3\gamma$;
  - A new method is proposed to calculate multi-photon decay directly, by summing over final and initial state polarizations.
  - The Dalitz analysis on lattice is suggested.
  - The new method is applied for $\eta_c \to 2\gamma$, and a most reliable result is obtained.
- **Outlook**
  - A new strategy is in progress for $J/\psi \to 3\gamma$, the large systematic error can be avoided.
  - The $\mathcal{T}_{a\to 0}(x, y)$ is our next target, be applied for the experiments to avoid the large systematic uncertainty.

End

Thank you!